111 MOST PROBABLE QUESTIONS
(EXPLANATIONS)

1. Real Number

1. (*d*) Given $p = 18a^2b^4$

$= 2 \times 3 \times 3 \times a \times a \times b \times b \times b \times b = 2 \times 3^2 \times a^2 \times b^4$

and $q = 20\, a^3b^2 = 2 \times 2 \times 5 \times a \times a \times a \times b \times b$

$= 2^2 \times 5 \times a^3 \times b^2$

$\therefore$ LCM $(p, q) = 2 \times 2 \times 3 \times 3 \times 5 \times a \times a \times a \times b \times b \times b \times b$

$= 2^2 \times 3^2 \times 5 \times a^3 \times b^4 = 180a^3b^4$ **(*1 M*)**

2. (*a*) We know that

LCM × HCF = product of the given numbers

$\Rightarrow 40 \times 252 \times k = 2520 \times 6600$

$\Rightarrow k = \frac{2520 \times 6600}{252 \times 40} = \frac{10 \times 660}{4}$

$\Rightarrow k = 10 \times 165 = 1650$ **(*1 M*)**

3. (*b*) The prime factorisation of 3750 is;

$3750 = 5^4 \times 2 \times 3$ **(*½ M*)**

$\therefore$ The exponent of 5 in the prime factorisation of 3750 is 4. **(*½ M*)**

4. (*c*) The time after which they again ring together = LCM of 20, 25, 30

5	20, 25, 30
2	4, 5, 6
	2, 5, 3

$\therefore$ LCM $(20, 25, 30) = 5^2 \times 2^2 \times 3 = 300$

$\therefore$ They ring again together after = 300 minutes

$= \frac{300}{60}$ hr = 5 hr

If they first ring together at 12 noon, they again ring together after 5hrs at 5:00 pm. **(*1 M*)**

5. Let $5 - 2\sqrt{3}$ is a rational number

$\therefore\ 5 - 2\sqrt{3} = \frac{a}{b}$, [where a and b are integers, $b \neq 0$] **(*½ M*)**

$2\sqrt{3} = 5 - \frac{a}{b} \Rightarrow \sqrt{3} = \frac{5}{2} - \frac{a}{2b} \Rightarrow \sqrt{3} = \frac{5b-a}{2b}$ **(*½ M*)**

Since, 'a' and 'b' are integers. Therefore $\frac{5b-a}{2b}$ is a rational number and so, $\sqrt{3}$ is also a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number So, our assumption is not true. **(*½ M*)**

Hence, $5 - 2\sqrt{3}$ is an irrational number. **(*½ M*)**

6. The prime factorisation of 72 and 120 is given by $72 = 2 \times 2 \times 2 \times 3 \times 3$ and $120 = 2 \times 2 \times 2 \times 3 \times 5$ **(*½ M*)**

H.C.F (72, 120) = Product of common factors with lowest power $= 2 \times 2 \times 2 \times 3 = 24$ **(*½ M*)**

L.C.M (72, 120) = Product of prime factors with highest power $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$ **(*½ M*)**

Hence, the H.C.F and L.C.M of 72 and 120 are 24 and 360 respectively. **(*½ M*)**

7. Let $\sqrt{5}$ be a rational number then it must be in the form of $\frac{p}{q}$, where $q \neq 0$ (p and q are co-prime) **(*½ M*)**

$\sqrt{5} = \frac{p}{q} \Rightarrow \sqrt{5} \times q = p$ **(*½ M*)**

Squaring on both sides,

$5q^2 = p^2$ ...(i) **(*½ M*)**

p^2 is divisible by 5

$p = 5c$

Squaring on both sides

$p^2 = 25c^2$ ...(ii) **(*½ M*)**

putting p^2 in eq (i)

$5q^2 = 25c^2 \Rightarrow q^2 = 5c^2$ **(*½ M*)**

So, q is divisible by 5.

Thus p and q have a common factor of 5. **(*½ M*)**

we have contradicted our assumption wrong.

Therefore $\sqrt{5}$ is an irrational number.

8. Assume $\sqrt{2}$ is rational. Then, it can be expressed as $\frac{a}{b}$, where a and b are coprime integers and $b \neq 0$

$\Rightarrow \sqrt{2} = \frac{a}{b}$

Squaring both sides,

$\Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$ ***(1 M)***

This implies a^2 is even, so a is even.

Let $a = 2k$,

$\Rightarrow (2k)^2 = 2b^2 \Rightarrow 4k^2 = 2b^2 \Rightarrow b^2 = 2k^2$ ***(1 M)***

Thus, b^2 is even, so b is even.

$\Rightarrow$ Both a and b being even contradicts the assumption that they are coprime. ***(1 M)***

Hence, $\sqrt{2}$ is irrational.

2. Polynomials

9. (*b*) Given $p(x) = 2x^2 - 9x + 5$

$$\alpha + \beta = \frac{-b}{a} = \frac{+9}{2} \Rightarrow \alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$(\alpha + \beta)^2 - 2\,\alpha\beta = \alpha^2 + \beta^2 \Rightarrow \frac{81}{4} - \frac{10}{2} = \alpha^2 + \beta^2$$

$$\Rightarrow \alpha^2 + \beta^2 = \frac{81 - 20}{4} = \frac{61}{4}$$ ***(1 M)***

10. (*d*) Let, $p(x) = 4x^2 - 3x - 7$

Here, $\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{3}{4}$ ...(*i*)

Also, $\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{-7}{4}$ ...(*ii*)

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{3}{4}}{\frac{-7}{4}}$ [From (*i*) and (*ii*)]

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-3}{7}$$ ***(1 M)***

11. (*a*) Given, polynomial $p(x) = x^2 + 3x + k$

It is also given that 2 is one of its zeroes.

$\therefore p(2) = 0 \Rightarrow (2)^2 + 3(2) + k = 0$

$\Rightarrow 4 + 6 + k = 0 \Rightarrow k = -10$

Hence, the value of k is -10. ***(1 M)***

12. (*d*) Let α and β are the zeroes of the polynomial $x^2 + (a + 1)x + b$

Here, $\alpha = 2$ and $\beta = -3$

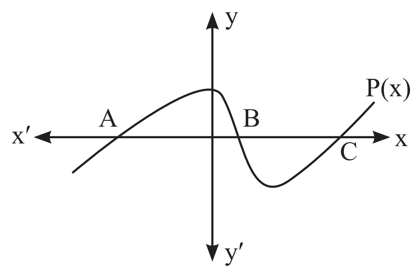
Sum of zeroes, $\alpha + \beta = \frac{-(a+1)}{1}$

$\Rightarrow 2 + (-3) = -a - 1 \Rightarrow a = 0$

Product of zeroes, $\alpha\beta = \frac{b}{1} \Rightarrow 2(-3) = b \Rightarrow b = -6$

(1 M)

13. (*c*)



Number of zeroes of a polynomial = Number of times intersects the graph at the x-axis.

$\therefore$ The no. of zeroes of the given polynomial = 3 ***(1 M)***

14. We know, quadrartic polynomial:

$p(x) = x^2 -$ (sum of zeroes) x + (product of zeroes)

Given, sum of zeroes = 0

and Product of zeroes = $\frac{-3}{5}$ ***(1 M)***

$\therefore$ The quadratic polynomial is;

$$x^2 - (0)x + \left(\frac{-3}{5}\right) = x^2 - \frac{3}{5}$$

Now, to find the zeroes of the polynomial;

$$x^2 - \frac{3}{5} = 0 \Rightarrow x^2 = \frac{3}{5} \Rightarrow x^2 = \left(\sqrt{\frac{3}{5}}\right)^2$$ ***(1 M)***

$$\Rightarrow x = \pm\sqrt{\frac{3}{5}} \Rightarrow x = \sqrt{\frac{3}{5}} \text{ or } x = -\sqrt{\frac{3}{5}}$$

$$\Rightarrow x = \sqrt{\frac{3}{5} \times \frac{5}{5}} \text{ or } x = -\sqrt{\frac{3}{5} \times \frac{5}{5}}$$

$$\Rightarrow x = \frac{\sqrt{15}}{5} \text{ or } x = \frac{-\sqrt{15}}{5}$$ ***(1 M)***

$\therefore$ The zeroes of the polynomial are $\frac{\sqrt{15}}{5}$ or $\frac{-\sqrt{15}}{5}$

15. Given quadratic polynomial:

$px^2 + qx + 1$, zeroes are α and β.

$\Rightarrow$ Sum of zeroes = $\alpha + \beta = -\frac{q}{p}$ ***(½ M)***

$\Rightarrow$ Product of zeroes = $\alpha\beta = \frac{1}{p}$ ***(½ M)***

New zeroes are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

Sum of new zeroes

$$= \frac{2}{\alpha} + \frac{2}{\beta} = 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) = 2\left(\frac{-\frac{q}{p}}{\frac{1}{p}}\right) = -2q,$$ ***(½ M)***

Product of new zeroes $= \frac{2}{\alpha}\cdot\frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{\frac{1}{p}} = 4p$ **(½ M)**

Hence, quadratic polynomial with new zeroes is

$x^2 - (\text{Sum})x + (\text{Product}) = x^2 - (-2q)x + 4p$

$= x^2 + 2qx + 4p$ **(1 M)**

16. (*i*) Given, equation $h = 25t - 5t^2$

Putting $h = 0$, we get

$25t - 5t^2 = 0 \Rightarrow 5t(5 - t) = 0 \Rightarrow t = 5$ or $t = 0$

Hence, zeroes are 5 & 0. **(1 M)**

(*ii*) Maximum height is achieved at the vertex of the this given parabola having $t = \frac{5}{2}$

$\therefore$ Putting $t = \frac{5}{2}$ in equation $h = 25t - 5t^2$ we get

$\therefore h = \frac{25\times5}{2} - \frac{5\times25}{4} = \frac{250-125}{4} = \frac{125}{4}$ m **(1 M)**

(*iii*) (*a*) To reach 30m, $h = 30$m

$30 = 25t - 5t^2 \Rightarrow 5t^2 - 25t + 30 = 0$

$\Rightarrow t^2 - 5t + 6 = 0 \Rightarrow t^2 - 3t - 2t + 6 = 0$ **(1 M)**

$\Rightarrow t(t-3) - 2(t-3) = 0 \Rightarrow (t-3)(t-2) = 0$

$\Rightarrow t = 3$, or $t = 2$

Hence ball took 2 seconds **(1 M)**

OR

(*iii*) (*b*) Given, $h = 20$

$20 = 25t - 5t^2 \Rightarrow 5t^2 - 25t + 20 = 0$

$\Rightarrow t^2 - 5t + 4 = 0 \Rightarrow t^2 - 4t - t + 4 = 0$ **(1 M)**

$\Rightarrow t(t-4) - 1(t-4) = 0 \Rightarrow (t-4)(t-1) = 0$

$\Rightarrow t = 4$ or $t = 1$ **(1 M)**

3. Pair of Linear Equation in Two Variables

17. (*b*) Given system of equations

$3x - y + 8 = 0$ and $6x - ky + 16 = 0$

For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{8}{16} \Rightarrow \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$

Hence for $k = 2$ equations have infinitely many solution **(1 M)**

18. (*d*) A pair of linear equation is inconsistent, if for two linear equation.

$a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c2 = 0$, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

given, two linear equation are;

$x + 2y = 3$ & $5x + ky + 7 = 0$ **(½ M)**

Here, $a_1 = 1$; $b_1 = 2$; $c_1 = -3$

$a_2 = 5$; $b_2 = k$; $c_2 = 7$

Now, for inconsistency, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{1}{5} = \frac{2}{k} \Rightarrow k = 10$

And, $\frac{a_1}{a_2} \neq \frac{c_2}{c_2} \Rightarrow \frac{1}{5} \neq \frac{-3}{7}$ [satisfied]

$\therefore k = 10$ **(½ M)**

19. Given, system of linear equations are

$2x + 3y = 7$ and

$(k+1)x + (2k-1)y = 4k + 1$

Since, the given system of equation is of the form

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = 3, c_1 = -7$

$a_2 = k + 1, b_2 = 2k - 1, c_2 = -(4k + 1)$ **(½ M)**

For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{2}{k+1} = \frac{3}{2k-1} = \frac{-7}{-(4k+1)}$ **(½ M)**

$\Rightarrow 2(2k-1) = 3(k+1)$ or $3(4k+1) = 7(2k-1)$

$\Rightarrow 4k - 2 = 3k + 3$ or $12k + 3 = 14k - 7$

$\Rightarrow k = 5$ or $k = 5$

Hence, value of k is 5. **(1 M)**

20. Consider, the fraction be $\frac{x}{y}$ **(½ M)**

Given, fraction becomes $\frac{1}{3}$ when 2 is subtracted from numerator.

$\frac{x-2}{y} = \frac{1}{3}$ **(½ M)**

$\Rightarrow 3x - 6 = y \Rightarrow 3x - y - 6 = 0$ ...(*i*)

Given, also fraction becomes $\frac{1}{2}$ when 1 is subtracted from denominator.

$\frac{x}{y-1} = \frac{1}{2}$ **(½ M)**

$\Rightarrow 2x = y - 1 \Rightarrow 2x - y + 1 = 0$ ...(*ii*)

Solving equation (*i*) and (*ii*), we get

$3x - y - 6 = 0$

$2x - y + 1 = 0$

$-\quad +\quad -$

$x - 7 = 0$

$\Rightarrow x = 7$ **(½ M)**

From equation (*i*) $3 \times 7 - y - 6 = 0$

$\Rightarrow 21 - y - 6 = 0 \Rightarrow y = 15$ **(½ M)**

Hence, fraction is $\frac{7}{15}$. **(½ M)**

21. Let the unit digit be x & the tens digit be y **(½ M)**

$\therefore$ original number = $10y + x$

Now, according to question, sum of digits is 8.

i.e., $x + y = 8$ ...(i) **(½ M)**

And, also the differences between the number & that formed by reversing the digits is 18.

If the digits are reversed, then, unit digit = y, tens digit = x

$\therefore$ The reversed number = $10x + y$. **(½ M)**

$\therefore$ According to question $(x + 10y) - (10x + y) = 18$

$\Rightarrow x + 10y - 10x - y = 18 \Rightarrow -9x + 9y = 18$

$\Rightarrow 9(-x + y) = 18 \Rightarrow -x + y = 2$ ...(ii) **(½ M)**

Adding equation (i) and (ii), we get

$2y = 10 \Rightarrow y = 5$

Putting $y = 5$ in (i), we get,

$x + 5 = 8 \Rightarrow x = 8 - 5 = 3$ **(½ M)**

$\therefore$ Original no. is: $10y + x = 10(5) + 3 = 50 + 3 = 53$ **(½ M)**

22. Let the monthly incomes of the two person be $9x$ and $7x$, and their monthly expenditure be $4y$ and $3y$ respectively

Given that each save ₹5,000.

So, $9x - 4y = 5000$ ...(i) **(½ M)**

$7x - 3y = 5000$ ...(ii) **(½ M)**

Multiply equation (i) by 3 and equation (ii) by 4,

$\Rightarrow 27x - 12y = 15000$ ...(iii)

$28x - 12y = 20000$ ...(iv)

Subtract equation (iii) from equation (iv)

$\Rightarrow (28x - 12y) - (27x - 12y) = 20000 - 15000$

$\Rightarrow x = 5000$ **(½ M)**

Substitute $x = 5000$ into equation (i)

$\Rightarrow 9(5000) - 4y = 5000$

$\Rightarrow 45000 - 4y = 5000 \Rightarrow 4y = 40000 \Rightarrow y = 10000$ **(½ M)**

Monthly incomes are

$\Rightarrow 9x = 9 \times 5000$ and $7x = 7 \times 5000 = 35000$ **(1 M)**

Hence, monthly incomes are ₹45,000 and ₹35,000.

23. (i) Given, Hockey ₹ x per student and cricket ₹ y per student. For school 'P', the total prize amount for hockey and cricket is ₹9500. The number of students awarded for hockey and cricket are 5 and 4 respectively.

Hence, $5x + 4y = 9500$ …(i) **(½ M)**

For school 'Q', the total prize amount for hockey and cricket is ₹7370. The number of students awarded are 4 and 3 respectively.

Hence, $4x + 3y = 7370$ …(ii) **(½ M)**

(ii) (a) Given equations are:

$$\begin{bmatrix} 5x + 4y = 9500 \\ 4x + 3y = 7370 \end{bmatrix} \begin{matrix} \times 3 \\ \times 4 \end{matrix}$$

$15x + 12y = 28500$ **(½ M)**

$16x + 12y = 29480$ **(½ M)**

$-x \quad = -980$ **(½ M)**

Hence, prize amount for hockey is ₹980 **(½ M)**

OR

(b) On substituting the value of x in (i), we get

$5(980) + 4y = 9500$ **(½ M)**

$\Rightarrow y = \dfrac{9500 - 4900}{4} = \dfrac{4600}{4} = 1150$ **(½ M)**

Hence, prize amount of cricket is more by

$1150 - 980 = ₹170$ **(1 M)**

(iii) If there are 2 students each from two games, then total prize money = $2x + 2y$

$= 2(980) + 2(1150) = ₹4260$ **(1 M)**

4. Quadratic Equations

24. (d) Given, quadratic equation is $ax^2 + bx + c = 0$

Condition for real and equal roots is $D = 0$

$\therefore b^2 - 4ac = 0$ $[\because D = b^2 - 4ac]$

$\Rightarrow b^2 = 4ac \Rightarrow c = \dfrac{b^2}{4a}$ **(1 M)**

25. We have, $x^2 - 2ax + (a^2 - b^2) = 0$

$\Rightarrow (x^2 - 2ax + a^2) - b^2 = 0$ **(½ M)**

$\Rightarrow (x - a)^2 - b^2 = 0$ $[\because a^2 - 2ab + b^2 = (a - b)^2]$

$\Rightarrow (x - a - b)(x - a + b) = 0$ **(½ M)**

$[\because a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow x - a - b = 0$ or $x - a + b = 0$

$\Rightarrow x = -(-a - b)$ or $x = -(-a + b)$ **(½ M)**

$\Rightarrow x = a + b$ or $x = a - b$ **(½ M)**

26. $\sqrt{2x+9} + x = 13 \Rightarrow \sqrt{2x+9} = 13 - x$

On squaring both sides, we get

$\left(\sqrt{2x+9}\right)^2 = (13 - x)^2 \Rightarrow 2x + 9 = 169 + x^2 - 26x$ **(1 M)**

$\Rightarrow x^2 - 26x - 2x + 169 - 9 = 0 \Rightarrow x^2 - 28x + 160 = 0$

$\Rightarrow x^2 - 20x - 8x + 160 = 0$ [Middle term factorisation]

$\Rightarrow x(x - 20) - 8(x - 20) = 0 \Rightarrow (x - 8)(x - 20) = 0$

$\therefore x = 8, 20$ **(1 M)**

27. Given,

$\Rightarrow p(x - 4)(x - 2) + (x - 1)^2 = 0$

$\Rightarrow p(x^2 - 4x - 2x + 8) + (x^2 - 2x + 1) = 0$

$\Rightarrow (p + 1)x^2 + (-6p - 2)x + 8p + 1 = 0$ **(1 M)**

Now, equation has real and equal roots

$\therefore D = 0$ [$\because D = b^2 - 4ac$]

$\Rightarrow b^2 - 4ac = 0$ ***(1 M)***

$\Rightarrow (-6p - 2)^2 - 4(p + 1)(8p + 1) = 0$

$\Rightarrow 36p^2 + 4 + 24p - 4(8p^2 + p + 8p + 1) = 0$

$\Rightarrow 36p^2 + 4 + 24p - 32p^2 - 36p - 4 = 0$

$\Rightarrow 4p^2 - 12p = 0 \Rightarrow p^2 - 3p = 0$

$\Rightarrow p(p - 3) = 0 \Rightarrow p = 0$ or $p = 3$.

Hence, the values of p are 0 and 3 ***(1 M)***

28. Let the uniform speed of the train be x *km/h*.

$\therefore$ The time taken to cover the distance 480 km $= \frac{480}{x} h$

Now the speed $(x - 8)$ km/h. ***(½ M)***

The time taken to cover the same distance $= \frac{480}{x-8} h$ ***(½ M)***

According to question, $\frac{480}{x-8} = \frac{480}{x} + 3$ ***(½ M)***

$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 3 \Rightarrow 480\left(\frac{x - x + 8}{(x-8)x}\right) = 3$ ***(½ M)***

$\Rightarrow x^2 - 8x = 160 \times 8 \Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0 \Rightarrow x(x - 40) + 32(x - 40) = 0$

$\Rightarrow (x - 40)(x + 32) = 0 \Rightarrow x = 40, -32 \Rightarrow x = -32$ not possible

$\therefore$ The speed is 40 km/h. ***(1 M)***

29. Let the length of piece be x m then rate per metre $= \frac{200}{x}$

New length $= (x + 5)$ m

New rate per metre $= \frac{200}{x+5}$ ***(1 M)***

Now, according to question

$\frac{200}{x} - \frac{200}{x+5} = 2 \Rightarrow \frac{200x + 1000 - 200x}{x(x+5)} = 2$ ***(1 M)***

$\Rightarrow 1000 = 2x(x + 5) \Rightarrow 2x^2 + 10x - 1000 = 0$

$\Rightarrow x^2 + 5x - 500 = 0 \Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x + 25) - 20(x + 25) = 0 \Rightarrow (x + 25)(x - 20) = 0$ ***(½ M)***

$\Rightarrow x = -25$ or $x = 20$ ***(½ M)***

length cannot be negative

So, $x = 20$ and rate per metre $= ₹\,\frac{200}{20} = ₹\,10$ ***(1 M)***

30. Given, $\frac{1}{(2x-3)} + \frac{1}{(x-5)} = 1\frac{1}{9} \Rightarrow \frac{1}{(2x-3)} + \frac{1}{(x-5)} = \frac{10}{9}$

$\Rightarrow \frac{(x-5) + (2x-3)}{(2x-3)(x-5)} = \frac{10}{9}$ [Taking LCM on LHS] ***(1 M)***

$\Rightarrow \frac{3x - 8}{(2x-3)(x-5)} = \frac{10}{9} \Rightarrow 9(3x - 8) = 10(2x - 3)(x - 5)$

[By using cross multiplication]

$\Rightarrow 27x - 72 = 10(2x^2 - 10x - 3x + 15)$

$\Rightarrow 27x - 72 = 10(2x^2 - 13x + 15)$

$\Rightarrow 27x - 72 = 20x^2 - 130x + 150$

$\Rightarrow 20x^2 - 130x - 27x + 150 + 72 = 0$ ***(2 M)***

[Arranging the terms on one side]

$\Rightarrow 20x^2 - 157x + 222 = 0$

$\Rightarrow 20x^2 - 37x - 120x + 222 = 0$ [Middle term factorisation]

$\Rightarrow x(20x - 37) - 6(20x - 37) = 0$

$\Rightarrow (x - 6)(20x - 37) = 0$

$\Rightarrow x - 6 = 0$ or $20x - 37 = 0 \Rightarrow x = 6$ or $x = \frac{37}{20}$

Hence, the values of x are 6 & $\frac{37}{20}$. ***(1 M)***

31. Given equation is $\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}$

$\Rightarrow \frac{(x-2)(x-5) + (x-4)(x-3)}{(x-3)(x-5)} = \frac{10}{3}$ ***(1 M)***

$\Rightarrow \frac{(x^2 - 7x + 10) + (x^2 - 7x + 12)}{(x-3)(x-5)} = \frac{10}{3}$

$\Rightarrow \frac{2x^2 - 14x + 22}{(x-3)(x-5)} = \frac{10}{3}$

$\Rightarrow 3(2x^2 - 14x + 22) = 10(x^2 - 8x + 15)$

$\Rightarrow 6x^2 - 42x + 66 = 10x^2 - 80x + 150$

$\Rightarrow 4x^2 - 38x + 84 = 0$ ***(1 M)***

$\Rightarrow 2x^2 - 19x + 42 = 0$ ***(½ M)***

$\because\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

when $ax^2 + bx + c = 0$, where a, b, c are constant and $a \neq 0$ ***(1 M)***

$\Rightarrow x = \frac{19 \pm \sqrt{361 - 336}}{4} = \frac{19 \pm 5}{4}$

$\Rightarrow x = 6$ or $x = 3.5$ ***(1½ M)***

Hence, the roots of the given equation are 6 and 3.5.

5. Arithmetic Progression

32. (*c*) Given $a = 7$, $a_n = 84$ and $S_n = \frac{2093}{2}$

As we know, $S_n = \frac{n}{2}[a + a_n]$

$\frac{2093}{2} = \frac{n}{2}[7 + 84] \Rightarrow 2093 = n[91]$

$\Rightarrow n = \frac{2093}{91} = 23$ ***(1 M)***

33. (*b*) Given A.P., –29, –26, –23, ..., 61

Here, $a = -29$, $d = -26 - (-29) = 3$

$\because a_n = a + (n-1)d$

$\therefore 16 = -29 + (n-1)3 \Rightarrow 45 = 3n - 3 \Rightarrow n = 16$

Hence, 16th term is 16. ***(1 M)***

34. Given A.P. is $\frac{-11}{2}, -3, \frac{-1}{2}, ..., \frac{49}{2}$

Here, $a = \frac{-11}{2}$ and $d = -3 + \frac{11}{2} = \frac{5}{2}$ ***(½ M)***

Given, $a_n = \frac{49}{2}$ ***(½ M)***

$\Rightarrow a + (n-1)d = a_n \Rightarrow \frac{-11}{2} + (n-1)\frac{5}{2} = \frac{49}{2}$

$\Rightarrow (n-1) = 12 \Rightarrow n = 13$ ***(1 M)***

35. Given sum of first *m* terms is *n* & sum of first *n* terms is *m*. Let the first term of the AP be *a* & the common difference be *d*.

$\therefore$ According to question,

$\therefore S_m = n \Rightarrow \frac{m}{2}[2a + (m-1)d] = n \Rightarrow 2a + (m-1)d = \frac{2n}{m}$ ...(*i*)

and $S_n = m \Rightarrow \frac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow 2a + (n-1)d = \frac{2m}{n}$ ...(*ii*) ***(1 M)***

Now, equation (*i*) – equation (*ii*), we get

$\Rightarrow (m-1)d - (n-1)d = \frac{2n}{m} - \frac{2m}{n}$

$\Rightarrow d[m-1-n+1] = \frac{2n^2 - 2m^2}{nm} \Rightarrow d(m-n) = \frac{2(n^2 - m^2)}{nm}$

$\Rightarrow d = \frac{2(n-m)(n+m)}{nm(m-n)} = \frac{-2(n+m)}{nm}$ ***(1 M)***

Putting, $d = \frac{-2(n+m)}{nm}$ in (*i*), we get

$2a + (m-1)\frac{(-2)(n+m)}{nm} = \frac{2n}{m} \Rightarrow 2a = \frac{2n}{m} + \frac{2(m-1)(n+m)}{nm}$

$\Rightarrow 2a = \frac{2n^2 + 2(mn + m^2 - n - m)}{nm}$

$\Rightarrow a = \left[\frac{n^2 + mn + m^2 - n - m}{mn}\right]$

Now, sum of first $(m + n)$ terms is

$S_{m+n} = \frac{(m+n)}{2}[2a + (m+n-1)d]$

$= \frac{(m+n)}{2}\left[2\left(\frac{n^2 + mn + m^2 - n - m}{mn}\right) + (m+n-1)\left(\frac{-2(m+n)}{nm}\right)\right]$

$= \frac{(m+n)}{2}\left[\frac{2(n^2 + mn + m^2 - n - m) - 2(m+n-1)(m+n)}{mn}\right]$

$= \frac{(m+n)}{2}\left[\frac{2n^2 + 2mn + 2m^2 - 2n - 2m - 2(m^2 + mn - m + nm + n^2 - n)}{nm}\right]$

$= \frac{(m+n)}{2}\left[\frac{2n^2 + 2mn + 2m^2 - 2n - 2m - 2m^2 - 4nm + 2m + 2n - 2n^2}{nm}\right]$

$= \frac{(m+n)}{2}\left[\frac{-2mn}{nm}\right] = -(m+n)$

$\Rightarrow S_{(m+n)} = -(m+n)$ Hence, proved. ***(1 M)***

36. Given, $a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ...(*i*)

and $a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ ...(*ii*)

On subtracting eq. (*ii*) from (*i*), we get

$a + (m-1)d - a - (n-1)d = \frac{1}{n} - \frac{1}{m}$

$\Rightarrow d(m-n) = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$ ***(1 M)***

Put the value of *d* in eq. (*i*) we get

$a + (m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$ ***(1 M)***

Now,

$a_{mn} = a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn}$

$a_{mn} = \frac{1}{mn}(1 + mn - 1) = \frac{mn}{mn} = 1 \Rightarrow a_{mn} = 1$ ***(1 M)***

37. Let the four consecutive numbers of AP be $a - 3d$, $a - d$, $a + d$, $a + 3d$

Now, according to question

$a - 3d + a - d + a + d + a + 3d = 32$

$\Rightarrow 4a = 32 \Rightarrow a = 8$ ...(*i*)

Now, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$ ***(1 M)***

$\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15} \Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$ ***(1 M)***

$\Rightarrow 8a^2 = 128d^2 \Rightarrow 16d^2 = a^2$

from equation (i) we have,

$16d^2 = 8^2 = 64 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$ ***(1 M)***

Now, $d = 2$ then $8 - 3 \times 2, 8 - 2, 8 + 2, 8 + 3 \times 2$

2, 6, 10, 14

and If $d = -2$ then $8 - 3 \times (-2), 8 - (-2), 8 + (-2), 8 + 3 \times (-2)$

14, 10, 6, 2 ***(1 M)***

38. (*i*) Numbers between 100 and 200 divisible by 9 are 108, 117, ... 198

Here, $a = 108$ and $d = 9$

$t_n = a + (n-1)d \Rightarrow 198 = 108 + (n-1)9$ **(½ M)**

$\Rightarrow 90 = (n-1)9 \Rightarrow 10 = n - 1 \Rightarrow n = 11$ **(1 M)**

Now, $S_n = \frac{n}{2}[a + l]$ [$\because l = t_n =$ *last term*]

$= \frac{11}{2}[108 + 198] = \frac{11 \times 306}{2} = 11 \times 153 = 1683$ **(1 M)**

(*ii*) Numbers between 100 and 200 which are not divisible by 9 are 99 i.e. $n = 99$.

101, 102, ...199 **(½ M)**

Now, $S_n = \frac{n}{2}[a + l] = \frac{99}{2}[101 + 199] = \frac{99}{2} \times 300$

$= 99 \times 150 = 14850$ **(1 M)**

$\therefore$ Sum of integers not divisible by 9 between 100 and 200

= Sum of all integers between 100 and 200 – Sum of integers divisible by 9 between 100 and 200

$= 14850 - 1683 = 13167$ **(1 M)**

39. (*i*) Given, first pole distance = 200 m

Interval between poles = 150 m

So, the series is 200 m, 350 m, 500 m ....

where, first term $a_1 = 200$ m,

And common difference $(d) = a_2 - a_1 = 350 - 200 = 150$ m

n^{th} term of a A.P. is $a_n = a + (n-1)d$

$\Rightarrow$ Distance of n^{th} pole $(a_n) = 200 + (n-1) \times 150$

Hence, for 10^{th} pole,

$\Rightarrow a_{10} = 200 + (10 - 1) \times 150$

$= 200 + 1350 = 1550$ m **(1 M)**

(*ii*) Distance of 15^{th} pole:

$\Rightarrow a_{15} = 200 + (15 - 1) \times 150$

$= 200 + 2100 = 2300$ m

$\Rightarrow$ Distance of 25^{th} pole:

$\Rightarrow a_{25} = 200 + (25 - 1) \times 150$

$= 200 + 3600 = 3800$ m

Hence, distance between 15^{th} and 25^{th} pole $= a_{25} - a_{15}$

$\Rightarrow 3800 - 2300 = 1500$ m **(1 M)**

(*iii*) (*a*) $\because$ Total length of ride = 5000 m

and, distance of 15^{th} pole from base = 2300 m

$\therefore$ Distance from top to 15^{th} pole:

$5000 - 2300 = 2700$ m

Since, speed = 5 m/s

Hence, time $= \frac{2700}{5} = 540$s **(2 M)**

OR

(*iii*) (*b*) Total distance = 5000 m

$\Rightarrow$ Distance covered by poles

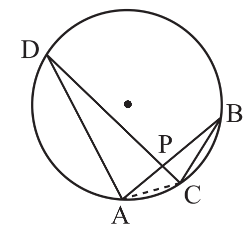
$= 5000 - 200 - 300 = 4500$ m

$\Rightarrow$ Number of intervals $= \frac{4500}{150} = 30$

Hence, total poles = 30 + 1 = 31 **(2 M)**

6. Triangles

40. (*d*) In ΔADP & ΔCBP



$\angle DPA = \angle BPC$ (vertically opposite angles)

$\angle ADP = \angle CBP$ (angles in same segment)

By *AA* – Similarity

$\Delta ADP \sim \Delta CBP$ **(1 M)**

41. (*b*) Since, $\frac{AB + BC + CA}{PQ + QR + RP} = \frac{56}{48} = \frac{7}{6}$

Also, for $\Delta ABC \sim \Delta PQR$

we have $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{7}{6}$

Hence $\frac{PQ}{AB} = \frac{6}{7}$ **(1 M)**

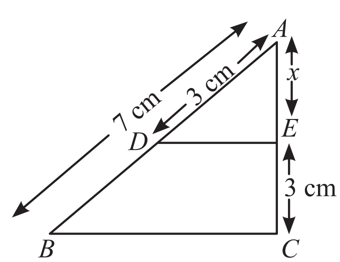
42. (*b*) Given, $\Delta ABC \sim \Delta QPR$

$\therefore \frac{AB}{QP} = \frac{BC}{PR} = \frac{AC}{QR}$

So, $\frac{AC}{BC} = \frac{QR}{PR} \Rightarrow \frac{6}{5} = \frac{3}{x} \Rightarrow x = 2.5$ cm **(1 M)**

43. (*b*) Given, $AD = 3$ cm, $AB = 7$ cm and $EC = 3$ cm.

Let $AE = x$ cm, then $AC = AE + EC = (x + 3)$ cm



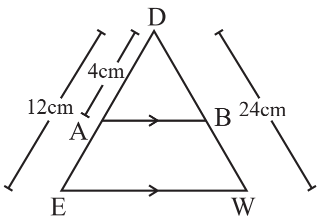
In ΔABC, $DE \parallel BC$

$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$ [By Thale's theorem]

$\Rightarrow \dfrac{3}{7} = \dfrac{x}{x+3} \Rightarrow 3(x+3) = 7x \Rightarrow 3x + 9 = 7x$

$\Rightarrow 7x - 3x = 9 \Rightarrow 4x = 9 \Rightarrow x = \dfrac{9}{4} = 2.25$ cm ***(1 M)***

44.



Given, $AD = 4$ cm, $DE = 12$ cm, $DW = 24$ cm ***(½ M)***

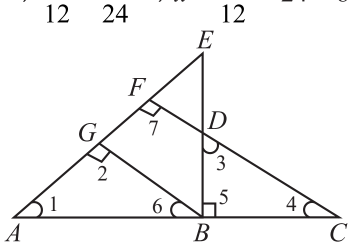
Let DB be x cm

Now, in ΔDEW, $AB \parallel EW$

$\therefore \dfrac{AD}{DE} = \dfrac{DB}{DW}$ [by Thale's Theorem]

$\Rightarrow \dfrac{4}{12} = \dfrac{x}{24} \Rightarrow x = \dfrac{4}{12} \times 24 = 8$ cm ***(½ M)***

45.



Given, $EB \perp AC$

$BG \perp AE$, $CF \perp AE$

We have to prove that

(*i*) $\Delta ABG \sim \Delta DCB$ ***(½ M)***

(*ii*) $\dfrac{BC}{BD} = \dfrac{BE}{BA}$

Proof: (*i*) In ΔABG and ΔDCB, ***(½ M)***

$BG \parallel CF$, as corresponding angles are equal

$\therefore \angle 2 = \angle 5$ [both are 90°]

$\angle 6 = \angle 4$ [corresponding angles]

$\therefore \Delta ABG \sim \Delta DCB$ [by AA similarity]

$\therefore \angle 1 = \angle 3$ [by CPCT] ...(*i*) ***(1 M)***

(*ii*) In ΔABE and ΔDBC,

$\angle 1 = \angle 3$ [from (*i*)]

$\angle ABE = \angle 5$ [both are 90°]

$\therefore \Delta ABE \sim \Delta DBC$ [both AA similarity]

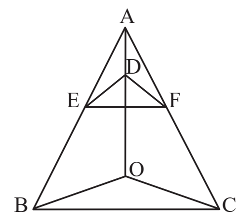
$\therefore \dfrac{BC}{BE} = \dfrac{BD}{BA}$

[$\because$ In similar triangles, the corresponding sides are proportional]

$\Rightarrow \dfrac{BC}{BD} = \dfrac{BE}{BA}$ Hence, proved. ***(1 M)***

46. Given, $DE \parallel OB$ and $EF \parallel BC$

now in ΔAOB, $DE \parallel OB$



$\dfrac{AE}{EB} = \dfrac{AD}{DO}$ (Thale's theorem) ...(*i*) ***(1 M)***

similarly in ΔABC, $EF \parallel BC$

$\dfrac{AE}{EB} = \dfrac{AF}{FC}$ (Thale's theorem) ...(*ii*) ***(1 M)***

From eqn (*i*) and (*ii*) we get

$\dfrac{AD}{DO} = \dfrac{AF}{FC}$

$DF \parallel OC$ (by converse of Thale's theorem) ***(1 M)***

47. We have to prove that $PQ \parallel AB$

Given, in ΔABC, $DP \parallel BC$

$\because \dfrac{AD}{BD} = \dfrac{AP}{PC}$ (By Thale's theorem)

$\Rightarrow \dfrac{AD}{AB} = \dfrac{AP}{AC}$ ...(*i*) ***(1 M)***

Similarly, $EQ \parallel AC$ in ΔABC

$\dfrac{BQ}{QC} = \dfrac{BE}{EA}$ *(By Thale's theorem)* ***(1 M)***

$\Rightarrow \dfrac{BQ}{BC} = \dfrac{BE}{AB}$

$BE = AD$ (given)

$\dfrac{BQ}{BC} = \dfrac{AD}{AB}$ ...(*ii*) ***(1 M)***

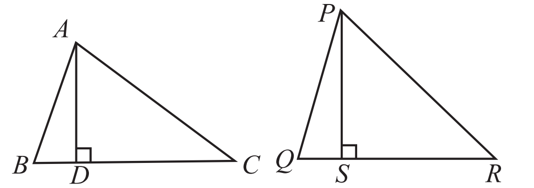
from eqn. (*i*) and (*ii*) we get

$\dfrac{BQ}{BC} = \dfrac{AP}{AC} \Rightarrow \dfrac{BQ}{QC} = \dfrac{AP}{PC}$

By converse of Thale's theorem

Hence, $PQ \parallel AB$ *proved* ***(1 M)***

48.



Given:

$\Delta ABC \sim \Delta PQR$ with corresponding sides in ratio 3 : 5, $AD \perp BC$ and $PS \perp QR$.

(*i*) In ΔADC and ΔPSR:

$\angle ADC = \angle PQR = 90°$,

$\angle C = \angle R$ ($\Delta ABC \sim \Delta PQR$) **(½ M)**

Hence by *AA* similarity, $\Delta ADC \sim \Delta PSR$. **(1 M)**

(*ii*) Since $\Delta ADC \sim \Delta PSR$.

$\therefore$ Ratio of corresponding side of ΔADC and ΔPSR is 3 : 5, **(½ M)**

Given $AD = 4$ cm, then, **(1 M)**

$\Rightarrow \dfrac{AD}{PS} = \dfrac{3}{5} \Rightarrow \dfrac{4}{PS} = \dfrac{3}{5} \Rightarrow PS = \dfrac{20}{3}$ cm **(1 M)**

(*iii*) Find ar(ΔABC) : ar(ΔPQR):

$\because$ Ratio of areas of similar triangles is square of ratio of corresponding sides.

Hence the ratio = ar(ΔABC) : ar(ΔPQR)

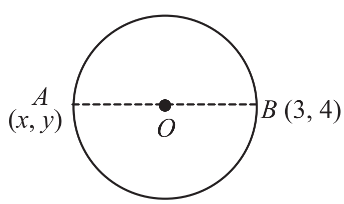
$= 3^2 : 5^2 = 9 : 25$ **(1 M)**

7. Coordinate Geometry

49. (*b*) The distance of the point (x, y) from y-axis is its x-coordinate.

Hence, the distance of the point (–4, 3) from y-axis is 4 units. **(1 M)**

50. Consider, the centre of the circle be O(–2, 2) and the co-ordinates of A be (x, y) and B be (3, 4)



By using mid-point formula, we get

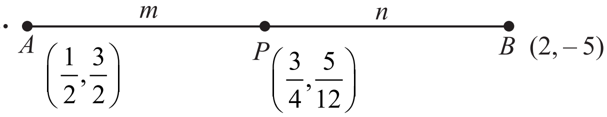
$$\left(\frac{x+3}{2}, \frac{y+4}{2}\right) = (-2, 2)$$

$\therefore \dfrac{x+3}{2} = -2 \Rightarrow x + 3 = -4 \Rightarrow x = -4 - 3 = -7$

And $\dfrac{y+4}{2} = 2 \Rightarrow y + 4 = 4 \Rightarrow y = 0$

Hence, coordinates of A are (–7, 0) **(1 M)**

51.



Let P divides AB internally in the ratio m:n

We have,

$$\therefore \quad P = \left(\frac{2m + \frac{n}{2}}{m+n}, \frac{-5m + \frac{3n}{2}}{m+n}\right)$$ **(1 M)**

and given $P = \left(\dfrac{3}{4}, \dfrac{5}{12}\right)$

Equating the corresponding co-ordinate from the point P, we get

$$\Rightarrow \frac{2m + \frac{n}{2}}{m+n} = \frac{3}{4} \text{ and } \frac{-5m + \frac{3n}{2}}{m+n} = \frac{5}{12}$$

$\therefore \dfrac{4m+n}{2(m+n)} = \dfrac{3}{4} \Rightarrow 16m + 4n = 6m + 6n$ **(½ M)**

$\Rightarrow 10m = 2n \Rightarrow \dfrac{m}{n} = \dfrac{2}{10} = \dfrac{1}{5}$

$\Rightarrow P$ divides AB in the ratio 1: 5 **(½ M)**

52. Given: Coordinates of A = (–1, 7)

Coordinates of B = (4, –3)

$AC = 2BC$

Section formula,

$\Rightarrow$ Coordinates of $C = \left(\dfrac{mx_2 - nx_1}{m-n}, \dfrac{my_2 - ny_1}{m-n}\right)$ **(1 M)**

Where,

$\dfrac{AC}{BC} = \dfrac{2}{1} = \dfrac{m}{n}$

$(x_1, y_1) = (-1, 7)$, (coordinates of A)

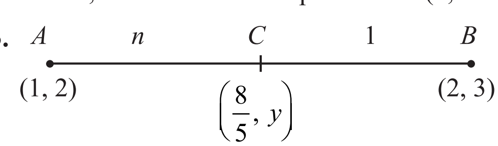
$(x_2, y_2) = (4, -3)$ (coordinates of B)

$\Rightarrow x_C = \dfrac{2(4) - 1(-1)}{2-1} = \dfrac{8+1}{1} = 9$

$\Rightarrow y_C = \dfrac{2(-3) - 1(7)}{2-1} = \dfrac{-6-7}{1} = -13$ **(1 M)**

Hence, the coordinates of point C are (9, –13).

53.



Let the ratio in which C divides AB be n:1. Applying section formula for x-coordinate, we get

$\Rightarrow x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2} \Rightarrow \dfrac{8}{5} = \dfrac{2(n) + 1(1)}{n+1}$ **(½ M)**

$\Rightarrow 8(n+1) = 5(2n+1) \Rightarrow 2n = 3 \Rightarrow n = \dfrac{3}{2}$

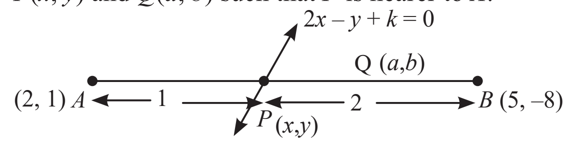
$\therefore$ Ratio $= \dfrac{n}{1} = \dfrac{\frac{3}{2}}{1} = \dfrac{3}{2}$ or 3 : 2 ***(1 M)***

Now, applying section formula for y coordinate

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \Rightarrow y = \frac{3\left(\frac{3}{2}\right)+2(1)}{\frac{3}{2}+1} = \frac{3(3)+2(2)}{3+2} \quad \text{(½ M)}$$

$\Rightarrow y = \dfrac{9+4}{5} = \dfrac{13}{5}$ ***(1 M)***

54. Let the points $A(2, 1)$ and $B(5, -8)$ is trisected at the points $P(x, y)$ and $Q(a, b)$ such that P is nearer to A.



Let, P divides AB internally in the ratio 1 : 2

Using section formula, coordinates of P are ***(1 M)***

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \quad \text{(½ M)}$$

$$= \left(\frac{1\times5+2\times2}{1+2}, \frac{1\times-8+2\times1}{1+2}\right)$$

$$(x, y) = \left(\frac{9}{3}, \frac{-6}{3}\right) = (3, -2)$$

Hence, $P \equiv (3, -2)$ ***(½ M)***

$\because$ P also lies on the line given by $2x - y + k = 0$

$\because$ $(3, -2)$ satisfies the equation $2x - y + k = 0$

$\therefore 2 \times 3 - (-2) + k = 0$ ***(½ M)***

$\Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$

Hence, value of k is -8. ***(½ M)***

55. Given, $P(x,y)$ is equidistant from $A(a+b, b-a)$ & $B(a-b, a+b)$.

So, $AP = BP$ ***(1 M)***

$$\Rightarrow \sqrt{(x-(a+b))^2 + (y-(b-a))^2} = \sqrt{(a-b-x)^2 + (a+b-y)^2}$$

(1 M)

Squaring and solving

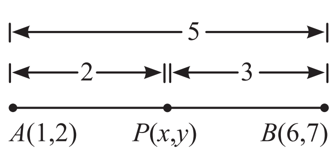
$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a)$

$= (a-b)^2 + x^2 - 2x(a-b) + (a+b)^2 + y^2 - 2y(a+b)$

$\Rightarrow \quad -2ax - 2bx - 2by + 2ay = -2ax + 2bx - 2ay - 2by$

$\Rightarrow \quad -4bx = -4ay \Rightarrow bx = ay$ *(Proved)* ***(1 M)***

56.



Let $P(x, y)$ be the any point on AB

Given $AP = \dfrac{2}{5}AB \Rightarrow \dfrac{AP}{AB} = \dfrac{2}{5} \quad \therefore PB = 5 - 2 = 3$ ***(1 M)***

So P divides AB internally in the ratio 2:3

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) \quad \text{(1 M)}$$

$$= \left(\frac{2\times6+3\times1}{2+3}, \frac{2\times7+3\times2}{2+5}\right) = \left(\frac{15}{5}, \frac{20}{5}\right)$$

$P(x, y) = (3, 4)$ ***(1 M)***

8. Introduction to Trigonometry

57. Given $2\sin(A+B) = \sqrt{3}$ & $\cos(A-B) = 1$

$\Rightarrow \sin(A+B) = \dfrac{\sqrt{3}}{2} \Rightarrow A + B = 60° \ldots.(i)$ ***(½ M)***

Also $\cos(A-B) = 1 \Rightarrow A - B = 0 \ldots.(ii)$ ***(½ M)***

Adding (i) & (ii) $A + B + A - B = 60°$

$\Rightarrow 2A = 60° \Rightarrow A = 30°$

From eqn (i) $B = 30°$ ***(1 M)***

58. $\text{L.H.S} = \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \dfrac{\sin A}{\cos A}\left(\dfrac{1-2\sin^2 A}{2\cos^2 A - 1}\right)$ ***(½ M)***

We know, $\sin^2\theta + \cos^2\theta = 1$

$$= \frac{\sin A}{\cos A}\left(\frac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - \left(\sin^2 A + \cos^2 A\right)}\right) \quad \text{(½ M)}$$

$$= \frac{\sin A}{\cos A}\left(\frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right) = \tan A = \text{R.H.S.} \quad \text{(1 M)}$$

Therefore, L.H.S. = R.H.S.

Hence, proved. ***(1 M)***

59. Given, $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

L.H.S. $= q(p^2 - 1) = (\sec q + \operatorname{cosec} q)\,[(\sin q + \cos q)^2 - 1]$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1] \quad \text{(1 M)}$$

$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)[1 + 2\sin\theta\cos\theta - 1] \quad \text{(1 M)}$$

$[\because \ \sin^2 q + \cos^2 q = 1]$

$= \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta = 2(\sin q + \cos q)$

$[\because \sin q + \cos q = p]$

$= 2p = $ R.H.S

Hence, L.H.S = R.H.S proved. ***(1 M)***

60. Given, $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

L.H.S $= 2((\sin^2\theta)^3 + (\cos^2\theta)^3) - 3[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta] + 1$ ***(1 M)***

$= 2\{(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)\}$

$- 3(1^2 - 2\sin^2\theta\cos^2\theta) + 1$ $\quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta) + 1$ ***(1 M)***

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1 = 3 - 3 = 0$

$\therefore$ LHS = RHS (proved) ***(1 M)***

61. Given, $\sin(A + 2B) = \dfrac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$

we know, $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 90° = 0$ ***(½ M)***

Now,

$\sin(A + 2B) = \dfrac{\sqrt{3}}{2} \Rightarrow \sin(A + 2B) = \sin 60°$ ***(½ M)***

$A + 2B = 60° \Rightarrow A = 60° - 2B$ ...(*i*)

and $\cos(A + 4B) = 0$

$\cos(A + 4B) = \cos 90° \Rightarrow A + 4B = 90°$ ...(*ii*) ***(1 M)***

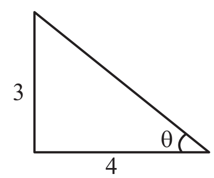
Put A in equation (*ii*)

$\Rightarrow 60° - 2B + 4B = 90° \Rightarrow 2B = 30° \Rightarrow B = 15°$ ***(½ M)***

Put B in equation (*i*), we get

$A = 60° - 2 \times 15° \Rightarrow A = 30°$ ***(½ M)***

62. Given, $4\tan\theta = 3 \Rightarrow \tan\theta = 3/4$,



$\because \tan\theta = \dfrac{\text{Perpendicular}}{\text{base}} = \dfrac{3}{4}$

$\therefore$ By Pythagoras theorem, we have

$h = \sqrt{(3)^2 + 4^2} = \sqrt{9 + 16} = 5$ ***(1 M)***

$\therefore \sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$

Now, $\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \dfrac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1}$ ***(1 M)***

$= \dfrac{12 - 4 + 5}{12 + 4 - 5} = \dfrac{13}{11}$ ***(1 M)***

63. L.H.S $= \dfrac{\cos A + \sin A - 1}{\cos A - \sin A + 1}$

Multiply numerator and denominator by $(\cos A - \sin A - 1)$

L.H.S $= \dfrac{(\cos A + \sin A - 1)}{(\cos A - \sin A + 1)} \times \dfrac{(\cos A - \sin A - 1)}{(\cos a - \sin A - 1)}$ ***(½ M)***

$= \dfrac{\cos^2 A - \cos A\sin A - \cos A + \sin A\cos A - \sin^2 A - \sin A - \cos A + \sin A + 1}{(\cos A - \sin A)^2 - 1^2}$ ***(½ M)***

$= \dfrac{\cos^2 A - \sin^2 A - 2\cos A + 1}{\cos^2 A + \sin^2 A - 2\sin A\cos A - 1}$

$[\because \sin^2\theta + \cos^2\theta = 1]$ ***(½ M)***

$= \dfrac{\cos^2 A + (1 - \sin^2 A) - 2\cos A}{1 - 2\sin A\cos A - 1}$ ***(½ M)***

$= \dfrac{\cos^2 A + \cos^2 A - 2\cos A}{-2\sin A\cos A} = \dfrac{2\cos^2 A - 2\cos A}{-2\sin A\cos A}$ ***(½ M)***

$= \dfrac{2\cos A(\cos A - 1)}{-2\cos A\sin A} = \dfrac{\cos A - 1}{-\sin A}$

$= \dfrac{1 - \cos A}{\sin A} = \dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}$

$= \text{cosec}A - \cot A$ = RHS ***(½ M)***

$\because$ LHS = RHS

Hence proved.

64. L.H.S $= \dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$

$\left[\because \tan A = \dfrac{\sin A}{\cos A}, \text{cosec}A = \dfrac{1}{\sin A}, \sec A = \dfrac{1}{\cos A}\right]$ ***(½ M)***

$= \dfrac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \dfrac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$ ***(½ M)***

$= \dfrac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \dfrac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\sin^2 A\cos^2 A}}$ ***(½ M)***

$= \dfrac{\sin^2 A}{\sin^2 A - \cos^2 A} + \dfrac{\cos^2 A}{\sin^2 A - \cos^2 A}$ ***(½ M)***

$= \dfrac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A}$ ***(½ M)***

$= \dfrac{1}{\sin^2 A - \cos^2 A}$ $\quad [\because \sin^2 A + \cos^2 A = 1]$ ***(½ M)***

Since, $\sin^2 A = 1 - \cos^2 A$

$= \dfrac{1}{1 - \cos^2 A - \cos^2 A}$ ***(½ M)***

$= \dfrac{1}{1 - 2\cos^2 A}$ = R.H.S ***(½ M)***

L.H.S. = R.H.S.

Hence proved.

65. L.H.S. $= \dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A}$ ***(1 M)***

$= \dfrac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos)}$ ***(1 M)***

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin A\cos A}{\sin^2 A - \cos^2 A} \quad \textbf{(1 M)}$$

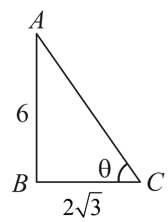
$$= \frac{1+1}{1-\cos^2 A - \cos^2 A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{2}{1-2\cos^2 A} = \text{R.H.S.} \quad \textbf{(1 M)}$$

Hence, L.H.S. = R.H.S. proved.

9. Some Applications of Trigonometry

66. (*a*) Given that,



Height of the pole = 6 m

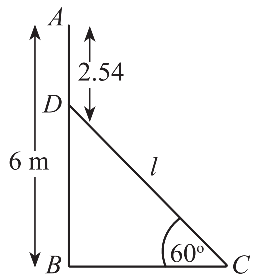
Length of shadow = $2\sqrt{3}$ m

Let θ be the elevation of the sun, then

$$\Rightarrow \tan\theta = \frac{AB}{BC} \Rightarrow \tan\theta = \frac{6}{2\sqrt{3}} = \sqrt{3} = \tan 60°$$

$\therefore \theta = 60°$ **(1 M)**

67.



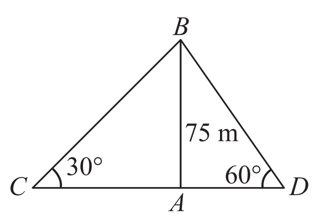
Let the length of ladder be l meters **(½ M)**

Now, In ΔDBC,

$$\sin 60° = \frac{BD}{DC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{6-2.54}{l}$$

$$\Rightarrow l = \frac{2\times 3.46}{\sqrt{3}} = \frac{6.92}{1.73} \Rightarrow l = 4 \text{ m} \quad \textbf{(½ M)}$$

68. **(½ M)**



In ΔBAC, $\tan 30° = \frac{AB}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{AC}$ $\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow AC = 75\sqrt{3}$ m ...(*i*) **(1 M)**

Now, in ΔABD, $\tan 60° = \frac{AB}{AD} \Rightarrow \sqrt{3} = \frac{75}{AD}$

$\Rightarrow AD = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{75\sqrt{3}}{3} = 25\sqrt{3}$ m ...(*ii*) **(½ M)**

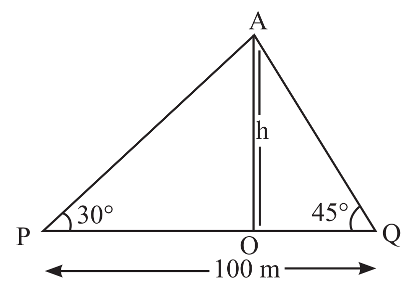
Now, $CD = AC + AD$

$= 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3}$ m [From (*i*) & (*ii*)] **(1 M)**

Hence, the distance between two men is $100\sqrt{3}$ m.

69. Let OA be the height of the tree

$OA = h$ m



(1 M)

In ΔPOA,

$\therefore \tan 30° = \frac{OA}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{OP} \Rightarrow OP = \sqrt{3}h$ ...(*i*)

and In ΔQOA,

$$\therefore \tan 45° = \frac{OA}{OQ}$$

$1 = \frac{h}{OQ} = OQ = h$ ...(*ii*) **(1 M)**

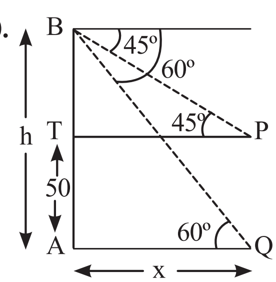
On adding eqn (*i*) and (*ii*), we get

$$OP + OQ = \sqrt{3}h + h \Rightarrow PQ = (\sqrt{3}+1)h$$

$$\Rightarrow 100 = 2.732\,h \Rightarrow h = \frac{100}{2.732} = 36.6 \text{ m}$$

Hence, height of tree is 36.6 m **(1 M)**

70.



In ΔBTP, $\tan 45° = \frac{BT}{PT} \Rightarrow 1 = \frac{h-50}{x}$

$\Rightarrow x = h - 50$ ...(*i*) **(1 M)**

In ΔBAQ, $\tan 60° = \frac{AB}{AQ} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(*ii*)

From eqn. (*i*) and (*ii*), we get

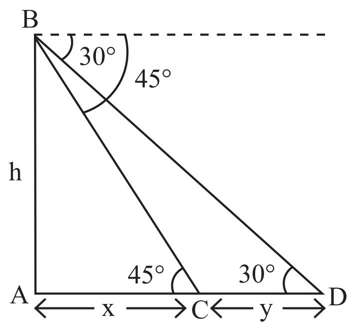
$$\Rightarrow h - 50 = \frac{h}{\sqrt{3}} \Rightarrow h - \frac{h}{\sqrt{3}} = 50 \Rightarrow h\left(\sqrt{3}-1\right) = 50\sqrt{3}$$

$$\Rightarrow h = \frac{50\sqrt{3}}{\sqrt{3}-1} \Rightarrow h = \frac{50 \times 1.732}{1.732 - 1} = 118.25 \text{ m}$$ **(1 M)**

From equation (*ii*) we get, $x = \frac{118.25}{\sqrt{3}} = \frac{118.25}{1.732} = 68.25$ m

Hence, height as the Tower = 118.25 m. and distance between tower and building = 68.25 m **(1 M)**

71.



Given, the angle of depression of the car at position *D* is 30° & at position *C* is 45°.

Clearly, the angle of elevation at *D* is 30° & at *C* is 45°.

Let the distances *AC* & *CD* be *x* & *y* respectively & the height of tower be *h*.

$\therefore$ In ΔABC:

$$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h$$ ...(*i*) **(1 M)**

and in ΔABD:

$$\tan 30° = \frac{h}{x+y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y} \Rightarrow x + y = h\sqrt{3}$$ ...(*ii*)

From, (*i*) and (*ii*), we get,

$x + y = x\sqrt{3} \Rightarrow y = x\sqrt{3} - x$ [subtracting *x* from both sides]

$= x(\sqrt{3} - 1)$ **(1 M)**

Therefore, the car took 12 minutes to reach from *D* to *C* i.e., the distance, $x(\sqrt{3} - 1)$.

$\therefore$ Speed of the car, $S = \frac{\text{distance}}{\text{time}} = \frac{x(\sqrt{3}-1)}{12}$ **(1 M)**

Now, time taken by car to reach from *C* to *D*, i.e., to cover *x* units is,

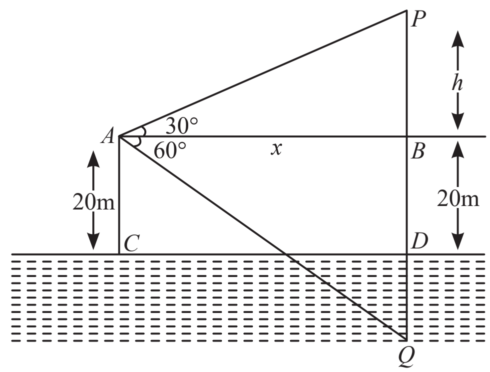
$$\text{Time taken} = \frac{\text{distance}}{\text{speed}} = \frac{x}{\frac{x(\sqrt{3}-1)}{12}} = \frac{12}{\sqrt{3}-1}$$

$$= \frac{12(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{12(\sqrt{3}+1)}{3-1} = \frac{12(\sqrt{3}+1)}{2} = 6(\sqrt{3}+1)$$

= 16.4 min (approx)

$\therefore$ The time taken by the car to reach the tower is 16.4 min. **(1 M)**

72. Let *P* be the position of the cloud which makes an angle of elevation from *A* is 30° and *Q* be the position of reflection of the cloud which makes angle of depression from *A* is 60°.



Here $AC = BD = 20$ m

$PD = 20 + h = DQ$ **(1 M)**

In ΔAPB, $\tan 30° = \frac{PB}{AB} = \frac{h}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h$$ ...(*i*) **(1 M)**

In ΔAQB

$$\tan 60° = \frac{BQ}{AB} = \frac{BD + DQ}{x} = \frac{20 + 20 + h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{40 + h}{x} \Rightarrow x = \frac{40 + h}{\sqrt{3}}$$ ...(*ii*) **(1 M)**

From eqn. (*i*) and (*ii*), we get

$$\frac{40+h}{\sqrt{3}} = \sqrt{3}h \Rightarrow 40 + h = 3h \Rightarrow h = 20$$

Put $h = 20$ in eqn (*i*), we get $x = 20\sqrt{3}$

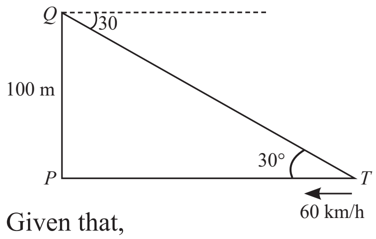
From the figure

$$AP = \sqrt{AB^2 + PB^2} = \sqrt{x^2 + h^2} = \sqrt{(20\sqrt{3})^2 + (20)^2}$$

$= 20\sqrt{3+1} = 20 \times 2 = 40$ m

$\therefore$ Distance of the cloud from A = 40 m. **(1 M)**

73. (*i*)



(1 M)

Given that,

Point Q is 100 m above P.

Ambulance moving towards P on the highway.

Angle of depression from Q to ambulance is 30°.

(*ii*) Let distance between ambulance and $P = x$

then, $\tan 30^\circ = \dfrac{100}{x} \Rightarrow x = \dfrac{100}{\tan 30^\circ} = 100\sqrt{3} \approx 173\,\text{m}$

(1 M)

(*iii*) (*a*) Initial distance $x_1 = 100\sqrt{3} \approx 173\,\text{m}$

Final distance x_2 = 100 m (since tan45° = 1)

$\Rightarrow$ Distance covered = 173 – 100 = 73m

Speed $= 60\ \text{km/h} = \dfrac{60 \times 1000}{3600} = \dfrac{50}{3}\ \text{m/s}$

Hence, time for angle of depression to change from 30°

to 45° $= \dfrac{73}{\frac{50}{3}} = \dfrac{219}{50} = 4.38\,s$ **(2 M)**

OR

(*iii*) (*b*) Let distance from T to $P = x$ m

then, $\tan 60^\circ = \dfrac{100}{x} \Rightarrow x = \dfrac{100}{\tan 60^\circ} = \dfrac{100}{\sqrt{3}} \approx 57.74$ m

Speed $= \dfrac{50}{3}\,\text{m/s}$

Hence, time for ambulance to reach P from point T with angle of depression 60°

$= \dfrac{57.74}{\frac{50}{3}} = \dfrac{173.22}{50} = 3.46\text{s}$ **(2 M)**

10. Circles

74. (*d*) In the given figure

OQ = OP, ∠QOP = 65°

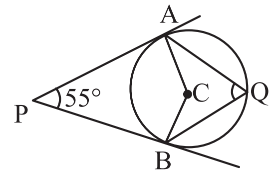
$\Rightarrow \angle PQO = \angle QPO$ (Angle opposite to equal sides)

$\therefore$ *In ΔPOQ, ∠PQO + ∠QPO + 65° = 180°*

$\Rightarrow \angle OPQ = \left(\dfrac{115}{2}\right)^\circ$

$\Rightarrow \angle QPT = \left(\dfrac{65}{2}\right)^\circ$ $\{\because \angle OPQ + \angle QPT = 90^\circ\}$ **(1 M)**

75. (*a*) Given, $\angle APB = 55^\circ$



Also, $\angle PAC = \angle PBC = 90^\circ$

[Tangent is ⊥ to radius]

Now, in quadrilateral *PACB*

$\angle APB + \angle PAC + \angle PBC + \angle ACB = 360^\circ$

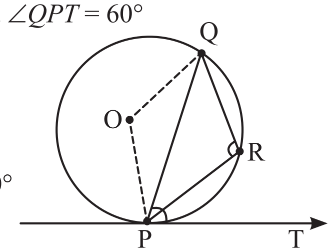
[∵ Sum of interior angles in quadrilateral = 360°]

$\therefore 55^\circ + 90^\circ + 90^\circ + \angle ACB = 360^\circ \Rightarrow \angle ACB = 125^\circ$

As we know, that the angle subtended by an arc at the centre is double the angle subtended by an arc at the remaining part of the circle.

$\therefore \angle ACB = 2\,\angle AQB \Rightarrow \angle AQB = \dfrac{125^\circ}{2} = 62\dfrac{1}{2}^\circ$ **(1 M)**

76. Given that PQ is chord and $\angle QPT = 60^\circ$



From the figure, we have

$\angle OPT = \angle QPO + \angle QPT$

$\Rightarrow 90^\circ = \angle QPO + 60^\circ$

$\Rightarrow \angle QPO = 90^\circ - 60^\circ = 30^\circ$

$\therefore \angle OQP = 30^\circ$

[∵ *OPQ* is isosceles triangle] **(½ M)**

In Δ *OPQ*

$\Rightarrow \angle QPO + \angle POQ + \angle OQP = 180^\circ$

[Sum of interior ∠'s of a triangle is 180°.]

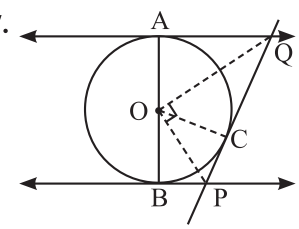
$\Rightarrow 30^\circ + \angle POQ + 30^\circ = 180^\circ$

$\Rightarrow \angle POQ = 180^\circ - 30^\circ - 30^\circ = 120^\circ$

$\therefore$ reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$\therefore \angle PRQ = \dfrac{1}{2} \times$ reflex $\angle POQ = \dfrac{1}{2} \times 240 = 120^\circ$ **(½ M)**

77.



Construction: Join *OC*

Proof:

In Δ*OQA* and Δ*OCQ*

OA = *OC* (Radii of the same circle)

QO = *QO* (Common side)

QA = *QC* (Tangents from same external point *A*)

$\therefore \Delta OAQ \cong \Delta OCQ$ (using SSS congruency) **(1 M)**

$\angle AOQ = \angle COQ$ …(*i*) [by C.P.C.T]

Similarly $\Delta OBP \cong \Delta OCP$

Therefore $\angle BOP = \angle COP$…(*ii*) [by C·P·C·T]

As line AOB is a straight line passing through centre O, therefore it can be considered as a diameter of the circle **(*1 M*)**

So $\angle AOQ + \angle COQ + \angle COP + \angle BOP = 180°$

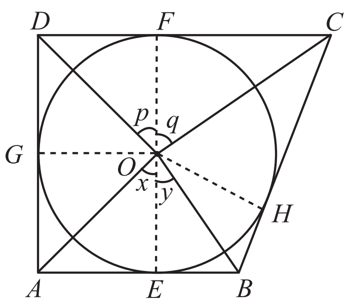
Now from equations (i) and equation (ii) we get

$2\angle COQ + 2\angle COP = 180° \Rightarrow \angle COQ + \angle COP = 90°$

$\therefore \angle POQ = 90°$ ($\therefore \angle COQ + \angle COP = \angle POQ$) **(*1 M*)**

Hence proved.

78.



In ΔAOG and ΔAOE

$AO = AO$ (common side)

$OE = OG$ (Radius of same circle)

$AE = AG$ (Length of tangents from same external point)

$\therefore \Delta AOE \cong \Delta AOG$ (*SSS Similarity*) **(*½ M*)**

$\angle AOE = \angle AOG = x$ ...(i)

Similarly

In ΔDOG and ΔDOF, $\angle DOF = \angle DOG = p$ ...(ii)

In ΔCOF and ΔCOH, $\angle COH = \angle COF = q$ ...(iii)

In ΔBOH and ΔBOE, $\angle BOE = \angle BOH = y$ ...(iv) **(*1 M*)**

Now $\angle AOE + \angle AOG + \angle DOG + \angle DOF + \angle COH + \angle COF + \angle BOE + \angle BOH = 360°$ **(*½ M*)**

$\Rightarrow x + x + p + p + q + q + y + y = 360°$

$\Rightarrow 2(x + p + q + y) = 360°$

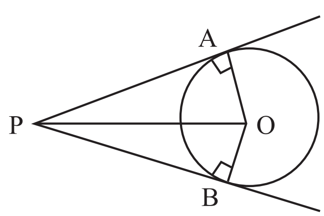
$\Rightarrow x + p + q + y = 180°$

$\Rightarrow (x + y) + (p + q) = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$ **(*1 M*)**

Hence proved.

79.



Given: A circle with center O & tangent PA & PB drawn to the circle from the external point P.

Construction: Join OA, OP & OB.

To prove: Length of tangents drawn are equal i.e., $PA = PB$ **(*1 M*)**

Proof:

We know that, tangents drawn to a circle is perpendicular to the radius of the circle at the point of contact.

$\therefore \angle OAP = \angle OBP = 90°$ ...(i) **(*1 M*)**

Now, In ΔOAP & ΔOBP;

$\angle OAP = \angle OBP$ [from (i)]

$OP = OP$ [common]

$OA = OB$ [both are radius of circle]

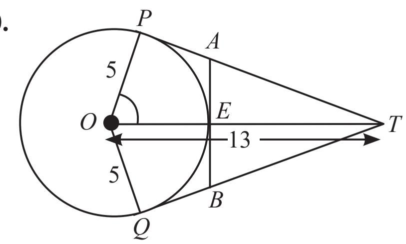
$\therefore$ by RHS congruency criteria,

$\Delta OAP \cong \Delta OBP$ **(*1 M*)**

Hence, $PA = PB$ [by C.P.C.T]

i.e., the length of tangents drawn from an external point to the circle are equal. **(*1 M*)**

80.



Given $OP = OQ = 5$ cm, $OT = 13$ cm

Also, $PT = TQ$ (tangent from same external point)

In ΔOPT, $\angle P = 90°$ **(*1 M*)**

$OT^2 = OP^2 + PT^2 \Rightarrow 13^2 = 5^2 + PT^2 \Rightarrow PT = 12\text{cm} = PQ$

Since length of tangents drawn from a point to a circle are equal.

Therefore

$AP = AE = x$ (let)

$\Rightarrow AP = AE = PT - AT = 12 - AT \Rightarrow AT = 12 - AP$

$\Rightarrow AT = (12 - x)$ cm

and $OT = OE + ET \Rightarrow 13 = 5 + ET \Rightarrow ET = 8$ cm **(*1 M*)**

Now, In ΔAET,

$AT^2 = AE^2 + ET^2 \Rightarrow (12 - x)^2 = x^2 + 8^2$

$\Rightarrow 144 - 24x + x^2 = x^2 + 64 \Rightarrow 24x = 80$

$\Rightarrow x = \frac{10}{3}$ cm **(*1 M*)**

Similarly, $BE = \frac{10}{3}$ cm

$\therefore AB = AE + BE = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ cm **(*1 M*)**

11. Areas Related to Circle

81. Given, $r = 5.2$ cm and perimeter of the sector = 16.4 cm.

Let AOB be the sector with center O.

$\therefore$ Perimeter of the sector = $AO + AB + OB$

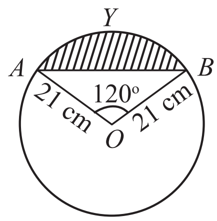
$\Rightarrow 16.4 = 5.2 + 5.2 + AB$

$\Rightarrow AB = 16.4 - 10.4 = 6$ cm **(*1 M*)**

$\therefore$ Area of the sector = $\frac{1}{2}rl = \frac{1}{2} \times 5.2 \times 6 = 15.6\text{ cm}^2$. **(*1 M*)**

82. Given, radius = 21 cm

Let $\theta = \angle AOB = 120°$



Area of segment AYB = Area of sector $OAYB$ – Area of ΔAOB **(½ M)**

Area of sector $OAYB = \frac{\theta}{360°}\times\pi r^2 = \frac{120°}{360°}\times\frac{22}{7}\times(21)^2$

$= \frac{1}{3}\times\frac{22}{7}\times 21\times 21 = 22\times 21 = 462\text{ cm}^2$ **(½ M)**

Now, for area of ΔAOB, draw $OP \perp AB$

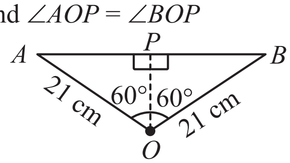
Note that, $OA = OB$ (Radii of the circle)

Therefore, by R.H.S congruence

$\Delta APO \cong \Delta BPO$ **(½ M)**

So, P is the mid-point of AB and $\angle AOP = \angle BOP$

$= \frac{1}{2}\times 120° = 60°$



Consider, $OP = x$ cm

From ΔOPA, $\frac{OP}{OA} = \cos 60° \Rightarrow \frac{x}{21} = \frac{1}{2} \Rightarrow x = \frac{21}{2}$ cm

Also, $\sin 60° = \frac{AP}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{21} \Rightarrow AP = \frac{21\sqrt{3}}{2}$ cm

$\therefore AB = 2AP = 21\sqrt{3}$ cm **(½ M)**

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\times AB\times OP = \frac{1}{2}\times 21\sqrt{3}\times\frac{21}{2}$

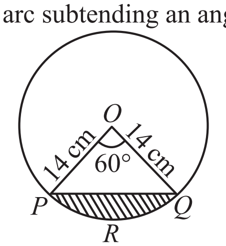
$= \frac{441\sqrt{3}}{4}\text{cm}^2$ **(½ M)**

$\therefore$ Area of segment $AYB = \left(462 - \frac{441}{4}\sqrt{3}\right)\text{cm}^2$

$= \left(\frac{1848 - 441\sqrt{3}}{4}\right)\text{cm}^2 = \frac{21}{4}(88 - 21\sqrt{3})\text{cm}^2 \cong 271.04\text{cm}^2$

(½ M)

83. Let PRQ be the arc subtending an angle of 60°



Area of the minor segment $PRQP$ **(1 M)**

= Area of the sector $OPRQ$ – Area of ΔOPQ

$= \pi r^2 \times \frac{\theta}{360} - \frac{1}{2}r^2\sin\theta$

$= \frac{22}{7}\times(14)^2\times\frac{60}{360} - \frac{1}{2}\times(14)^2\sin 60°$

$= 22\times 2\times 14\times\frac{1}{6} - 7\times 14\times\frac{\sqrt{3}}{2}$

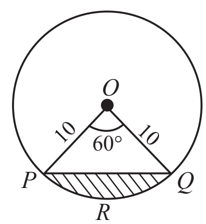
$= \frac{308}{3} - 49\sqrt{3} \approx 17.89\text{ cm}^2$ **(1 M)**

Area of major segment = Area of circle – Area of minor segment $PRQP$.

$= \pi r^2 - 17.89 = \frac{22}{7}\times 14\times 14 - 17.89$

$= 616 - 17.89 \approx 598.11\text{ cm}^2$ **(1 M)**

84. Given, Radius of circle = 10 cm



Angle of minor sector at centre = 60°

Now, Area of sector $= \frac{\theta}{360°}\times\pi r^2$

area of circle = πr^2 **(1 M)**

Area of minor segment PRQ

= Area of sector $OPRQ$ – Area of ΔOPQ

$= \frac{60°}{360°}\times 3.14\times 10^2 - \frac{\sqrt{3}}{4}\times(\text{side})^2$ **(1 M)**

$= 52.33 - \frac{\sqrt{3}}{4}(10)^2 = 52.33 - 43.30 \approx 9.03\text{ cm}^2$ **(1 M)**

Area of major sector = Area of circle – Area of minor segment PRQ

$= \pi r^2 - 9.03 = 3.14\times 10\times 10 - 9.03 \approx 304.97\text{ cm}^2$ **(1 M)**

85. (*i*) $\because$ Circumference of each ring = 44 m

Circumference formula, $2\pi r = 44$

$\Rightarrow r = \frac{44}{2\pi} = \frac{22}{\pi} = 7\text{ m}$ **(1 M)**

Hence, radius of each circular ring is 7 m.

(*ii*) ΔOAB is equilateral.

$\Rightarrow$ Each angle in an equilateral triangle is 60°.

Hence, $\angle AOB = 60°$ **(1 M)**

(*iii*) (*a*) Area of sector OAB

$= \frac{60}{360}\times\pi r^2 = \frac{1}{6}\times\pi\times 7^2 = \frac{49\pi}{6}\text{m}^2$

Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times 7^2 = \frac{49\sqrt{3}}{4} \text{m}^2$

Hence, area of shaded region

$$R_1 = \pi(7)^2 - 2\left(\frac{49\pi}{6} - \frac{49\sqrt{3}}{4}\right)$$

$$\Rightarrow 49\pi - \frac{49\pi}{3} + \frac{49\sqrt{3}}{2}$$

$$\Rightarrow \frac{98\pi}{3} + \frac{49\sqrt{3}}{2} = 145.06 \text{m}^2$$ ***(2 M)***

OR

(*iii*) (*b*) Each unshaded region is a segment of the circle.

Length of rope around one unshaded region =

$$\Rightarrow \text{Arc length} = \frac{60}{360} \times 44 = \frac{44}{6} \approx 7.33 \text{m}$$

Hence, total length for all unshaded regions

= 8 × 7.33 ≈ 58.64 m ***(2 M)***

12. Surface Areas and Volumes

86. (*d*) For new cuboid formed

$A = 2(lb + bh + hl)$ $(l = 20, b = 10, h = 10)$

= 2 (200 + 100 + 200) = 1000 cm^2

For cube of 10 cm length $A = 10 \times 10 = 100 \text{ cm}^2$

Hence, Assertion is not true but reason is true. ***(1 M)***

87. Given,

Curved surface area of a right circular cylinder = 176 cm^2

$\Rightarrow 2\pi rh = 176 \Rightarrow \pi rh = 88$ ...(*i*)

also given, vol. of cylinder = 1232 cm^3

$\Rightarrow \pi r^2 h = 1232$ ...(*ii*)

On dividing equ.(*ii*) from (*i*), we get ***(1 M)***

$$\Rightarrow \frac{\pi r^2 h}{\pi rh} = \frac{1232}{88} \Rightarrow r = 14 \text{ cm}$$

Put the value of r in equ. (*i*), we get

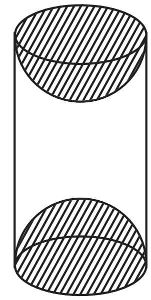
$\Rightarrow \pi \times 14 \times h = 88$

$$\Rightarrow \frac{22}{7} \times 14 \times h = 88 \Rightarrow h = 2 \text{ cm}$$ ***(1 M)***

88. Given, height of the cylinder = 10 cm

radius of base = 3.5 cm

Total surface area of the article = curved surface area of the cylinder + 2 × surface area of hemisphere. ...(*i*)



Now, the curved scarface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 3.5 \times 10 = 220 \text{ cm}^2$$ ***(1 M)***

Now surface area of the hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times (3.5)^2$

$$= 2 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} = 77 cm^2$$ ***(1 M)***

∴ Total surface area of the article = 220 + 2 × 77 = 374 cm^2 ***(1 M)***

89. Given, Radius of conical heap = 12 m

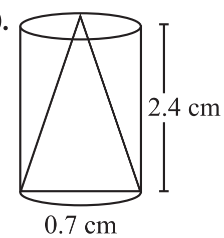
$$\because \text{ Volume of rice} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (12)^2 \times 3.5 = 528 \text{ m}^3$$ ***(1 M)***

Area of canvas cloth required = πrl

$$l = \sqrt{12^2 + (3.5)^2} = 12.5 \text{ m}$$ ***(1 M)***

Now, area of canvas required $= \frac{22}{7} \times 12 \times 12.5 = 471.4 \text{ m}^2$ ***(1 M)***

90.



Given, height of cylinder = 2.4 cm,

radius of cylinder = 0.7 cm

Now, a right circular cone is cut out from the cylinder with same height & radius ***(1 M)***

∴ Total surface area of the remaining solid = (curved surface area of cylinder) + (curved surface area of cone) + (area of top of the cylinder).

$= 2\pi rh + \pi r\ell + \pi r^2$

Now, slant height of cone, $\ell = \sqrt{h^2 + r^2}$

$$= \sqrt{(2.4)^2 + (0.7)^2} \text{cm} = \sqrt{5.76 + 0.49} \text{ cm}$$

$$= \sqrt{6.25} \text{ cm} = 2.5 \text{ cm}$$ ***(1 M)***

$\therefore$ Total surface area of remaining solid

$= 2\pi rh + \pi r\ell + \pi r^2 = \pi r\,(2h + \ell + r)$

$= \pi \times 0.7\,(2(2.4) + 2.5 + 0.7) = \frac{22}{7} \times 0.7 \times 8 = 17.6 \text{ cm}^2$

Hence, the total surface area of the remaining solid is 17.6 cm^2. ***(1 M)***

91. Given; Side of a Cube = 6 cm

Total surface area of cube $= 6 \times (\text{Side})^2 = 216 \text{ cm}^2$

Area coverd on the face of cube by circular part of hemisphere $= \pi r^2 = \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} = \frac{77}{8}$ ***(1 M)***

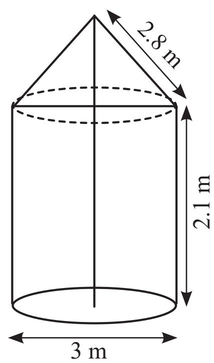
curved surface area of hemisphere

$= 2\pi r^2 = 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} = \frac{77}{4}$ ***(1 M)***

Now total surface area = Surface area of cube – area of circular part + area of hemisphere

$= 216 - \frac{77}{8} + \frac{77}{4} = \frac{1728 - 77 + 154}{8} = 225.625 \text{ cm}^2$ ***(1 M)***

92. Given dimensions of cylindrical part,



Height, $h_1 = 2.1$ m; Diameter, $d_1 = 3$m

Dimensions of conical part,

Slant height, $l = 2.8$ m;

Diameter of cone d_2

= diameter of cylinder $d_1 = 3$ cm ***(1 M)***

Now, Area of canvas needed = curved surface area of tent

$\therefore$ Area of canvas needed = curved surface area of cone + curved surface area of cylinder.

$= \pi rl + 2\pi rh \Rightarrow \pi\frac{d_2}{2}l + 2\pi\frac{d_1}{2}h$

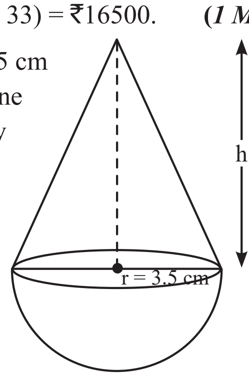
$= \frac{22}{7}\times\frac{3}{2}\times[2.8 + 2\times 2.1] = 33 \text{ m}^2$ ***(1 M)***

$\therefore$ The area of canvas needed = 33 m^2

Given, cost of canvas is ₹500 /m^2

$\therefore$ Total cost of canvas = ₹(500 × 33) = ₹16500. ***(1 M)***

93. r = radius of the hemisphere = 3.5 cm



and let, 'h' is the height of the cone

Total volume of solid wooden toy

= volume of cone

+ volume of hemisphere

$\Rightarrow 166\frac{5}{6}\text{cm}^3 = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$\Rightarrow \frac{1001}{6} = \frac{1}{3}\pi\times r^2(h + 2r)$

$\Rightarrow \frac{1001}{6} = \frac{1}{3}\times\frac{22}{7}\times(3.5)^2(h+7)$ ***(1 M)***

$\Rightarrow \frac{1001}{77} = h + 7 \Rightarrow h = \frac{462}{77} \Rightarrow h = 6$ cm ***(1 M)***

Height of toy = $h + r = 6 + 3.5 = 9.5$ cm

Surface area of hemisphere $= 2\pi r^2 = 2\pi(3.5)^2 = 77 \text{ cm}^2$

Cost of painting the hemispherical part of the toy

= 77 × 10 = ₹ 770 ***(1 M)***

13. Statistics

94. (*a*) Since the median is the middle value of the data set when it is arranged in ascending order, increasing every value by the same amount won't change its position relative to other values but the median of the new data increases by 2. ***(1 M)***

95.

Cost of Living Index	No. of Weeks (f)	cf
1400 – 1550	8	8
1550 – 1700	15	23
1700 – 1850	21	44
1850 – 2000	8	52
	$N = \sum f = 52$	

Here, $N = 52$ ***(½ M)***

$\Rightarrow \frac{N}{2} = \frac{52}{2} = 26$

$\therefore$ 26 will lie in the class interval 1700 – 1850.

$\therefore$ Median class is 1700 – 1850. ***(½ M)***

96. From the given table,

Maximum frequency is 25 for shoes size 5 ***(½ M)***

Hence, modal size of shoes is 5 ***(½ M)***

97.

C.I	f_i	x_i	$x_i f_i$
3 – 5	5	4	20
5 – 7	10	6	60
7 – 9	10	8	80
9 – 11	7	10	70
11 – 13	8	12	96
Total	$\Sigma f_i = 40$		$\Sigma x_i f_i = 326$

(1 M)

From the above table we have,

$\Sigma f_i = 40$ and $\Sigma x_i f_i = 326$

$\because$ Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{326}{40} = 8.15$ ***(1 M)***

98.

C.I	f_i
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

Highest frequency = 12

Modal class = 60 – 80

$l = 60, f_0 = 10, f_1 = 12, f_2 = 6$ and $h = 20$ ***(1 M)***

$\because$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$ ***(½ M)***

$= 60 + \left(\frac{12-10}{2\times12-10-6}\right)\times 20 = 60 + \frac{2}{8}\times 20 = 60 + 5 = 65$

(½ M)

99.

Pocket money in ₹	No. of students
0 – 20	2
20 – 40	2
40 – 60	3
60 – 80	12
80 – 100	18
100 – 120	5
120 – 140	2

(½ M)

Modal class is 80 – 100

$l = 80, f_1 = 18, f_0 = 12, f_2 = 5, h = 20$

Now, mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}\times h$ ***(½ M)***

mode $= 80 + \frac{18-12}{2\times18-12-5}\times 20$ ***(½ M)***

$= 80 + \frac{6\times20}{19} = 86.32$(approx)

Hence, required pocket money = ₹86.32 (approx) ***(½ M)***

100. We will use the mid-point of each interval to represent the marks for calculation.

Interval	Class mark (x_i)	No. of students (f_i)
0–10	5	12
10–20	15	23
20–30	25	34
30–40	35	25
40–50	45	6

(1 M)

Mean $= \frac{\sum f_i x_i}{\sum f_i}$ ***(½ M)***

$= \frac{(5\times12)+(15\times23)+(25\times34)+(35\times25)+(45\times6)}{12+23+34+25+6}$

(½ M)

$= \frac{60+345+850+875+270}{100} = \frac{2400}{100} = 24$ ***(1 M)***

101.

Class Interval	Frequency f_i	x_i	$d_i = x_i - 55$	$f_i d_i$
10 – 20	8	15	–40	–320
20 – 30	7	25	–30	–210
30 – 40	12	35	–20	–240
40 – 50	23	45	–10	–230
50 – 60	11	55 = A	0	0
60 – 70	13	65	10	130
70 – 80	8	75	20	160
80 – 90	6	85	30	180
90 – 100	12	95	40	480
	$\Sigma f_i = 100$			$\Sigma f_i d_i = -50$

(2 M)

Let $A = 55$ [Assumed Mean]

$\therefore$ Mean $= A + \frac{\Sigma fidi}{\Sigma fi} = 55 + \frac{(-50)}{100} = 55 - 0.5 = 54.5$

$\therefore$ The mean is 54.5. ***(1 M)***

102.

Marks	No. of Student (f_i)	x_i	$d_i = x_i - 55$	$f_i d_i$
0–10	1	5	–50	–50
10–20	3	15	–40	–120
20–30	7	25	–30	–210
30–40	10	35	–20	–200
40–50	15	45	–10	–150
50–60	x	55 = A	0	0

60–70	9	65	10	90
70–80	27	75	20	540
80–90	18	85	30	540
90–100	y	95	40	40y
	$\sum f_i = 90 + x + y$			$\sum f_i d_i = 440 + 40y$

(2 M)

Here, $h = 10$, $A = 55$

We know, Mean = $A + \frac{\sum f_i d_i}{\sum f_i} \Rightarrow 59 = 55 + \frac{440 + 40y}{90 + x + y}$

$\Rightarrow \frac{440 + 40y}{90 + x + y} = 4 \Rightarrow 440 + 40y = 360 + 4x + 4y$

$\Rightarrow 4x - 36y = 80 \Rightarrow x - 9y = 20$ ...(*i*)

and given $\sum f_i = 120$

$\Rightarrow 90 + x + y = 120 \Rightarrow x + y = 30$ ...(*ii*) ***(1 M)***

Adding equation (*i*) + 9 × (*ii*), we get

$10x = 290 \Rightarrow x = 29$

From equation (*ii*)

$29 + y = 30 \Rightarrow y = 1$ ***(1 M)***

14. Probability

103. (*d*) Let A be the event of winning the game and $\overline{A}$ be the event of not winning (losing) the game.

Given $P(A) = 0.79$

We know that $P(A) + P(\overline{A}) = 1$

$\Rightarrow \quad 0.79 + P(\overline{A}) = 1$

$\Rightarrow \quad P(\overline{A}) = 1 - 0.79 = 0.21$ ***(1 M)***

104. Total number of possible outcomes in an English alphabet = 26

Total number of favourable outcomes of chosen letter is a consonant = 21

$\therefore \quad P(E) = \frac{21}{26}$ ***(1 M)***

105. Given numbers –3, –2, –1, 0, 1, 2, 3

Square of the given numbers = 9, 4, 1, 0, 1, 4, 9 ***(½ M)***

Favourable numbers = 4, 1, 0, 1, 4

$\therefore$ The number of favourable number $n(E) = 5$

Total number $n(S) = 7$ ***(½ M)***

$\therefore$ Probability of $x^2 \le 4 = \frac{n(E)}{n(S)} = \frac{5}{7}$ ***(1 M)***

106. (*i*) We know, when a die is thrown, total possible outcomes = 6

$\because$ Prime numbers are 2, 3 and 5

$\therefore$ Total number of prime numbers = 3

$\therefore$ P(getting a prime number)

$= \frac{\text{Total no. of favourable outcomes}}{\text{Total no. of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$ ***(1 M)***

(*ii*) Since, numbers between 2 and 6 are 3, 4, and 5

$\therefore$ Total number between 2 and 6 = 3

$\therefore$ P(Getting a number between 2 and 6) $= \frac{3}{6} = \frac{1}{2}$ ***(1 M)***

107. Total number of days in December = 31

(*i*) Anil can be born on any of the 31 days.

Ashraf must be born on the same day as Anil.

$\because$ Probability $= \frac{\text{Number of favourables outcomes}}{\text{Total number of possible outcomes}}$

Hence, P(same date) $= \frac{1}{31}$ ***(1 M)***

(*ii*) Probability they have difference dates of birth,

Anil can be born on any of the 31 days

Ashraf must be born on one of the remaining 30 days.

$\because$ Probability $= \frac{\text{Number of favourables outcomes}}{\text{Total number of possible outcomes}}$

Hence, P(different date) $= \frac{30}{31}$ ***(1 M)***

108. The possible outcomes

= {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} ***(1 M)***

$\therefore$ Total no. of possible outcomes = 8

(*i*) Number of favourable outcomes at least two heads

= {HHT, HHH, HTH, THH} = 4

$\therefore$ Probability of at least 2 heads $= \frac{4}{8} = \frac{1}{2}$ ***(1 M)***

(*ii*) Number of favourable outcomes at most two heads

= {HHT, HTH, HTT, THH, THT, TTH, TTT} = 7

$\therefore$ Probability of at most 2 heads $= \frac{7}{8}$ ***(1 M)***

109. The event is that two dies are thrown together

$\therefore$ The sample space is,

{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1),
(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2),
(1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3),
(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4),
(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5),
(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)}

$\therefore$ Total no. of outcomes = 36.

(*i*) No. of outcomes with even sum = 18

$\therefore$ P(even sum) $= \frac{\text{No. of outcomes with even sum}}{\text{Total no. of outcomes}}$

$= \frac{18}{36} = \frac{1}{2}$ ***(2 M)*** (*ii*) No. of outcomes with even product = 27

$$\therefore P(\text{even product}) = \frac{\text{No. of outcomes with even product}}{\text{Total no. of outcomes}}$$

$$= \frac{27}{36} = \frac{3}{4}$$ **(2 M)**

110. Chances of arrow = {1, 2, 3, 4, 5, 6, 7, 8}

$n(s) = 8$

we know,

$$\text{Probability} = \frac{\text{No. of favourable outcome}}{\text{Total no. of outcomes}}$$ **(1 M)**

(*i*) Let A be a event of an odd number.

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$ **(1 M)**

(*ii*) Let B be a Event of a number greater than 3

$B = \{4, 5, 6, 7, 8\}$

$$n(B) = 5 \Rightarrow P(B) = \frac{5}{8}$$ **(1 M)**

(*iii*) Let C be a event of a number less than 9

$C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$n(C) = 8 \Rightarrow P(C) = \frac{8}{8} = 1$$ **(1 M)**

111. (*i*) Total number of balls = 15

Number of ball bears numbers 8 = 1

$$P(\text{drawing a ball bears number 8}) = \frac{1}{15}$$ **(1 M)**

(*ii*) Even number balls are 2, 4, 6, 8, 10, 12 and 14

Total number of even balls = 7 **(1 M)**

$$P(\text{drawing a ball bears an even number}) = \frac{7}{15}$$ **(1 M)**

OR

Number of ball bears a number which is a multiple of 3 are 3, 6, 9, 12 and 15 **(1 M)**

$\therefore$ Total number of ball bears a number, which is a multiple of 3 = 5.

$$P(\text{drawing a ball bears a number having a multiple of 3}) = \frac{5}{15} = \frac{1}{3}$$ **(1 M)**

(*iii*) Number of solid coloured balls = 8

Number of solid coloured balls having an even number = 4

$$P(\text{drawing a solid coloured and bears an even number ball}) = \frac{4}{15}$$ **(1 M)**