

1. Arithmetic Sequences

1

Arrangement of numbers in an order is called sequence.

The numbers in the sequence are the terms of the sequence

Three sequences are given below . These are familiar to you

1, 2, 3... sequence of natural numbers

2, 4, 6... sequence of even numbers

1, 3, 5, 7... sequence of odd numbers.

Worksheet 1

(1) 3, 6, 9... is the sequence of the multiples of 3

(a) What is the smallest two digit term of the sequence?

(b) Is 12345 a term of this sequence? How do you know this?

(2) (a) Write the sequence of numbers 1 more than the multiples of 3

(b) Is 125 a term of this sequence . How do you know this?

(3) $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \dots$ is a sequence in which numbers in the numerator are natural numbers in the order.

(a) Write next three terms of this sequence

(b) What is the first integer term of this sequence?

(c) At what position the first integer terms appears in this sequence?

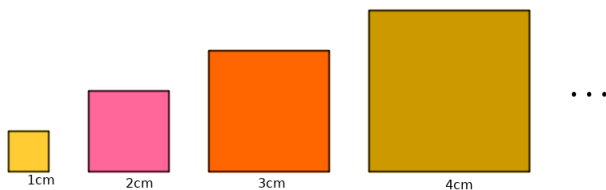
(d) At what position 100 becomes a term of the sequence?

(4) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \dots$ is an arrangement of numbers.

(a) Write each number as the product of an integer and an irrational number

(b) What is the 10 th term of this sequence?

(5) This is the sequence of squares with sides 1cm, 2cm, 3cm in the order



(a) Write the sequence of perimeters

(b) What is the perimeter of 10 th square?

(c) Describe this sequence in your own words

- (1) (a) 12
(b) This is a multiple of 3.
12345 is a term of this sequence.
- (2) (a) 4, 7, 10 ...
(b) $125 = 3 \times 41 + 2$.
125 is not 1 more than multiple of 3. This is not a term
- (3) (a) $\frac{4}{7}, \frac{5}{7}, \frac{6}{7} \dots$
(b) $\frac{7}{7} = 1$
(c) 7th term is 1
(d) 700th term is 100
- (4) (a) $\sqrt{3}$
 $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$
 $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$
(b) $10\sqrt{3}$
- (5) (a) 4, 8, 12 ...
(b) 40
(c) This is the sequence of the multiples of 4

1. Arithmetic Sequences

1

Natural numbers are the building blocks of the entire mathematical system.

Reflections on the numbers and their relationships lead to number sequence .

$1, 2, 3 \dots$ is the sequence of natural numbers. All number sequences are generated from the sequence of natural numbers.

Worksheet 2

(1) Look at the sequence of square numbers $1, 4, 9, 16 \dots$

- (a) What is the 10^{th} term of this sequence?
- (b) At what position 441 becomes a term of this sequence?
- (c) What is the largest three digit term of the sequence?

(2) Look at the sequence $0, 3, 8, 15 \dots$

- (a) What relationship can you observe in this sequence?
- (b) What would be the 10 th term by your finding?
- (c) What is the largest three digit term of this sequence?

(3) Look at the pattern made by natural numbers

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      1
     2 3 4
    5 6 7 8 9
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- (a) Write the number of numbers in each line as a sequence
 - (b) Which number comes in the right end of 20^{th} line?
 - (c) Which number comes in the left end of 20^{th} line?
- (4) Multiply the natural numbers by 3 and add 5
- (a) Write the sequence numerically
 - (b) What is its 20 th term?
- (5) $1, 11, 21, 31 \dots$ is a sequence in which terms end in 1
- (a) Write two more terms of this sequence
 - (b) What is the largest two digit term of this sequence?
 - (c) How many two digit terms are there in this sequence?

(1) (a) $10^2 = 100$

(b) $21^2 = 441$

(c) $961 \implies 31^2 = 961$

(2) Look at the sequence 0, 3, 8, 15...

(a) I think this is the sequence of numbers 1 less than perfect squares in the order

(b) $10^2 - 1 = 99$

(c) 960

(3) (a) 1, 3, 5, 7...

(b) $20^2 = 400$

(c) $19^2 + 1 = 362$

(4) Multiply the natural numbers by 3 and add 5

(a) 8, 11, 14...

(b) $3 \times 20 + 5 = 65$

(5) 1, 11, 21, 31... is a sequence in which terms end in 1

(a) 41, 51

(b) 91

(c) 9

1. Arithmetic Sequences

1

- ★ The sequence starting from a number and adding a number again and again is called Arithmetic Sequence
- ★ The number adding repeatedly is called common difference.

Worksheet 3

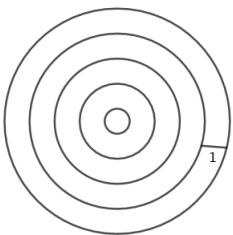
- (1) Manju is baking a cake today. She bakes the cake every fifth day. Today is a thursday.
- Write the number of days from today thursday comes
 - Write the number days from today the baking days occurs
 - How many days will it be before she next bakes a cake on a Thursday ?
- (2) First term of an arithmetic sequence is 7 and common difference 3
- Write the sequence.
 - Divide the terms of the sequence by the common difference 3. Write the remainders as a sequence.
 - Is 123 a term of this sequence? How do you know this ?
- (3) Look at the pictures given below. The coloured triangles are drawn by joining the mid points of the sides of its just outer triangle.

Perimeter of first triangle is 120



- Write perimeter of coloured triangles as a sequence
 - Is this an arithmetic sequence ?
- (4) Circles having same centre are called concentric circles.

Circles are drawn with equal spacing. Inner circle has perimeter π and space between two adjacent circles is 1.



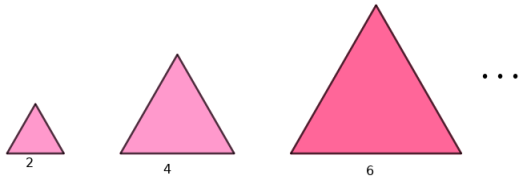
- What is the radius of the smallest circle ?

(b) Write the radius of circles as a sequence

2

(c) Write the sequence of perimeters . Is this an arithmetic sequence? If so what is the common difference?

(5) This is the sequence of equilateral triangles of sides 2cm, 4cm, 6cm ...



(a) Write the altitude of the triangles as a sequence. Is this an arithmetic sequence? If so , what is its common difference?

(b) What is the altitude of 10 th triangle ?

(c) Write the sequence of perimeters. Is this an arithmetic sequence ?

Answers

(1) (a) 7, 14, 21 ...

(b) 5, 10, 15 ...

(b) 35 days .

(2) (a) 7, 10, 13 ...

(b) 1, 1, 1 ...

(c) When 123 is divided by 3 the remainder is 0 , not 1.
123 is not a term of the sequence.

(3) (a) We know the length of line joining the mid points of two sides of a triangle is half the third side . So perimeter also halved

Sequence: 120, 60, 30 ...

(b) No. This sequence is obtained by repeated division by a number , not by adding a number again and again .

(4) (a) $\frac{1}{2}$

(b) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2} \dots$

(c) $\pi, 3\pi, 5\pi \dots$

This is an arithmetic sequence having common difference 2π

(5) (a) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3} \dots$

This is an arithmetic sequence. Common difference is $\sqrt{3}$

(b) $10\sqrt{3}$

(c) 6, 12, 18 ...

This is the sequence of multiples of 6. So its is an arithmetic sequence with common difference 6.

Worksheet 4

- (1) a) Write the sequence of natural numbers leaves the remainder 2 on dividing by 5.
b) Is this an arithmetic sequence ? If so , what is its common difference
c) What is the smallest three digit term of the sequence?
(d) Describe the sequence in other words
- (2) Consider the natural numbers $1, 2, 3, 4 \dots$. On adding two numbers at a time from the beginning we get a sequence of sums
 $1 + 2, 3 + 4, 5 + 6 \dots$. This is the sequence $3, 7, 11 \dots$
a) Is this an arithmetic sequence? If so what is the common difference ?
b) Write the sequence of numbers adding three natural numbers as shown above
Is this an arithmetic sequence ? If so what is the common difference ?
- (3) $7, 10, 13 \dots$ is an arithmetic sequence.
a) What is the common difference?
b) If 1 is added to each term of this sequence then will the resulting sequence an arithmetic sequence ?If so what is its common difference?
c) What about the common difference of the sequence if all terms are multiplied by 2?
- (4) $1, 4, 9, 16 \dots$ is the sequence of perfect squares.This is not an arithmetic sequence.
a) Write the difference between the terms of above as another sequence.
b) Is this an arithmetic sequence . If so what is its common difference
- (5) Look at the arithmetic sequences given below
 $2, 4, 6 \dots$
 $1, 3, 5 \dots$
a) What is the common measure of these sequences ?
b) Write another sequence by adding the terms in the same position of these sequences
c) Is this an arithmetic sequence? If so what is the common difference?

Answers

- (1) a) $2, 7, 12, \dots$

b) Yes. Common difference is 5

c) 102

d) This is the sequence of numbers having 2 or 7 in ones' place.

(2) a) Yes. Common difference is 4

b) $1 + 2 + 3, 4 + 5 + 6, 7 + 8 + 9 \dots$

$6, 15, 24 \dots$

Common difference is $3^2 = 9$

(3) a) Common difference is 3

b) $8, 11, 14 \dots$

Common difference of $8, 11, 14 \dots$ is 3.

c) $16, 22, 28 \dots$. Common difference is 6. Common difference become doubled

(4) a) $3, 5, 7 \dots$

b) Yes. Common difference is 2

(5) a) Common difference of both sequences is 1

b) $3, 7, 11 \dots$

c) Common difference = 4. $\rightarrow (2 + 2 = 4)$

1. Arithmetic Sequences

1

- ★ In any arithmetic sequence, the change of two terms is the product of the change in positions and a fixed number
- ★ In any arithmetic sequence, the change in terms is proportional to the change in position. Common difference is the proportionality constant.

Worksheet 5

- (1) a, b, c are in arithmetic sequence. d is its common difference. If $a - b = k(c - a)$ then find k .
- (2) The difference between 5^{th} term and 15^{th} term is 30.
- (a) What is the common difference?
 - (b) What is the difference between 5^{th} term and 25^{th} term?
 - (c) What should be subtracted from 11 th term to get 1st term?
- (3) Common difference of an arithmetic sequence is 4
- (a) What is the difference between 11 th term and 1 st term ?
 - (b) Write another term positions having the same difference
 - (c) What is the difference between sum of first 10 terms and sum of next 10 terms?
- (4) 2^{nd} and 5^{th} terms of an arithmetic sequence are 20 and 32
- (a) What is the difference between 20 th term and 26th term ?
 - (b) What is the first term of the sequence ?
 - (c) Write the sequence
- (5) 5 th term of an arithmetic sequence is 10 and 10 th term is 5
- (a) Find the common difference of the sequence?
 - (b) What is the first term of the sequence ?
 - (c) What is the 15 th term?

Answers

$$-d = k \times (2d) = k = \frac{-1}{2}$$

- (2) a) 10 times common difference is 30.
Common difference is 3
- b) 60
- c) 30 should be subtracted.

- (3) (a) Difference is 10 times common difference. It is $10 \times 4 = 40$ 2
(b) Difference between 12th term and 2nd term is also 40
(c) Pairing the terms having same difference, the difference of sums is same as sum of differences.
It is $10 \times 40 = 400$

(4) Given difference is 12. It is three times common difference.

Common difference is 4

- (a) Difference between 20th and 26th term is $2 \times 12 = 24$
(b) First term = $20 - 4 = 16$
(c) Sequence : 16, 20, 24...
- (5) (a) Five times common difference is -5 . Common difference is -1
(b) First term = 5th term $- 4 \times$ common difference
It is $10 - (-5) = 15$
(c) 15th term = 10th term + five times common difference
 $15^{\text{th}} \text{ term} = 5 + (-5) = 0$

1. Arithmetic Sequences

1

- ★ In any arithmetic sequence, the change of two terms is the product of the change in positions and a fixed number
- ★ In any arithmetic sequence, the change in terms is proportional to the change in position. Common difference is the proportionality constant.

Worksheet 6

(1) Fill in the circles with suitable numbers forming arithmetic sequences.

a) $5, \bigcirc, 9, 11, \bigcirc$

b) $16, 23, \bigcirc, 37, \bigcirc$

c) $23, \bigcirc, 49, \bigcirc, 75, 88$

d) $17, \bigcirc, \bigcirc, 47, \bigcirc$

(2) Fifth term of an arithmetic sequence is 12 and common difference 2

a) What is its 10 th term?

b) What is the first term?

(3) Third term of an arithmetic sequence is 24 and sixth term 60

a) What is the common difference of the sequence?

b) What is the difference between 10 th term 19 th term?

c) Find the first term of the sequence.

(4) 4 th term of an arithmetic sequence is 45 and 5 th term is 56

a) What is the common difference ?

b) What is the first term of the sequence?

c) Write the sequence?

(5) 4th term of an arithmetic sequence is 65 and 5 th term is 54

a) What is the common difference?

b) What is the first term?

c) Find the 10 th term of the sequence.

Answers

(1) Fill in the circles with suitable numbers forming arithmetic sequences.

a) Common difference is $11 - 9 = 2$

Second term = $5 + 2 = 7$, Fifth term = $11 + 2 = 13$

b) Common difference is $23 - 16 = 7$

Third term = 30, Fifth term = 44

c) Common difference = $88 - 75 = 13$

Second term = 36, Fourth term = 62

d) $3 \times$ common difference is $47 - 17 = 30$, common difference 10

Second term 27, third term 37, fifth term 57

(2) a) 10 th term = fifth term + $5 \times$ common difference

= $12 + 5 \times 2 = 22$

b) First term = fifth term - $4 \times$ common difference

First term = $12 - 4 \times 2 = 4$

(3) a) Difference between 3 rd term and 6 th term is $60 - 24 = 36$

It is 3 times common difference. Common difference is 12

b) Difference between 10 th term and 19 th term is 9 times common difference .

$9 \times$ common difference = $9 \times 12 = 108$

c) First term = third term - $2 \times$ common difference

= $24 - 2 \times 12 = 0$

(4) a) Common difference = $56 - 45 = 11$

b) First term = 4 th term - $3 \times$ common difference .

First term = $45 - 3 \times 11 = 45 - 33 = 12$

c) First term = 4 th term - $3 \times$ common difference

First term = $45 - 36 = 9$

(5) a) Common difference = $54 - 65 = -11$

b) First term = Fourth term - $3 \times$ common difference

= $65 - 3 \times -11 = 65 + 33 = 98$

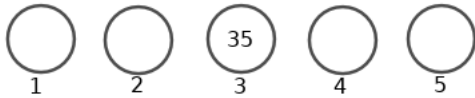
c) Tenth term = Fifth term + $5 \times$ common difference = -1

(3) Sum of the fifth and seventh terms of an arithmetic sequence is 40

2

- (a) What is the 4th and 8th terms of the sequence?
- (b) What is the sum of 1st term and 11th term?
- (c) What is the sum of 5th, 6th and 7th terms?

(4) The numbers in the circles form an arithmetic sequence



- (a) What is the sum of numbers in second and fourth circles?
 - (b) What is the sum of numbers in first and fifth circles?
 - (c) What is the sum of all numbers in these circles ?
- (5) A flower carpet is made by arranging the flowers in 17 concentric circles. Concentric circles are the circles having same centre .

The number of flowers in the circles are in arithmetic sequence.

9th circle has 45 flowers and first circle , which is the inner most circle contains 5 flowers.

- (a) How many flowers are there in outermost circle?
- (b) What is the difference between the number of flowers in two adjacent circles
- (c) What is the total number of flowers in 8th, 9th and 10th circles.

Answers

(1) Choose the correct answer

- (a) $3p$
- (b) 20
- (c) 50

- (2) (A) \rightarrow (iv)
(B) \rightarrow (iii)

(3) Sum of the fifth and seventh terms of an arithmetic sequence is 40

- (a) 40
- (b) 40
- (c) $40 + 20 = 60$

- (4) (a) 70
(b) 70

(c) $70 + 70 + 35 = 175$

- (5) A flower carpet is made by arranging the flowers in 17 concentric circles. Concentric circles are the circles having same centre .

The number of flowers in the circles are in arithmetic sequence.

9 th circle has 45 flowers and first circle , which is the inner most circle contains 5 flowers.

- (a) 85

9 th circle is the middle circle. So middle term is 45.

Sum of first and 17 th is 90. That is 17 th term is 85

- (b) Difference between first term and 9 th term is 40. It is 8 times common difference. Common difference is 5

- (c) 135

1. Arithmetic Sequences

1

- ★ When odd number of consecutive terms are considered there will be a middle term. For example , if number of terms is 11, sixth term will be the middle term
- ★ Sum of these terms will be the product of middle term and number of terms
- ★ If even number of terms are considered complete pairing of terms equidistant from both ends is possible. Pair sum equal , sum of the terms is equal to pair sum and number of pairs.

Worksheet 8

- (1) Fifth term of an arithmetic sequence is 24.
- (a) What is the sum of first 9 terms
 - (b) If each term increased by 1 then what is the sum of first 9 terms?
- (2) First term of an arithmetic sequence is 6 and common difference 4
- (a) What is the 13 th term?
 - (b) Find the sum of first 25 terms?
- (3) $-12, -8, -4 \dots$ is an arithmetic sequence
- (a) What is its 4th term?
 - (b) What is the sum of first 7 terms?
- (4) Choose the correct from left side and right side and write them as statements
- (a) 10 th term of an arithmetic sequence is 100 . The sum of first 19 terms is (i) 1000
 - (b) Common difference of an arithmetic sequence is 5 and third term is 0 . Sum of first 9 terms is (ii) 100
 - (c) Sum of the first 17 terms of an arithmetic sequence is 272
The 9 th term is (iii) 1900
(iv) 90
(iv) 16
- (5) Fifth term of an arithmetic sequence is 10 and tenth term is 5
- (a) What is the common difference ?
 - (b) What is 15 th term?
 - (c) What is sum of first 29 terms

Answers

- (1) (a) $24 \times 9 = 216$

(b) 225

2

(2) (a) 13th term = 1st term + 12 times common difference

Thirteenth term is $6 + 12 \times 4 = 54$

(b) $54 \times 25 = 1350$

(3) (a) 0

(b) $0 \times 7 = 0$

(4) (a) \rightarrow (iii)

(b) \rightarrow (iv)

(c) \rightarrow (v)

(5) (a) -1

(b) 0

(c) 0

1. Arithmetic Sequences

1

- ★ When odd number of consecutive terms are considered there will be a middle term.
- ★ Sum of these terms will be the product of middle term and number of terms
- ★ If even number of terms are considered complete pairing of terms equidistant from both ends is possible. Pair sum equal, sum of the terms is equal to pair sum and number of pairs.
- ★ In an arithmetic sequence, if the sum of two positions is equal to the sum of other two positions, then the sum of the terms at each pair is the same

Worksheet 9

- (1) The sum of first 9 terms of an arithmetic sequence is 144 and the sum of next 6 terms is 231.
- (a) What is the 5th term of the sequence ?
 - (b) What is its 8th term ?
 - (c) What is the common difference of the sequence?
 - (d) Write the sequence
- (2) There are 20 terms in an arithmetic sequence. Sum of the first and last terms is 67.
- (a) What is the sum of 2nd and 19th term ?
 - (b) If the 10th term is 32, What is the 11th term ?
 - (c) What is the common difference of the sequence ?
 - (d) What is the first term ?
- (3) 7th term of an arithmetic sequence is 18 and its 18th term is 7.
- (a) What is the common difference of the sequence ?
 - (b) Find the 25th term of the sequence.
 - (c) What is the sum of first 49 terms of the sequence ?
 - (d) What is the sum of first 50 terms of the sequence ?
- (4) The sum of the first 5 terms of an arithmetic Sequence is 65 and the sum of the first 9 terms is 189.
- (a) What is the 3rd term of the sequence ?
 - (b) What is the 5th term of the sequence ?
 - (c) What is the common difference of the sequence ?
- (5) Consider the arithmetic sequence 4, 7, 10, ...

(a) What is its 13 th term ?

2

(b) What is the sum of its first 25 terms ?

(c) Find the sum of first 25 terms of the arithmetic sequence 8, 14, 20, ...

Answers

(1) (a) Fifth term is $\frac{144}{9} = 16$

(b) Sum of first 15 terms = $144 + 231 = 375$

8 th term is $\frac{375}{15} = 25$

(c) Difference between 8 th term and 5 th term is 3 times common difference. That is 3 times common difference is $25 - 16 = 9$, common difference is 3

(d) First term = fifth term - 4 times common difference

First term = $16 - 4 \times 3 = 4$

(2) There are 20 terms in an arithmetic sequence. Sum of the first and last terms is 67.

(a) 67

(b) $67 - 32 = 35$

(c) 3

(d) First term = $32 - 9 \times 3 = 5$

(3) (a) 18^{th} term - 7^{th} term = 11 times common difference

$-11 = 11 \times$ common difference. Common difference is -1

(b) 25^{th} term = $18^{th} + 7 \times -1 = 7 + (-7) = 0$

(c) Whenever 49 terms are considered , 25 th term comes in the middle. Sum of first 49 terms = 0

(d) Sum of first 50 terms = sum of first 49 terms + 50^{th} term.

50th term is 25 th term + $25 \times -1 = -25$

Sum of first 50 terms = $0 + -25 = -25$

(4) (a) Third term = $\frac{65}{5} = 13$

(b) Fifth term = $\frac{189}{9} = 21$

(c) Two times common difference is $21 - 13 = 8$, common difference = 4

(5) (a) 13 th term = $4 + 12 \times 3 = 40$

(b) $40 \times 25 = 1000$

(c) $2 \times 1000 = 2000$