Measure of Central Tendency.

- mean
  - Arithmetic mean
  - Geometric mean
  - Harmonic mean
- median
- mode

① **Arithmetic mean :** Arithmetic mean of a set of observations is their sum divided by the number of observations.
e.g.- the arithmetic mean $\bar{x}$ of n observations $x_1, x_2, \ldots, x_n$ is given by - $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ (for ungrouped Data)

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \quad \text{(for grouped data)}$$

$i = 1, 2, \ldots, n$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i x_i \qquad N = \sum_{i=1}^{n} f_i$$

e.g.

① Find the arithmetic mean of the following frequency distribution -

x :	1	2	3	4	5	6	7
f :	5	9	12	17	14	10	6

→

x	f	fx
1	5	5
2	9	18
3	12	36
4	17	68
5	14	70
6	10	60
7	6	42
	$\Sigma f = 73$	$\Sigma fx = 299$

$$\therefore \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{299}{73} = 4.09$$

(ii) Find the arithmetic mean of the following data.
2, 3, 5, 7, 8, 10

→ $\Sigma x_i = 2+3+5+7+8+10 = 35$
$n = 6$
$\therefore \bar{x} = 35/6 = 5.83$

② **Geometric mean :** Geometric mean of a set of n observations is the n^{th} root of their product. Thus, the geometric mean G, of n observations x_i; $i = 1, 2, \ldots, n$ is given by : $G = (x_1 \cdot x_2 \cdots x_n)^{1/n}$

Taking logarithm of both sides, we have

$$\log G = \frac{1}{n}(\log x_1 + \log x_2 + \cdots + \log x_n)$$

$$= 1/n \sum_{i=1}^{n} \log x_i$$

$$G = \text{Anti}\log\left(\frac{1}{n}\sum_{i=1}^{n} \log x_i\right)$$

(for grouped data)

$$G = \left(x_1^{f_1} \cdot x_2^{f_2} \cdots x_n^{f_n}\right)^{1/N}, \text{ where } N = \sum_{i=1}^{n} f_i$$

$$\log G = \frac{1}{N} \log \left(x_1^{f_1} \cdot x_2^{f_2} \cdots x_n^{f_n}\right)$$

$$= \frac{1}{N}\left(f_1 \log x_1 + f_2 \log x_2 + \cdots + f_n \log x_n\right)$$

$$= \frac{1}{N} \sum_{i=1}^{n} f_i \log x_i$$

$$G = \text{Antilog}\left(\frac{1}{N} \sum_{i=1}^{n} f_i \log x_i\right) \quad \text{(for grouped data)}$$

Ex-1. Find the geometric mean of 2, 8, & 32.

$$n = 3, \quad x_1 = 2, \quad x_2 = 8, \quad x_3 = 32$$

$$G = (2 \times 8 \times 32)^{1/3}$$

$$= (512)^{1/3} = \underline{\underline{8}}$$

$\bar{x} =$

Ex-2.

class interval	midpoint (x_i)	frequency	$\log_{10}(x_i)$	$f_i \cdot \log_{10}(x_i)$
10 - 20	15	2	$\log(15) \approx 1.1761$	$2 \times 1.1761 = 2.3522$
20 - 30	25	5	$\log(25) \approx 1.3979$	$5 \times 1.3979 = 6.9895$
30 - 40	35	3	$\log(35) \approx 1.5441$	$3 \times 1.5441 = 4.6323$
Total		$N = 10$		$\sum f_i \log(x_i) = 13.974$

$$\therefore G = \text{Antilog}\left(\frac{1}{N} \cdot \sum_{i=1}^{n} f_i \log(x_i)\right)$$

$$= \text{Antilog}(1.3974)$$

$$= 10^{1.3974} \approx 24.97.$$

Harmonic mean: It is the reciprocal of the arithmetic mean of the reciprocals of the given values.

$$H = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} (1/x_i)} \quad \text{(ungrouped data)}$$

$$H = \frac{1}{\frac{1}{N} \sum_{i=1}^{n} (f_i/x_i)} \quad \text{(grouped data)}$$

$$N = \sum^{n} f_i$$

Example ① Find the harmonic mean of 2, 4 & 8.

↳ $n = 3$

$$HM = \frac{3}{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)}$$

$$= \frac{3}{\frac{4+2+1}{8}} = \frac{24}{7} \simeq 3.43$$

Ex. 2

class interval	frequency (f_i)
10 - 20	2
20 - 30	3
30 - 40	5
40 - 50	2

↳

class interval	midpoint (x_i)	frequency (f_i)	f_i/x_i
10 - 20	15	2	$2/15 = 0.1333$
20 - 30	25	3	$3/25 = 0.12$
30 - 40	35	5	$5/35 = 0.1429$
40 - 50	45	2	$2/45 = 0.0444$
		$N = 12$	$\Sigma f_i/x_i = 0.4406$

$$\therefore HM = \frac{12}{0.4406}$$

$$\simeq \underline{\underline{27.2356}}$$