1) Discrete uniform Distribution—

A discrete variable X is said to follow discrete uniform distribution if its probability function is

$$P(X=x) = P(x) = \frac{1}{n} \quad \text{for } x = 1, 2, \dots, n$$
$$0 \quad \text{otherwise}$$

example — The outcomes of rolling a fair die or drawing cards successively from a well shuffled deck of cards follow uniform distribution. Probability at every point remains the same.

Bandhan Bank

Date

(2) Bernouli Distribution —

A random variable X marked by only two values 1 & 0 or 1 & -1 with probability of occurance p & q ($q = 1-p$) ~~with probability of~~ respectively where $P(x=1) = p$ & $P(x=0) = q$ is called Bernouli Distribution

$$f(x) = p^x q^{1-x} \quad \text{where } 0 \le p \le 1 \text{ \& } x = 0, \text{ or } 1.$$

only parameter of Bernouli Distribution is p.

- Mean of Bernouli distribution - p
- Variance of 〃 〃 - pq
- M.G.F of Bernoulis distribution $= (q + pe^t)$

(3) Binomial Distribution — (James Bernouli) 1713

n (finite) Bernouli trials be conducted with probability 'p' of a success & 'q' of a failure. The probability of x success out of n trials is given by —

$$f(x) = {}^nC_x \cdot p^x q^{n-x}$$

where $x = 0, 1, 2, \ldots n$ $\quad 0 \le p \le 1$ & $p + q = 1$

Denoted $b(n,p)$ or $b(x; n, p)$. Binomial Expension $(p+q)^n$

- It has two parameters p & n.
- Mean of binomial distribution is np.

- variance of binomial distribution = npq
- m.g.f of binomial distribution is $(q+pe^t)^n$
- characteristics function of b(n,p) is $(q+pe^{ti})^n$
- 1st four moment -

$\mu = K_1 = np$, $\mu_2 = K_2 = npq$, $\mu_3 = K_3$ (measure of skewness) $= npq(q-p)$

$\mu_4 = K_4 + 3K_2^2 = npq[1+3pq(n-2)]$

- Additive / reproductive - $X \sim b(n_1, p)$, $Y \sim b(n_2, p)$, then ~~$X+Y \sim b(n_1+n_2, p)$~~

$X+Y \sim b(n_1+n_2, p)$

③ Poission distribution — (Simon Denis poisson 1937)

$P(X=x) = e^{-\lambda} \lambda^x / x!$ for $x = 0,1,2,\dots$

0 otherwise. Denoted by $P(x;\lambda)$

- It has only one parameter λ.
- Mean is λ & variance also λ it is only distribution of which mean & variance are equal.
- M.G.F of poission distribution - $e^{\lambda(e^t-1)}$
- characteristics function of - " " - $e^{\lambda(e^{it}-1)}$
- 1st 3 moments are equal $\mu_1' = \mu_2' = \mu_3' = \lambda$ & $\mu_4 = \lambda + 3\lambda^2$
- All the cumulants are equal.
- If n is large & p is small in Binomial distribution then it tends to poission distribution.

- If X & Y are two poisson variates with means λ_1 & λ_2, the conditional distribution of X given (X+Y) is binomial.
- Poisson distribution with unit mean, mean deviation about mean is $2/e$

④ Negative binomial Distribution -

A random variable X, the number of failures before the r^{th} success occures in a random experiment.

$$P_X\{nb(x;r,p)\} = {}^{x+r-1}C_{r-1} \cdot p^r \cdot q^x \text{ for. } x = 0,1,2,\ldots$$

$$r \geq 0;\ 0 \leq p < 1.$$

also,

$$P_X\{nb(x;r,p) = {}^{-r}C_x\, p^r q^x$$

- Negative binomial distribution also known as pascal's distribution.
- Number of success is fixed & number of trials are random variables.
- Mean is $r(1-p)/p$ & variance $= r(1-p)/p^2$
- frequency curve is J-shaped with max. frequency at $= n \pm 1 + \frac{r-1}{p}$,
- M.G.F is $(Q-Pe^t)^{-r}$ where $p = 1/Q$, $q = P/Q$, $Q - P = 1$.
- Moments & cumulants $= \mu_1 = k_1 = rP$, $\mu_2 = k_2 = rPQ$, $\mu_3 = rPQ(P+Q)$, $\mu_4 = k_4 + 3k_2^2$