⑤ Geometric Distribution—

The probability of success 'p' at each trial remains the same, the probability that there are x failuers before the 1st success is given by $p.q^x$.

$$f(x) = P(X=x) = p.q^x$$

for, $x = 0,1,2,\ldots$ & $0 \leq p \leq 1$;

0 otherwise

- It is a particular case of negative binomial distribution for $n=1$,
- Mean is q/p, variance is: q/p^2
- M.G.F is $p/(1-qe^t)$
- $x=0$ is the mode of geometric distribution.
- Median is $-\log 2/\log(1-p)$
- If the event 'E' has not occured before the time k, then $Y = X-k$ is the additional time required for E to occure. The distribution of $Y=t$ for $X \geq k$ is pq^t which is independent of k, the waiting time k is forgotten.

It is known as the lack of memory or memoryless property.

⑥ Normal Distribution - (De-Moivre 1733)

It is also called Gaussian distribution.

Normal distribution is the maximally used probability distribution in the theory of statistics.

⇒ p.d.f of μ (mean) & σ^2 (variance)

$$f_X(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\cdot\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for } -\infty < x < \infty$$

$-\infty < \mu < \infty$ & $\sigma > 0$

X- is said standarized normal variate & is denoted as $X \sim N(0,1)$. Also, p.d.f $f_X(x)$ is called standarized normal distribution. If $X \sim N(\mu,\sigma^2)$ & we make a transformation, $Z = \frac{X-\mu}{\sigma}$.

- Mean = Median = mode , Normal curve is unimodal.
- Bell sheped curve & is symmetrical about the line $x=\mu$.
- The mode of the normal curve lies at the point $x=\mu$.
- The area under the curve within its range $-\infty$ to $+\infty$
- All odd order moments of the normal distribution are 0.
- Mean $=\mu$. $\mu_2 = \sigma^2$, $\mu_3 = 0$, $\mu_4 = 3\sigma^4 = 3\mu_2^2$
  $\beta_1 = 0$, $\gamma_1 = 0$. $\beta_2 = 3$, $\gamma_2 = \beta_2 - 3 = 0$
- $Q.D = Q_3 - Q_1/2 = 2/3\,\sigma$.
- $M.D = 4/5\,\sigma$

- Max. Probability. $\max[P(x)] = \frac{1}{\sigma\sqrt{2\pi}}$
  σ - increase - $P(x)$ decreases & curve become more flat.
- m.g.f - $N(x; \mu, \sigma^2)$
  $$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$
- characteristics function - $\phi_X(t)$
- The points of inflexion of the normal curve are given by $x = \mu \pm \sigma$ & at this point
  $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-1/2}$$