

9) Gamma Distribution -

$$f_X(x; \alpha, n) = \begin{cases} \frac{\alpha^n}{\Gamma(n)} \cdot e^{-\alpha x} \cdot x^{n-1} & \text{for } x > 0 \text{ or } n, \alpha > 0 \\ 0 & \text{otherwise} \end{cases}$$

It is denoted by $\gamma(\alpha, n)$

- mean of gamma distribution is γ_α
- variance of gamma variate is γ_{α^2}
- If $\alpha < 1$, $\text{var} > \text{mean}$; if $\alpha > 1$, $\text{var} < \text{mean}$
- m.g.f = $(1 - t/\alpha)^{-n}$
- $\mu_1 = \kappa_1 = \gamma_\alpha$, $\mu_2 = \kappa_2 = \gamma_{\alpha^2}$, $\mu_3 = \kappa_3 = 2^n/\alpha^3$, $\mu_4 = \kappa_4 = 3\kappa_2^2$
 $\beta_1 = 4/n$, $\gamma_1 = 2/\sqrt{n}$, $\beta_2 = 3 + 6/n$, $\gamma_2 = 6/n$
- characteristics functions - $(1 - it/\alpha)^{-n}$
- As $n \rightarrow \alpha$, gamma distribution tends to normal distribution
- If $X \sim \gamma(\alpha, n)$ the mode of gamma distribution is $\frac{n-1}{\alpha}$ for $n > 1$, again if $n < 1$ mode is 0.
- If X & Y are two gamma variates distributed as $\gamma(n_1)$ & $\gamma(n_2)$ respectively - $(X+Y) \sim \gamma(n_1+n_2)$
 The variable $X/(X+Y)$ is distributed as $\beta_1(n_1, n_2)$
- The variate X/Y is distributed as $\beta_{II}(n_1, n_2)$