



RAJALAKSHMI ENGINEERING COLLEGE
DEPARTMENT OF AUTOMOBILE ENGINEERING

UNIT II TIRES

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Tire forces and moments, Tire structure, Longitudinal and Lateral force at various slip angles, rolling resistance, Tractive and cornering property of tire. Performance of tire on wet surface. Ride property of tires. Magic formulae tire model, Estimation of tire road friction. Test on Various road surfaces. Tire vibration.



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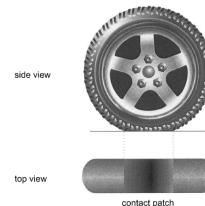


Figure 13-1: The contact patch of a tire

The origin of the reference axes is at the center of the contact patch.

The X axis is defined by the intersection of the ground plane with the mean plane of the wheel.

Axis Z is perpendicular to the ground and points upwards.

To ensure a right handed axes, the Y axes points rightwards.

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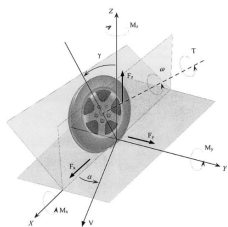


Figure 13-2: Tire forces and moments

The force the tire receives from the road is assumed to be at the center of the contact patch

The lateral force F_y is the force along the Y axis, the longitudinal force F_x is the force along the X axis and the normal or vertical force F_z is the force along the Z axis.

The moment along the Z axis M_z is called the aligning moment.

The moment along the X axis M_x is called the overturning moment and

the moment along the Y axis M_y is called the rolling resistance moment.

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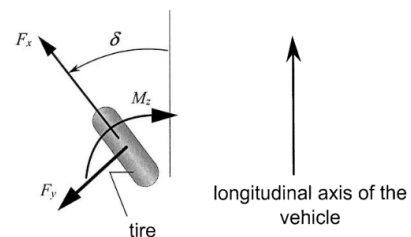


Figure 13-3: Lateral and longitudinal forces and restoring moment

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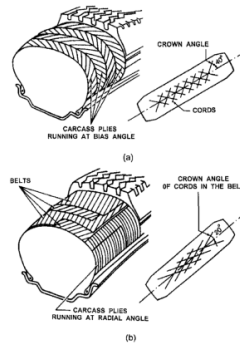


Figure 13-4. Tire construction: a) Bias-ply tire and b) Radial-ply tire⁷

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- tire "P185/70 R14 87S,"
- "P" indicates a passenger car tire;
- "185" is the nominal width of the cross section in millimeters;
- "70" is the aspect ratio, which is the ratio of the height of the sidewall to the cross-sectional width;
- "R" stands for radial-ply tire;
- "14" is the rim diameter in inches;
- "87" is a code indicating the maximum load the tire can carry at its maximum rated speed;
- "S" is a speed rating which indicates the maximum speed that the tire can sustain without failure,
- S-112 mph (180 km/h), T-118 mph (190 km/h), H-130 mph (210 km/h), V-149 mph (240 km/h), Z-149 mph (240 km/h) or more.
- Traction and temperature capabilities are indicated on a scale from A to C, A being the best and C the worst.

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•The **traction rating** is based on straight-line stopping ability on a wet surface.

•The **temperature rating** is an index of the tire's ability to withstand the heat that high speeds, heavy loads, and hard driving generate.

•**Tread-wear index** is an indication of expected tire life. It is rated against a reference tire with an index of 100. For instance, a tread-wear rating of 420 means that the tire should last 4.2 times as long as the reference tire.

•A tread- wear index of 180 is considered to be quite low and an index of 500, quite high.

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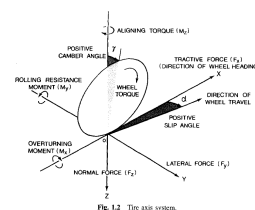


Fig. 1.2 The axis system.

Slip angle a is the angle formed between the direction of wheel travel and the line of intersection of the wheel plane with the road surface.

Camber angle is the angle formed between the XZ plane and the wheel plane.

The lateral force at the tire-ground contact patch is a function of both the slip angle and the camber angle

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LONGITUDINAL TIRE FORCE AT SMALL SLIP RATIOS

Slip ratio

The difference between the actual longitudinal velocity at the axle of the wheel V_x and the equivalent rotational velocity $r_{eff}\omega_w$ of the tire is called longitudinal slip. In other words, longitudinal slip is equal to $(r_{eff}\omega_w - V_x)$. Longitudinal slip ratio is defined as

$$\sigma_x = \frac{r_{eff}\omega_w - V_x}{V_x} \text{ during braking} \quad (13.1)$$

$$\sigma_x = \frac{r_{eff}\omega_w - V_x}{r_{eff}\omega_w} \text{ during acceleration} \quad (13.2)$$

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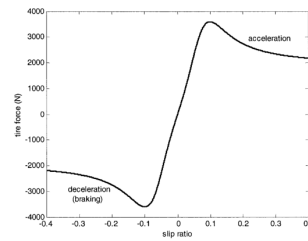


Figure 13-5. Longitudinal tire force as a function of slip ratio

$$F_{xf} = C_{\sigma f} \sigma_{xf}$$

$$F_{xr} = C_{\sigma r} \sigma_{xr}$$

where $C_{\sigma f}$ and $C_{\sigma r}$ are called the longitudinal tire stiffness parameters of the front and rear tire respectively. It should be noted that longitudinal slip

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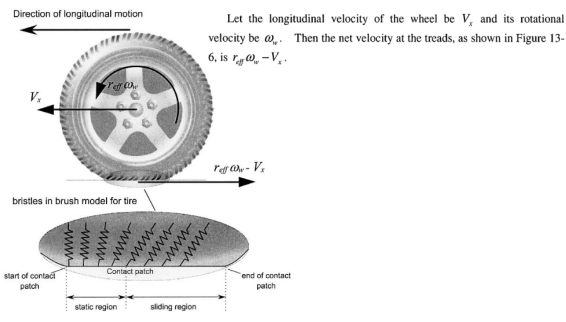


Figure 13-6. Longitudinal force in a driving wheel

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the front wheels in a front-wheel drive vehicle. In this case, since the wheel is a driving wheel, $r_{eff}\omega_w > V_x$. Hence the net velocity of the treads is in a direction opposite to that of the longitudinal velocity of the vehicle. Assume that the slip $r_{eff}\omega_w - V_x$ is small. Then there is a region of the contact patch where the tread elements do not slide with respect to the ground (called the "static region" in Figure 13-6). As the tire rotates and a tread element enters the static region of the contact patch, its tip which is in contact with the ground must have zero velocity. This is because there is no sliding in the static region of the contact patch. The upper part of the tread element moves with a velocity of $r_{eff}\omega_w - V_x$. Hence the tread element will bend forward as

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static region of the contact patch. The upper part of the tread element moves with a velocity of $r_{eff}\omega_w - V_x$. Hence the tread element will bend forward as shown in Figure 13-6 and the bending will be in the direction of the longitudinal direction of motion of the vehicle. The maximum bending deflection of the tread is proportional to the slip velocity $r_{eff}\omega_w - V_x$ and to the time duration for which the tread element remains in the contact patch. The time duration in the contact patch is inversely proportional to the rotational velocity $r_{eff}\omega_w$. Hence the maximum deflection of the tread element is proportional to the ratio of slip to absolute velocity i.e. proportional to the slip ratio $\frac{r_{eff}\omega_w - V_x}{r_{eff}\omega_w}$.

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LATERAL TIRE FORCE AT SMALL SLIP ANGLES

Experimental results show that for small "slip angle", the lateral force on a tire is proportional to the slip angle at the tire.

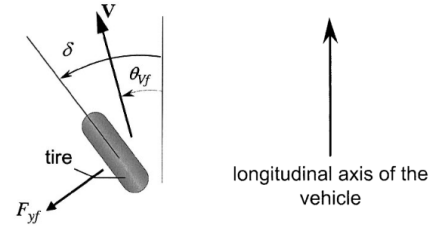


Figure 13-7. Tire slip angle and lateral force at front tire

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LATERAL TIRE FORCE AT SMALL SLIP ANGLES

Slip Angle

The slip angle of a tire is defined as the angle between the orientation of the tire and the orientation of the velocity vector of the wheel (see Figure 13-7). In Figure 13-7, the slip angle of the front wheel is

$$\alpha_f = \delta - \theta_{vf} \quad (13.5)$$

where θ_{vf} is the angle that the velocity vector at the front wheel makes with the longitudinal axis of the vehicle and δ is the front wheel steering angle. The rear wheel slip angle is similarly given by

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LATERAL TIRE FORCE AT SMALL SLIP ANGLES

$$\alpha_r = -\theta_{vr} \quad (13.6)$$

where θ_{vr} is the angle that the velocity vector at the rear wheel makes with the longitudinal axis of the vehicle.

Note that if a vehicle is not being steered and is traveling straight ahead, then the velocity angle at the tire and the steering angle are both zero, resulting in zero slip angle.

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LATERAL TIRE FORCE AT SMALL SLIP ANGLES

A rough physical explanation of why the lateral tire force is proportional to slip angle is as follows. In the static region of the contact patch, the tip of each tread is in contact with the ground and remains stationary. The top of the tread therefore moves with respect to the tip of the tread resulting in tread deformation. As seen in Figure 13-7, if the velocity at the wheel is V_w , the lateral component of the velocity is $V_w \sin(\alpha)$. The magnitude of lateral deflection of the tread is proportional both to the lateral velocity and to the magnitude of time spent by the tread in the contact patch. Since the lateral velocity is proportional to velocity and slip angle while the amount of time in the contact patch is inversely proportional to the rotational velocity, the lateral tread deflection is effectively proportional only to the slip angle.

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The lateral force on the tire depends on the magnitude of lateral deflection of the treads in the contact patch. Hence, for small slip angles, the lateral force is proportional to slip angle.

A more detailed explanation of the relationship between lateral force and slip angle can be found in section 13.6.

The lateral tire force for the front wheels of the vehicle can therefore be written as

$$F_{yf} = C_\alpha (\delta - \theta_{vf}) \quad (13.7)$$

where the proportionality constant C_α is called the cornering stiffness, δ is the front wheel steering angle and θ_{vf} is the front tire velocity angle.

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Similarly, the lateral tire force for the rear wheels of the vehicle can be written as

$$F_{yr} = C_\alpha (-\theta_{vr}) \quad (13.8)$$

where C_α is the cornering stiffness and θ_{vr} is the rear tire velocity angle.

The ratio of the lateral velocity to the longitudinal velocity at each wheel can be used to calculate the velocity angle at that wheel. Hence the following relations can be used to calculate θ_{vf} and θ_{vr} :

$$\tan(\theta_{vf}) = \frac{V_y + \ell_f \dot{\psi}}{V_x} \quad (13.9)$$

$$\tan(\theta_{vr}) = \frac{V_y - \ell_r \dot{\psi}}{V_x} \quad (13.10)$$

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where V_y is the lateral velocity at the c.g. of the vehicle, V_x is the longitudinal velocity at the c.g. of the vehicle, $\dot{\psi}$ is the yaw rate of the vehicle and ℓ_f and ℓ_r are the longitudinal distances from the c.g. to the front and rear wheels respectively.

Using small angle approximations,

$$\theta_{vf} = \frac{V_y + \ell_f \dot{\psi}}{V_x} \quad (13.11)$$

$$\theta_{vr} = \frac{V_y - \ell_r \dot{\psi}}{V_x} \quad (13.12)$$

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$$F_{yf} = C_{\alpha} \left(\delta - \frac{V_y + \ell_f \dot{\psi}}{V_x} \right) \quad (13.13)$$

$$F_{yr} = C_{\alpha} \left(-\frac{V_y - \ell_r \dot{\psi}}{V_x} \right) \quad (13.14)$$

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ROLLING RESISTANCE OF TIRES

- The rolling resistance of tires on hard surfaces is primarily caused by the **hysteresis in tire materials** due to the deflection of the carcass while rolling.
- Friction between the tire and the road caused by sliding, the resistance due to air circulating inside the tire, and the fan effect of the rotating tire on the surrounding air also contribute to the rolling resistance of the tire, but they are of secondary importance.
- When a tire is rolling, the carcass is deflected in the area of ground contact.
- As a result of tire distortion, **the normal pressure in the leading half of the contact patch is higher than that in the trailing half.**
- The center of normal pressure is shifted in the direction of rolling. This shift produces a moment about the axis of rotation of the tire, which is the **rolling resistance moment.**

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ROLLING RESISTANCE OF TIRES

- In a free-rolling tire, the applied wheel torque is zero; therefore, a horizontal force at the tire-ground contact patch must exist to maintain equilibrium.
- This resultant horizontal force is generally known as the **rolling resistance.**
- The ratio of the rolling resistance to the normal load on the tire is defined as the **coefficient of rolling resistance.**

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ROLLING RESISTANCE OF TIRES

- Thicker treads and sidewalls and an increased number of carcass plies** tend to increase the rolling resistance because of greater hysteresis losses.
- Tires made of **synthetic rubber** compounds generally have higher rolling resistance than those made of natural rubber.
- Tires made of **butyl rubber compounds**, which are shown to have better traction and road holding properties, have an even higher rolling resistance than those made of conventional synthetic rubber.
- It is found that the rolling resistance of tires with tread made of **synthetic rubber** compounds and that made of **butyl rubber compounds** are approximately 1.06 and 1.35 times that made of natural rubber compounds, respectively

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ROLLING RESISTANCE OF TIRES

- Surface conditions also affect the rolling resistance. On hard, smooth surfaces, the rolling resistance is considerably lower than that on a rough road.
- On wet surfaces, a higher rolling resistance than on dry surfaces is usually observed.
- Inflation pressure affects the flexibility of the tire. Depending on the deformability of the ground, the inflation pressure affects the rolling resistance of the tire in different manners.
- On hard surfaces, the rolling resistance generally decreases with the increase in inflation pressure.
- This is because, with **higher inflation pressure, the deflection of the tire decreases, with consequent lower hysteresis losses**

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ROLLING RESISTANCE OF TIRES

- Rolling resistance is also affected by driving speed because of **the increase of work in deforming the tire and of vibrations in the tire structure with the** increase in speed.
- For a given tire under a particular operating condition, there exists a **threshold speed** above which the phenomenon popularly known as **standing waves** will be observed.
- Standing waves are formed because owing to high speed, the tire tread does not recover immediately from distortion originating from tire deflection after it leaves the contact surface, and the residual deformation initiates a wave.
- The amplitude of the wave is the greatest immediately on leaving the ground, and is **damped out in an exponential manner** around the circumference of the tire.
- The formation of the standing wave greatly **increases energy losses**, which in turn cause considerable heat generation that could lead to tire failure.
- This places an upper limit on the safe operating speed of tires.

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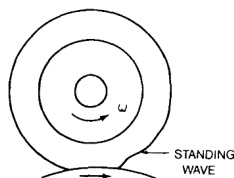
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ROLLING RESISTANCE OF TIRES

Fig. 1.10 Formation of standing waves of a tire at high speeds.



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ROLLING RESISTANCE OF TIRES

- Tire temperature affects the rolling resistance in two ways: one is by changing the **temperature of the air** in the tire cavity, and thereby changing the operating **inflation pressure**; and the other is by changing the **stiffness and hysteresis of the rubber compounds**.
- The rolling resistance at a **shoulder temperature of -10°C is approximately 2.3 times that at 60°C for the tire examined**.
- The effect of tire diameter on the coefficient of rolling resistance- **the effect of tire diameter is negligible on hard surfaces (concrete), but is considerable on deformable or soft ground**.
- The rolling resistance coefficient of **truck tires is usually lower than that of passenger car tires** on road surfaces. This is primarily due to the higher inflation pressure used in truck tires (typically 620-827 kPa or 90-120 psi as opposed to 193-248 kPa or 28-36 psi for passenger car tires)

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

When a pneumatic tire is not subject to any force perpendicular to the wheel plane (i.e., side force), it will move along the wheel plane.

If, however, a **side force F_s** is applied to a tire, a lateral force will be developed at the contact patch, and the tire will move along a path at an angle with the wheel plane, as OA shown

The angle is usually referred to as the **slip angle**, and the phenomenon of side slip is mainly due to the **lateral elasticity of the tire**.

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

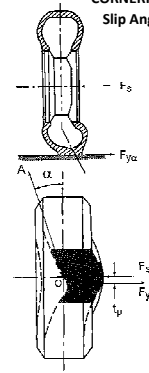


Fig. 1.22 Behavior of a tire subject to a side force. (Reproduced with permission from *Mechanics of Pneumatic Tires*, edited by S.R. Clark, Monograph 122, National Bureau of Standards, 1971)

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

The lateral force developed at the tire-ground contact patch is usually called the **cornering force when the camber angle of the wheel is zero**.

The relationship between the cornering force and the slip angle is of fundamental importance to the **directional control and stability of road vehicles**.

When the tire is moving at a uniform speed in the direction of OA, the side force applied at the wheel center and the cornering force developed in the ground plane are usually **not collinear**

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

At small slip angles, the cornering force in the ground plane is normally behind the applied side force, giving rise to a torque (or couple), which tends to align the wheel plane with the direction of motion.

This torque is called the **aligning or self-aligning torque**, and is one of the primary restoring moments which help the steered tire return to the original position after negotiating a turn.

The distance t , between the side force and the cornering force is called the **pneumatic trail**, and

the product of the cornering force and the pneumatic trail determines the self-aligning torque.

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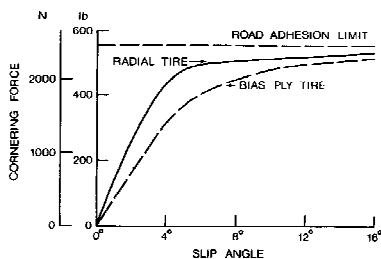


Fig. 1.23 Cornering characteristics of a bias-ply and a radial-ply car tire. (Reproduced with permission from *Mechanics of Pneumatic Tires*, edited by S.K. Clark, Monograph 122, National Bureau of Standards, 1971.)

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

for slip angles below 4° , the cornering force is approximately proportional to the slip angle.

Beyond that, the cornering force increases at a lower rate with an increase of the slip angle, and it reaches a maximum value where the tire begins sliding laterally.

For passenger car tires, the maximum cornering force may occur at a slip angle of about 18° , while for racing car tires, the cornering force may peak at approximately 6° .

cornering force of a bias-ply tire increases more slowly with an increase of the slip angle than that of a radial-ply tire.

These characteristics are considered to be more suited to two-wheeled vehicles, such as motorcycles.

A more gradual increase of the cornering force with the slip angle enables the driver to exercise better control over a two-wheeled vehicle.

This is one of the reasons why bias-ply tires are used for motorcycles

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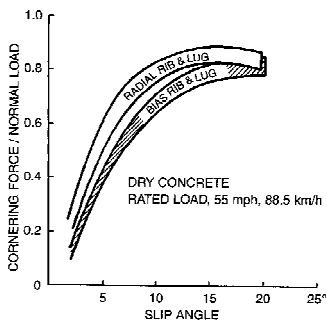


Fig. 1.24 Cornering characteristics of bias ply and radial-ply truck tires on dry concrete. (Reproduced with permission from reference 1.8.)

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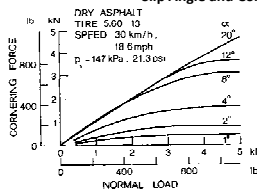


Fig. 1.25 Effect of normal load on the cornering characteristics of a car tire. (Reproduced with permission from *Mechanics of Pneumatic Tires*, edited by S.K. Clark, Monograph 122, National Bureau of Standards, 1971.)

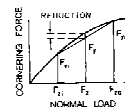


Fig. 1.26 Effect of lateral load transfer on the cornering capability of a pair of tires on an axle.

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

The **normal load** on the tire strongly influences the cornering characteristics.

for a **given slip angle**, the **cornering force** generally **increases** with an increase of the **normal load**.

However, the **relationship** between the cornering force and the normal load is **nonlinear**.

Thus, the **transfer of load** from the inside to the outside tire during a turning maneuver **will reduce the total cornering force** that a pair of tires can develop.

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

Consider a pair of tires on a beam axle, each with **normal load F_z** ,

*The **cornering force per tire with normal load F_z is F_y** , for a given slip angle.*

If the vehicle undergoes a steady-state turn, owing to lateral load transfer,

*the normal load on the inside tire will be reduced to **F_{zi}** and that on the outside tire will be **increased to F_{zo}** .*

*As a result, the total cornering force of the two tires **will be the sum of F_{yi} and F_{yo}** which is **less than $2F_y$** .*

This implies that for a pair of tires on a beam axle to develop the required amount of cornering force to balance a given centrifugal force during a turn, the lateral load transfer results in an increase in the slip angle of the tires.

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

tires, a parameter called cornering stiffness C_α is used. It is defined as the derivative of the cornering force $F_{y\alpha}$ with respect to slip angle α evaluated at zero slip angle:

$$C_\alpha = \left. \frac{\partial F_{y\alpha}}{\partial \alpha} \right|_{\alpha=0} \quad (1.37)$$

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

To evaluate the effect of the normal load on the cornering ability of tires, a parameter called the **cornering coefficient**, which is defined as the **cornering stiffness per unit normal load**,

It shows that the cornering coefficient decreases with an increase in the normal load.

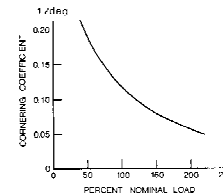


Fig. 1.28 Effect of normal load on the cornering coefficient of a tire. (Reproduced with permission from reference 1.12.)

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CORNERING PROPERTIES OF TIRES Slip Angle and Cornering Force

Inflation pressure usually has a moderate effect on the cornering properties of a tire.

In general, the cornering stiffness of tires increases with an increase of the inflation pressure.

Figure shows a comparison of the cornering coefficients at different inflation pressures of a radial-ply, a bias-belted, and a bias-ply passenger car tire

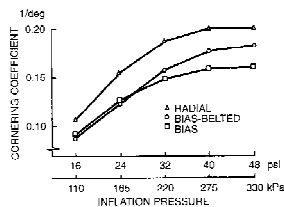


Fig. 1.29 Variation of cornering coefficient with inflation pressure for radial-ply, bias-ply, and bias-belted car tires. (Reproduced with permission of the Society of Automotive Engineers from reference 1.11.)

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CORNERING PROPERTIES OF TIRES Slip Angle and Aligning Torque

a plot of the cornering force versus the aligning torque for a passenger car tire at various slip angles and under different normal loads

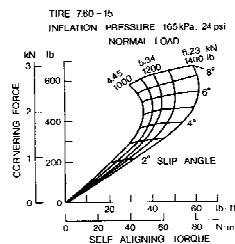


Fig. 1.30 Variation of self-aligning torque with cornering force of a car tire under various normal loads. (Reproduced with permission of the Society of Automotive Engineers from reference 1.20)

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CORNERING PROPERTIES OF TIRES Slip Angle and Aligning Torque

variations of the aligning torque with the slip angle and the normal load for a bias-ply truck tire and for a radial-ply truck tire

with a given normal load, the aligning torque first increases with an increase of the slip angle.

It reaches a maximum at a particular slip angle, and then decreases with a further increase.

mainly caused by the sliding of the tread in the trailing part of the contact patch at high slip angles, which results in shifting the point of application of the cornering force forward

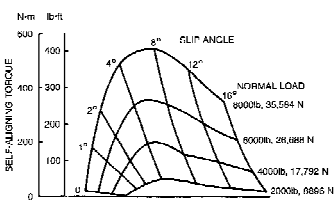


Fig. 1.41 Variation of self-aligning torque with normal load and slip angle for a bias-ply truck tire, 10.00/20/A. (Reproduced with permission from reference 1.8.)

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CORNERING PROPERTIES OF TIRES Slip Angle and Aligning Torque

SUMMARY:

- effect of a driving torque is to increase the aligning torque for a given slip angle, while a braking torque has the opposite effect.
- Inflation pressure and normal load also have noticeable effects on the aligning torque because they affect the size of the tire contact patch.
- Higher normal load and lower inflation pressure result in longer tire contact length, and hence pneumatic trail.
- This causes an increase in the aligning torque

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CORNERING PROPERTIES OF TIRES

Camber and Camber Thrust

Camber is the inclination of the wheel plane from a plane perpendicular to the road surface when viewed from the fore and aft directions of the vehicle

Its main purpose is to achieve axial bearing pressure and to decrease the king-pin offset.

Camber on passenger cars is between 1/2 and 1.

High camber angles promote excessive tire wear

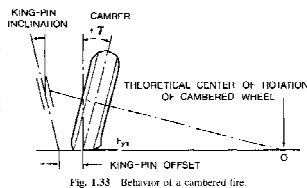


Fig. 1.33 Behavior of a cambered tire.

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CORNERING PROPERTIES OF TIRES

Camber and Camber Thrust

Camber causes a lateral force developed on the contact patch. This lateral force is usually referred to as camber thrust

A free-rolling tire with a camber angle would revolve about point *O*. However, the cambered tire in a vehicle is constrained to move in a straight line.

A lateral force in the direction of the camber is, therefore, developed in the ground plane. The camber thrust acts ahead of the wheel center, and therefore forms a small camber torque.

camber thrust is approximately one-fifth the value of the cornering force obtained from an equivalent slip angle for a bias-ply tire and somewhat less for a radial-ply tire.

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CORNERING PROPERTIES OF TIRES

Camber and Camber Thrust

a parameter called "camber stiffness" is defined as the derivative of the camber thrust with respect to the camber angle evaluated at zero camber angle.

the normal load and inflation pressure have an influence on the camber stiffness.

The total lateral force of a cambered tire operating at a slip angle is the sum of the cornering force and the camber thrust

$$C_{\gamma} = \left. \frac{\partial F_{yy}}{\partial \gamma} \right|_{\gamma=0}$$

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CORNERING PROPERTIES OF TIRES

Camber and Camber Thrust

$$F_y = F_{ya} \pm F_{y\gamma} \quad (1.39)$$

If the cornering force and the camber thrust are in the same direction, the positive sign should be used in the above equation. For small slip and camber angles, the relationship between the cornering force and the slip angle and that between the camber thrust and the camber angle are essentially linear; the total lateral force of a cambered tire at a slip angle can, therefore, be determined by

$$F_y = C_{\alpha} \alpha \pm C_{\gamma} \gamma \quad (1.40)$$

As discussed previously, the lateral forces due to slip angle and camber angle produce an aligning torque. The aligning torque due to slip angle, however, is usually much greater.

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PERFORMANCE OF TIRES ON WET SURFACES

The performance of tires on wet surfaces depends on the surface texture, water depth, tread pattern, tread depth, tread material, and operating mode of the tire (i.e., free rolling, braking, accelerating, or cornering).

To achieve acceptable performance on wet surfaces, maintaining effective contact between the tire tread and the road is of importance

necessity of removing water from the contact area as much as possible.

the tire tread should have a suitable pattern to facilitate the flow of fluid from the contact area, and the surface of the pavement should have an appropriate texture to facilitate drainage as well.

To provide good skid resistance, road surfaces must fulfill two requirements:

- an open macrotexture to facilitate gross draining,
- microharshness to produce sharp points that can penetrate the remaining water film

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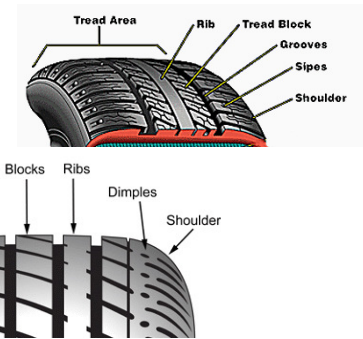
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PERFORMANCE OF TIRES ON WET SURFACES

smooth tire



a tire with ribs, and a tire with ribs and sipes



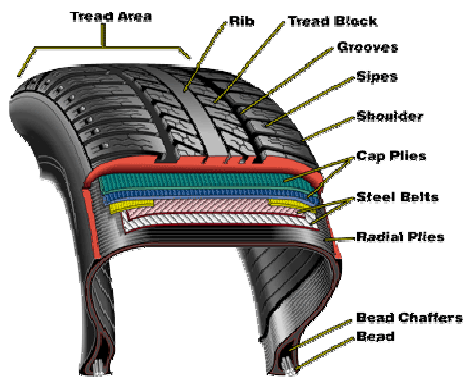
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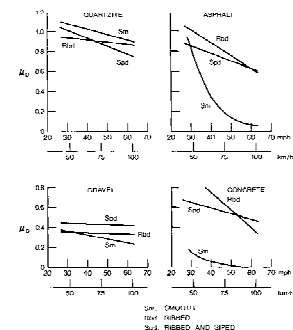


Fig. 147 Effect of road texture on the peak value of road adhesion coefficient in wet wet surfaces, (Reprinted with permission from *Handbook of Pneumatic Tires* edited by S.B. Clark, McGraw-Hill, National Bureau of Standards, 1971.)

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PERFORMANCE OF TIRES ON WET SURFACES

the variations of the peak values and the sliding values of the coefficient of road adhesion with speed for a smooth tire, a tire with ribs, and a tire with ribs and sipes on wet quartzite, asphalt, gravel, and concrete

difference in the coefficient of road adhesion between patterned tires, including the ribbed and siped tires, and smooth tires on wet asphalt and concrete surfaces

The tread pattern increases the value of the coefficient of road adhesion and reduces its speed dependency.

there is little pattern effect on wet quartzite surfaces, and a high level of road adhesion is maintained over the entire speed range.

Thus, it can be concluded that the advantages of a patterned tire over a smooth tire are pronounced only on badly drained surfaces

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PERFORMANCE OF TIRES ON WET SURFACES

tread pattern can function satisfactorily on a wet road only when the grooves and sipes constitute a reservoir of sufficient capacity, and that its effectiveness decreases with the wear of the tread or the tread depth.

The decline in value of the coefficient of road adhesion with the decrease of tread depth is more pronounced on smooth than on rough roads as rough roads can provide better drainage

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PERFORMANCE OF TIRES ON WET SURFACES

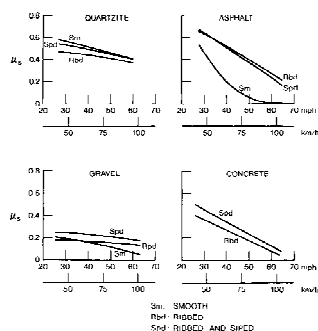


Fig. 1-40 Effect of road design on the sliding value of road adhesion coefficient μ_s over wet surfaces. (Reproduced with permission from *Mechanics of Pneumatic Tires*, edited by S.K. Chak, Monograph 122, National Bureau of Standards, 1971.)

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PERFORMANCE OF TIRES ON WET SURFACES - HYDROPLANING

When a pneumatic tire is braked over a flooded surface, the motion of the tire creates hydrodynamic pressure in the fluid.

The hydrodynamic pressure acting on the tire builds up as the square of speed of the tire, and tends to separate the tire from the ground.

At low speeds, the front part of the tire rides on a wedge or a film of fluid. This fluid film extends backward into the contact area as the speed of the tire increases.

At a particular speed, the hydrodynamic lift developed under the tire equals the vertical load, the tire rides completely on the fluid, and all contact with the ground is lost.

This phenomena is usually referred to as hydroplaning

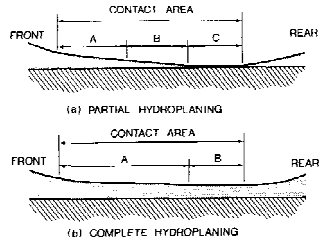
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PERFORMANCE OF TIRES ON WET SURFACES - HYDROPLANING



A - UNBROKEN WATER FILM
B - REGION OF PARTIAL BREAKDOWN OF WATER FILM
C - CONTACT ZONE

Fig. 1.49 Hydroplaning of a tire on flooded surfaces. (Reproduced with permission from *Mechanics of Pneumatic Tires*, edited by S.K. Clark, Monograph 122, National Bureau of Standards, 1971.)

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PERFORMANCE OF TIRES ON WET SURFACES - HYDROPLANING

For smooth or close-patterned tires that do not provide escape paths for water

for patterned tires on flooded surfaces with a fluid depth exceeding the groove depth in the tread, the **speed at which hydroplaning occurs** may be determined based on the theory of hydrodynamics.

It can be assumed that the lift component of the hydrodynamic force is proportional to the tire ground contact area A , fluid density ρ , and the square of the vehicle speed

$$F_h \propto \rho_f A V^2$$

The most important effect of hydroplaning is the reduction in the coefficient of road adhesion between the tire and the ground.

This affects braking, steering control, and directional stability.

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PERFORMANCE OF TIRES ON WET SURFACES

the following formula was proposed by Horne and Joyner for a hydroplaning speed V_p :

$$V_p = 10.35 \sqrt{p_i} \text{ mph} \quad (1.89)$$

the **hydroplaning speed** for a tire at an inflation pressure of 193 kPa (28 psi) is approximately 88 kmph which is well within the normal operating range for passenger cars.

For **heavy trucks**, the inflation pressure is usually in the range 620-827 kPa (90-120 psi)., the **hydroplaning speed** V_p for a tire at an inflation pressure of 620 kPa (90 psi) is approximately 158 kmph (98 mph), which is beyond the normal range of operating speed for heavy trucks

This would suggest that **hydroplaning may not be possible for heavy truck tires** under normal circumstances.

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RIDE PROPERTIES OF TIRES

Supporting the weight of the vehicle and **cushioning** it over surface irregularities are two of the basic functions of a pneumatic tire.

When a normal load is applied to an inflated tire, the tire progressively deflects as the load increases.

static load-deflection relationship for a 5.60 X 13 bias-ply tire at various inflation pressures

The type of diagram is usually referred to as a **lattice plot**, in which the origin of each load-deflection curve is displaced along the deflection axis by an amount proportional to the inflation pressure.

The relationship between the load and the inflation pressure for constant deflection can also be shown in the lattice plot

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RIDE PROPERTIES OF TIRES- Bias Ply

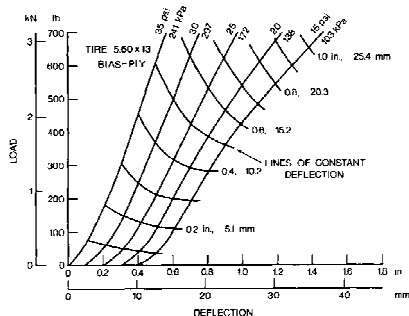


Fig. 1.53 Static load-deflection relationship of a bias-ply car tire. (Reproduced with permission of the Council of the Institution of Mechanical Engineers from reference 1.35.)

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RIDE PROPERTIES OF TIRES- Radial Ply

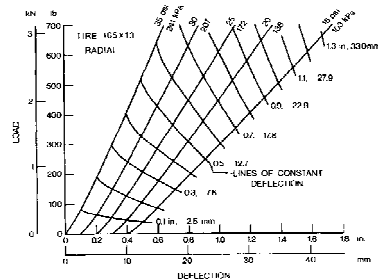


Fig. 1.54 Static load-deflection relationship of a radial-ply car tire. (Reproduced with permission of the Council of the Institution of Mechanical Engineers from reference 1.35.)

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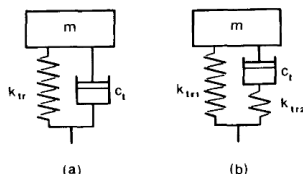
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RIDE PROPERTIES OF TIRES

shows the interrelationship among the static load, inflation pressure, and deflections for a 165 X 13 radial-ply passenger car tire.

cushioning characteristics of a pneumatic tire – mathematical models.

The most widely used and simplest model representing the fundamental mode of vibration of the pneumatic tire consists of a **mass element** and a **linear spring in parallel with a viscous damping element**, "viscoelastic" model



(a) A linear model and (b) a viscoelastic model for tire vibration analysis.

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RIDE PROPERTIES OF TIRES

Depending upon the test conditions, three distinct types of tire vertical stiffness may be defined:

- Static,
- Non Rolling Dynamic,
- Rolling Dynamic Stiffness

Static Stiffness:

The static vertical stiffness of a tire is determined by the slope of the static load-deflection curves.

It has been found that for a given inflation pressure, the load-deflection characteristics for both radial- and bias-ply tires are more or less linear, except **at relatively low values of load.**

the tire vertical stiffness is independent of load in the range of practical interest.

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RIDE PROPERTIES OF TIRES

Nonrolling Dynamic Stiffness:

The dynamic stiffness of a nonrolling tire -drop test.

In this test, the tire with a certain load is allowed to fall freely from a height at which the tire is just in contact with the ground. Consequently, the tire remains in contact with the ground throughout the test.

The transient response of the tire is recorded.

A typical amplitude decay trace is shown .

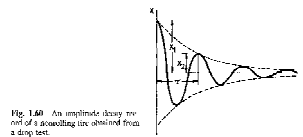


Fig. 1.60 An amplitude decay trace of a nonrolling tire obtained from a drop test.

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RIDE PROPERTIES OF TIRES

The values of the equivalent viscous damping coefficient and the dynamic stiffness k , of the tire can then be determined from the decay trace using the well-established theory of free vibration for a single-degree-of-freedom system

$$c_{eq} = \sqrt{\frac{4m^2\omega_d^2\delta^2/(\delta^2 + 4\pi^2)}{1 - [\delta^2/(\delta^2 + 4\pi^2)]}} \quad (1.91)$$

d

$$k_z = \frac{m\omega_d^2}{1 - \delta^2/(\delta^2 + 4\pi^2)} \quad (1.92)$$

ω_d is the damped natural frequency of the tire with mass m , and can be identified from the amplitude decay trace shown in Fig. 1.60.

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RIDE PROPERTIES OF TIRES

ω_d is the damped natural frequency of the tire with mass m , and can be identified from the amplitude decay trace shown in Fig. 1.60.

$$\omega_d = 2\pi/\tau \quad (1.93)$$

where τ is the period of damped oscillation shown in Fig. 1.60.

δ is the logarithmic decrement, which is defined as the natural logarithm of the ratio of any two successive amplitudes, such as x_1 and x_2 , shown in Fig. 1.60.

$$\delta = \ln(x_1/x_2) \quad (1.94)$$

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RIDE PROPERTIES OF TIRES

The drop test may also be performed utilizing a tire endurance testing machine consisting of a beam pivoted at one end, which carries the test tire loaded against a drum.

To initiate the test, the beam is displaced and the system is set in angular oscillation about the pivot of the beam.

A decay trace for the amplitude of angular displacement is recorded.

A set of equations for this torsional system, similar to that for a single-degree-of-freedom linear system described above, can be derived for determining the equivalent damping coefficient and nonrolling dynamic stiffness for the tire from the decay trace.

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RIDE PROPERTIES OF TIRES

Rolling Dynamic Stiffness

The rolling dynamic stiffness is usually determined by measuring the response of a rolling tire to a known harmonic excitation.

The response is normally measured at the hub, and the excitation is given at the tread.

By examining the ratio of output to input and the phase angle, it is possible to determine the dynamic stiffness and the damping coefficient of a rolling tire.

An alternative method for determining the dynamic stiffness of a tire is to measure its resonant frequency when rolling on a drum or belt.

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RIDE PROPERTIES OF TIRES

the dynamic stiffness of car tires decreases sharply as soon as the tire is rolling.

However, beyond a speed of approximately 20 km/h (12 mph), the influence of speed becomes less important.

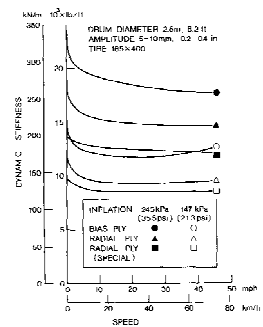


Fig. 14-1 Effect of speed on rolling dynamic stiffness of car tire. (Reproduced with permission from *Mechanics of Pneumatic Tires*, edited by S.N. Clark, Monograph 122, National Bureau of Standards, 1971.)

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THE MAGIC FORMULA TIRE MODEL

large slip angles and large slip ratios

The Magic Formula tire model (Pacejka and Bakker, 1993) provides a method to calculate lateral and longitudinal tire forces and aligning moment for a wide range of operating conditions including large slip angle and slip ratios as well as combined lateral and longitudinal force generation.

In the simpler case where only either lateral or longitudinal force is being generated, the force generated Y can be expressed as a function of the input variable X as follows:

$$Y(X) = y(x) + S_y$$

with

$$y(x) = D \sin[C \arctan\{Bx - E(Bx - \arctan Bx)\}]$$

$$x = X - S_h$$

where

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Y is the output variable: longitudinal force F_x or lateral force F_y or aligning moment M_z

X is the input variable: slip angle α or slip ratio σ_x .

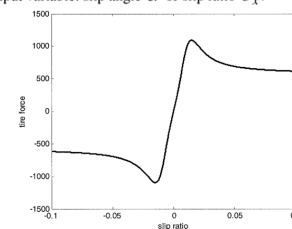


Figure 13-8 Magic Formula tire force curve

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THE MAGIC FORMULA TIRE MODEL

The model parameters B , C , D , E , S_v and S_h have the following nomenclature:

B stiffness factor
 C shape factor
 D peak value
 E curvature factor

S_h horizontal shift

S_v vertical shift

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This empirical formula is capable of producing characteristics that closely match measured curves for the side force and the longitudinal force as functions of their respective slip quantities: slip angle and longitudinal slip ratio

For small slip angles and small slip ratio values, the linear tire force relation between Y and X can be approximated by

$$Y = (BCD) X$$

The quantity BCD in equation represents the

cornering stiffness or the longitudinal tire stiffness

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THE MAGIC FORMULA TIRE MODEL

The Magic Formula, in its basic form, can be used to fit experimental tire data for characterizing the relationships between the cornering force and slip angle, self-aligning torque and slip angle, or braking effort and skid

$$y(x) = D \sin \{ C \arctan [Bx - E(Bx - \arctan Bx)] \}$$

$$Y(X) = y(x) + S_v$$

$$x = X + S_h$$

Equation I & II

where $Y(X)$ represents cornering force, self-aligning torque, or braking effort, and X denotes slip angle or skid.

Coefficient B is called stiffness factor, C shape factor, D peak factor, and E curvature factor. S_h and S_v are the horizontal shift and vertical shift, respectively

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THE MAGIC FORMULA TIRE MODEL

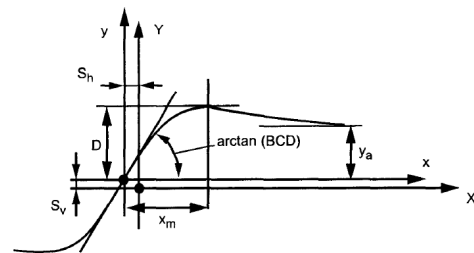


Fig. 1.43 Characteristics of the Magic Formula for fitting tire test data. (From Pacejka, H.B. and Besselink, I.J.M. (1997), *Proceedings of the 2nd International Colloquium on Tyre Models for Vehicle Dynamic Analysis*, pp. 234–249, © Swets & Zeitlinger. Used with permission.)

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THE MAGIC FORMULA TIRE MODEL

produces a curve that passes through the origin, $x = y = 0$, and reaches a maximum at $x = x_m$ as shown in Fig. Beyond that it decreases and finally approaches an asymptote Y_a .

For given values of the coefficients, the curve shows an anti-symmetric shape with respect to the origin, $x = y = 0$.

To allow the curve to have an offset with respect to the origin, two shifts S_h and S_v are introduced. Consequently, a new set of coordinates X and Y , respectively, is established.

This enables the effects of **ply-steer, conicity, or rolling resistance on cornering force, self-aligning torque, or braking effort** to be taken into account.

if Fig. 1.43 represents the **cornering force and slip angle** relationship of a tire, then coefficient **D** represents the **peak value** with respect to x, y coordinates and

the product **BCD** corresponds to the **slope of the curve at the origin**, representing the **cornering stiffness** of the tire

The Magic Formula is capable of producing characteristics that closely match measured data

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THE MAGIC FORMULA TIRE MODEL

of the normal load and/or camber angle of the tire [1.24]. For instance, peak factor D may be expressed as a function of normal load F_z as follows:

$$D = a_1 F_z^2 + a_2 F_z \quad (1.80)$$

where F_z is in kN, and a_1 and a_2 are empirical coefficients.

For cornering stiffness (i.e., the initial slope of the cornering force–slip angle curve):

$$BCD = a_3 \sin [a_4 \arctan (a_5 F_z)] \quad (1.81)$$

where a_3, a_4 , and a_5 are empirical coefficients.

For aligning stiffness (i.e., the initial slope of the self-aligning torque–slip angle curve) or longitudinal stiffness (i.e., the initial slope of the braking effort–skid curve):

$$BCD = \frac{a_3 F_z^2 + a_4 F_z}{e^{a_5 F_z}} \quad (1.82)$$

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THE MAGIC FORMULA TIRE MODEL

The shape factor C appears to be practically independent of F_z , and the average values for the particular car tire tested may be taken as follows (based on the data shown in Table 1.6).

For the cornering force–slip angle relationship, $C = 1.30$

For the self-aligning torque–slip angle relationship, $C = 2.40$

For the braking effort–skid relationship, $C = 1.65$

The stiffness factor B can be derived from

$$B = \frac{BCD}{CD} \quad (1.83)$$

The curvature factor E as a function of normal load F_z is given by

$$E = a_6 F_z^2 + a_7 F_z + a_8 \quad (1.84)$$

where a_6, a_7 , and a_8 are empirical coefficients.

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THE MAGIC FORMULA TIRE MODEL

TABLE 1.6 Values of the Coefficients in the Magic Formula for a Car Tire (Slip Angle in Degrees and Skid in Minus %)

	Load, F_z , kN	B	C	D	E	S_h	S_v	BCD
F_y , N	2	0.244	1.50	1936	-0.132	-0.280	-118	780.6
	4	0.239	1.19	3650	-0.678	-0.049	-156	1038
	6	0.164	1.27	5237	-1.61	-0.126	-181	1091
	8	0.112	1.36	6677	-2.16	0.125	-240	1017
M_z , N · m	2	0.247	2.56	-15.53	-3.92	-0.464	-12.5	-9.820
	4	0.234	2.68	-48.56	-0.46	-0.082	-11.7	-30.45
	6	0.164	2.46	-112.5	-2.04	-0.125	-6.00	-45.39
	8	0.127	2.41	-191.3	-3.21	-0.009	-4.22	-58.55
F_x , N	2	0.178	1.55	2193	0.432	0.000	25.0	605.0
	4	0.171	1.69	4236	0.619	0.000	70.6	1224
	6	0.210	1.67	6090	0.686	0.000	80.1	2136
	8	0.214	1.78	7711	0.783	0.000	104	2937

Source: Reference 1.24.

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THE MAGIC FORMULA TIRE MODEL

TABLE 1.7 Values of Coefficients a_1 to a_8 for a Car Tire (F_z in kN)

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
F_x , N	-22.1	1011	1078	1.82	0.208	0.000	-0.354	0.707
M_x , N · m	-2.72	-2.28	-1.86	-2.73	0.110	-0.070	0.643	-4.04
F_{z0} , N	-21.3	1144	49.6	226	0.069	-0.006	0.056	0.486

Source: Reference 1.24.

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THE MAGIC FORMULA TIRE MODEL

Camber angle γ is found to have an influence on the relationships between cornering force and slip angle and self-aligning torque and slip angle, in the form of horizontal and vertical shifts, S_h and S_v [1.24]. The additional shifts due to camber angle γ may be expressed by

$$\begin{aligned}\Delta S_h &= a_9 \gamma \\ \Delta S_v &= (a_{10} F_z^2 + a_{11} F_z) \gamma\end{aligned}\quad (1.85)$$

where a_9 , a_{10} , and a_{11} are empirical coefficients.

The change in stiffness factor ΔB is obtained by multiplying B by $(1 - a_{12} |\gamma|)$:

$$\Delta B = (1 - a_{12} |\gamma|)B \quad (1.86)$$

where a_{12} is an empirical coefficient.

The value of the self-aligning torque at high slip angles will change due to this change in stiffness factor B . To compensate for this effect, the curvature factor E for self-aligning torque M_z must be divided by $(1 - a_{13} |\gamma|)$.

The values of coefficients a_9 to a_{13} for the same tire as in Table 1.6 are given in Table 1.8.

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THE MAGIC FORMULA TIRE MODEL

TABLE 1.8 The Values of Coefficients a_9 to a_{13} for a Car Tire (Camber Angle in Degrees)

	a_9	a_{10}	a_{11}	a_{12}	a_{13}
F_x , N	0.028	0.000	14.8	0.022	0.000
M_z , kN	0.015	-0.066	0.945	0.030	0.070

Source: Reference 1.24.

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THE MAGIC FORMULA TIRE MODEL

When brakes are applied during a turning maneuver, the tires on a vehicle **develop both slip angles and skids**.

Under these circumstances, Eqs. I and II are inadequate for characterizing tire behavior.

To characterize the **combined effects of slip angle and skid on the cornering force, self-aligning torque, or braking effort**, empirical weighting functions G are introduced.

which when **multiplied by the original functions given in Eqs. I and II produce the interactive effects of skid on cornering force and self-aligning torque, or of slip angle on braking effort**.

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When the tire operates **only with slip angle or skid**, the weighting functions **G** take the **value of one**.

However, when a tire operates under a given slip angle and at the same time its skid gradually increases, then the weighting function for cornering force *may first show a slight increase in magnitude, then reach its peak, followed by a continuous decrease*.

The weighting function **G** takes the following form:

$$G = D' \cos[C' \arctan(B'x)]$$

where B' , C' , and D' are empirical coefficients, and x is either slip angle or skid.

if Equation represents the weighting function for determining the effect of skid on the cornering force **at a given slip angle, then x in Equation represents the skid of the tire**.

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The model parameters B , C , D , E , S_y and S_h have the following nomenclature:

B stiffness factor

C shape factor

D peak value

E curvature factor

S_h horizontal shift

S_y vertical shift

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THE MAGIC FORMULA TIRE MODEL

- 1) The coefficient D represents the peak value of the tire force (or moment)
- 2) The product BCD corresponds to the slope at the origin ($x = y = 0$).
- 3) The value y_s is the asymptotic value of the output y at large values of x .
- 4) The shape factor C controls the limits of the range of the sine function appearing in the formula and thereby determines the shape of the resulting curve.

$$C = \frac{2}{\pi} \sin^{-1} \left(\frac{y_s}{D} \right)$$

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- 8) Wheel camber can give rise to a considerable offset of the F_y versus α curves. Such a shift may be accompanied by a significant deviation from the pure anti-symmetric shape of the original curve (Pacejka and Bakker, 1993). To accommodate such an asymmetry, the curvature factor E is made dependent on the sign of the abscissa (x).

$$E = E_o + \Delta E \operatorname{sgn}(x) \quad (13.66)$$

Also the difference in shape that is expected to occur in the F_x vs σ characteristic between the driving and braking ranges can be taken care of by this modification (Pacejka and Bakker, 1993).

- 9) The asymptotic value which y approaches at large slip values equals

$$y_s = D \sin \left(\frac{\pi}{2} C \right) \quad (13.67)$$

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THE MAGIC FORMULA TIRE MODEL

Example 1.2. Using the Magic Formula, estimate the braking effort developed by a tire with a normal load of 6 kN (1349 lb), at a skid of -25%, and having empirical coefficients B , C , D , E , S_h , and S_v shown in Table 1.6.

$$y(x) = D \sin \{C \arctan [Bx - E(Bx - \arctan Bx)]\} \quad (1.78)$$

$$Y(X) = y(x) + S_v$$

$$x = X + S_h \quad (1.79)$$

Solution. For this case, the variables Y and X in the Magic Formula, Eqs. 1.78 and 1.79, represent the braking effort F_x and skid i_x , respectively. It should be noted that skid i_x in the Magic Formula is expressed in percentage and considered to be a negative value and that the value of the arctan function should be expressed in radians.

$$F_x = D \sin [C \arctan (B(i_x + S_h) - E\{B(i_x + S_h) - \arctan [B(i_x + S_h)]\})] + S_v$$

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THE MAGIC FORMULA TIRE MODEL

TABLE 1.6 Values of the Coefficients in the Magic Formula for a Car Tire (Slip Angle in Degrees and Skid in Minus %)

Load, F_z , kN	B	C	D	E	S_h	S_v	BCD
F_x , N							
4	0.171	1.69	4236	0.619	0.000	70.6	1224
6	0.210	1.67	6090	0.686	0.000	80.1	2136
8	0.214	1.78	7711	0.783	0.000	104	2937

Using the appropriate values of the empirical coefficients for a normal load of 6 kN (1349 lb) given in Table 1.6, the braking effort at a skid of -25% is calculated as follows:

$$\begin{aligned} F_x &= 6090 \sin [1.67 \arctan (0.210 (-25 + 0) - 0.686(0.210(-25 + 0) \\ &\quad - \arctan[0.210(-25 + 0)]))] \\ &\quad + 80.1 \\ &= 6090 \sin [1.67 \arctan [-5.25 - 0.686(-5.25 + 1.3826)]] \\ &\quad + 80.1 = -5433 \text{ N } (-1221 \text{ lb}) \end{aligned}$$

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ESTIMATION OF TIRE ROAD FRICTION

Definition of tire-road friction coefficient

Let F_x , F_y , and F_z be the longitudinal, lateral, and normal (vertical) forces acting on a tire. The normalized traction force for the tire, ρ , is defined as:

$$\rho = \frac{\sqrt{F_x^2 + F_y^2}}{F_z} \quad (14.1)$$

If we consider only longitudinal motion, and assume that the lateral force F_y can be neglected, then

$$\rho = \frac{F_x}{F_z} \quad (14.2)$$

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ESTIMATION OF TIRE ROAD FRICTION

The normalized traction force for the tire be a function of **both slip ratio and the tire-road friction coefficient**.

The tire road-friction coefficient on any given road surface is defined as the maximum value that normalized traction force can achieve on that surface for any slip ratio value.

For a given normal force F_z , the **longitudinal tire force F_x initially** increases as slip ratio is increased and at an optimum value of slip ratio can reach a maximum value equal to μF_z .

If μ is equal to 1, the maximum longitudinal force generated can be as much as the normal force F_z and this happens at an optimum value of slip ratio.

If μ is less than 1, then the maximum longitudinal force that can be generated will only be a fraction of the normal force F_z .

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ESTIMATION OF TIRE ROAD FRICTION

Benefits of tire-road friction estimation

made "road-adaptive," i.e., the control algorithms can be modified to account for the external road conditions if the actual tire-road friction coefficient information is available in real time. Active safety control systems in many vehicles get benefited.

In adaptive cruise control system, road condition information from friction coefficient estimation can be used to adjust the longitudinal spacing headway from the preceding vehicle that the ACC vehicle should maintain.

In the case of vehicle stability control systems, the value of tire-road friction coefficient is needed for estimating the target value of yaw rate for the vehicle.

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ESTIMATION OF TIRE ROAD FRICTION slip-slope based approach to friction estimation

At low values of slip ratio, the normalized longitudinal force is **proportional to the slip ratio**

The slope of the relation between normalized longitudinal force and slip ratio at low values of slip ratio is called the "**slip slope**."

The basic idea behind the use of slip-slope for friction coefficient estimation is that at low slip ratios the **tire-road friction coefficient is proportional to the slip-slope**.

Thus by estimating slip-slope, the tire road friction coefficient can be estimated.

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LONGITUDINAL VEHICLE DYNAMICS AND TIRE MODEL FOR FRICTION ESTIMATION

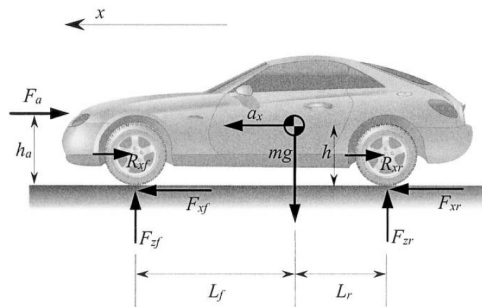


Figure 14-1. Vehicle longitudinal dynamics schematic diagram

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ESTIMATION OF TIRE ROAD FRICTION

Consider a **bicycle type model** (the difference between right and left tires is ignored) shown in Figure.

Ignoring the road gradient and wind speed, the longitudinal dynamics can be represented as

$$ma_x = F_{xf} + F_{xr} - R_x - D_a V_x^2 \quad (14.5)$$

where m is the total mass of the vehicle, a_x is the longitudinal acceleration/deceleration, F_{xf} and F_{xr} are the front and rear wheel traction/braking forces, $R_x = R_{xf} + R_{xr} = C_{roll}mg$ is the rolling resistance force with C_{roll} being the rolling resistance coefficient, D_a is the aerodynamic drag force constant and V_x is the longitudinal velocity. Let

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ESTIMATION OF TIRE ROAD FRICTION

L_f be the distance from c.g. to the front axle; L_r the distance from c.g. to the rear axle and $L = L_f + L_r$ be the wheelbase of the vehicle.

The total longitudinal tire force F_x therefore can be calculated as follows:

$$F_x = F_{xf} + F_{xr} = m[a_x + |R_x| + D_a V_x^2], \text{ if } a_x \geq 0 \quad (\text{acceleration}) \quad (14.6)$$

$$F_x = F_{xf} + F_{xr} = m[a_x - |R_x| - D_a V_x^2], \text{ if } a_x < 0 \quad (\text{deceleration}) \quad (14.7)$$

the vehicle longitudinal **acceleration/deceleration** is measured by using an accelerometer and the total vehicle longitudinal force can be obtained based on above equations- **Determination of the Longitudinal force**

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ESTIMATION OF TIRE ROAD FRICTION

Determination of the normal force

It is important to calculate the value of the normal force **for each tire** as accurately as possible for the **friction estimation algorithm**.

The weight of the vehicle contributes the major part of the normal forces on the tires.

Longitudinal acceleration and deceleration forces acting on the vehicle redistribute the normal forces between the tires.

If the vehicle is traveling in a **straight line** on level road, the normal forces at the front and rear tires can be calculated using a **static force model** of the vehicle

$$F_{xf} = \frac{mgL_r - ma_x h - D_a V^2 h_a}{L} \quad F_{xr} = \frac{mgL_f + ma_x h + D_a V^2 h_a}{L}$$

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ESTIMATION OF TIRE ROAD FRICTION

Thus an acceleration of the vehicle causes the normal forces on the front tires to decrease and the normal forces on the rear tires to increase.

The above normal force calculation method is based on a **static force model** and ignores the influence of the vibrations of the suspension. - reasonable estimate of the normal force, especially when the road surface is fairly paved and not bumpy.

However, if the road surface is **very bumpy**, a **dynamic normal force estimation method** incorporating the suspension dynamics will provide a more accurate calculation of the normal force

Note also that during **cornering**, the **normal forces of the right and left tires** on both front and rear axle **are different** due to vehicle roll moment.

However, since we are using a bicycle model, the **differences in the left and right tires cannot be considered in this formulation**.

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ESTIMATION OF TIRE ROAD FRICTION

Tire model

Formula model. As the figure shows, $\rho = \frac{F_x}{F_z}$ is an increasing function of slip ratio σ_x until a critical slip value, where ρ reaches a value equal to μ and then starts decreasing slowly.

The longitudinal force generated at each tire is known to depend on the **longitudinal slip ratio, the tire-road friction coefficient, and the normal force applied at the tire**.

Figure shows the **traction and braking force vs. slip ratio** relationship for a variety of road surfaces computed using the **Magic Tire Road Friction Measurement Formula model**

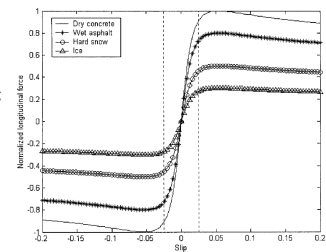


Figure 14-2. Longitudinal force vs. slip computed using Magic Formula model

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ESTIMATION OF TIRE ROAD FRICTION

In the **slip-slope model** used for tire road friction coefficient identification, we merely assume that the **slip-slope is proportional to the tire road friction coefficient at small slip ratios**.

Longitudinal force ratio p is modeled as being proportional to the slip ratio, with the **proportionality constant being a function of the tire road friction coefficient**.

At high values of slip ratio, the force ratio is constant and independent of the slip ratio.

The constant value of the **force ratio is a function of the friction coefficient**.

Hence the tire is modeled at high slip ratios with a constant Longitudinal force ratio P , **with the value of the constant being dependent on the tire-road friction coefficient**

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

•unified slip-slope based friction coefficient estimation method for front or rear-wheel drive as well as all-wheel drive vehicles in both traction and braking situations.

•the normalized longitudinal force generated at an individual tire is proportional to its slip ratio in the low-slip region (or the linear part of the friction curve) for any given road surface and normal force.

•This relationship can be described as:

$$\rho = \frac{F_x}{F_z} = K\sigma_x \quad (14.9)$$

individual tire,

where K is the slip-slope, whose value changes with road surface conditions and could be used to predict the tire-road friction coefficient μ .

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

For an all-wheel drive vehicle, the linear relationships between slip and normalized longitudinal force for the front and rear tires can be written as:

$$\rho_f = \frac{F_{x_f}}{F_{z_f}} = K_f \sigma_{xf} \quad (14.10)$$

$$\rho_r = \frac{F_{x_r}}{F_{z_r}} = K_r \sigma_{xr} \quad (14.11)$$

$$F_x = F_{xf} + F_{xr}$$

all-wheel drive vehicle

where, F is the total vehicle longitudinal tire force.

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

calculated as described in section 14.2.1. K_f and K_r are the slip-slopes of the front and rear tires whose values are determined by the front and rear tire properties and road surface characteristics. Combining the above three equations, we get

$$F_x = F_{xf} + F_{xr} = K_f F_{zf} \sigma_{xf} + K_r F_{zr} \sigma_{xr} \quad (14.13)$$

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

If we assume that the front and rear tires are on the same road surface condition, which is true for many driving situations, then the difference between the values of K_f and K_r is mainly dominated by the tire properties (including the tire type and number of tires for front and rear axles), which are independent of the road surface condition. Therefore, K_f and K_r can be related as:

$$K_f = \alpha K_r \quad (14.14)$$

where, α is a ratio coefficient determined by the front and rear tire properties and independent of road surface condition. Thus, the relationship

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

properties and independent of road surface condition. Thus, the relationship between total force and slip ratios can be written as:

$$F_x = F_{xf} + F_{xr} = K_f F_{zf} \sigma_{xf} + K_r F_{zr} \sigma_{xr} = K_r (\alpha F_{zf} \sigma_{xf} + F_{zr} \sigma_{xr}) \quad (14.15)$$

all-wheel drive vehicle

where, F_x , F_{zf} , F_{zr} , σ_{xf} and σ_{xr} can be measured or calculated in real-time, and α can be determined experimentally for each vehicle. For

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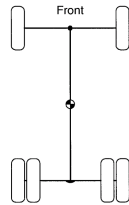
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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking



all-wheel drive vehicle

Figure 14-3. Chassis tire configuration example

example, if the vehicle chassis configuration is as shown in Figure 14-3, with two tires on the front axle and four tires on the rear axle (which is the configuration of the SAFELOW used in Wang, et. al., 2004), and all tires are exactly the same, then $\alpha = \frac{1}{2}$. If the front tires are different from the rear tires in terms of wear level and tread pattern, then the value of α could be experimentally determined as some value less than 0.5. But, its value will

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

If the vehicle is rear-wheel drive instead of all-wheel drive, then $\alpha = 0$ during acceleration by ignoring the traction force of the front tires. During braking α can be chosen as a specific value determined by the chassis configuration. If the vehicle is front-wheel drive, the equations for K_f can be derived similarly as:

front-wheel drive,

$$F_x = F_{xf} + F_{xr} = K_f F_{zf} s_{xf} + K_r F_{zr} s_{xr} = K_f (F_{zf} s_{xf} + \frac{1}{\alpha} F_{zr} s_{xr})$$

where $\frac{1}{\alpha} = 0$ or $\alpha = \infty$ during acceleration and α is a different specific value determined by the chassis configuration during braking.

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Friction coefficient estimation for both traction and braking

$$F_x = F_{xf} + F_{xr} = K_f F_{zf} \sigma_{xf} + K_r F_{zr} \sigma_{xr} = K_r (\alpha F_{zf} \sigma_{xf} + F_{zr} \sigma_{xr}) \quad (14.15)$$

The equation (14.15) can be rewritten into a standard parameter identification format as:

$$y(t) = \varphi^T(t) \theta(t) \quad (14.17)$$

where, $y(t) = F_x$ is the system output, $\theta(t) = K_r$ is the unknown parameter, and $\varphi(t) = \alpha F_{zf} \sigma_{xf} + F_{zr} \sigma_{xr}$ is the measured regression vector.

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ESTIMATION OF TIRE ROAD FRICTION

Friction coefficient estimation for both traction and braking

The only unknown parameter K_r can be identified in real-time using a parameter identification approach

Once the slip-slope K_r is identified, it can be related to the road surface condition or the maximum friction coefficient, by a classification function.

Since the above method incorporates both front and rear tire forces and slip ratios, it can be used to identify the friction coefficient for both traction and braking situations on rear or front-wheel drive as well as all-wheel drive vehicle

Note: at low slip ratios, the slip-slope is used to identify the friction coefficient

while at high slip ratios, the magnitude of the normalized longitudinal force itself is used to identify the friction coefficient.

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ESTIMATION OF TIRE ROAD FRICTION

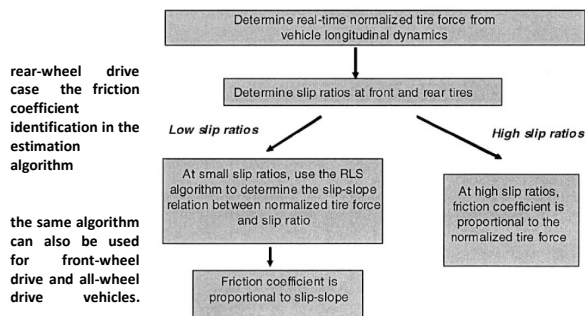


Figure 14-4. Summary of algorithm for tire-road friction estimation

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TESTS ON VARIOUS ROAD SURFACE

14.6.1 System hardware and software

The vehicle used to conduct the experiments was a full sized snowplow (referred to as SAFEPLow) manufactured by Navistar International Truck Company as shown in Figure 14-6.



Figure 14-6. The SAFEPLow used for the experiments

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The main parameters of the SAFEFLOW related to the friction coefficient identification algorithm are listed in Table 14.1.

Table 14-1. SAFEFLOW main parameters

Parameter	Total Mass (Kg)	L_f (m)	L_r (m)	Height of C.G. (m)	Vehicle Front Area (m^2)
Value	9834	2.339	2.716	1.2	6.0

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In order to experimentally implement the designed friction coefficient estimator in real-time, the SAFEFLOW was equipped with a differential GPS system, an accelerometer, and ABS wheel speed sensors. The Mathworks xPC system was used to serve as the real-time system and included a host PC (TOSHIBA 4200 laptop) and a target PC (DELL GX110). Details of the experimental hardware can be found in Wang, Alexander and Rajamani (2004).

After designing the estimation algorithm, extensive experimental tests were carried out to improve and verify the estimation system performance.

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TESTS ON VARIOUS ROAD SURFACE

Tests on dry concrete road surface

The experiments for this part were carried out on the dry concrete testing track. The track was a well paved concrete surface and completely dry. So, the friction level of the surface was very good.

Acceleration(Traction)Test

The slip-slope based friction coefficient estimation method was evaluated for acceleration with different starting speeds.

At the beginning of the test, the speed of the vehicle was kept constant for about 6 seconds to allow the low-pass filters to initialize and the Kalman filter to estimate the accelerometer bias.

After that, the vehicle starts accelerating. Figure shows the slip-slope estimation results for acceleration with the starting speed at 20 mph

As the results indicate, the slip-slope for the acceleration on dry concrete converges to a value of about 9.8.

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TESTS ON VARIOUS ROAD SURFACE

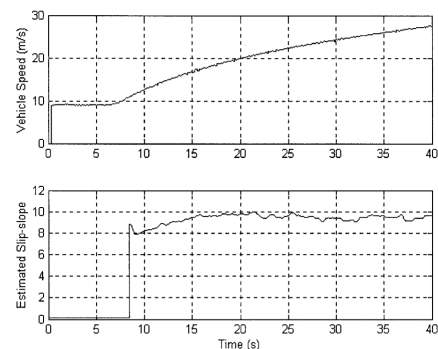


Figure 14-7. Acceleration starting at 20mph on dry concrete surface

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TESTS ON VARIOUS ROAD SURFACE

Combination of Acceleration and Braking Test

Experiments for a situation of sequential acceleration and braking were also studied on the same dry concrete surface.

Figure 14-8 shows one of the test results.

The SAFEFLOW first stays at a constant speed for a while to let the filters initialize and estimate the accelerometer bias, then it starts accelerating for about 20 seconds and then gently brakes for about 10 seconds.

As the result indicates, the slip-slope consistently converges to about the same value around 9.8.

From the experimental results, it was observed that for acceleration and braking in the small slip region, the slip-slope consistently converges to some value around 9.8 on this surface.

Therefore, we can use this slip-slope value to classify the road surface as dry concrete, dry concrete like surface or to classify the friction coefficient as being close to 1.

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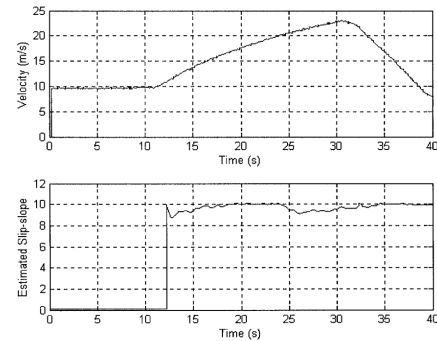


Figure 14-8. Slip-slope estimation during acceleration and braking

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Tests on concrete surface with loose snow covering



Figure 14-9. The road surface used to conduct the experiments for this section

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TESTS ON VARIOUS ROAD SURFACE

Tests on concrete surface with loose snow covering

The experiments for this part were carried out also on the concrete test track after a heavy snow day.

The right side lane in the figure was used for the testing.

Since the road surface is slightly slippery, the friction coefficient is expected to be noticeably less than 1.

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TESTS ON VARIOUS ROAD SURFACE

Tests on concrete surface with loose snow covering

Combination of Acceleration and Braking Test

Figure 14-10 shows one of the test results.

The SAFELOW first stays at a constant speed for a while to let the filters initialize and estimate the accelerometer bias, then it starts accelerating for about 16 seconds and then gently brakes for about 16 seconds.

As the result indicates, the **slip-slope** consistently converges to about the same value around 7.0.

From the experimental results, we can see that for acceleration and braking in the small slip region, the slip-slope consistently converges to some value around 7.0, which is quite different from the slip-slope value (9.8) obtained on dry concrete surface.

Therefore, we can use this slip-slope value to classify the road surface as a slightly slippery surface or to classify the friction coefficient as being about 0.7.

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TESTS ON VARIOUS ROAD SURFACE

Tests on concrete surface with loose snow covering

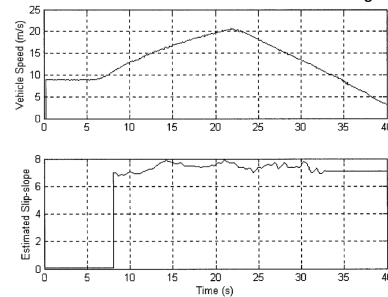


Figure 14-10. Acceleration and braking on surface with light covering snow

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Tests on surface consisting of two different friction levels



Figure 14-11. The track used to conduct the experiments for this section

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TESTS ON VARIOUS ROAD SURFACE

Tests on surface consisting of two different friction levels

The purpose of the experiments in this section is to test the system's transient response performance and its ability to detect a sudden change of the road friction level.

The test track consists of two surfaces with different friction levels-dry asphalt surface and icy surface.

Figure 14-11 shows a photograph of the transitional part of the track.

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TESTS ON VARIOUS ROAD SURFACE

Tests on surface consisting of two different friction levels

Combination of Acceleration and Braking Test

Figure 14-12 shows experimental results from a test in which the SAFELOW accelerates on the dry asphalt surface and brakes through the transitional part of the track.

As before, on the **dry asphalt surface**, tires work in the low-slip linear region and the slip-slope is used to classify the surface friction level for both acceleration and braking.

However, the **wheels lock up once the vehicle reaches the icy surface** when performing braking.

Thus, the **slip ratio will be almost as high as 100%** and the tires are working in the nonlinear region of the tire force characteristics.

Therefore, the normalized force is used to classify the surface friction level.

As the result indicates, the friction coefficient estimate promptly converges to a value of about **0.22** once the vehicle reaches the icy surface.

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TESTS ON VARIOUS ROAD SURFACE

Tests on surface consisting of two different friction levels

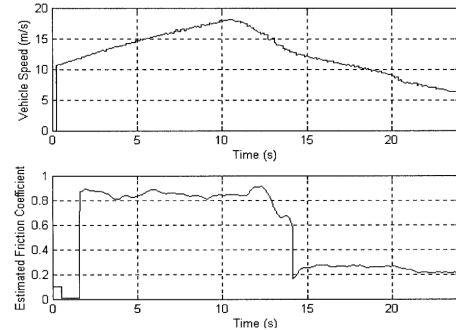


Figure 14-12. System response when braking through the transitional part

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TESTS ON VARIOUS ROAD SURFACE

Hard braking test

It is used to further verify the system performance in high-slip region (nonlinear part of the force-slip curves).

The SAFELOW performs **hard braking on a dry concrete surface**

Figure 14-13 shows a test result.

Since the tires are working outside the linear region, the **normalized force is directly used to classify the surface friction level**.

As the result indicates, the system behaves well even in the nonlinear region.

The estimated **friction coefficient converges to about 0.96** once the vehicle starts hard braking.

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TESTS ON VARIOUS ROAD SURFACE

Hard braking test

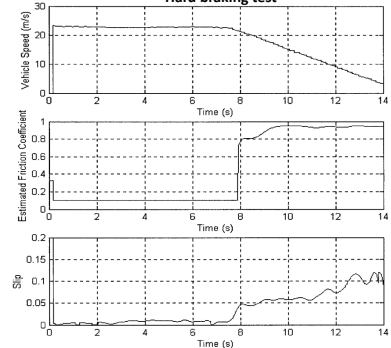


Figure 14-13. Test results for hard braking

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