



RAJALAKSHMI ENGINEERING COLLEGE
DEPARTMENT OF AUTOMOBILE ENGINEERING

UNIT III VERTICAL DYNAMICS

9

Human response to vibration, Sources of Vibration. Design and analysis of Passive, Semi-active and Active suspension using Quarter car, half car and full car model. Influence of suspension stiffness, suspension damping, and tire stiffness. Control law for LQR, H-Infinite, Skyhook damping. Air suspension system and their properties.

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1



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Ride quality:

Ride quality is concerned with the **sensation or feel** of the passenger in the environment of a moving vehicle.

Variety of sources:

- surface irregularities,
- aerodynamic forces,
- vibrations of the engine and driveline,
- Non uniformities (imbalances) of the tire wheel assembly.

surface irregularities - ranging from potholes to surface elevation profile

Excitations by **aerodynamic forces** are applied **directly to the vehicle body**,

Engine and driveline vibrations are transmitted through **engine transmission mounts**.

Excitations resulting from **mass imbalances** and dimensional and stiffness variations of the tire wheel assembly are transmitted to the **vehicle body through the suspension**.

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2



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HUMAN RESPONSE TO VIBRATION methods for assessing human tolerance to vibration

1. Subjective Ride Measurements.

- trained jury to rate the ride comfort, on a relative basis, of different vehicles driven over a range of road surfaces.
- comparison of the ride quality of different vehicles.
- Disadv:** The degree of difference in ride quality, however, cannot be quantitatively determined by this type of subjective evaluation.

2. Shake Table Tests.

- quantitatively study** human response to vibration,
- human response to **sinusoidal excitation**. It is intended to **identify zones of comfort (or discomfort)** for humans in terms of vibration amplitude, velocity, or acceleration in a given direction (such as foot-to-head, side-to-side, or back-to-chest) over a specific frequency range.

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3



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HUMAN RESPONSE TO VIBRATION

3. Ride Simulator Tests.

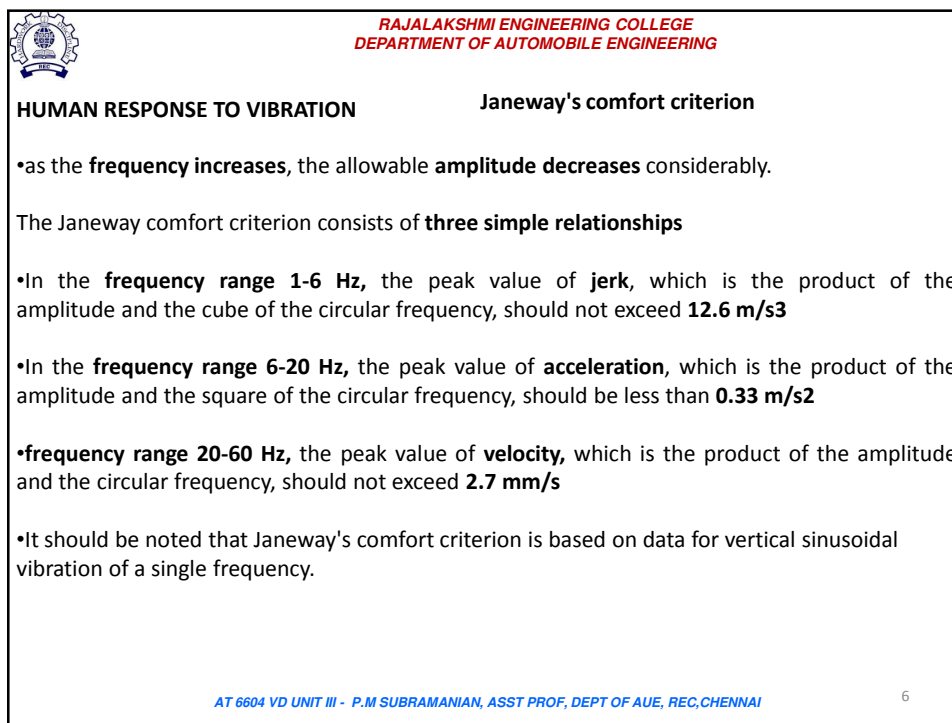
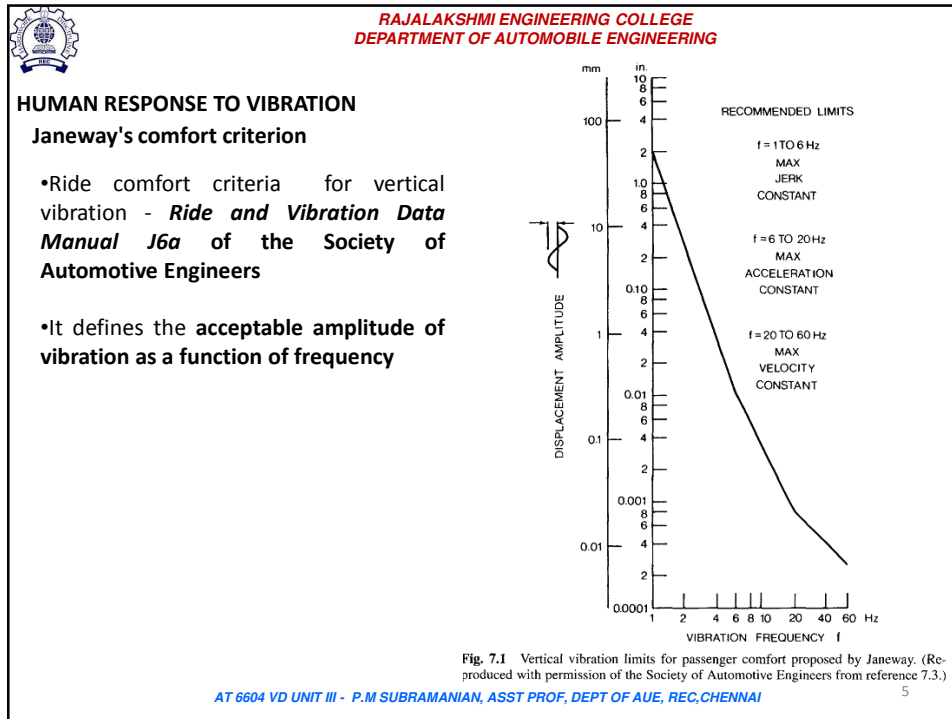
- ride simulators** are used to replicate the vibration of the vehicle traveling over different road surfaces.
- an actual vehicle body is mounted on **hydraulic actuators**, which reproduce vehicle motions in pitch, roll, and bounce (or heave).
- Road inputs are fed into the actuators.
- Using the simulator, it is possible to establish a **human tolerance limit in terms of vibration parameters**.

4. Ride Measurements in Vehicles

- on-the road ride measurements**,
- This test method attempts to correlate the response in qualitative terms, such as "unpleasant" or "intolerable," with **vibration parameters measured at the location situated under actual driving conditions**.

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HUMAN RESPONSE TO VIBRATION

- **International Standard ISO 2631**- a general guide for defining human tolerance to whole-body vibration
- evaluation of vibrational environments in transport vehicles as well as in industry.
- Three distinct limits for whole-body vibration in the frequency range **1-80 Hz**
 1. **Exposure limits**, which are related to the preservation of safety (or health)
 2. **Fatigue or decreased proficiency boundaries**, which are related to the preservation of working efficiency and apply to such tasks as **driving a road vehicle or a tractor**.
 3. **Reduced comfort boundaries**, which are concerned with the preservation of comfort and in transport vehicles are related to such functions as **reading, writing, and eating in a vehicle**.

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7



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HUMAN RESPONSE TO VIBRATION

- vibrations in the frequency range below **1 Hz** -associated with symptoms such as motion sickness,
- A severe discomfort boundary and a reduced comfort boundary for various exposure times in the frequency range of **0.1-1 Hz** have been recommended by the International Organization for Standardization (ISO).
- The "**absorbed power**," which is the product of **vibration force and velocity** transmitted to the human body, has also been proposed as a parameter of significance **in evaluating human response to vibration** .
- It is a measure of the **rate at which vibrational energy is absorbed by a human**, and it has been used to define human tolerance to vibration .
- the tolerance limit is taken as **6 W** absorbed power at the driver's position, and **the ride-limiting speed** is that speed at which the driver's average absorbed power over the total elapsed time reaches a sustained level of **6 W**.

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8



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HUMAN RESPONSE TO VIBRATION

- establishing the ride comfort criteria were obtained using sinusoidal inputs, whereas the actual vehicle vibration is usually of a **random nature**.
- Hence, the ride comfort criteria thus far proposed may require revision as new data become available.
- With defining a specific ride comfort criterion, the **designer should then select an appropriate suspension system** to ensure that the level of vehicle vibration is below the specified limits when operating over a particular range of environments.

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Full, half and quarter car suspension models

An automotive suspension supports the vehicle body on the axles. A “full car” model of a suspension with 7 rigid body degrees of freedom is shown in Figure 10-1. The vehicle body is represented by the “sprung mass” m while the mass due to the axles and tires are represented by the “unsprung” masses m_{u1} , m_{u2} , m_{u3} and m_{u4} . The springs and dampers between the sprung and unsprung mass represent the vehicle suspension. The vertical stiffness of each of the 4 tires are represented by the springs k_{t1} , k_{t2} , k_{t3} and k_{t4} .

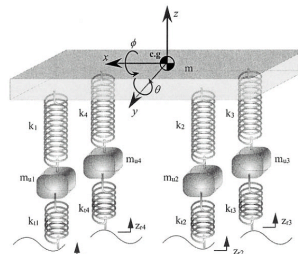


Figure 10-1. Full car automotive suspension model

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Full car automotive suspension model

The seven degrees of freedom of the full car model are the heave z , pitch θ and roll ϕ of the vehicle body and the vertical motions of each of the four unsprung masses. The variables z_{r1} , z_{r2} , z_{r3} and z_{r4} are the road profile inputs that excite the system.

11

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Half car automotive suspension model

Figure 10-2. Half car automotive suspension model

A “half car” model with four degrees of freedom is shown in Figure 10-2. In the half car model, the pitch and heave motions of the vehicle body (θ and z) and the vertical translation of the front and rear axles (z_{u1} and z_{u2}) are represented.

12



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Quarter-car active automotive suspension

A two-degree-of-freedom "quarter-car" automotive suspension system is shown in Figure .

It represents the automotive system at each wheel i.e. the motion of the axle and of the vehicle body at any one of the four wheels of the vehicle.

The suspension itself is shown to consist of a **spring**, **dumper**, and an **active force actuator**

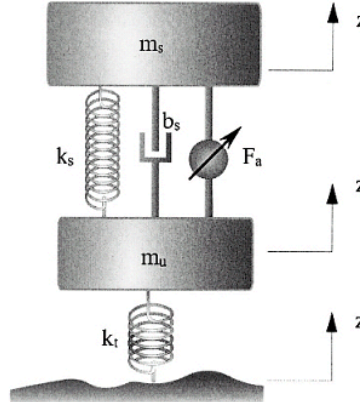


Figure 10-3. Quarter-car active automotive suspension

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Quarter-car active automotive suspension

The vertical stiffness of the tire is represented by the spring k_t .

The variables z_s , z_u and z_r represent the vertical displacements from static equilibrium of the sprung mass, unsprung the road respectively.

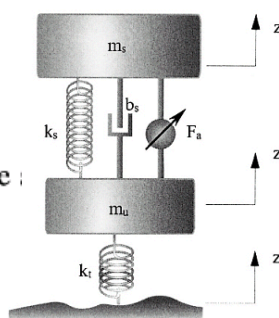


Figure 10-3. Quarter-car active automotive suspension

The active force can be set to zero in a passive suspension.

The sprung mass m , represents the quarter car equivalent of the vehicle body mass.

The unsprung mass mu represents the equivalent mass due to the axle and tire.

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14



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Suspension function:

1.To isolate a car body from road disturbances in order to provide good ride quality. Ride quality - vertical acceleration of the passenger locations.

In the case of the quarter car suspension, sprung mass acceleration can be used to quantify ride quality.

2.To keep good road holding: -cornering, braking and traction abilities- the variations in normal tire loads are to be minimized.

- This is because the lateral and longitudinal forces generated by a tire depend directly on the normal tire load.

- Since a tire roughly behaves like a spring in response to vertical forces, variations in normal tire load can be directly related to vertical tire deflection($z_u - z_r$) .

- The road holding performance of a suspension can therefore be quantified in terms of the **tire deflection performance**.

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Suspension function:

3. To provide good handling:

The roll and pitch accelerations of a vehicle during cornering, braking and traction are measures of good handling.

Half-car and full-car models can be used to study the pitch and roll performance of a vehicle. A good suspension system should ensure that roll and pitch motion are minimized.

4.To support the vehicle static weight

This task is performed well if the rattle space requirements in the vehicle are kept small.

In the case of the quarter car model, it can be quantified in terms of the maximum suspension deflection ($z_s - z_u$) undergone by the suspension.

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16



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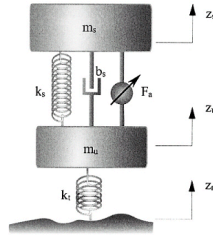


Figure 10-3. Quarter-car active automotive suspension

The equations of motion of the two-degree-of-freedom quarter-car suspension shown in Figure 10-3 are

$$m_s \ddot{z}_s + b_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = F_a \quad (10.8)$$

$$m_u \ddot{z}_u + b_t (\dot{z}_u - \dot{z}_r) + k_t (z_u - z_r) - b_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) = -F_a \quad (10.9)$$

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In standard second-order matrix form, the system can be represented as

$$M\ddot{z} + C\dot{z} + Kz = H_1 z_r + H_2 \dot{z}_r + H_3 F_a \quad (10.10)$$

or

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \begin{Bmatrix} z_s \\ z_u \end{Bmatrix} + \begin{bmatrix} b_s & -b_s \\ -b_s & b_s + b_t \end{bmatrix} \begin{Bmatrix} \dot{z}_s \\ \dot{z}_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ k_t \end{Bmatrix} z_r + \begin{Bmatrix} 0 \\ b_t \end{Bmatrix} \dot{z}_r + \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} F_a \quad (10.11)$$

where $M = \begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}$, $K = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix}$ and the other matrices are as defined in equation (10.11).

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18



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The state space model of the quarter-car active automotive suspension system can be written as (Yue, et. al., 1988)

$$\dot{x} = Ax + BF_a + L\dot{z}_r \quad (10.12)$$

where

$x_1 = z_s - z_u$ is the suspension deflection (rattle space)

$x_2 = \dot{z}_s$ is the absolute velocity of sprung mass

$x_3 = z_u - z_r$ tire deflection

$x_4 = \dot{z}_u$ absolute velocity of unsprung mass

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19



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$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{(b_s + b_t)}{m_u} \end{bmatrix}, \quad B = \begin{Bmatrix} 0 \\ 1/m_s \\ 0 \\ -1/m_u \end{Bmatrix} \text{ and}$$

$$L = \begin{Bmatrix} 0 \\ 0 \\ -1 \\ \frac{b_t}{m_u} \end{Bmatrix}$$

In the case of a passive suspension, the active force F_a is set to zero.

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20

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Example for State Space Model

Figure3 shows a basic two-degree-of freedom system representing the model of a quarter-car and representing in ref. [1]. The model consists of the sprung mass M2 and the unsprung mass M1. The tire is modelled as a linear spring with stiffness Kt. The suspension system consists of a passive spring Ka and a damper Ca in parallel with an active control force u. The passive elements will guarantee a minimal level of performance and safety, while the active element will be

Figure3: Active Suspension for Quarter car Model

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21

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Example for State Space Model

From the Figure3 and Newton's law, we can obtain the dynamic equations as the following:

For M₁,

$$F = Ma$$

$$K_t(X_w - r) - K_a(X_s - X_w) - C_a(\dot{X}_s - \dot{X}_w) - U_a = M_1 \ddot{X}_w$$

$$\ddot{X}_w = \frac{K_t(X_w - r) - K_a(X_s - X_w) - C_a(\dot{X}_s - \dot{X}_w) - U_a}{M_1} \quad (1)$$

For M₂,

$$F = Ma$$

$$-K_a(X_s - X_w) - C_a(\dot{X}_s - \dot{X}_w) + U_a = M_2 \ddot{X}_s$$

$$\ddot{X}_s = \frac{-K_a(X_s - X_w) - C_a(\dot{X}_s - \dot{X}_w) + U_a}{M_2} \quad (2)$$

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Example for State Space Model

Let the state variables are:

$$\begin{aligned} X_1 &= X_s - X_w \\ X_2 &= \dot{X}_s \\ X_3 &= X_w - r \\ X_4 &= \dot{X}_w \end{aligned} \quad (3)$$

Where,

$X_s - X_w$ = Suspension travel

$\dot{X}_s$ = Car Body Velocity

$\ddot{X}_s$ = Car Body Acceleration

$X_w - r$ = Wheel Deflection

$\dot{X}_w$ = Wheel Velocity

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Example for State Space Model

Therefore in state space equation, the state variables are established in equation (3). Therefore, equations (1) and (2) can be written as below

$$\dot{X}(t) = Ax(t) + Bu(t) + f(t) \quad (4)$$

Where,

$$\begin{aligned} \dot{X}_1 &= \dot{X}_s - \dot{X}_w \approx X_2 - X_4 \\ \dot{X}_2 &= \ddot{X}_s \\ \dot{X}_3 &= \dot{X}_w - \dot{r} \approx X_4 - \dot{r} \\ \dot{X}_4 &= \ddot{X}_w \end{aligned} \quad (5)$$

Rewrite equation (4) into the matrix form yield

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_a}{M_2} & -\frac{C_a}{M_2} & 0 & \frac{C_a}{M_2} \\ 0 & 0 & 0 & 1 \\ -\frac{K_a}{M_1} & \frac{C_a}{M_1} & -\frac{K_t}{M_1} & -\frac{C_a}{M_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} U_a + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r} \quad (6)$$

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Design and Analysis of Passive Automotive Suspensions – Quarter Car Suspension

The following three transfer functions are of interest and their attenuation will be used to judge the effectiveness of the suspension system :

a) Acceleration transfer function

$$H_A(s) = \frac{\ddot{z}_s(s)}{\dot{z}_r(s)} \quad (10.13)$$

b) Rattle space transfer function

$$H_{RS}(s) = \frac{z_s(s) - z_u(s)}{\dot{z}_r(s)} \quad (10.14)$$

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c) Tire deflection transfer function

$$H_{TD}(s) = \frac{z_u(s) - z_r(s)}{\dot{z}_r(s)} \quad (10.15)$$

The following values of parameters are typical for a passenger sedan: $k_s = 16000$, $b_s = 1000$, $m_s = 250$, $m_u = 45$, $k_t = 160000$, $b_t = 0$. The tire damping b_t is assumed to be negligible. We will assume that the active force F_A is zero. For now, we are only analyzing a passive suspension system.

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26



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The two undamped natural frequencies of the quarter-car suspension system ω_1 and ω_2 are determined by solving

$$\det(-\omega^2 M + K) = 0 \quad (10.16)$$

where the matrices M and K are as defined in equation (10.11). Hence

$$\det \begin{bmatrix} k_s - m_s \omega^2 & -k_s \\ -k_s & k_s + k_t - m_u \omega^2 \end{bmatrix} = 0$$

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$$\omega_1 = \sqrt{\frac{k_s}{m_s}} \quad (10.19)$$

and

$$\omega_2 = \sqrt{\frac{k_t}{m_u}} \quad (10.20)$$

For the typical parameters discussed earlier, the approximate natural frequencies turn out to be

$$f_1 = \frac{\omega_1}{2\pi} = 1.27 \text{ Hz and } f_2 = \frac{\omega_2}{2\pi} = 9.49 \text{ Hz.}$$

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The mode shapes ϕ_1 and ϕ_2 corresponding to the two natural frequencies can be obtained using $[-\omega_1^2 M + K]\phi_1 = 0$ and $[-\omega_2^2 M + K]\phi_2 = 0$. Let the modal matrix be $\tilde{P} = [\phi_1 \ \phi_2]$.

The mode shapes ϕ_1 and ϕ_2 can be mass-normalized so that the mass-normalized modal matrix $\tilde{P} = [\phi_1 \ \phi_2]$ satisfies

$$\tilde{P}^T M \tilde{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad (10.21)$$

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The mass-normalized modal matrix for the quarter-car suspension system is found to be

$$\tilde{P} = \begin{bmatrix} -0.0632 & 0.002 \\ -0.006 & -0.149 \end{bmatrix} \quad (10.22)$$

From the mode shapes in equation (10.22), one can see that the mode corresponding to the first natural frequency predominantly consists of sprung mass motion. This mode is therefore called *the sprung mass mode*. The mode corresponding to the second natural frequency is called *the unsprung mass mode*.

We also find

$$\tilde{P}^T K \tilde{P} = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} = 4\pi^2 \begin{bmatrix} (1.21)^2 & 0 \\ 0 & (9.96)^2 \end{bmatrix} \quad (10.23)$$

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$$m_s \ddot{z}_s + b_s \dot{z}_s + k_s z_s = b_s \dot{z}_r + k_s z_r \quad \text{when } |z_s| \gg |z_u| \quad (10.30)$$

and

$$m_u \ddot{z}_u + b_s \dot{z}_u + k_t z_u = k_t z_r \quad \text{when } |z_u| \gg |z_s| \quad (10.31)$$

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Design and Analysis of Passive Automotive Suspensions – Quarter Car Suspension

The following figures show the decoupled 1 dof approximations to the quarter car suspension system:

Sprung mass mode approximation:

Valid when $|z_s| \gg |z_u|$

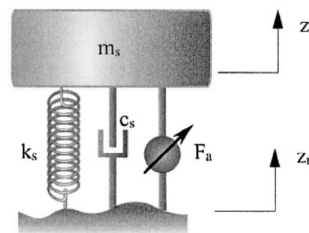


Figure 10-6. Sprung mass mode

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32


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Unsprung mass mode approximation :

Valid when $|z_u| \gg |z_s|$

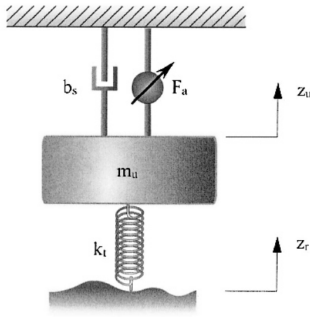


Figure 10-7. Unsprung Mass Mode

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33


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The following approximate transfer functions can then be obtained:

From the sprung mass mode

The equation of motion of the 1dof system shown in Figure 10-6 is

$$m_s \ddot{z}_s + b_s \dot{z}_s + k_s z_s = k_s z_r + b_s \dot{z}_r$$

This approximation leads to the transfer function

$$H_A(s) \approx \frac{(b_s s + k_s)s}{(m_s s^2 + b_s s + k_s)} \quad \text{if } |z_s| \geq |z_u| \quad (10.32)$$

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34



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From the unsprung mass mode

The equation of motion of the 1dof system shown in Figure 10-7 is

$$m_u \ddot{z}_u + k_t(z_u - z_r) + b_s \dot{z}_u + k_s z_u = 0$$

This leads to the relations

$$Z_u(s) = \frac{k_t}{m_u s^2 + b_s s + k_s + k_t} Z_r(s) \quad \text{if } |z_u| \geq |z_s| \quad (10.33)$$

and

$$H_{TD}(s) \approx \frac{-m_u s^2 - b_s s - k_s}{m_u s^3 + b_s s^2 + (k_s + k_t)s} \quad \text{if } |z_u| \geq |z_s| \quad (10.34)$$

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35



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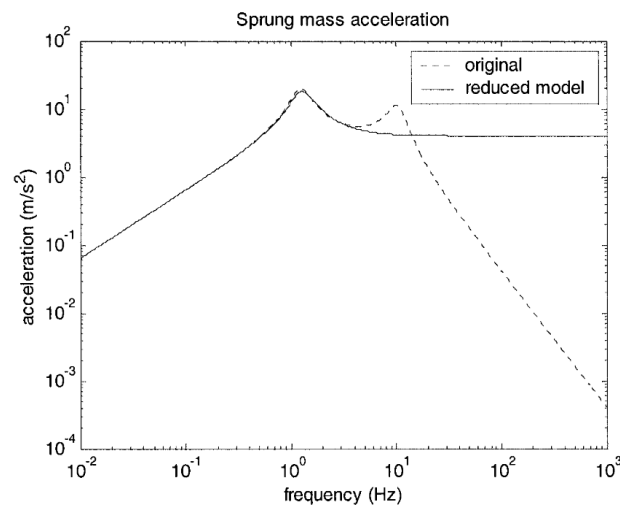
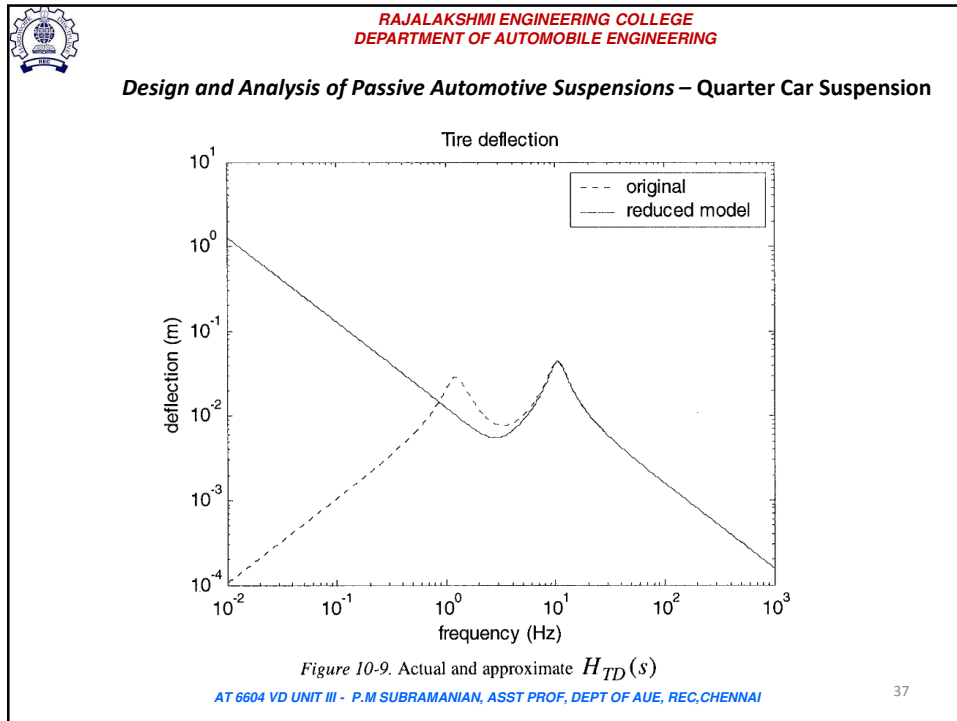


Figure 10-8. Actual and approximate $H_A(s)$

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36



37

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10.6 ANALYSIS OF VIBRATIONS IN THE SPRUNG MASS MODE

As discussed in the previous section, the approximate transfer functions for vibrations in the sprung mass mode are

$$\frac{1}{s} H_A(s) = \frac{z_s}{z_r} = \frac{k_s + b_s s}{m_s s^2 + b_s s + k_s} \quad (10.37)$$

and

$$s H_{RS}(s) = \frac{z_s - z_r}{z_r} \approx - \frac{m_s s^2}{m_s s^2 + b_s s + k_s} \quad (10.38)$$

By inspection of the simple second order transfer functions in equations (10.37) and (10.38) above, it is clear that changes in the suspension stiffness k_s and in the suspension damping b_s will lead to the changes in the transfer function $H_A(s)$ and $H_{RS}(s)$ as shown in Table 10.1.

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38



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Table 10-1. Influence of Suspension Parameters on Sprung Mass Vibrations

Suspension Change	Influence	Impact on Ride Quality	Impact on Rattle Space
Reduced suspension stiffness k_s	Decrease in the value of the first natural frequency ω_1	GOOD Improved sprung mass acceleration transfer function at high frequencies	BAD Increased suspension deflection at low frequencies
Increased suspension damping b_s	Better damping at the first natural frequency ω_1	GOOD Reduces or eliminates the first resonant peak resulting in highly improved ride quality at the first resonant frequency.	GOOD Reduces or eliminates the first resonant peak in the suspension deflection transfer function resulting in improved suspension deflection performance at the first resonant frequency.
		BAD Deteriorates ride quality at high frequencies by causing a slower roll-off and resulting in high frequency "harshness"	BAD This change has no detrimental effects on the suspension deflection transfer function

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39



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From the Table, it can be seen that a softer suspension (lower k_s) leads to an improvement in ride quality by reducing the first resonant frequency and hence causing the roll-off in the transfer function $H_A(s)$ to start at a lower frequency. However, a softer suspension leads to increased suspension deflection at low frequencies, thus increasing rattle space requirements.

An increase in suspension damping b_s reduces or eliminates the resonant peak corresponding to the sprung mass natural frequency. Thus the ride quality transfer function $H_A(s)$ will be significantly improved at the sprung mass frequency. However, due to the impact of b_s on the numerator in equation (10.37), the higher damping introduces high frequency harshness in $H_A(s)$ by causing a slower roll-off.

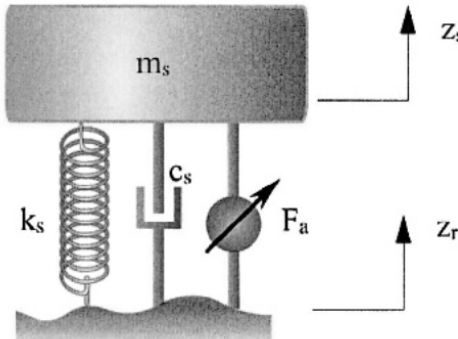
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40



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Design and Analysis of Passive Automotive Suspensions – Quarter Car Suspension Sky Hook Damper



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41



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Consider again the decoupled sprung mass mode model of Figure 10-6. If the damping b_s were placed between the sprung mass and inertial ground, instead of being placed between the sprung mass and the road, the resonant peak in the ride quality transfer function would be damped without causing the slower roll off at high frequencies. Thus significant ride quality improvement at the sprung mass frequency could be obtained without any high frequency harshness. Such a damper placed between the sprung mass and inertial ground is called a “sky-hook” damper. While the benefits of a sky-hook damper are clear, it is obviously not directly realizable in a passive suspension system. In the case of an active suspension system, the equivalent effect of a sky-hook damper can be obtained by controlling a hydraulic actuator placed between the sprung and unsprung masses (Redfield and Karnopp, 1989).

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42



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10.7 ANALYSIS OF VIBRATIONS IN THE UNSPRUNG MASS MODE

For the case where $|z_u| \gg |z_s|$ (in the unsprung mass mode), the quarter car system can be replaced by the 1 dof system shown in Figure 10-7 earlier. As seen earlier, the tire deflection transfer function in this case can be approximated by

$$\frac{z_u - z_r}{z_r} = \frac{-m_u s^2}{m_u s^2 + b_s s + k_t} \quad (10.39)$$

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43



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Influence of tire stiffness on road holding

By examining the simple second order transfer function in equation (10.39), it is clear that an increase in tire stiffness reduces tire deflection by reducing the low frequency asymptote of $H_{TD}(s)$. Table 10.2 summarizes the influence of an increase in tire stiffness.

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44



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Table 10-2. Influence of Suspension Parameters on Unsprung Mass Vibrations

Suspension Change	Influence	Impact on Road Holding
Increased tire stiffness k_t	Increase in the value of the second natural frequency ω_2. Reduction in the low frequency asymptote of the tire deflection transfer function.	GOOD Improves tire deflection transfer function by reducing its low frequency asymptote

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45



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10.9 HALF-CAR AND FULL-CAR SUSPENSION MODELS

Consider a two degree of freedom half-car model of an automot suspension system, as shown in Figure 10-19 below. The two degrees freedom are the pitch θ and the heave (vertical translation) z .

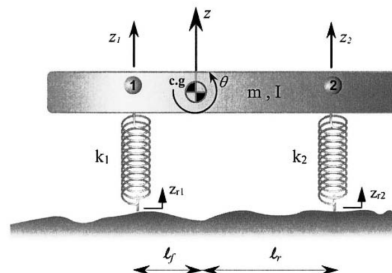


Figure 10-19. Two-degree of freedom half car model

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46



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The displacements of the car body at the two suspension locations are related to z and ϕ by

$$z_1 = z + \ell_f \phi \quad (10.40)$$

and

$$z_2 = z - \ell_r \phi \quad (10.41)$$

Using Newton's laws, we have

$$m\ddot{z} + k_1(z_1 - z_{r1}) + k_2(z_2 - z_{r2}) = 0 \quad (10.42)$$

and

$$I\ddot{\phi} + k_1(z_1 - z_{r1})\ell_f + k_2(z_2 - z_{r2})\ell_r = 0 \quad (10.43)$$

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47



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Substituting for z_1 and z_2 from equations (10.40) and (10.41), the equations of motion turn out to be:

$$\begin{aligned} & \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_1\ell_f - k_2\ell_r \\ k_1\ell_f - k_2\ell_r & k_1\ell_f^2 + k_2\ell_r^2 \end{bmatrix} \begin{bmatrix} z \\ \phi \end{bmatrix} \\ &= \begin{bmatrix} k_1 & k_2 \\ k_1\ell_f & -k_2\ell_r \end{bmatrix} \begin{bmatrix} z_{r1} \\ z_{r2} \end{bmatrix} \end{aligned} \quad (10.44)$$

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48



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The standard procedure for decoupling the equations of motion can be followed by calculating the natural frequencies and mode shapes.

First consider the special case where the moment of inertia is given by

$$I = m \ell_f \ell_r \quad (10.45)$$

In this special case, the natural frequencies (obtained by using $\det(-\omega^2 M + K) = 0$) turn out to be

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49



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$$\omega_1^2 = \frac{k_1}{m \frac{\ell_r}{\ell_f + \ell_r}} \quad \text{and} \quad \omega_2^2 = \frac{k_2}{m \frac{\ell_f}{\ell_f + \ell_r}} \quad (10.46)$$

The mass-normalized mode shapes are

$$\tilde{\phi}_1 = \frac{1}{\sqrt{m(\ell_f + \ell_r)}} \begin{bmatrix} \sqrt{\ell_r} \\ 1 \\ \sqrt{\ell_r} \end{bmatrix} \quad \text{and} \quad \tilde{\phi}_2 = \frac{1}{\sqrt{m(\ell_f + \ell_r)}} \begin{bmatrix} \sqrt{\ell_f} \\ 1 \\ -\sqrt{\ell_f} \end{bmatrix} \quad (10.47)$$

$$\text{and } \tilde{P} = [\tilde{\phi}_1 \quad \tilde{\phi}_2]$$

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50



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Thus, the special case $I = m\ell_f\ell_r$ corresponds to a system where the coupled 2-dof system of Figure 10-19 can be exactly represented by the decoupled masses at the front and rear of the system. In this case the front and rear suspensions can be designed independently.

It turns out that for a typical passenger sedan, the relationship $I = m\ell_f\ell_r$ is approximately satisfied. Some typical values of parameters for a passenger sedan are

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51



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$$m = 1460$$

$$\ell_f = 1.4 \text{ meters}$$

$$\ell_r = 1.4 \text{ meters}$$

$$I_\theta = 2460 \text{ kg m}^2$$

$$m\ell_f\ell_r = 2862 \text{ kg m}^2$$

$$\text{Thus } m\ell_f\ell_r \approx I_\theta.$$

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52



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Since the motions at the front and rear are decoupled, independent design of the front and rear suspensions is adequate to control both heave and pitch motions due to road irregularities.

Similarly, the roll and heave vibrations of the automotive suspension can also be analyzed using a half car model. The corresponding roll-heave parameters for a typical passenger sedan are

$$I_{\phi} = 660 \text{ kg m}^2$$

$$\ell_f = 0.761 \text{ meters}$$

$$\ell_r = 0.761$$

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53



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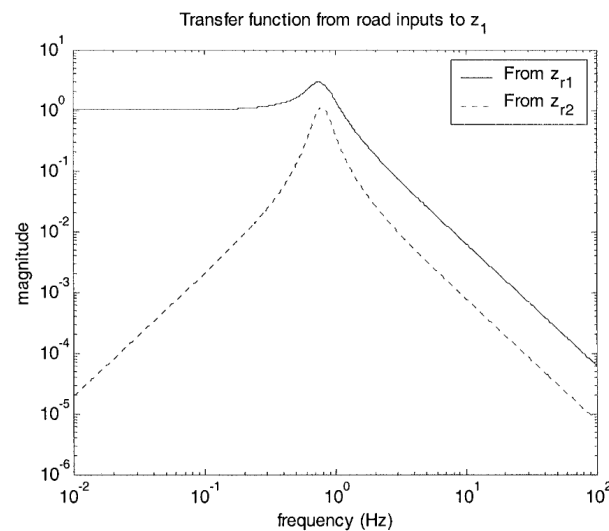


Figure 10-21. Transfer function from the two road inputs to z_1

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54



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decoupled. The vertical motion of the sprung mass at each wheel is significantly influenced by the road input at that wheel. In summary, quarter car models are adequate to design suspensions when the influence of road irregularities is being considered. However, it must here be noted that, full car models are needed when the influence of cornering on vehicle roll and the influence of braking and longitudinal acceleration on vehicle pitch are to be considered. When only the influence of road irregularities is being considered, a quarter car model is adequate.

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55



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A two-degree-of-freedom “quarter-car” automotive suspension system is shown in Figure 11-1 below. It represents the automotive system at each wheel i.e. the motion of the axle and of the vehicle body at any one of the four wheels of the vehicle. The suspension itself is shown to consist of a spring k_s , a damper b_s and an active force actuator F_a . The sprung mass

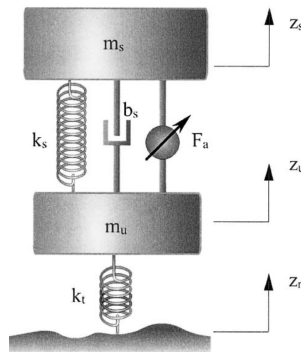


Figure 11-1. Quarter-car active automotive suspension

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56



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The sprung mass

m_s represents the quarter-car equivalent of the vehicle body mass. The unsprung mass m_u represents the equivalent mass due to the axle and tire. The vertical stiffness of the tire is represented by the spring k_t . The variables z_s , z_u and z_r represent the vertical displacements from static equilibrium of the sprung mass, unsprung mass and the road respectively.

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57



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The equations of motion of the two-degree-of-freedom quarter-car suspension shown in Figure 11-1 are

$$m_s \ddot{z}_s + b_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = F_a \quad (11.1)$$

$$m_u \ddot{z}_u + k_t (z_u - z_r) - b_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) = -F_a \quad (11.2)$$

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58



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The state space model of the quarter-car active automotive suspension system can be written as (Yue, et. Al., 1988)

$$\dot{x} = Ax + BF_a + L\dot{z}_r \quad (11.3)$$

where

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$$

$x_1 = z_s - z_u$ is suspension deflection (rattle space)

$x_2 = \dot{z}_s$ is the absolute velocity of sprung mass

$x_3 = z_u - z_r$ is tire deflection

$x_4 = \dot{z}_u$ is the absolute velocity of unsprung mass

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59



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$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{(b_s + b_t)}{m_u} \end{bmatrix}, B = \begin{Bmatrix} 0 \\ 1/m_s \\ 0 \\ -1/m_u \end{Bmatrix} \quad \text{and} \quad L = \begin{Bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{Bmatrix} \quad (11.4)$$

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60



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Design and Analysis of Active Automotive Suspensions –Quarter Car Suspension

feedback control uses signals measured from the system response to a disturbance to drive a control actuator so as to attenuate the response.

11.2.1 Transfer functions of interest

The following three transfer functions are of interest and their attenuation will be used to judge the effectiveness of the suspension system :

d) Acceleration transfer function

$$H_A(s) = \frac{\ddot{z}_s(s)}{\ddot{z}_r(s)} \quad (11.5)$$

e) Rattle Space transfer function

$$H_{RS}(s) = \frac{z_s(s) - z_u(s)}{\dot{z}_r(s)} \quad (11.6)$$

f) Tire deflection transfer function

$$H_{TD}(s) = \frac{z_u(s) - z_r(s)}{\dot{z}_r(s)} \quad (11.7)$$

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61



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Design and Analysis of Active Automotive Suspensions –Quarter Car Suspension – LQR FORMULATION

The main objective of this section is to design LQR controller for the active suspension system. In optimal control, the attempts to find controller that can provide the best possible performance. The LQR approach of vehicle suspension control is widely used in background of many studies in vehicle suspension control. . The strength of LQR approach is that in using it the factors of the performance index can be weighted according to the designer's desires or other constraints. In this study, the LQR method is used to improve the road handling and the ride comfort for a quarter car model. LQR control approach in controlling a linear active suspension system and representing in ref. ^[21].

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62



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Design and Analysis of Active Automotive Suspensions –Quarter Car Suspension –
LQR FORMULATION

11.2.2 Use of the LQR formulation and its Relation to H_2 - optimal control

Consider the following plant

$$\dot{x} = Ax + B_1 d + B_2 u \quad A \in R^{n \times n}, B_1 \in R^n, B_2 \in R^n \quad (11.8)$$

$$z = C_1 x + D_{12} u \quad C_1 \in R^{m \times n}, D_{12} \in R^{m \times 1} \quad (11.9)$$

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63



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LQR FORMULATION

where $d \in R$ is a disturbance input, assumed to be zero-mean white noise of unit intensity, $u \in R$ is the control input and the variables in $z \in R^m$ constitute the ones to be minimized. In the case of the active suspension problem, the variables in z consist of the sprung mass acceleration, the suspension deflection and the tire deflection. Assume that the pair (A, C_1) is detectable, the pair (A, B_2) is stabilizable and that $D_{12}^T D_{12} > 0$.

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64



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**Design and Analysis of Active Automotive Suspensions –Quarter Car Suspension –
LQR FORMULATION**

If the control design problem for the above system is posed as that of minimizing the variance of the output z , for the input d being white noise, then this control design problem is called the H_2 optimal control problem (Levine, 1996).

It turns out that the solution to the H_2 optimal control problem is the same as the solution to the linear quadratic regulator (LQR) problem (Levine, 1996). In the LQR problem, the controller is to be designed so that the following performance index is minimized:

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65



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LQR FORMULATION**

(Levine, 1996). In the LQR problem, the controller is to be designed so that the following performance index is minimized:

$$J = \int_0^{\infty} z^T z dt = \int_0^{\infty} [x^T C_1^T C_1 x + 2x^T C_1^T D_{12} u + u^T D_{12}^T D_{12} u] dt \quad (11.10)$$

for all initial conditions $x_0 = x(0)$.

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66



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LQR FORMULATION**

The solution to the LQR problem is

$$\begin{aligned} u &= -(D_{12}^T D_{12})^{-1} B_2^T P x - (D_{12}^T D_{12})^{-1} (C_1^T D_{12})^T x \\ &= -(D_{12}^T D_{12})^{-1} \left[B_2^T P + (C_1^T D_{12})^T \right] x \end{aligned} \quad (11.11)$$

with the matrix P being given by the positive semi-definite solution to the Riccati equation

, the LQR solution of equation (11.11) will be used as the solution to the H_2 optimal control problem of minimizing the variance of the variables z in the presence of the white noise disturbance d .

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67



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**Design and Analysis of Active Automotive Suspensions –Quarter Car Suspension –
LQR FORMULATION**

11.2.3 LQR formulation for active suspension design

The original research on which the results presented in sections 11.2 – 11.7 of this chapter are largely based are credited to the doctoral dissertation of Tetsuro Butsuen (Butsuen, 1989).

Define the following quadratic performance index.

$$J = \left[\int_0^\infty \ddot{z}_s^2 + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^2 + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^2 dt \right] \quad (11.14)$$

where the weighting factors ρ_1 , ρ_2 , ρ_3 and ρ_4 can be chosen so as to emphasize appropriate variables of interest.

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68



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LQR FORMULATION

11.2.4 Performance studies of the LQR controller

The performance of the LQR controller has been studied for different values of the weights ρ_1 , ρ_2 , ρ_3 and ρ_4 (Butsuen, 1989). The sprung mass acceleration can be heavily penalized without penalizing any of the other variables in the performance index by choosing the weights on the other variables to be very small: $\rho_1=0.4$, $\rho_2=0.16$, $\rho_3=0.4$ and $\rho_4 = 0.16$.

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69



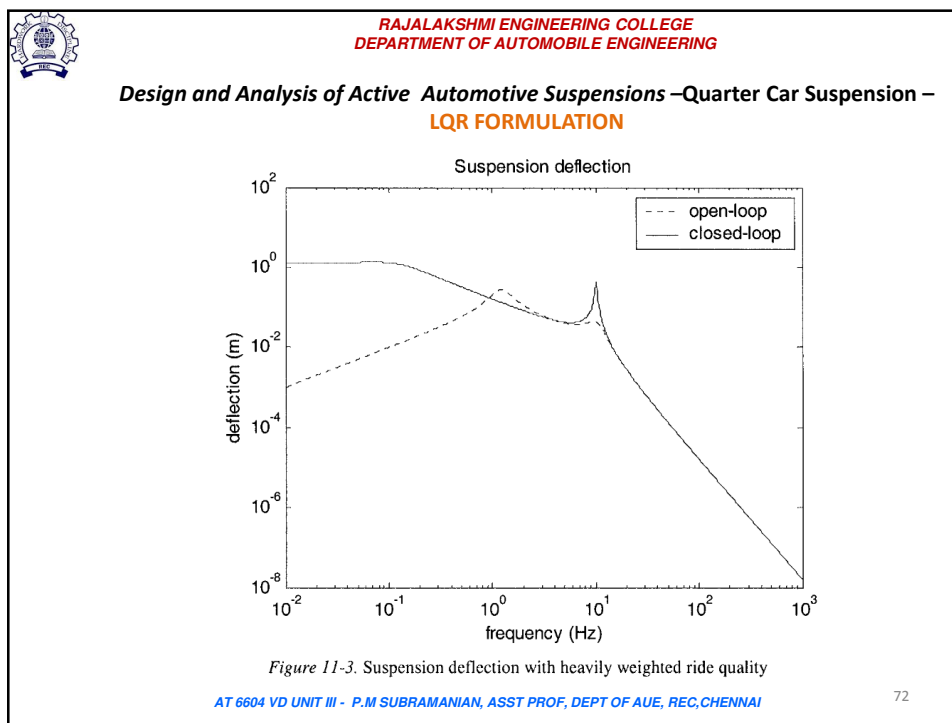
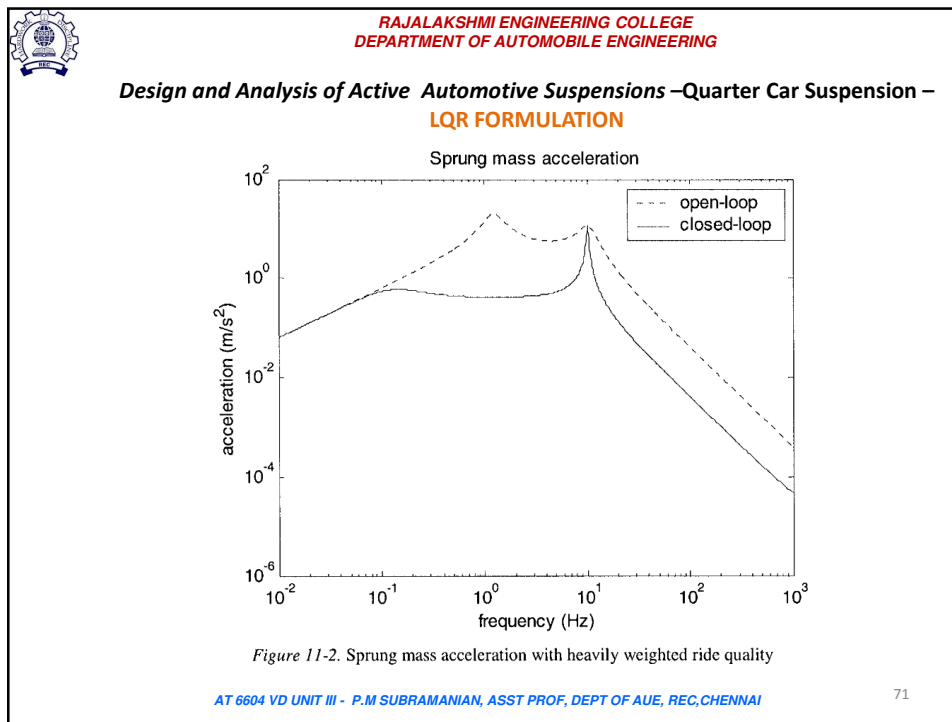
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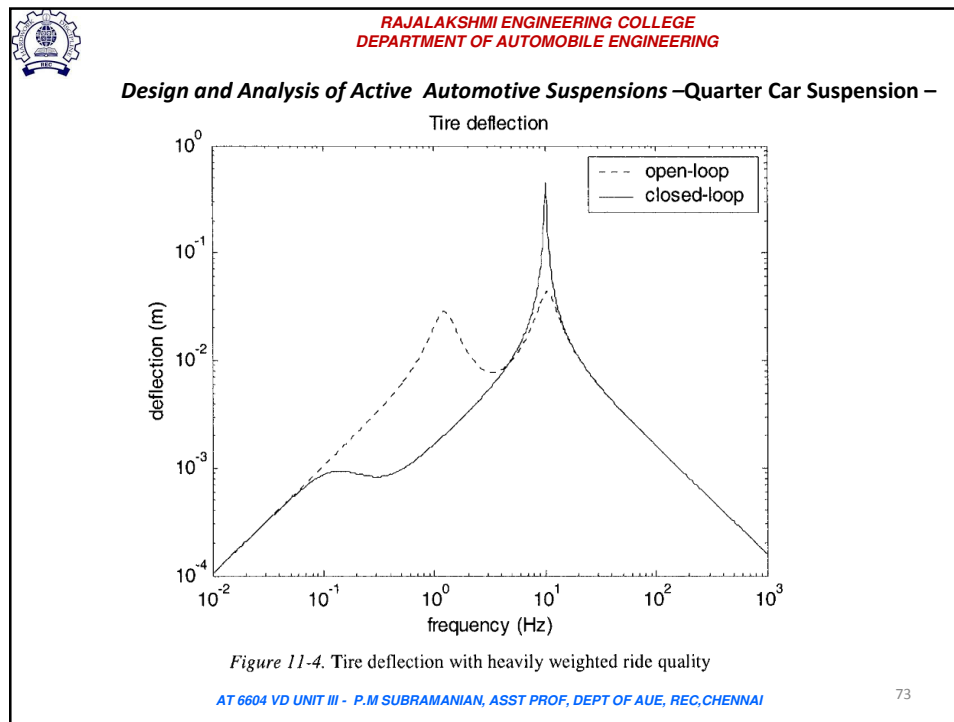
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LQR FORMULATION

The performance of the corresponding controller can be seen in the following figures (Figures 11-2, 11-3 and 11-4). One can see that the sprung mass acceleration is reduced considerably over a broad frequency range. However, its value is unchanged compared to the passive suspension at one particular frequency - the unsprung mass resonant frequency of 10 Hz. No matter how heavily the sprung mass acceleration is weighted, its performance cannot be improved at that frequency.

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70





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SKYHOOK DAMPING

11.7 PERFORMANCE OF A SIMPLE VELOCITY FEEDBACK CONTROLLER

Since very little performance improvement can be obtained at the unsprung mass resonant frequency (10 Hz), it might be best to concentrate on improving performance at the sprung mass resonant frequency (1.2 Hz). Almost all of the performance improvement at the sprung mass resonant frequency can be obtained by using a simple velocity feedback control law also known as “sky-hook” damping, defined as follows (Karnopp, 1986):

$$F_a = -k_2 \dot{z}_s \quad (11.50)$$

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74



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SKYHOOK DAMPING**

This control law is simpler, does not require full-state feedback and provides almost all the performance improvement that the earlier full state feedback LQR control law could provide. Note that the absolute (i.e. inertial) sprung mass velocity is being used in the skyhook damping control law.

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75



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SKYHOOK DAMPING**

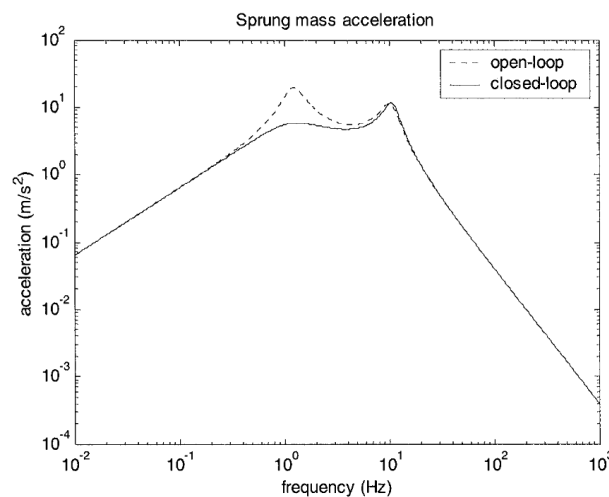
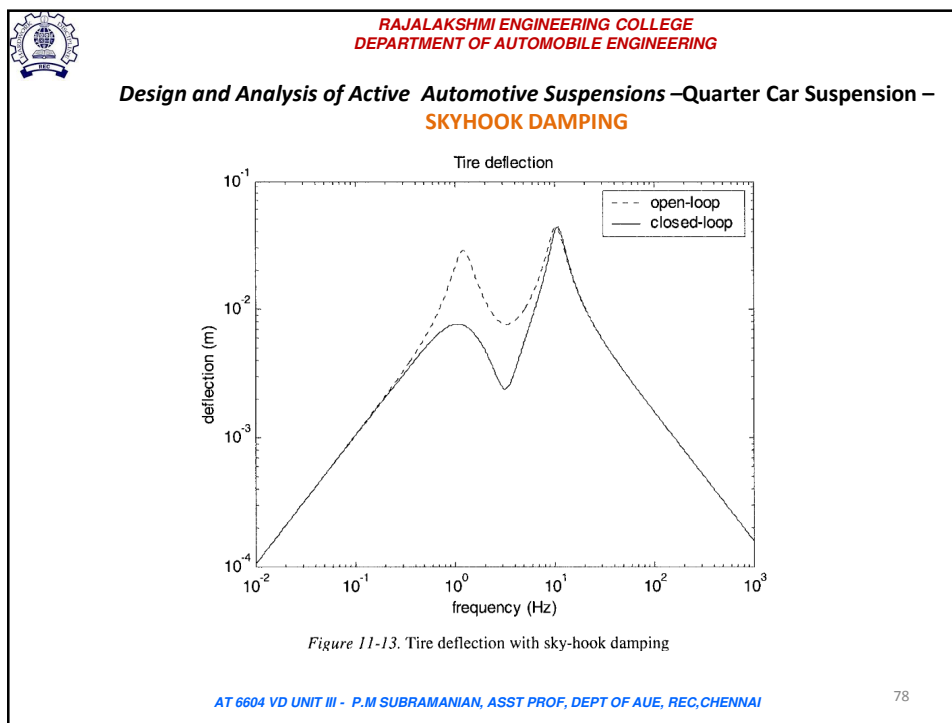
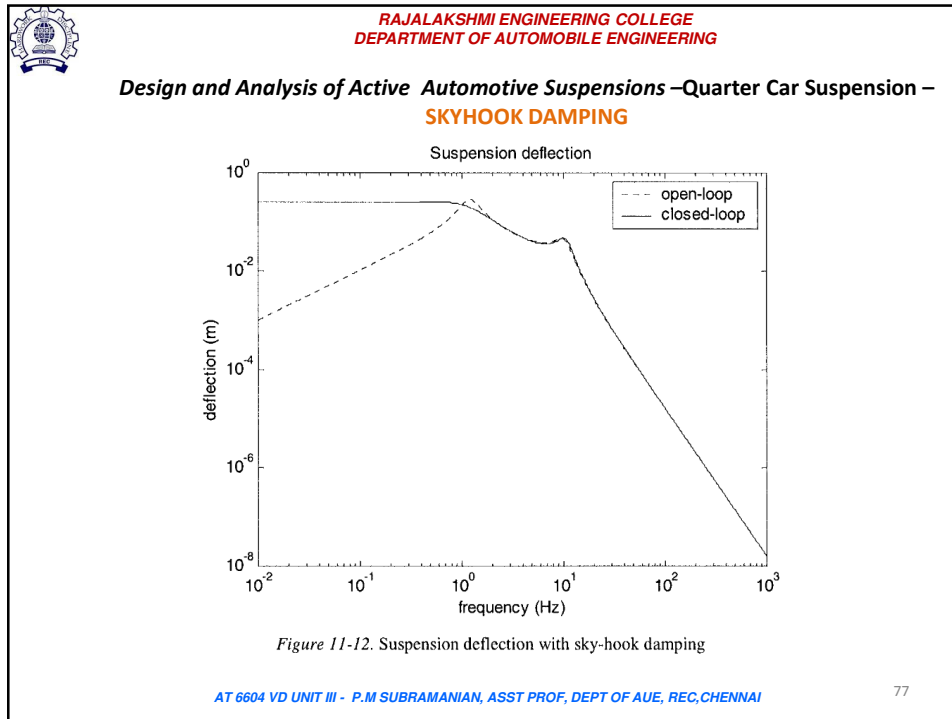


Figure 11-11. Sprung mass acceleration with sky-hook damping

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76





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- 1) Achieve significant reduction in sprung mass acceleration at the sprung mass frequency.
- 2) Simultaneously achieve significant reduction in suspension deflection and tire deflection at the sprung mass natural frequency.
- 3) Avoid any deterioration in all three transfer functions at the unsprung mass natural frequency.
- 4) Avoid high frequency harshness by ensuring that the sprung mass acceleration rolls off at 20 dB/decade at high frequencies.
- 5) If possible, ensure that the suspension deflection transfer function does not have a constant low frequency asymptote.

Section 11.7 showed that the “sky-hook damping” controller provides almost all of the achievable performance described above. The sky-hook damping controller can therefore be effectively used in place of a full state feedback controller.

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79



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A semi-active suspension system utilizes a variable damper or other variable dissipation component in the automotive suspension. An example of a variable dissipator is a twin tube viscous damper in which the damping coefficient can be varied by changing the diameter of the orifice in a piston.

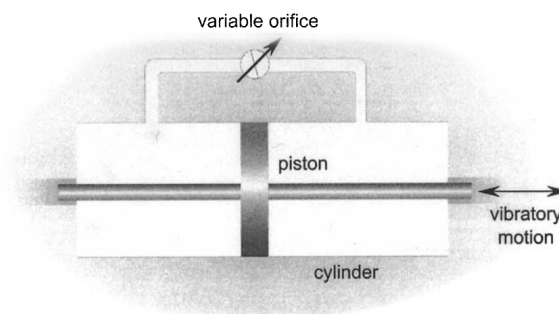


Figure 12-1. Schematic of a variable orifice damper

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80



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Figure 12-1 shows the schematic of a twin tube variable orifice damper in which the orifice diameter can be varied with electronic control. As the piston moves inside the cylinder, it causes fluid flow through the orifice. A larger orifice provides less dissipative resistance while a smaller orifice provides increased dissipative resistance.

To use the variable orifice damper as a semi-active actuator, the opening of the orifice is determined in real time by feedback control laws. Thus the damping provided by the device is varied in real time by feedback control.

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81



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Another example of a semi-active dissipator is a magneto rheological (MR) damper which uses MR fluid. A MR damper is shown in Figure 12-2. Magnetorheological (MR) fluids are materials that respond to an applied magnetic field with a change in rheological behavior. Typically, this change is manifested by the development of a yield stress that monotonically increases with applied magnetic field. The dissipative force provided by the damper can be controlled by controlling the electromagnetic field.

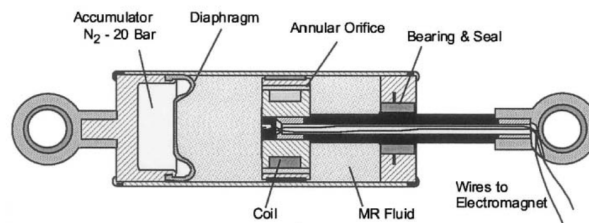


Figure 12-2. Commercial Linear MR Fluid-based Damper⁵

The dissipative force as a function of velocity across the piston of the MR damper is shown in Figure 12-3 for different values of current in the electromagnetic coil. Thus, with different levels of current, different levels of dissipative force can be obtained.

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82



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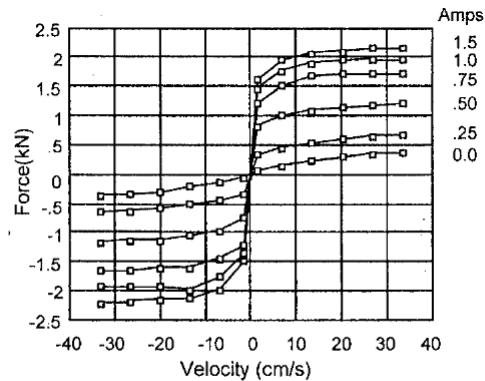


Figure 12-3. Performance curves for the linear MR damper⁶

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83



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Such a suspension system that utilizes a variable dissipator which is controlled in real-time is called a semi-active suspension.

Advantages of a Semi-Active Suspension System

Compared to fully active suspension systems, semi-active systems consume significantly less power. The power consumption in a semi-active system is only for purposes of changing the real-time dissipative force characteristics of the semi-active device. For example, power is used to change the area of the piston orifice in a variable opening damper or to change the current in the electromagnetic coil of a MR damper. External power is not directly used to counter vibratory forces. Another advantage of semi-active systems over active systems is that they cannot cause the suspension system to become unstable. This is due to the fact that they do not actively supply energy to the vibratory suspension system but only dissipate energy from it.

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84


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SEMI-ACTIVE SUSPENSION MODEL

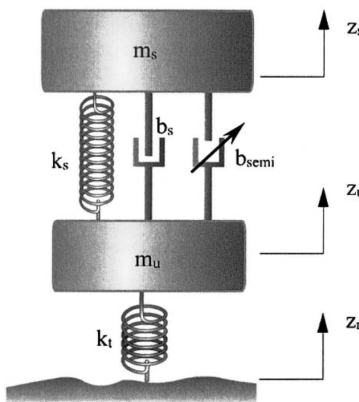


Figure 12-4. Quarter-car active automotive suspension

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85


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A quarter car semi-active suspension system is shown in Figure 12-4. The variable damper $b_{semi}(t)$ is constrained to be between the following values:

$$0 \leq b_{semi}(t) \leq b_{max} \quad (12.1)$$

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86



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The equations of motion of the two-degree-of-freedom quarter-car suspension shown in Figure 12-4 are

$$m_s \ddot{z}_s + b_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = -b_{semi}(t)(\dot{z}_s - \dot{z}_u) \quad (12.2)$$

$$m_u \ddot{z}_u + k_t (z_u - z_r) - b_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) = b_{semi}(t)(\dot{z}_s - \dot{z}_u) \quad (12.3)$$

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87



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The state space model of the quarter-car active automotive suspension system can be written as (Yue, et. al., 1988)

$$\dot{x} = A_0 x + B F_{semi} + L \dot{z}_r \quad (12.4)$$

$$= A_0 x + N x b_{semi} + L \dot{z}_r \quad (12.5)$$

$$= A_0 x - B b_{semi} (x_2 - x_4) + L \dot{z}_r \quad (12.6)$$

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88



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where the variables are as follows

$x_1 = z_s - z_u$	suspension deflection (rattle space)
$x_2 = \dot{z}_s$	absolute velocity of sprung mass
$x_3 = z_u - z_r$	tire deflection
$x_4 = \dot{z}_u$	absolute velocity of unsprung mass
F_{semi}	semi-active force $(= -b_{semi}(\dot{z}_s - \dot{z}_u))$
$x_2 - x_4$	relative suspension velocity

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89



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$$A_0 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix}, \quad B = \begin{Bmatrix} 0 \\ 1/m_s \\ 0 \\ -1/m_u \end{Bmatrix}$$

$$, N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{m_s} & 0 & \frac{1}{m_s} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_u} & 0 & -\frac{1}{m_u} \end{bmatrix} \text{ and } L = \begin{Bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{Bmatrix}$$

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90



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12.3.1 Problem formulation

For the plant given by equation (12.5), assume that the road input $\dot{z}_r$ is white noise with intensity γ . Assume that the passive system is by itself stable i.e.

$$\text{Re}\{\lambda(A_0)\} < 0 \quad (12.7)$$

The performance index of this system is the same as that of the active system in the previous chapter and is given by

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} E \left[\int_0^T \dot{z}_s^2 + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^2 + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^2 \right] \quad (12.8)$$

where $\rho_1, \rho_2, \rho_3, \rho_4$ are weighting factors as before.

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91



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Table 12-1. Summary of Semi-Active Control Law

SUMMARY OF SEMI-ACTIVE CONTROL LAW		
Symbol	Nomenclature	Equation
F_a	Optimal fully-active force	$F_a = -Kx + k_s \dot{z}_s + b_s (\dot{z}_s - \dot{z}_u)$ See equations (11.16) and (11.17) in Chapter 11
b_{semi}	Semi-active (variable) damping coefficient	$b_{semi}^* = 0$ if $-F_a (\dot{z}_s - \dot{z}_u) \leq 0$ $b_{semi}^* = -\frac{F_a}{x_2 - x_4}$ if $0 < -\frac{F_a}{\dot{z}_s - \dot{z}_u} \leq b_{max}$ $b_{semi}^* = b_{max}$ if $-\frac{F_a}{x_2 - x_4} > b_{max}$ and $F_{semi} = -b_{semi}^* (x_2 - x_4)$

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92



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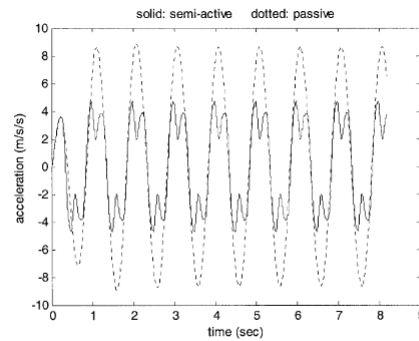


Figure 12-7. Acceleration at 1 Hz

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93



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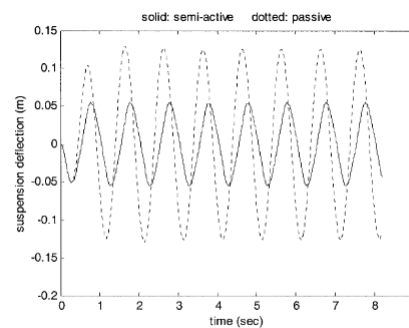


Figure 12-8. Suspension deflection at 1 Hz

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94



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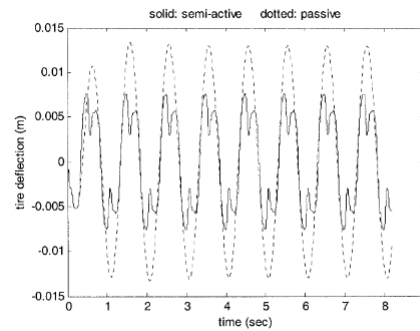


Figure 12-9. Tire deflection at 1 Hz

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95



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SKY HOOK DAMPING

12.7.2 Sky hook damping

This section demonstrates the performance of a semi-active system which attempts to provide the equivalent of sky-hook damping. The desired semi-active force is defined as

$$F_a = -4000\dot{z}_s \quad (12.44)$$

The semi-active damping coefficient is determined as before, using

$$b^* = 0 \quad \text{if} \quad -F_a(x_2 - x_4) \leq 0 \quad (12.45)$$

$$b^* = -\frac{F_a}{x_2 - x_4} \quad \text{if} \quad 0 < -\frac{F_a}{x_2 - x_4} \leq b_{\max} \quad (12.46)$$

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96



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SKY HOOK DAMPING

$$b^* = b_{\max} \quad \text{if} \quad -\frac{F_a}{x_2 - x_4} > b_{\max} \quad (12.47)$$

The performance of this system and its comparison with the standard passive system is shown in Figures 12-16, 12-17 and 12-18. One can see that the sprung mass acceleration, suspension deflection and tire deflection are all significantly improved at the first natural frequency. There is no difference in performance at the second natural frequency. The performance achieved in sky-hook damping is similar to that achieved in section 12.7.1. It is also clear from Figure 12-18 that the switching nature of the semi-active system introduces some high frequency variations in the sprung mass acceleration transfer function.

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97



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SKY HOOK DAMPING

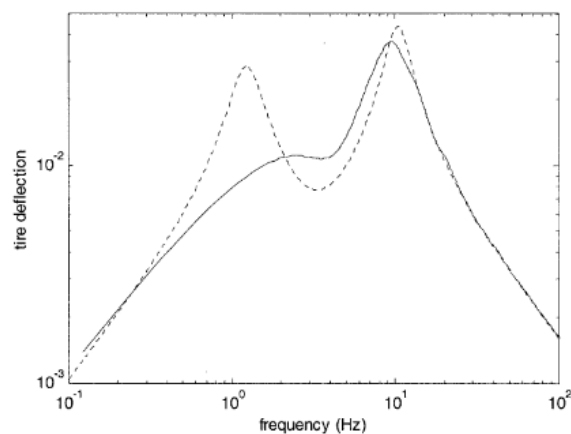
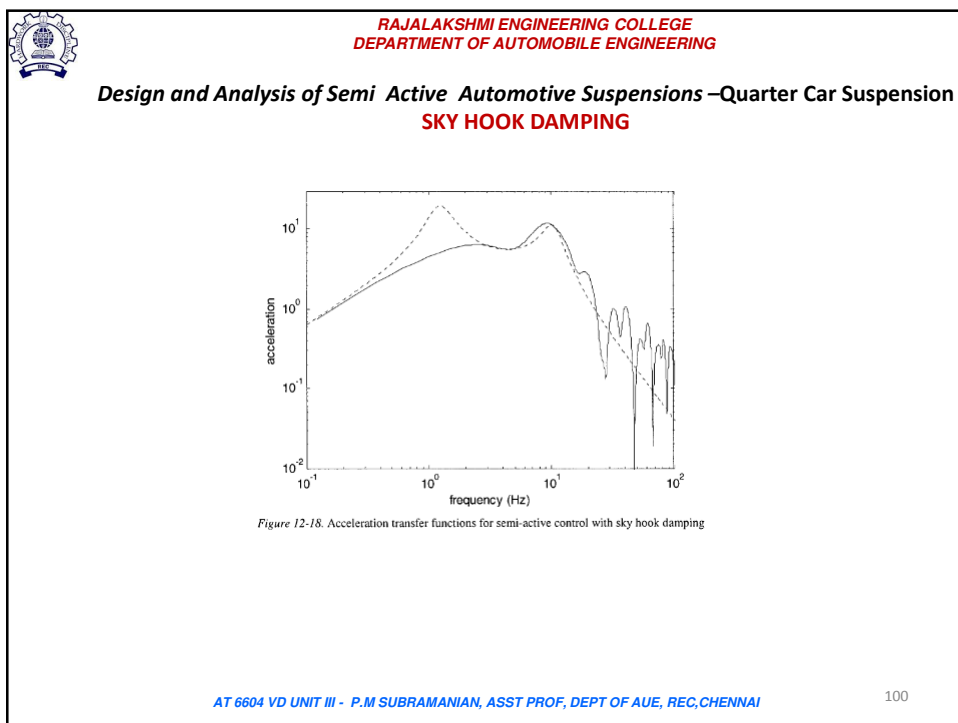
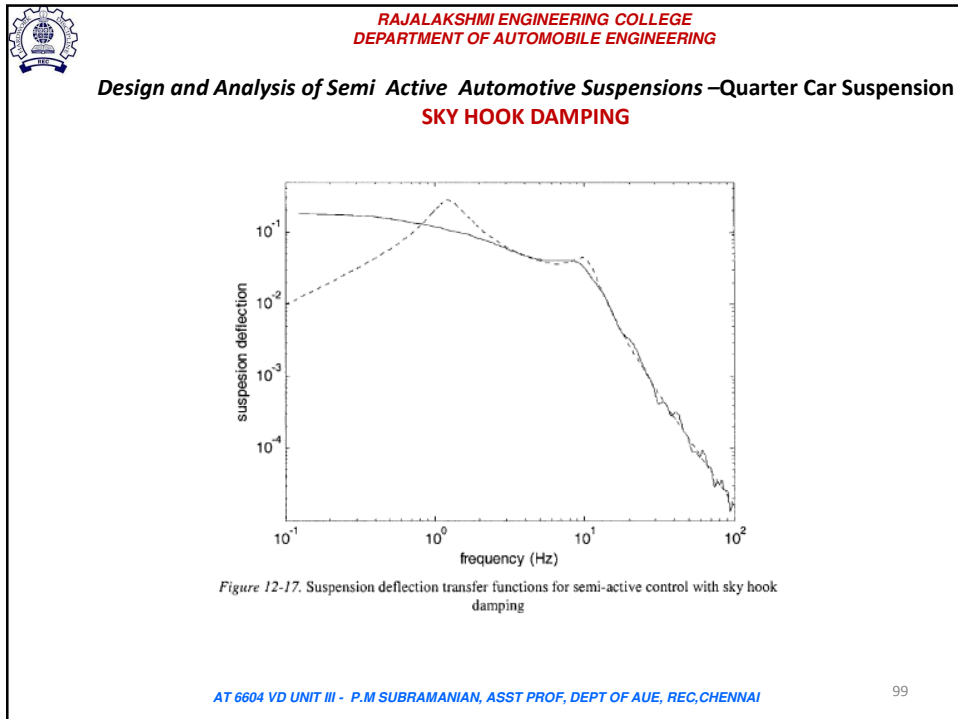


Figure 12-16. Tire deflection transfer functions for semi-active control with sky hook damping

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98





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AIR SUSPENSION

Benefits of Air Suspension Systems



1. By changing the bellows pressure, depending on the load carried on the vehicle, the distance between the road surface and the vehicle's superstructure addresses the same level. This means that the boarding or loading height, and the headlight settings, remain constant.
2. Spring comfort remains almost unchanged across the whole of the loading range; again this is achieved by changing the bellows pressure. The passenger on a motor coach will always perceive the same pleasant type of oscillations. Sensitive loads can thus be carried without being severely damaged. The well-known "jumping" of an unladen or partially laden trailer no longer occurs if an air suspension system is used.
3. The stability of the steering system and the transfer of the braking forces are improved since all wheels always have good adhesion to the road surface.
4. The pressure in the air bellows, depending on the load the vehicle carries, is ideal for use in controlling automatic load-sensitive braking.
5. In the area of control for interchangeable platforms, air suspension systems are an excellent basis for cost-effective loading and unloading of containers.
6. The kneeling effect often required for routine buses can easily be achieved by venting the nearside air bellows.

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101



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AIR SUSPENSION

Purpose

Depending on the levelling valve's control, the air suspension bellows are designed to take up the required pressure in the bellows' volume, depending on the load carried on the vehicle.

Air suspension bellows are used as elastic constructional elements between the axle and the vehicle's superstructure. Since its internal friction is less than that of mechanical suspension systems, the air-sprung vehicle has to have shock-absorbers fitted.

Design types

Today the following variants are mainly used:

Twin Concertina Bellows



Twin concertina bellows show a favourable ratio of height versus spring travel, i. e. this type of bellows permits the lowest installation height.

The beaded heels around the bellows' openings are held by metallic bead rings which are screwed against supporting consoles or plates. This causes part of the bellows' heels to be deformed, thereby achieving a sealing effect.

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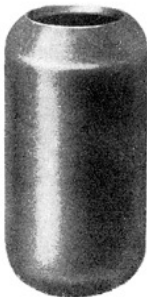
102



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AIR SUSPENSION

Rolling Tube Bellows



Rolling tube bellows achieve an excellent cushioning effect and offer exceptionally good lateral movement. For this reason they are particularly suitable for use in buses and passenger cars but are also used on lorries and trailers.

In the course of their cushioning action, these bellows roll on a cylindrical or similar piston whose shape essentially effects the cushioning characteristic. This allows the natural frequency to be varied and the best possible suspension for the vehicle to be achieved. For this purpose, rolling tube bellows require no additional volume. The air volume in the piston can also be used for cushioning.

These bellows are fairly easy to install and to seal. The bellows' heels are pushed onto conical fittings and assume their intended position when connected to the air line.

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103



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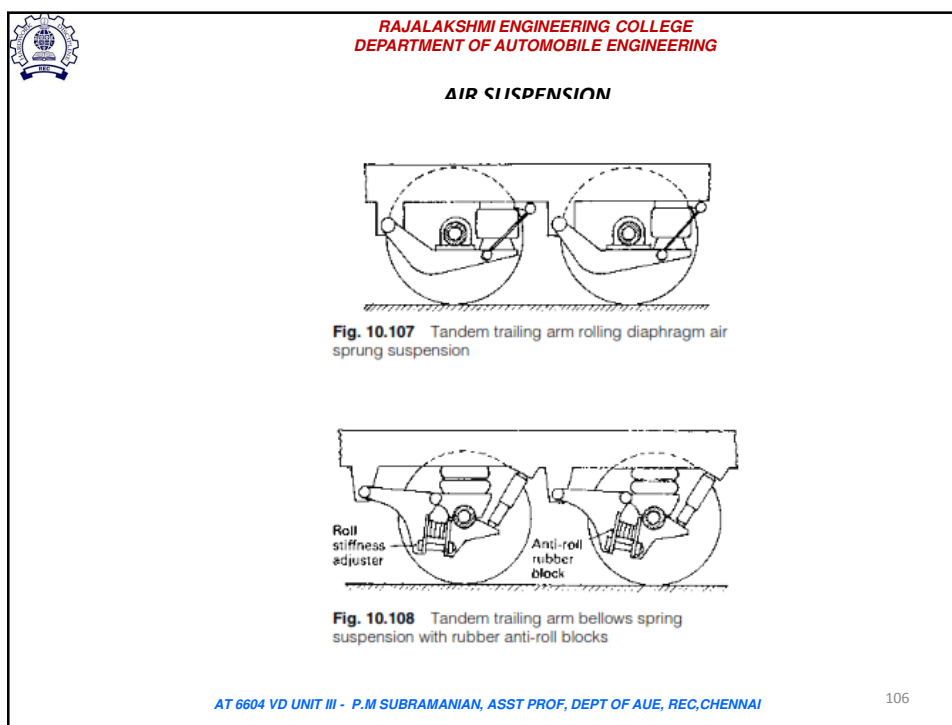
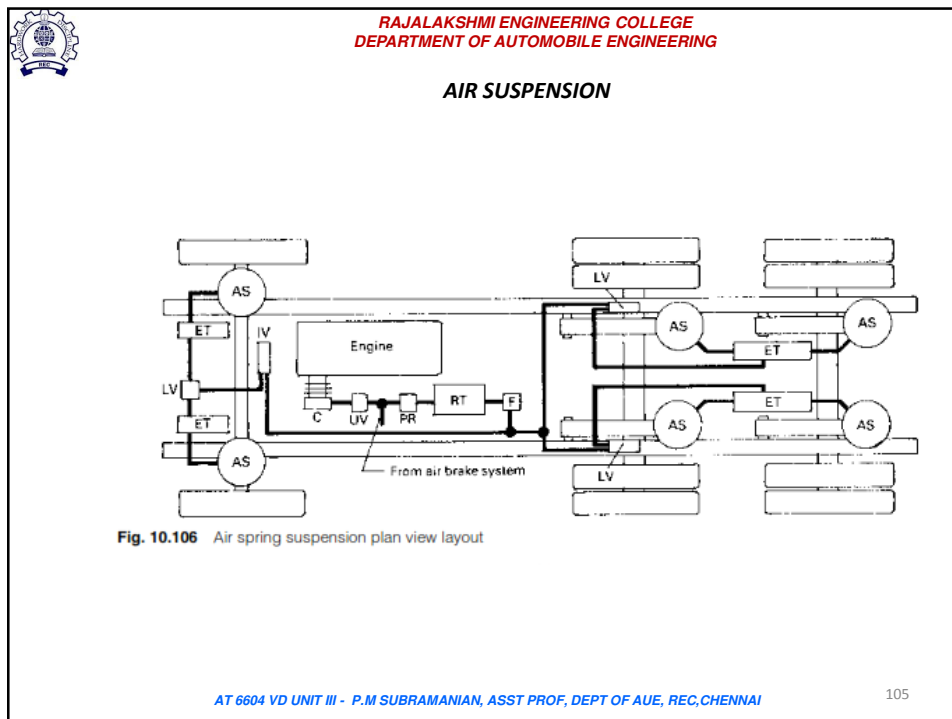
AIR SUSPENSION

10.15 Air suspensions for commercial vehicles

A rigid six wheel truck equipped with pairs of air springs per axle is shown in Fig. 10.106. The front suspension has an air spring mounted between the underside of each chassis side-member and the transverse axle beam, and the rear tandem suspension has the air springs mounted between each trailing arm and the underside of the chassis (Figs 10.107 and 10.108).

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104





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AIR SUSPENSION

Air from the engine compressor passes through both the unloader valve and the pressure regulator valve to the reservoir tank. Air is also delivered to the brake system reservoir (not shown). Once the compressed air has reached some pre-determined upper pressure limit, usually between 8 and 8.25 bar, the unloader valve exhausts any further air delivery from the pump directly to the atmosphere, thereby permitting the compressor to 'run light'. Immediately the air supply to the reservoir has dropped to a lower limit of 7.25 bar, the unloader valve will automatically close its exhaust valve so that air is now transferred straight to the reservoir to replenish the air consumed. Because the level of air pressure demanded by the brakes is greater than that for the suspension system, a pressure regulator valve is incorporated between the unloader valve and suspension reservoir valve, its function being to reduce the delivery pressure for the suspension to approximately 5.5 bar.

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107



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AIR SUSPENSION

Air now flows from the suspension reservoir through a filter and junction towards both the front and rear suspensions by way of a single

central levelling valve at the front (Fig. 10.109) and a pair of levelling valves on each side of the first tandem axle. These levelling valves are bolted to the chassis, but they are actuated by an arm and link rod attached to the axles. It is the levelling valves' function to sense any change in the chassis to axle height and to increase or decrease the air pressure supply passing to the air springs, thereby raising or reducing the chassis height respectively. The air pressure actually reaching the springs may vary from 5.5 bar fully laden down to 2.5 bar when the vehicle is empty.

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108

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AIR SUSPENSION

Fig. 10.109 Air spring suspension front view layout

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109

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AIR SUSPENSION

To improve the quality of ride, extra volume tanks can be installed in conjunction with the air springs to increase the volume of air in the system. This minimizes changes in overall pressure and reduces the spring rate (spring stiffness), thus enabling the air springs to provide their optimum frequency of spring bounce.

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110



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AIR SUSPENSION

An additional feature at the front end of the suspension is an isolating valve which acts both as a junction to split the air delivery to the left and right hand air springs and to permit air to pass immediately to both air springs if there is a demand for more compressed air. This valve also slows down the transfer of air from the outer spring to the inner spring when the body rolls while the vehicle is cornering.