JEE MAINS

LIMITS, CONTINUITY & DERIVATIVES

with DPP + TEST

100 mins ka DAILY DOSE 3.0



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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

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(High weightage)

01	• Average Number of Questions in JEE Main	• 2
02	• Input Vs Output	• High input High output
03	• Prerequisites Knowledge	• Basics of Function, domain & range
04	• Easiest Topics	• Factorisation, Rationalisation, L'H Rule • Continuity at a point
05	• Most Repeated Topics	• 1^{∞} Form, ∞/∞ Form, L'H Rule, Trigonometric Limits • Continuity at a point • Problems involving GIF

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- Key Highlights of Chapter

- Easy to Moderate Level Questions
- Continuity and differentiability requires understanding of limits and its format & formulae.
- G.I.F are important as it is used in good questions.
- Continuity and differentiability problems are mainly based on single concepts(definition based).

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LIMITS, CONTINUITY & DERIVATIVES	2024 (20 Papers)	2023 (24 Papers)	2022 (22 Papers)	2021 (26 Papers)	2020 (16 Papers)	2019 (16 Papers)
	47	31	44	52	31	34

TOPICS	Trigonometric limits	Algebraic Limits	Limits of Special Functions ([ ], { }, | |)	Continuity/discontinuity of function at a point or in interval	Differentiation (MOD)	Differentiability	Misc. Problems
Number of Qs in JEE Main 2024	4	6	3	13	11	2	8
Number of Qs in JEE Main 2023	4	3	2	4	6	4	8
Number of Qs in JEE Main 2022	6	6	2	9	10	9	2
Number of Qs in JEE Main 2021	9	7	1	15	9	6	5

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LIMITS, CONTINUITY & DERIVATIVES

- Limit at a Point
- Concept of Limit (LHL & RHL)
- Properties of Limits
- Indeterminate form
- Methods of Solving Limits
  - Direct Substitution Method
  - Factorisation
  - Rationalisation
  - Trigonometric Limits
  - Standard Limits
  - Logarithmic & Exponential Limits
  - 1^{∞} Form, ∞/∞ Form
  - L'H Rule
- Continuity
- Differentiability
- Method of Differentiation

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LIMIT AT A POINT

Left Hand Limit (LHL)

$\lim_{x \to a^-} f(x)$ = exists & finite

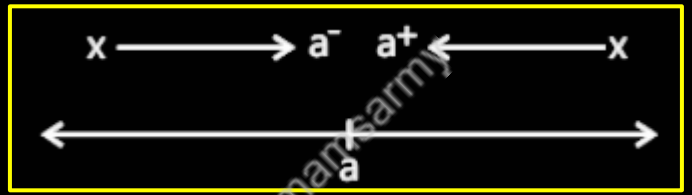
a^- : left of a

Right Hand Limit (RHL)

$\lim_{x \to a^+} f(x)$ = exists & finite

a^+ : right side of a

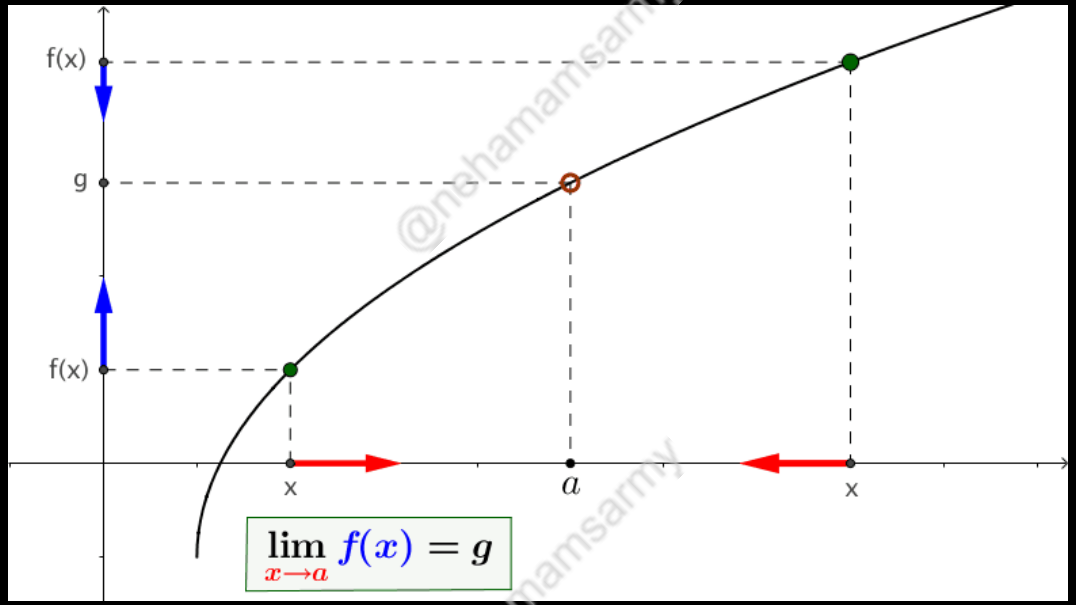
$$\lim_{x \to a} f(x) = l$$



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Limit of a function

The fact that the limit of f(x) exists at x=a means that the graph of f(x) approaches the same value from both sides of x=a.



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USE LHL RHL for:-

→ PIECEWISE Defined (Fn has a split)

→ [ ], { }, | |, $\sqrt{\ }$

$$f(x) = \begin{cases} g(x) & x \le a \\ h(x) & x \ge a \end{cases}$$

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$$f(x)=\begin{cases}\frac{x-x}{x}=0, & x>0\\ \frac{x-(-x)}{x}=2, & x<0\\ 1, & x=0\end{cases}$$

$\lim_{x\to 0^+} f(x)=0$

$\lim_{x\to 0^-} f(x)=2$

$\Rightarrow \lim_{x\to 0} f(x)$ does not exist

Q. Let $f(x)=\begin{cases}\frac{x-|x|}{x}, & x\neq 0 \quad (x>0,\ x<0)\\ 1, & x=0\end{cases}$ then

A. $\lim_{x\to 0^+} f(x)=1$

B. $\lim_{x\to 0^-} f(x)=0$

C. $\lim_{x\to 0^+} f(x)\neq \lim_{x\to 0^-} f(x)$

D. $\lim_{x\to 0^+} f(x)$ does not exist

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Solution:

If $x > 0$, $|x| = x$, so

$$\lim_{x \to 0+} \frac{x - |x|}{x} = \lim_{x \to 0+} \frac{x - x}{x} = 0$$

for $x < 0$, $|x| = -x$, so

$$\lim_{x \to 0-} \frac{x - |x|}{x} = \lim_{x \to 0-} \frac{x + x}{x} = 2$$

Thus $\lim_{x \to 0+} f(x) \neq \lim_{x \to 0-} f(x)$

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Q. Let $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ then

A $\lim_{x \to 0^+} f(x) = 1$

B $\lim_{x \to 0^-} f(x) = 0$

C $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

D $\lim_{x \to 0^+} f(x)$ does not exist

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Q. $\lim_{x \to 2} \frac{|x-2|}{x-2} =$

$\lim_{x \to 2^-} \frac{-(x-2)}{x-2} = -1$

$\lim_{x \to 2^+} \frac{(x-2)}{x-2} = 1$

A. 1

B. -1

C. Does not exist

D. None of these

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Solution:

$$\text{L.H.L.} = \lim_{x \to 2^-} \frac{|x-2|}{x-2} = \lim_{h \to 0} \frac{|2-h-2|}{2-h-2} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$\text{and, R.H.L.} = \lim_{x \to 2^+} \frac{|x-2|}{x-2} = \lim_{h \to 0} \frac{|2+h-2|}{2+h-2} = \lim_{h \to 0} \frac{h}{h} = 1 \qquad \text{.....(ii)}$$

From (i) and (ii) L.H.L. $\neq$ R.H.L. *i.e.* $\lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exist.

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Q. $\lim_{x \to 2} \frac{|x-2|}{x-2} =$

A 1

B -1

C Does not exist

D None of these

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Indeterminate form

$$\frac{0}{0} \qquad \frac{\infty}{\infty} \qquad \infty - \infty \qquad \infty \times 0 \qquad 0^0 \qquad \infty^0 \qquad 1^\infty$$

$\infty + \infty \to \infty$

- **Note:** In the indeterminate forms '0' and '1' are not exact value, both are approaching(or tending) to their corresponding values.

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NOTE:

$\infty + \infty \to \infty$, $\infty - 0 \to \infty$, $\infty \times \infty \to \infty$

$(\infty)^{\infty} \to \infty$, $0^{\infty} \to 0$, $\frac{0}{\infty} \to 0$.

$\frac{1}{\infty} = 0^{+}$, $\frac{1}{0^{+}} \equiv \infty$

$\frac{1}{-\infty} = 0^{-}$, $\frac{1}{0^{-}} \equiv -\infty$

These are not indet.

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ALGEBRA OF LIMITS

Let $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$

$$\lim_{x\to a}(f \pm g)(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}(fg)(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left(\frac{f}{g}\right)(x) = \frac{l}{m}; m \neq 0$$

$$\lim_{x\to a} kf(x) = k\lim_{x\to a} f(x) = kl$$

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Algebra of Limit TABLE

f(x)	g(x)	f(x) ± g(x)	f(x).g(x) and f(x)/g(x)
Exist	Exist	Exist	Exist
Exist	✗ Do Not Exist	Do Not Exist	✓ May Exist
✗ Do Not Exist	✗ Do Not Exist	May Exist	May Exist

$f(x) = x^2 - 4$

$\lim_{x \to 2} f(x)$ exists

$\lim_{x \to 2} g(x)$ ✗

$g(x) = \frac{1}{x^2+x-6}$

$f(x) \cdot g(x) = \frac{x^2-4}{x^2+x-6} = \lim_{x \to 2} f(x) \cdot g(x)$

$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)(x+3)} = 4/7$ ✓

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Ex: $f(x) = x^2 - 4, \quad g(x) = \frac{1}{x^2+x-6}$

$\lim_{x\to 2} f(x) = 2^2 - 4 = 0$ (*exist*)

$\lim_{x\to 2} g(x) = \frac{1}{2^2+2-6} = \frac{1}{0}$ (*do not exist*)

Now $\lim_{x\to 2} \underbrace{f(x)\cdot g(x)}_{h(x)} = \lim_{x\to 2} \frac{x^2-4}{x^2+x-6} = \frac{4}{5}$ (*exist*)

$x \to 2$

$h(x) \to \frac{4}{5}$

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METHODS OF SOLVING LIMIT

indet. form by DS

1. Direct Substitution
2. Factorisation
3. Rationalisation
4. Trigo. limits
5. Log & Exponential limits
6. 1^∞ form
7. $\frac{\infty}{\infty}$ form
8. Expansions
9. L'H Rule ♥

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① DIRECT SUBSTITUTION (DS) DUMDAAR

If by directly putting the value of $x = a$, we get $f(x) =$ finite no. (ℓ)

$\ell \rightarrow$ LIMIT (END of THE STORY —x—)

Q. Evaluate $\lim_{t \to -3} \frac{6 + 4t}{t^2 + 1}$, if it exists.

$$\frac{6+4(-3)}{(-3)^2+1} = \frac{-6}{10}$$

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$$\lim_{t \to -3} \frac{6+4t}{t^2+1} = \frac{-6}{10} = \boxed{-\frac{3}{5}}$$

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② FACTORISATION

- $\lim_{x \to a} \frac{f(x)}{g(x)}$ By DS $x = a$, if we get $\frac{0}{0}, \frac{\infty}{\infty}, \ldots$ NOT FINITE
- Factorise $f(x)$ and $g(x)$
- $(x-a)$ will be a factor, (Cancel IT!)
- CULPRIT
- Now, go back to DS ① ✱

Q. The value of $\lim_{x \to 2} \frac{3^{x/2} - 3}{3^x - 9}$ is

0/0

$$\lim_{x \to 2} \frac{3^{x/2} - 3}{(3^{x/2})^2 - (3)^2} = \frac{3^{x/2} - 3}{(3^{x/2} - 3)(3^{x/2} + 3)}$$

$$= \frac{1}{6}$$

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Solution:

$$\lim_{x\to 2}\frac{3^{x/2}-3}{(3^{x/2})^2-(3)^2}=\lim_{x\to 2}\frac{(3^{x/2}-3)}{(3^{x/2}-3)(3^{x/2}+3)}=\frac{1}{6}$$

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Q. $\lim_{x \to 2} \dfrac{3^x + 3^{3-x} - 12}{3^{-\frac{x}{2}} - 3^{1-x}}$ is equal to

$x \to 2$

$t \to 3^{x/2} = 3^{2/2} = 3^1$

$(3^{x/2})^2$

$3^{x/2} = t$ (let)

$\frac{3^1}{3^x}$

$$\lim_{t \to 3} \frac{t^2 + \frac{27}{t^2} - 12}{\frac{1}{t} - \frac{3}{t^2}}$$

$$\lim_{t \to 3} \frac{t^4 - 12t^2 + 27}{t - 3} = \lim_{t \to 3} \frac{(t-3)(t+3)(t^2-3)}{(t-3)}$$

$= 6(6)$

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$$\text{Let, } 3^{\frac{x}{2}} = t, \; x \to 2 \Rightarrow t \to 3$$

$$\lim_{t\to 3} \frac{t^2 + \frac{27}{t^2} - 12}{\frac{1}{t} - \frac{3}{t^2}} = \lim_{t\to 3} \frac{t^4 + 27 - 12t^2}{t-3}$$

$$\lim_{t\to 3} \frac{(t^2-3)(t+3)(t-3)}{(t-3)} = 6 \times 6 = 36$$

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Q. $\lim_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$ is equal

$(1-x)(1+x+x^2)$

$\lim_{x \to 1} \left(\frac{1+x+x^2-3}{1-x^3} \right)$

$\lim_{x \to 1} \left(\frac{x^2+x-2}{1-x^3} \right)$ $\left(\frac{0}{0}\right)$

$\lim_{x \to 1} \frac{-(x-1)(x+2)}{(1-x)(1+x+x^2)}$

$= \frac{-3}{3} = -1$

A. 1

B. -1

C. 2

D. $-\frac{1}{2}$

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Solution:

$$\text{For } x \neq 1, f(x) = \frac{1}{1-x} - \frac{3}{1-x^3}$$

$$= \frac{1}{(1-x)}\left[1 - \frac{3}{1+x+x^2}\right]$$

$$= \frac{1}{(1-x)} \cdot \frac{x^2+x-2}{(1+x+x^2)} = \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$$

$$= -\frac{x+2}{1+x+x^2}$$

$$\text{so} \quad \lim_{x \to 1} f(x) = -\lim_{x \to 1} \frac{x+2}{1+x+x^2} = -1 .$$

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Q. $\lim_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$ is equal

A. 1

B. -1

C. 2

D. $-\frac{1}{2}$

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③ RATIONALISATION

$\frac{f(x)}{g(x)}$ one of them or both involve $\sqrt{\ }$ ★

We Rationalize & go back to DS ①

Q. $\lim_{x \to 3} \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$ equals $\left(\frac{0}{0}\right)$

$\lim_{x \to 3} \frac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{(x-2)-(4-x)}$ → ②

$2 \lim_{x \to 3} \frac{(x-3)}{2(x-3)} = 1$

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Solution:

$$\lim_{x\to 3}\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}=\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{\left(\sqrt{x-2}\right)^2-\left(\sqrt{4-x}\right)^2}$$

$$=\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(2x-6)}=\lim_{x\to 3}\frac{\sqrt{x-2}+\sqrt{4-x}}{2}=\frac{1+1}{2}=1$$

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$$\lim_{x\to 3} \frac{(\sqrt{3x}-3)}{(\sqrt{2x-4}-\sqrt{2})} \times \frac{(\sqrt{2x-4}+\sqrt{2})}{(\sqrt{2x-4}+\sqrt{2})} \frac{(\sqrt{3x}+3)}{(\sqrt{3x}+3)}$$

$$= \lim_{x\to 3} \frac{3x-9}{2x-4-2} \times \left(\frac{2\sqrt{2}}{6\sqrt{3}}\right)$$

$$\frac{\sqrt{2}}{3} \lim_{x\to 3} \frac{3(x-3)}{2(x-3)}$$

$$= \frac{1}{\sqrt{2}}$$

JEE Main 2017

Q. $\lim\limits_{x\to 3} \dfrac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to

$\dfrac{3-3}{\sqrt{2}-\sqrt{2}} = \dfrac{0}{0}$

A $\dfrac{1}{\sqrt{2}}$

B $\dfrac{1}{2\sqrt{2}}$

C $\dfrac{\sqrt{3}}{2}$

D $\sqrt{3}$

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Given,

$$\lim_{x\to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$$

On rationalization we get,

$$\lim_{x\to 3} \frac{(3x-9)\times(\sqrt{2x-4}+\sqrt{2})}{\{(2x-4)-2\}\times(\sqrt{3x}+3)}$$

$$= \lim_{x\to 3} \frac{3(x-3)}{2(x-3)} \times \frac{\sqrt{2x-4}+\sqrt{2}}{(\sqrt{3x}+3)}$$

$$= \frac{3}{2} \times \frac{2\sqrt{2}}{6} = \frac{1}{\sqrt{2}}.$$

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JEE Main 2017

Q. $\lim\limits_{x \to 3} \dfrac{\sqrt{3x} - 3}{\sqrt{2x - 4} - \sqrt{2}}$ is equal to

A $\dfrac{1}{\sqrt{2}}$

B $\dfrac{1}{2\sqrt{2}}$

C $\dfrac{\sqrt{3}}{2}$

D $\sqrt{3}$

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Q. Find the value of $\lim_{x \to 2} \dfrac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$

$$\lim_{x \to 2} \frac{1+\sqrt{2+x} - 3}{\left(\sqrt{1+\sqrt{2+x}} + \sqrt{3}\right)(x-2)}$$

Harmless!

$2\sqrt{3}$

$$\lim_{x \to 2} \frac{\sqrt{2+x} - 2}{(x-2)}$$

$\frac{x-2}{2-2} = \frac{0}{0}$

$$\lim_{x \to 2} \frac{(2+x)-4}{(x-2)\left(\sqrt{2+x}+2\right)}$$

$\times \frac{1}{2\sqrt{3}} = \frac{1}{8\sqrt{3}}$

4

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$$\because \lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}=\lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}\times\frac{\sqrt{1+\sqrt{2+x}}+\sqrt{3}}{\sqrt{1+\sqrt{2+x}}+\sqrt{3}}$$

$$=\lim_{x\to 2}\frac{\sqrt{2+x}-2}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)}$$

$$=\lim_{x\to 2}\frac{\sqrt{2+x}-2}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)}\times\frac{\sqrt{2+x}+2}{\sqrt{2+x}+2}$$

$$=\lim_{x\to 2}\frac{(x-2)}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{2+x}+2\right)}$$

$$=\lim_{x\to 2}\frac{1}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{2+x}+2\right)}$$

$$=\frac{1}{\left(\sqrt{1+\sqrt{2+2}}+\sqrt{3}\right)\left(\sqrt{2+2}+2\right)}$$

$$=\frac{1}{2\sqrt{3}\times 4}$$

$$=\frac{1}{8\sqrt{3}}$$

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JEE-Main 2019, 12th April -II

Q. $\lim\limits_{x\to 0} \dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is:

$\left(\dfrac{0}{0}\right)$

$\lim\limits_{x\to 0} \dfrac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}{x^2+2\sin x+1-\sin^2 x+x-1}$ $\quad \sqrt{2}$

$2\lim\limits_{x\to 0} \dfrac{x+2\sin x}{x^2+x+2\sin x-\sin^2 x}$ $\left(\dfrac{0}{0}\right)$

L'H

$2\lim\limits_{x\to 0} \dfrac{(1+2\cos x)}{2x+1+2\cos x-2\sin x\cos x}$

$2\left(\dfrac{3}{1+2-0}\right) = 2$

A. 2

B. 6

C. 3

D. 1

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Solution:

$$\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$$

$$=\lim_{x\to 0}\frac{x+2\sin x}{x^2+2\sin x+1-\sin^2 x+x-1}$$

$$\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)$$

$$=\lim_{x\to 0}\frac{x+2\sin x}{x^2+2\sin x-\sin^2 x+x}\cdot(2)$$

Applying $L'H$ Rule

$$=\lim_{x\to 0}\frac{2\cdot(1+2\cos x)}{2x+2\cos x-2\sin x\cos x+1}$$

$$=\frac{2(3)}{2+1}=2$$

Hence the correct answer is option (A)

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JEE-Main 2019, 12th April -II

Q. $\lim\limits_{x\to 0}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is:

A 2

B 6

C 3

D 1

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Trigonometric limits :

$$\frac{sinx}{x} \ll 1 \; and \; \frac{x}{sinx} \gg 1$$

$$\frac{tanx}{x} \gg 1 \; and \; \frac{x}{tanx} \ll 1$$

(i) $\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} \frac{x}{\sin x}$

(ii) $\lim_{x \to 0} \frac{\tan x}{x} = 1 = \lim_{x \to 0} \frac{x}{\tan x}$

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Trigonometric limits :

(i) $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1 = \lim\limits_{x\to 0}\frac{x}{\sin x}$

(ii) $\lim\limits_{x\to 0}\frac{\tan x}{x} = 1 = \lim\limits_{x\to 0}\frac{x}{\tan x}$

(iii) $\lim\limits_{x\to 0}\frac{\sin^{-1} x}{x} = 1 = \lim\limits_{x\to 0}\frac{x}{\sin^{-1} x}$

(iv) $\lim\limits_{x\to 0}\frac{\tan^{-1} x}{x} = 1 = \lim\limits_{x\to 0}\frac{x}{\tan^{-1} x}$

(v) $\lim\limits_{x\to 0}\cos x = 1$

(vi) $\lim\limits_{x\to a}\sin^{-1} x = \sin^{-1} a$, where $|a| \le 1$

(vii) $\lim\limits_{x\to a}\cos^{-1} x = \cos^{-1} a$, where $|a| \le 1$

(viii) $\lim\limits_{x\to a}\tan^{-1} x = \tan^{-1} a$

(ix) $\lim\limits_{x\to\infty}\frac{\sin x}{x} = \lim\limits_{x\to\infty}\frac{\cos x}{x} = 0$

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Trigonometric limits :

$$\lim_{x\to 0}\frac{\sin x}{2x}=\frac{1}{2}$$

$$\star\ \lim_{x\to 0}\frac{1-\cos(x)}{x^2}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{2\sin^2(x/2)}{x^2}=2\left(\frac{\sin x/2}{2\,x/2}\right)^2$$

$$\frac{2}{4}=\frac{1}{2}$$

$$as\ x\to a\Rightarrow f(x)\to 0$$

$$\lim_{x\to a}\frac{\sin(f(x))}{f(x)}=1$$

$f(x)\to 0$

$$\star\ \lim_{x\to a}\frac{\tan(f(x))}{f(x)}=1$$

$$\lim_{x\to a}\frac{1-\cos(f(x))}{(f(x))^2}=\frac{1}{2}$$

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JEE Main 2019

Q. $\lim\limits_{x\to 0} \dfrac{x\cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :

$\lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1$

$\lim\limits_{x\to 0} \dfrac{x}{\tan x} = 1$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

$\lim\limits_{x\to 0} \dfrac{4x \cdot \tan^2 2x / x^2}{\tan 4x \cdot \dfrac{\sin^2 x}{x^2}} \times \dfrac{1}{4}$

$\left(\dfrac{\tan 2x}{2x}\right)^2 \times 4 \times \dfrac{1}{4}$

$= 1$

A. 0

B. 2

C. 4

D. 1

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Solution:

$$\lim_{x\to 0} \frac{x \cot 4x}{\sin^2 x \cdot \cot^2 2x} = \lim_{x\to 0} \frac{x \cdot \tan^2 2x}{\sin^2 x \cdot \tan 4x}$$

$$= \lim_{x\to 0} \left(\frac{x}{\sin x}\right)^2 \cdot \left(\frac{\tan 2x}{2x}\right)^2 \cdot \left(\frac{4x}{\tan 4x}\right) \cdot \frac{4}{2^2} = 1$$

@nehamamsarmy

JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

NEHA MA'AM'S ARMY

JEE Main 2019

Q. $\lim\limits_{x \to 0} \dfrac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :

A 0

B 2

C 4

D 1

@nehamamsarmy

NEHA MA'AM'S ARMY

Q. $\lim_{x \to 0} \dfrac{(1 - cos\, 2x) \sin 5x}{x^2 \sin 3x} =$

$$\lim_{x \to 0} \frac{2\sin^2 x}{x^2} \cdot \frac{\sin 5x}{5x} \times 5x \Big/ \frac{\sin 3x}{3x} \times 3x$$

$$= 2 \times \frac{5}{3}$$

A. $\frac{10}{3}$

B. $\frac{3}{10}$

C. $\frac{6}{5}$

D. $\frac{5}{6}$

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NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 0}\frac{2\sin^2 x\ \sin 5x\ \ 3x\ \ 5x}{x^2\qquad \sin 3x\ \ 3x\ \ 5x} = \lim_{x\to 0}\frac{2\sin^2 x}{x^2}\cdot\frac{3x}{\sin 3x}\cdot\frac{\sin 5x}{5x}\cdot\frac{5x}{3x} = 2.\frac{5}{3} = \frac{10}{3}$$

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NEHA MA'AM'S ARMY

Q. $\lim_{x \to 0} \frac{(1 - cos\, 2x) \sin 5x}{x^2 \sin 3x} =$

A $\frac{10}{3}$

B $\frac{3}{10}$

C $\frac{6}{5}$

D $\frac{5}{6}$

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NEHA MA'AM'S ARMY

JEE Main 2013

Q. The value of $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

$\to 4$

$= 4 \lim_{x \to 0} \frac{2 \sin^2 x}{x \tan 4x \times 4x}$

$= 2 \lim_{x \to 0} 1$

$= 2$

#sabtasawal.

A 1

B 2

C $-\frac{1}{4}$

D $\frac{1}{2}$

@nehamamsarmy

NEHA MA'AM'S ARMY

Solution:

$$\text{Given } \lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 x(3+\cos x)}{x \tan 4x}$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \left(\frac{4x}{\tan 4x}\right)(3+\cos x)\times\frac{1}{2}$$

$$= 2$$

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NEHA MA'AM'S ARMY

JEE Main 2013

Q. The value of $\lim\limits_{x \to 0} \dfrac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

A 1

B 2

C $-\dfrac{1}{4}$

D $\dfrac{1}{2}$

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NEHA MA'AM'S ARMY

राष्ट्रीय परीक्षा एजेंसी
National Testing Agency
Excellence in Assessment

Q. The value of $\lim\limits_{x\to 0} \dfrac{1-\cos^3(\sin x)}{\sin x \sin(\sin x)\cos(\sin x)}$ is equal to

$\sin x = t$

$\lim\limits_{t\to 0} \dfrac{1-\cos^3 t}{t \sin t \cos t} \to 1 \to 3$

$\lim\limits_{t\to 0} \dfrac{(1-\cos t)(1+\cos t+\cos^2 t)}{t^2 \dfrac{\sin t}{t}} \to 1$

$\dfrac{1}{2}$

A. $\frac{3}{2}$

B. 1

C. 0

D. 2

NEHA MA'AM'S ARMY

Solution:

$$\text{Let, } \sin x = t$$

$$\Rightarrow \lim_{t \to 0} \frac{1-\cos^3 t}{t \sin t \cos t}$$

$$\Rightarrow \lim_{t \to 0} \frac{(1-\cos t)}{t^2} \times \left(\frac{t}{\sin t}\right) \times \frac{(1+\cos t+\cos^2 t)}{\cos t}$$

$$\Rightarrow \frac{1}{2} \times 1 \times \frac{(1+1+1)}{1} = \frac{3}{2}$$

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NEHA MA'AM'S ARMY

राष्ट्रीय परीक्षा एजेंसी
National Testing Agency
Excellence in Assessment

Q. The value of $\lim\limits_{x \to 0} \dfrac{1 - \cos^3(\sin x)}{\sin x \, \sin(\sin x) \cos(\sin x)}$ is equal to

A $\frac{3}{2}$

B 1

C 0

D 2

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NEHA MA'AM'S ARMY

NTA PAPER 124

Q. If $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} = \lambda$, then $8\lambda =$

TRY

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NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$$

$$=\lim_{x\to 0}\frac{x\left(\frac{2\tan x}{1-\tan^2 x}\right)-2x\tan x}{(2\sin^2 x)^2}$$

$$=\lim_{x\to 0}\frac{2x\tan^3 x}{4\sin^4 x(1-\tan^2 x)}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{x^4}{\sin^4 x}\times\frac{\tan^3 x}{x^3}\times\frac{1}{(1-\tan^2 x)}\right)$$

$$=\frac{1}{2}=0.5$$

$$8\lambda=4$$

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NEHA MA'AM'S ARMY

JEE Main 2021

Q. The value of $\lim_{\theta \to 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$ is equal to:

$$\lim_{\theta \to 0} \frac{\tan(\pi(1-\sin^2\theta))}{\sin(2\pi\sin^2\theta)}$$

$$= \lim_{\theta \to 0} \frac{\tan(\pi\sin^2\theta)}{\pi\sin^2\theta} \cdot (\sin^2\theta)\pi \times \frac{1}{\sin(2\pi\sin^2\theta)} \times 2\pi\sin^2\theta$$

$$= 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

A. $-\frac{1}{2}$

B. $-\frac{1}{4}$

C. 0

D. $\frac{1}{4}$

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NEHA MA'AM'S ARMY

Solution:

We have to find $L = \lim_{\theta \to 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$

Using $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow L = \lim_{\theta \to 0} \frac{\tan(\pi(1-\sin^2 \theta))}{\sin(2\pi \sin^2 \theta)}$$

$$\Rightarrow L = \lim_{\theta \to 0} \frac{\tan(\pi - \pi \sin^2 \theta)}{\sin(2\pi \sin^2 \theta)}$$

Using $\tan(\pi - \theta) = -\tan\theta$, we get

$$\Rightarrow L = \lim_{\theta \to 0} \frac{-\tan(\pi \sin^2 \theta)}{\sin(2\pi \sin^2 \theta)}$$

$$\Rightarrow L = \lim_{\theta \to 0} -\left(\frac{\tan(\pi \sin^2 \theta)}{\pi \sin^2 \theta}\right)\left(\frac{2\pi \sin^2 \theta}{\sin(2\pi \sin^2 \theta)}\right) \times \frac{1}{2}$$

Using, $\lim_{x \to 0}\left(\frac{\sin x}{x}\right) = 1$ & $\lim_{x \to 0}\left(\frac{\tan x}{x}\right) = 1$, we get

$L = -\frac{1}{2}$.

NEHA MA'AM'S ARMY

JEE Main 2021

Q. The value the limit $\lim_{\theta \to 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$ is equal to:

A -½

B -¼

C 0

D ¼

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NEHA MA'AM'S ARMY

EXPONENTIAL and LOGARITHMIC LIMITS

① $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a = \ln a$

② $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

③ $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

④ $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$

⑤ $\lim_{x \to a} f(x)^{g(x)}$ (1^∞ form)

** Tree Monkey TRICK

$= e^{\lim_{x \to a} (f(x)-1)g(x)}$

NEHA MA'AM'S ARMY

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$$

Q. $\lim\limits_{x\to 0}\dfrac{4^x-9^x}{x\left(4^x+9^x\right)}=$

$\frac{0}{0}$

$$\lim_{x\to 0}\frac{(4^x-1)-(9^x-1)}{x(2)}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{4^x-1}{x}-\frac{9^x-1}{x}\right)$$

$$=\frac{1}{2}(\ln 4-\ln 9)$$

$$=\frac{1}{2}\ln\left(\frac{4}{9}\right)=\ln\left(\frac{4}{9}\right)^{1/2}$$

A. $\log\left(\frac{3}{2}\right)$

B. $\log\left(\frac{2}{3}\right)$

C. $\log\left(\frac{4}{3}\right)$

D. $\log 2$

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Solution:

$$\lim_{x\to 0}\frac{\left(4^x-1\right)-\left(9^x-1\right)}{x}\times\frac{1}{\left(4^x+9^x\right)}$$

$$=\frac{\lim_{x\to 0}\left(\frac{4^x-1}{x}\right)-\lim_{x\to 0}\left(\frac{9^x-1}{x}\right)}{\lim_{x\to 0}(4^x+9^x)}$$

$$=\frac{\log 4-\log 9}{2}=\frac{1}{2}\log\left(\frac{4}{9}\right)=\log\left(\frac{2}{3}\right)$$

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Q. $\lim\limits_{x \to 0} \dfrac{4^x - 9^x}{x\left(4^x + 9^x\right)} =$

A $\log\left(\dfrac{3}{2}\right)$

B $\log\left(\dfrac{2}{3}\right)$

C $\log\left(\dfrac{4}{3}\right)$

D $\log 2$

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NEHA MA'AM'S ARMY

JEE Main 2019

Q. If $\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$

then k is

$\lim_{x\to a}\frac{x^n-a^n}{x-a} = n\cdot a^{n-1}$

$\lim_{x\to 1}\frac{x^4-1^4}{x-1} = 4(1)^{4-1} = 4 = \lim_{x\to k}\frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}} = \frac{3k^2}{2k} = \frac{3k}{2}$

$4 = \frac{3k}{2}$

$\Rightarrow k = \frac{8}{3}$

A 3/2

B 4/3

C 3/8

D 8/3

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$$\Rightarrow 4(1)^{4-1} = \lim_{x \to k} \frac{(x-k)(x^2+k^2+kx)}{(x-k)(x+k)}$$

$$\Rightarrow 4 = \lim_{x \to k} \frac{(x^2+k^2+kx)}{(x+k)}$$

$$\Rightarrow 4 = \frac{k^2+k^2+k^2}{2k}$$

$$\Rightarrow 4 = \frac{3k^2}{2k}$$

$$\Rightarrow 4 = \frac{3}{2}k$$

$$\Rightarrow k = \frac{8}{3}$$

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JEE Main 2019

Q. If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$

then k is

A 3/2

B 4/3

C 3/8

D 8/3

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$\lim_{x \to 0} \frac{e^x - 1}{x}$

$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$= \lim_{\substack{x \to 0 \\ x^2 \to 0}} \left(\frac{e^{x^2} - 1}{x^2} \right) + \lim_{x \to 0} \left(\frac{1 - \cos x}{x^2} \right)$

$1 \qquad 1/2$

Q. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$ is equal to

$\left(\frac{0}{0}\right)$

A 1/2

B 3/2

C 2/3

D 2

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Solution:

$$\lim_{x\to 0}\frac{e^{x^2}-1+1-\cos x}{x^2}=\lim_{x\to 0}\left(\frac{e^{x^2}-1}{x^2}+\frac{2\sin^2 x/2}{4(x/2)^2}\right)$$

$$=1+\frac{1}{2}=\frac{3}{2}$$

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NEHA MA'AM'S ARMY

Q. $\lim_{x \to 0} \dfrac{e^{x^2} - \cos x}{x^2}$ is equal to

A 1/2

B 3/2

C 2/3

D 2

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NEHA MA'AM'S ARMY

JEE(Main): 8th April 2023 -I

Q. $\lim\limits_{x\to 0}\left(\left(\frac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to

$\hookrightarrow 1$

$(1+\cos 3x)(1-\cos 3x)\sin^3 4x$ — (2)

$\lim\limits_{x\to 0}\frac{(\ln(2x+1))^5}{\left(\frac{1-\cos 3x}{(3x)^2}\right)\left(\frac{\sin^3 4x}{(4x)^3}\right)(3x)^2(4x)^3\left(\frac{\ln(1+2x)}{2x}\right)^5(2x)^5}$ — ($\frac{1}{2}$, 1, 1)

$= \frac{3^2\times 4^3}{2^5}\times\frac{1}{2} = \frac{3^2\times 2^6}{2^5} = 18$

$\lim\limits_{x\to 0}\frac{\sin x}{x}=1$

$\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

$\lim\limits_{x\to 0}\frac{\ln(1+2x)}{x}=1$

A 24

B 9

C 18

D 15

NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 0}\left(\left(\frac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{\log_e((2x+1))^5}\right)\right)$$

$(1-\cos 3x)(1+\cos 3x)$

$$\lim_{x\to 0}\left[\frac{1-\cos^2 3x}{9x^2}\right]\frac{9x^2}{\cos^3 4x}\cdot\frac{\left(\frac{\sin 4x}{4x}\right)^3\times 64x^3}{\left[\frac{\ln(1+2x)}{2x}\right]^5\times 32x^5}$$

$$\lim_{x\to 0} 2\left(\frac{1}{2}\times\frac{9}{1}\times\frac{1\times 64}{1\times 32}\right)=18$$

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NEHA MA'AM'S ARMY

JEE(Main): 8th April 2023 -I

Q. $\lim\limits_{x \to 0} \left(\left(\dfrac{1 - \cos^2(3x)}{\cos^3(4x)} \right) \left(\dfrac{\sin^3(4x)}{(\log_e(2x+1))^5} \right) \right)$ is equal to

A 24

B 9

C 18

D 15

@nehamamsarmy

NEHA MA'AM'S ARMY

Q. If $\lim_{x \to 0} (1 + a\sin x)^{\operatorname{cosec} x} = 3$, then a is

1^∞

$e^{\lim_{x \to 0} \operatorname{cosec} x (a \sin x)}$

$e^a = 3$

$a = \ln 3$

A ln2

B ln3

C ln4

D e^3

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NEHA MA'AM'S ARMY

Solution:

Given $3 = \lim_{x\to 0}(1 + a\sin\ x)^{\text{cosec}\ x}[1^{\infty}$ form $]$

Put $\sin\ x = y$ $\therefore$ when $x \to 0, y \to 0$

$\therefore \text{Lim}_{x\to 0}\ (1 + a\sin\ x)^{\text{cosec}\ x} = \text{Lim}_{y\to 0}\ (1 + ay)^{1/y} = e^{a}$

$\therefore e^{a} = 3 \Rightarrow a = \log_e 3 = \ln 3.$

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NEHA MA'AM'S ARMY

Q. If $\lim_{x \to 0} (1 + a\sin x)^{\text{cosec } x} = 3$, then a is

A. ln2

B. ln3

C. ln4

D. e^3

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NEHA MA'AM'S ARMY

JEE Main 2020: 6 Sep - I

Q. $\lim\limits_{x\to 0}\left(\dfrac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

1^∞

$e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$

$e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{-4x^2}{7x^2+2}\right)}$

$= e^{\frac{-4}{2}} = e^{-2}$

A. $\frac{1}{e}$

B. $\frac{1}{e^2}$

C. e^2

D. e

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NEHA MA'AM'S ARMY

Solution:

$$\text{Let } L = \lim_{x \to 0} \left(\frac{3x^2+2}{7x^2+2} \right)^{\frac{1}{x^2}}$$

$$= e^{\lim_{x \to 0} \frac{1}{x^2} \left\{ \frac{3x^2+2}{7x^2+2} - 1 \right\}}$$

$$= e^{\lim_{x \to 0} \left\{ \frac{-4x^2}{7x^2+2} \right\}}$$

$$= e^{\frac{-4}{2}}$$

$$= e^{-2}$$

$$= \frac{1}{e^2}$$

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JEE Main 2020: 6 Sep - I

Q.
is equal to $\lim_{x \to 0} \left(\frac{3x^2 + 2}{7x^2 + 2} \right)^{\frac{1}{x^2}}$

A $\frac{1}{e}$

B $\frac{1}{e^2}$

C e^2

D e

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NEHA MA'AM'S ARMY

TRICK

IMPORTANT $\frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m}{b_0 + b_1 x + b_2 x^2 + \cdots + b_n x^n}$$

Most Asked

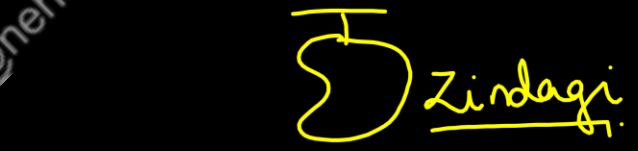
As $x \to \infty$

$\frac{1}{x} \to 0$

- $m = n$: $\frac{a_m}{b_n}$
- $m < n$: 0
- $m > n$: $\to \infty$ ($a_m b_n > 0$)
- $m > n$: $\to -\infty$ ($a_m b_n < 0$)

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NEHA MA'AM'S ARMY



$$\lim_{n\to\infty} \frac{(4n^3+n+4n^2)}{n^3+5n^2+5n-2}$$

$\frac{1}{n}\to 0$

$$\lim_{\frac{1}{n}\to 0} \frac{n^3\left(4+\frac{1}{n^2}+\frac{4}{n}\right)}{n^3\left(1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}\right)} = 4$$

Q. $\lim_{n\to\infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)} =$

N → deg 3

D → deg 3

$\frac{4}{1}$ Trick!!

A 0

B 2

C 4

D ∞

NEHA MA'AM'S ARMY

$$\lim_{n\to\infty} \frac{n\,(2n+1)^2}{(n+2)\,(n^2+3n-1)} = \lim_{n\to\infty} \frac{4n^3+4n^2+n}{n^3+5n^2+5n-2}$$

$$= \lim_{n\to\infty} \frac{n^3\left(4+\frac{4}{n}+\frac{1}{n^2}\right)}{n^3\left(1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}\right)} = 4$$

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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

NEHA MA'AM'S ARMY

Q. $\lim_{n\to\infty} \dfrac{n(2n+1)^2}{(n+2)(n^2+3n-1)} =$

A. 0

B. 2

C. 4

D. ∞

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NEHA MA'AM'S ARMY

Q. $\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right]$ is equal to

$\lim_{n\to\infty}\left(\frac{\sum_{k=1}^{n} k}{1-n^2}\right)$

$\lim_{n\to\infty}\frac{n(n+1)}{2(1-n^2)} = \frac{1}{-2}$

A) 0

B) $-\frac{1}{2}$

C) $\frac{1}{2}$

D) 1

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NEHA MA'AM'S ARMY

Solution:

$$\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots\ldots+\frac{n}{1-n^2}\right]=\lim_{n\to\infty}\frac{\sum n}{1-n^2}=\frac{1}{2}\lim_{n\to\infty}\frac{n^2+n}{1-n^2}=-\frac{1}{2}$$

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NEHA MA'AM'S ARMY

Q. $\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right]$ is equal to

A 0

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 1

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NEHA MA'AM'S ARMY



$$\frac{\sum_{k=1}^{100} x^{10}\left(1+\frac{k}{x^{10}}\right)^{10}}{x^{10}\left(1+\left(\frac{10}{x}\right)^{10}\right)}$$

$$\frac{\sum_{k=1}^{100} 1}{1} = 100$$

Q. $\lim_{x\to\infty} \dfrac{(x+1)^{10}+(x+2)^{10}+\cdots+(x+100)^{10}}{x^{10}+10^{10}}$

$\frac{1}{x}\to 0$

is equal to

$\frac{\infty}{\infty}$ Trick

$$\frac{x^{10}(1+1+\cdots+1)}{1\cdot x^{10}}$$ (100 terms)

$= 100$

A. 100

B. 0

C. 1

D. 10

NEHA MA'AM'S ARMY

Solution:

$$f(x) = \frac{(x+1)^{10} + (x+2)^{10} + \cdots + (x+100)^{10}}{x^{10} + 10^{10}}$$

$$= \frac{\left(1+\frac{1}{x}\right)^{10} + \left(1+\frac{2}{x}\right)^{10} + \cdots + \left(1+\frac{100}{x}\right)^{10}}{1 + \left(\frac{10}{x}\right)^{10}}$$

$$\text{As } \frac{1}{x} \to 0, \text{for } x \to \infty \text{ so } \lim_{x\to\infty} f(x) = \frac{1+1\cdots+1-(100 \text{ times})}{1}$$

$$= 100.$$

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NEHA MA'AM'S ARMY

Q. $\lim_{x\to\infty} \dfrac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$

is equal to

A 100

B 0

C 1

D 10

@nehamamsarmy

JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

NEHA MA'AM'S ARMY

Rationalise

$$\lim_{x\to\infty}\frac{\left(x^2-x+1-a^2x^2\right)}{\sqrt{x^2-x+1}+ax}=b$$

$$\lim_{x\to\infty}\frac{\left((1-a^2)x^2-x+1\right)}{\sqrt{x^2-x+1}+ax}$$

$(1-a^2)\to 0$

$\frac{\infty}{\infty}$

$a^2-1=0 \Rightarrow a=\pm 1$

$a\neq -1$

$\boxed{a=1}$

JEE MAIN 2021

Q. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1},-ax\right)=b$ $(\infty-\infty)$

then the ordered pair(a,b) is :

$a=-1$ $\quad \infty+\infty\to\infty$

$$\lim_{x\to\infty}\frac{-x+1}{\sqrt{x^2-x+1}+x}=b$$

Taking $b=\frac{-1}{2}$

A $\left(1,\frac{1}{2}\right)$

B $\left(-1,\frac{1}{2}\right)$

C $\left(-1,-\frac{1}{2}\right)$

D $\left(1,-\frac{1}{2}\right)$

NEHA MA'AM'S ARMY

Solution:

$\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \quad (\infty-\infty)$ form

$(\Rightarrow a>0\ )$

Now, $\lim_{x\to\infty}\dfrac{\left(x^2-x+1-a^2x^2\right)}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\dfrac{\left(1-a^2\right)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\dfrac{\left(1-a^2\right)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow 1-a^2=0\Rightarrow a=1$

Now, $\lim_{x\to\infty}\dfrac{-x+1}{\times\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow \dfrac{-1}{1+a}=b\Rightarrow b=-\dfrac{1}{2}$

$(a,b)=\left(1,-\dfrac{1}{2}\right)$

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NEHA MA'AM'S ARMY

JEE MAIN 2021

Q. If $\lim_{x \to \infty} \left(\sqrt{x^2 - x + 1}, - ax \right) = b$

then the ordered pair(a,b) is :

A $\left(1, \frac{1}{2}\right)$

B $\left(-1, \frac{1}{2}\right)$

C $\left(-1, -\frac{1}{2}\right)$

D $\left(1, -\frac{1}{2}\right)$

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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

JEE(Main): 25th Jan 2023 -I

Q. The value of $Lim_{n\to\infty} \frac{(1+2-3)+(4+5-6)+\ldots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is

$N^r = (3n-2)+(3n-1)-3n$

$= 3(n-1)$

$3 \lim_{n\to\infty} \frac{\sum_{K=1}^{n}(K-1)}{\sqrt{\quad}-\sqrt{\quad}}$

$3 \lim_{n\to\infty} \frac{\left(\frac{n(n+1)}{2}-n\right)}{\sqrt{\quad}-\sqrt{\quad}}$

$\frac{\infty}{\infty}$ TRICK

N&D are of deg 2

$3\left(\frac{\frac{1}{2}}{\sqrt{2}-1}\right)$

$= \frac{3}{2(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

A. $\frac{\sqrt{2}+1}{2}$

B. $3(\sqrt{2}+1)$

C. $\frac{3}{2}(\sqrt{2}+1)$

D. $\frac{3}{2\sqrt{2}}$

NEHA MA'AM'S ARMY

Solution:

$$\lim_{n\to\infty}\frac{(1+2+4+5+\ldots+(3n-2)+(3n-1)-3+6+\ldots+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\text{Let } N^r=\sum_{n=1}^{\infty}(3n-2)+(3n-1)-3n$$

$$=\sum_{n=1}^{\infty}(3n-3)$$

$$=\frac{3n(n+1)}{2}-3n=\frac{3}{2}(n^2-n)$$

$$\frac{3}{2}\lim_{n\to\infty}\frac{n^2\left(1-\frac{1}{n}\right)}{n^2\left(\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right)}$$

$$=\frac{3}{2(\sqrt{2}-1)}=\frac{3}{2}(\sqrt{2}+1)$$

NEHA MA'AM'S ARMY

JEE(Main): 25th Jan 2023 -I

Q. The value of

$$Lim_{n\to\infty}\frac{1+2-3+4+5-6+\ldots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$ is

A $\frac{\sqrt{2}+1}{2}$

B $3(\sqrt{2}+1)$

C $\frac{3}{2}(\sqrt{2}+1)$

D $\frac{3}{2\sqrt{2}}$

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NEHA MA'AM'S ARMY

VIDEO SOL.

JEE(Main): 6 April 2024 -II

Q. $\lim_{n\to\infty} \dfrac{(1^2-1)(n-1)+(2^2-2)(n-2)+\ldots+\left((n-1)^2-(n-1)\right)\cdot 1}{\left(1^3+2^3+\ldots+n^3\right)-\left(1^2+2^2+\ldots+n^2\right)}$

is equal to :

A $\frac{2}{3}$

B $\frac{1}{3}$

C $\frac{3}{4}$

D $\frac{1}{2}$

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Solution:

$$\lim_{n\to\infty}\frac{\sum_{r=1}^{n-1}(r^2-r)(n-r)}{\sum_{r=1}^{n}r^3-\sum_{r=1}^{n}r^2}$$

$$\lim_{n\to\infty}\frac{\sum_{r=1}^{n-1}\left(-r^3+r^2(n+1)-nr\right)}{\left(\frac{n(n+1)}{2}\right)^2-\frac{n(n+1)(2n+1)}{6}}$$

$$\lim_{n\to\infty}\frac{\left(\frac{((n-1)n)}{2}\right)^2+\frac{(n+1)(n-1)n(2n-1)}{6}-\frac{n^2(n-1)}{2}}{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}-\frac{2n+1}{3}\right)}$$

$$\lim_{n\to\infty}\frac{\frac{n(n-1)}{2}\left(\frac{-n(n-1)}{2}+\frac{(n+1)(2n-1)}{3}-n\right)}{\frac{n(n+1)}{2}\frac{3n^2+3n-4n-2}{6}}$$

$$\lim_{n\to\infty}\frac{(n-1)\left(-3n^2+3n+2\left(2n^2+n-1\right)-6\right)}{(n+1)(3n^2-n-2)}$$

$$\lim_{n\to\infty}\frac{(n-1)(n^2+5n-8)}{(n+1)(3n^2-n-2)}=\frac{1}{3}$$

NEHA MA'AM'S ARMY

JEE(Main): 6 April 2024 -II

Q. $\lim_{n\to\infty} \dfrac{(1^2-1)(n-1)+(2^2-2)(n-2)+\ldots+\left((n-1)^2-(n-1)\right)\cdot 1}{\left(1^3+2^3+\ldots+n^3\right)-\left(1^2+2^2+\ldots+n^2\right)}$

is equal to :

A $\frac{2}{3}$

B $\frac{1}{3}$

C $\frac{3}{4}$

D $\frac{1}{2}$

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BEST FRIEND

L'HOSPITAL RULE

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}, \frac{\infty}{\infty}$$

Both $f(x)$ & $g(x)$ are cts. & diff. at $x = a$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Continue the process till you get a finite ans.

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NEHA MA'AM'S ARMY

$$\lim_{x \to 1} \frac{1 + \log x - x}{1 - 2x + x^2}$$

(0/0)

L'H

$$\lim_{x \to 1} \frac{\frac{1}{x} - 1}{-2 + 2x}$$

L'H

$$\lim_{x \to 1} \frac{\frac{-1}{x^2}}{2} = \frac{-1}{2}$$

A 1

B 0

C -1

D -1/2

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NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 1}\frac{1+\log x-x}{1-2x+x^2}=\lim_{x\to 1}\frac{1/x-1}{-2+2x}$$

(using L'Hôpital Rule)

$$=\lim_{x\to 1}-\frac{1}{2}\frac{(1-x)}{x(1-x)}=-\frac{1}{2}\lim_{x\to 1}\frac{1}{x}=-\frac{1}{2}$$

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NEHA MA'AM'S ARMY

Q. $\lim_{x \to 1} \frac{1 + \log x - x}{1 - 2x + x^2}$ equals

A 1

B 0

C -1

D -1/2

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NEHA MA'AM'S ARMY

Q. $\lim\limits_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$ equals

$\left(\frac{0}{0}\right)$

$(x^n)' = nx^{n-1}$

$(x^{-n})' = -nx^{-n-1}$

$\lim\limits_{x \to 0} \dfrac{e^x + xe^x - \frac{1}{1+x}}{2x}$ (L'H)

$\lim\limits_{x \to 0} \dfrac{e^x + e^x + xe^x + \frac{1}{(1+x)^2}}{2}$ (L'H)

$\dfrac{2+1}{2}$

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{2}$

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NEHA MA'AM'S ARMY

Solution:

Let $y = \lim_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$ $\left(\dfrac{0}{0}\text{ form}\right)$

Applying L–Hospital's rule, $y = \lim_{x \to 0} \dfrac{e^x + xe^x - \dfrac{1}{1+x}}{2x}$ $\left(\dfrac{0}{0}\text{ form}\right)$

$$y = \lim_{x \to 0} \frac{1}{2}\left[e^x + e^x + xe^x + \frac{1}{(1+x)^2}\right] = \lim_{x \to 0} \frac{1}{2}[1+1+0+1] = \frac{3}{2}$$

NEHA MA'AM'S ARMY

Q. $\lim_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$ equals

A $\frac{2}{3}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{3}{2}$

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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

NEHA MA'AM'S ARMY

JEE MAIN 2022 25th July - II

$$\lim_{x\to\frac{\pi}{4}} \frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$$ is equal to

(A) 14
(B) 7
(C) $14\sqrt{2}$
(D) $7\sqrt{2}$

Trigo manipulation

$$\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)\sqrt{2}$$

$$=\sqrt{2}\cos\left(x-\pi/4\right)$$

$$\lim_{x\to\pi/4}\frac{8\sqrt{2}-(\sqrt{2})^7\cos^7(x-\pi/4)}{\sqrt{2}\left(1-\sin 2x\right)}$$

$$8\lim_{x\to\pi/4}\frac{\left(1-\cos^7(x-\pi/4)\right)}{1-\sin 2x}$$

$x-\frac{\pi}{4}=t$

$x\to\pi/4$

$t\to 0$

$\sin 2\left(\frac{\pi}{4}+t\right)=\sin\left(\frac{\pi}{2}+2t\right)$

$=\cos 2t$

$$8\lim_{t\to 0}\frac{\left(1-\cos^7 t\right)}{1-\cos 2t}$$

(L'H)

$$8\lim_{t\to 0}\frac{7\cos^6 t\sin t}{2\sin(2t)}$$

$$8\lim_{t\to 0}\frac{7\cos^6 t\sin t}{4\sin t\cos t}$$

$$14\lim_{t\to 0}\cos^5(t)=14$$

NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to\frac{\pi}{4}} \frac{8\sqrt{2}-(\sqrt{2})^7\left(\cos\left(x-\frac{\pi}{4}\right)\right)^7}{\sqrt{2}\,(1-\sin 2x)} \qquad x=\frac{\pi}{4}+h$$

$$= \lim_{h\to 0} \frac{8\sqrt{2}(1-\cos^7 h)}{\sqrt{2}\,(1-\cos 2h)} \qquad \text{L-Hospital}$$

$$= \lim_{h\to 0} 8\,\frac{-7\cos^6 h\,(-\sin h)}{\sin 2h\times 2}$$

$$= \lim_{h\to 0} \frac{8\times 7\sin h\cos^6 h}{4\sin h\cos h}$$

$$= \lim_{h\to 0} \frac{8\times 7}{4}\cos^5 h = 14$$

NEHA MA'AM'S ARMY

JEE MAIN 2022 25th July - II

$$\lim_{x \to \frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2}\sin 2x}$$ is equal to

A 14

B 7

C $14\sqrt{2}$

D $7\sqrt{2}$

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NEHA MA'AM'S ARMY

(L'H)

$$\lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2}}{9x^2}$$

$$\lim_{x\to 0} \frac{(1+x^2) - \sqrt{1-x^2}}{9x^2\sqrt{1-x^2}(1+x^2)}$$

(1)

(L'H)

$$\frac{1}{9}\lim_{x\to 0} \frac{\left(2x + \frac{2x}{2\sqrt{1-x^2}}\right)}{(2x)\sqrt{1-x^2} + \frac{x^2(-2x)}{2\sqrt{1-x^2}}}$$

JEE Main 2021 (18 Mar Shift 1)

Q. If $\lim_{x\to 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3}$ is equal to L, then the value of (6L+1) is

$$\frac{1}{9}\lim_{x\to 0} \frac{x\left(2\sqrt{1-x^2} + \frac{1}{x}\right)}{x\left(2(1-x^2) - x^2\right)}$$

$$\frac{\frac{1}{9}(3)}{2} = \frac{1}{6} = L$$

$$\Rightarrow 6L = 1$$

$$6L + 1 = 2$$

A. 1/6

B. 1/2

C. 6

D. 2

NEHA MA'AM'S ARMY

Solution:

$$\text{Given } \lim_{x\to 0} \frac{\sin^{-1}x - \tan^{-1}x}{3x^3}$$

$$\Rightarrow \lim_{x\to 0} \frac{\left(x+\frac{x^3}{3!}\ldots\right)-\left(x-\frac{x^3}{3}\ldots\right)}{3x^3}$$

$$\Rightarrow \lim_{x\to 0} \frac{\left(1+\frac{3x^2}{3!}\ldots\right)-\left(1-\frac{3x^2}{3}\ldots\right)}{9x^2} \text{ (Use L'Hospital rule } \frac{0}{0} \text{ form)}$$

$$\Rightarrow \lim_{x\to 0} \frac{\left(\frac{6x}{3!}+\ldots\right)-\left(-\frac{6x}{3}+\ldots\right)}{18x} \text{ (Use L'Hospital rule } \frac{0}{0} \text{ form)}$$

$$\Rightarrow \lim_{x\to 0} \frac{\left(\frac{6}{3!}+\ldots\right)-\left(-\frac{6}{3}+\ldots\right)}{18} = \frac{1+2}{18} = \frac{1}{6}$$

$$\text{Given that, } \lim_{x\to 0} \frac{\sin^{-1}x - \tan^{-1}x}{3x^3} = L$$

$$\therefore \frac{1}{6} = L \Rightarrow 6L = 1$$

So, $6L + 1 = 2$

NEHA MA'AM'S ARMY

JEE Main 2021 (18 Mar Shift 1)

Q. If $\lim_{x\to 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3}$ is equal to L, then the value of (6L+1) is

A ⅙

B ½

C 6

D 2

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NEHA MA'AM'S ARMY

JEE Main 2016

Q. Let $P = \lim_{x \to 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$, then $\log P$ is equal to

1^∞ form

$$P = e^{\lim_{x \to 0} \frac{1}{2x}(\tan^2 \sqrt{x})}$$

(L'H)

$$= e^{\frac{1}{2}\lim_{x \to 0}\left(2\tan\sqrt{x}\cdot \sec^2\sqrt{x}\cdot \frac{1}{2\sqrt{x}}\right)}$$

$$P = e^{\frac{1}{2}(1)}$$

$$\log P = \frac{1}{2}$$

A $\frac{1}{2}$

B $\frac{1}{4}$

C 2

D 1

NEHA MA'AM'S ARMY

Solution:

$$\ln p = \lim_{x \to 0^+} \frac{1}{2x} \ln \left(1 + \tan^2 \sqrt{x}\right)$$

$$\lim_{x \to 0^+} \frac{1}{x} \ln \left(\sec \sqrt{x}\right)$$

Applying L hospital's rule :

$$= \lim_{x \to 0^+} \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sec \sqrt{x}.2\sqrt{x}}$$

$$= \lim_{x \to 0^+} \frac{\tan \sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$$

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JEE Main 2016

Q. Let $P = \lim_{x \to 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$, then $\log P$ is equal to

A $\frac{1}{2}$

B $\frac{1}{4}$

C 2

D 1

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NEHA MA'AM'S ARMY

Newton-Leibnitz's formula in evaluating the limits:

$$\text{if} \quad l(x) = \int_{g(x)}^{h(x)} f(t)\,dt$$

$$\frac{dl}{dx} = f\left(h\left(x\right)\right).\frac{d\ h(x)}{dx} - f\left(g\left(x\right)\right).\frac{d\ g(x)}{dx}$$

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NEHA MA'AM'S ARMY

JEE(Main): 30th Jan 2023 -I

Q. $\lim_{x\to 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6+1} dt$ is equal to _____.

$$48 \lim_{x\to 0} \frac{\int_0^x \frac{t^3}{t^6+1}}{x^4} \quad \frac{0}{0}$$

L'H

$$\lim_{x\to 0} \left(\frac{\frac{x^3}{x^6+1}}{4x^3} \right)$$

$= 12$

NEHA MA'AM'S ARMY

Solution:

$$48 \lim_{x \to 0} \frac{\int_0^x \frac{t^3}{t^6+1} dt}{x^4} \left(\frac{0}{0}\right)$$

Applying L'Hospitals Rule

$$48 \lim_{x \to 0} \frac{x^3}{x^6+1} \times \frac{1}{4x^3}$$

$$= 12$$

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VIDEO SOL.

JEE(Main): 30 Jan 2024 -I

Q. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \to R$ be a differentiable function such that $f(0)=\frac{1}{2}$. If the $\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$, then $8\alpha^2$ is equal to :

A 2

B 16

C 4

D 1

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Solution:

$$\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{\left(\frac{e^{x^2}-1}{x^2}\right)\times x^2}$$

$$\lim_{x\to 0}\frac{\int_0^x f(t)dt}{x} \qquad \left(\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}=1\right)$$

$$=\lim_{x\to 0}\frac{f(x)}{1} \quad \text{(using L Hospital)}$$

$$f(0)=\frac{1}{2}$$

$$\alpha=\frac{1}{2}$$

$$8\alpha^2=2$$

NEHA MA'AM'S ARMY

JEE(Main): 30 Jan 2024 -I

Q. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \to R$ be a differentiable function such that $f(0)=\frac{1}{2}$. If the $\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$, then $8\alpha^2$ is equal to :

A 2

B 16

C 4

D 1

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Maclaurin Series

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\cdots+\frac{f^{(n)}(0)}{n!}x^n+\cdots=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k$$

let $f(x)=\sin x \Rightarrow f(0)=0$

$f'(x)=\cos x \Rightarrow f'(0)=1$

$f''(x)=-\sin x \Rightarrow f''(0)=0$

$f'''(x)=-\cos x \Rightarrow f'''(0)=-1$

$\Rightarrow \sin x = 0+1(x)+0(x^2)-1\frac{(x^3)}{3!}$

$= x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$

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सबसे Imp. Wali

EXPANSIONS

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

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JEE MAIN 2022 26th July - II

Q Let $\beta = \lim_{x \to 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x \left(\frac{e^{3x}-1}{3x}\right) \times 3x}$ for some $\alpha \in \mathbb{R}$.

Then the value of $\alpha + \beta$ is

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\beta = \lim_{x \to 0} \frac{\alpha x - (e^{3x} - 1)}{3\alpha x^2}$ $\left(\frac{0}{0}\right)$

L'H

$\beta = \lim_{x \to 0} \frac{\alpha - 3e^{3x}}{6\alpha x}$ $\left(\frac{\alpha - 3}{0}\right)^{0}$

(L'H) $\alpha = 3$ ✓

$\beta = \lim_{x \to 0} \frac{-9e^{3x}}{18} \Rightarrow \beta = -\frac{1}{2}$

$\alpha + \beta = 5/2$.

A $\frac{14}{5}$

B $\frac{3}{2}$

C $\frac{5}{2}$

D $\frac{7}{2}$

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Solution:

$$\beta = \lim_{x\to 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x(e^{3x} - 1)}$$

$$\Rightarrow \beta = \lim_{x\to 0} \frac{\alpha x - e^{3x} + 1}{\alpha x \frac{(e^{3x} - 1)}{3x} \times 3x}$$

$$\Rightarrow \beta = \lim_{x\to 0} \frac{1 + \alpha x - \left(1 + 3x + \frac{9x^2}{2!} + \cdots\right)}{3\alpha x^2}$$

$$\Rightarrow \beta = \lim_{x\to 0} \left(\frac{\alpha - 3}{6\alpha x} - \frac{9x}{6\alpha x}\right)$$

$$\alpha - 3 = 0, \beta = -\frac{3}{2\alpha}$$

$$\alpha = 3\,, \beta = -\frac{1}{2} \Rightarrow \alpha + \beta = \frac{5}{2}$$

NEHA MA'AM'S ARMY

JEE MAIN 2022 26th July - II

Q Let $\beta = \lim\limits_{x \to 0} \dfrac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$ for some $\alpha \in \mathbb{R}$.

Then the value of $\alpha + \beta$ is

A $\dfrac{14}{5}$

B $\dfrac{3}{2}$

C $\dfrac{5}{2}$

D $\dfrac{7}{2}$

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$$\lim_{x\to 0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{4x^2}$$

L'H rule

$$\frac{1}{2}\lim_{x\to 0}\left(\frac{ae^{ax}+b\sin bx-\frac{c}{2}\left(1e^{-cx}+x(-c)e^{-cx}\right)}{2x}\right)$$

(L'H)

$$\frac{a+0-\frac{c}{2}(1+0)}{0}\Rightarrow a-\frac{c}{2}=0\Rightarrow \boxed{2a=c}$$

$$\frac{1}{2}\lim_{x\to 0}\left(\frac{a^2e^{ax}+b^2\cos bx-a\left(-2ae^{-2ax}-2a\left(1e^{-cx}+x(-c)e^{-cx}\right)\right)}{2}\right)$$

$$\frac{1}{4}\left(a^2+b^2-a(-2a)\right)=\frac{1}{4}\left(b^2+5a^2\right)=17$$

JEE(Main): 13th April 2023 -II

Q. If $\lim_{x\to 0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{\frac{1-\cos(2x)}{(2x)^2}\times(2x)^2}=17$

then $5a^2+b^2$ is equal to

(A) 76

(B) 72

(C) 64

(D) 68

NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 0}\frac{e^{ax}-\cos bx-\frac{cxe^{-cx}}{2}}{1-\cos 2x}=17$$

On expansion

$$\lim_{x\to 0}\frac{\left(1+ax+\frac{(ax)^2}{2!}+\ldots\right)-\left(1-\frac{(bx)^2}{2!}+\ldots\right)-\frac{cx}{2}\left(1-cx+\frac{(cx)^2}{2!}\right)}{\left(\frac{1-\cos 2x}{(2x)^2}\right)\times(2x)^2}=17$$

$$\lim_{x\to 0}\frac{x\left(a-\frac{c}{2}\right)+x^2\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)}{\frac{1}{2}(4x^2)}=17$$

For limit to be exist

$$a-\frac{c}{2}=0\Rightarrow c=2a$$

$$\Rightarrow \frac{\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}}{2}=17$$

$$\Rightarrow \frac{a^2}{2}+\frac{b^2}{2}+\frac{4a^2}{2}=34$$

$$\Rightarrow 5a^2+b^2=68$$

NEHA MA'AM'S ARMY

JEE(Main): 13th April 2023 -II

Q. If $\lim_{x \to 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1 - \cos(2x)} = 17$,

then $5a^2 + b^2$ is equal to

A 76

B 72

C 64

D 68

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$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$e^{-x^2/2} = 1 + \left(\frac{-x^2}{2}\right) + \frac{\left(\frac{-x^2}{2}\right)^2}{2!} + \frac{\left(\frac{-x^2}{2}\right)^3}{3!} + \cdots$

Q. $\lim_{x \to 0} \frac{e^{-x^2/2} - \cos x}{x \cdot x^3 \cdot \frac{\sin x}{x}} =$

$\lim_{x \to 0} \frac{e^{-x^2/2} - \cos x}{x^4}$

A 1/4

B 1/6

C 1/12

D 1/8

@nehamamsarmy

NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 0}\frac{e^{-x^2/2}-\cos x}{x^3\sin x}$$

$$=\lim_{x\to 0}\frac{1+\frac{1}{2}\left(-\frac{x^2}{2}\right)^2+\left(-\frac{x^2}{2}\right)+\ldots.-\left(1-\frac{x^2}{21}+\frac{x^4}{31}\right)}{x^4\cdot\left(\frac{\sin x}{x}\right)}$$

$\to 1$

$$=\lim_{x\to 0}\frac{x^4\left(\frac{3-1}{24}\right)+\ldots\ldots\ldots}{x^4}=\frac{1}{12}$$

NEHA MA'AM'S ARMY

Q. $\lim_{x \to 0} \dfrac{e^{-x^2/2} - \cos x}{x^3 \sin x} =$

A 1/4

B 1/6

C 1/12

D 1/8

@nehamamsarmy

NEHA MA'AM'S ARMY

BINOMIAL APPROXIMATION

$(1+x)^n \approx 1+nx$

OR

$1+nx+\frac{n(n-1)}{2}x^2$

$x \to 0$

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NEHA MA'AM'S ARMY

Q. Let $f(x) = \dfrac{\sqrt{x^2+1} - \sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4+1}}$, $\lim\limits_{x\to\infty} f(x) = 1$

$(1+x)^n = 1 + nx$ as $x \to 0$

$\lim\limits_{x\to\infty} f(x) =$

$\lim\limits_{x\to\infty} \dfrac{x\left(1+\frac{1}{x^2}\right)^{\frac{1}{2}} - x^{2/3}\left(1+\frac{1}{x^2}\right)^{1/3}}{x\left(1+\frac{1}{x^4}\right)^{1/4} - x^{4/5}\left(1+\frac{1}{x^4}\right)^{1/5}}$

$\lim\limits_{\frac{1}{x}\to 0}$ (multiply numerator and denominator by $x^{-1/3}$)

$\dfrac{\left(1+\frac{1}{2}\left(\frac{1}{x^2}\right)\right) - x^{-1/3}\left(1+\frac{1}{3}\left(\frac{1}{x^2}\right)\right)}{\left(1+\frac{1}{4}\left(\frac{1}{x^4}\right)\right) - x^{-1/5}\left(1+\frac{1}{5}\left(\frac{1}{x^4}\right)\right)}$

$= \lim\limits_{\frac{1}{x}\to 0} \dfrac{1 - x^{-1/3}}{1 - x^{-1/5}} = \dfrac{1-0}{1-0} = 1$

NEHA MA'AM'S ARMY

Solution:

$$f(x) = \frac{x\left(1+1/x^2\right)^{1/2} - x^{2/3}\left(1+1/x^2\right)^{1/3}}{x\left(1+1/x^4\right)^{1/4} - x^{4/5}\left(1+1/x^4\right)^{1/5}}$$

$$= \frac{\left(1+\frac{1}{2}\frac{1}{x^2}+0\left(\frac{1}{x^2}\right)\right) - \frac{1}{x^{1/3}}\left(1+\frac{1}{3x^2}+0\left(\frac{1}{x^2}\right)\right)}{\left(1+\frac{1}{4}\frac{1}{x^4}+0\left(\frac{1}{x^4}\right)\right) - \frac{1}{x^{1/5}}\left(1+\frac{1}{5x^4}+0\left(\frac{1}{x^4}\right)\right)}$$

Since $\lim_{x\to\infty} \frac{1}{x^n} = 0$ for all $n > 0$, so $\lim_{x\to\infty} f(x) = \frac{1-0.1}{1-0.1} = 1.$

NEHA MA'AM'S ARMY

JEE MAIN 2022 25th July - I

Q If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to :

Try!

A 4

B -8

C -4

D 8

@nehamamsarmy

NEHA MA'AM'S ARMY

Given, $\lim_{n\to\infty} \sqrt{n^2 - n - 1} + \alpha n + \beta = 0$

Now rewriting the expression we get,

$$\lim_{n\to\infty} (n^2 - n - 1)^{\frac{1}{2}} + \alpha n = -\beta$$

$$\Rightarrow \lim_{n\to\infty} n\left(1 - \frac{1}{n} - \frac{1}{n^2}\right)^{\frac{1}{2}} + \alpha n = -\beta$$

Now on using binomial approximation $(1+x)^n = 1 + nx$ we get,

$$\lim_{n\to\infty} n\left(1 - \frac{1}{2n} - \frac{1}{2n^2}\right) + \alpha n = -\beta$$

$$\Rightarrow \lim_{n\to\infty} n - \frac{1}{2} - \frac{1}{2n} + \alpha n = -\beta$$

$$\Rightarrow \lim_{n\to\infty} n(1+\alpha) - \frac{1}{2} - \frac{1}{2n} = -\beta$$

Now for limit to exist $(1+\alpha) = 0 \Rightarrow \alpha = -1$

Now on putting the value of α we get,

$$\lim_{n\to\infty} n(1+\alpha) - \frac{1}{2} - \frac{1}{2n} = -\beta$$

$$\Rightarrow \lim_{n\to\infty} -\frac{1}{2} - \frac{1}{2n} = -\beta$$

$$\Rightarrow -\frac{1}{2} - 0 = -\beta$$

$$\Rightarrow \beta = \frac{1}{2}$$

So, $8(\alpha+\beta) = 8(-1+\frac{1}{2}) = 8 \times \frac{-1}{2} = -4$

NEHA MA'AM'S ARMY

JEE MAIN 2022 25th July - I

Q If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to :

A 4

B -8

C -4

D 8

@nehamamsarmy

NEHA MA'AM'S ARMY

JEE(Main): 27 Jan 2024 -I

Q. If

$$a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4} \text{ and } b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}},$$

then the value of ab^3 is:

Consider $(1+x^4)^{\frac{1}{2}} = 1+\frac{1}{2}x^4$

$\left(1+\left(1+\frac{1}{2}x^4\right)\right)^{\frac{1}{2}}$

$= \sqrt{2}\left(1+\frac{1}{4}x^4\right)^{\frac{1}{2}}$

$= \sqrt{2}\left(1+\frac{1}{8}x^4\right)$

$(1+x^n) \approx 1+nx$

$\therefore a = \lim_{x\to 0} \frac{\sqrt{2}+\frac{\sqrt{2}}{8}x^4-\sqrt{2}}{x^4}$

$a = \frac{1}{4\sqrt{2}}$

$b = \lim_{x\to 0} \frac{(\sin^2 x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{2-(1+\cos x)}$

$= \lim_{x\to 0} 2\sqrt{2}\,\frac{\sin^2 x}{(1-\cos x)}$

$b = \lim_{x\to 0} 2\sqrt{2}\,(1+\cos x)$

$b = 4\sqrt{2}$

$\therefore ab^3 = \frac{1}{4\sqrt{2}} \times 4^3 \times 2\sqrt{2}$

A. 32

B. 25

C. 36

D. 30

NEHA MA'AM'S ARMY

Solution:

$$a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x^4} - 1}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)}$$

$$= \lim_{x\to 0} \frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)\left(\sqrt{1+x^4} + 1\right)}$$

Applying limit $a = \frac{1}{4\sqrt{2}}$

$$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$$

$$= \lim_{x\to 0} \frac{\left(1-\cos^2 x\right)\left(\sqrt{2} + \sqrt{1+\cos x}\right)}{2-(1+\cos x)}$$

$$b = \lim_{x\to 0} (1+\cos x)\left(\sqrt{2} + \sqrt{1+\cos x}\right)$$

Applying limits $b = 2\left(\sqrt{2}+\sqrt{2}\right) = 4\sqrt{2}$

Now, $ab^3 = \frac{1}{4\sqrt{2}} \times \left(4\sqrt{2}\right)^3 = 32$

NEHA MA'AM'S ARMY

JEE(Main): 27 Jan 2024 -I

Q. If

$$a = \lim_{x \to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4} \text{ and } b = \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}},$$

then the value of ab^3 is:

A 32

B 25

C 36

D 30

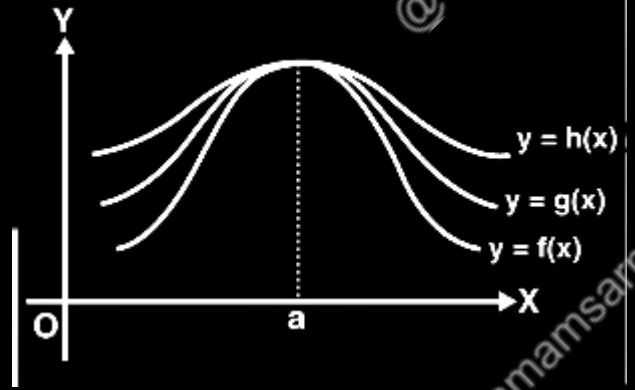
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NEHA MA'AM'S ARMY

Sandwich(Squeeze Theorem)

Statement: Let f, g and h be real functions such that $f(x) \leq g(x) \leq h(x)$ for all x in the common domain of definition. For some real number a, if

$\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$ then $\lim_{x \to a} g(x) = l$



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NEHA MA'AM'S ARMY

Q. If $2x \le g(x) \le x^2-x+2$ for all x, evaluate $\lim_{x \to 1} g(x)$

$\lim_{x\to 1}(2x) \le \lim_{x\to 1} g(x) = \lim_{x\to 1}(x^2-x+2)$

$2 \le g \le 2$

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Solution:

We use the Sandwich theorem here. Since $2x \leq g(x) \leq x^2 - x + 2$, we have

$$\lim_{x \to 1} 2x \leq \lim_{x \to 1} g(x) \leq \lim_{x \to 1} (x^2 - x + 2),$$

therefore

$$2 \leq \lim_{x \to 1} g(x) \leq 2$$

and hence

$$\lim_{x \to 1} g(x) = 2.$$

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NEHA MA'AM'S ARMY

JEE Main 2010

Q. Let f: R → R be a positive increasing function with

$$\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1. \text{ Then } \lim_{x\to\infty} \frac{f(2x)}{f(x)} =$$

$x \le 2x \le 3x$

$f(x) \le f(2x) \le f(3x)$

$\Rightarrow \lim_{x\to\infty} 1 \le \lim_{x\to\infty} \frac{f(2x)}{f(x)} \le \lim_{x\to\infty} \frac{f(3x)}{f(x)}$

$1 \le \lim \frac{f(2x)}{f(x)} \le 1$

$\lim \frac{f(2x)}{f(x)} = 1$

(A) 2/3

(B) 3/2

(C) 3

(D) 1

NEHA MA'AM'S ARMY

Solution:

$f(x)$ is a positive increasing function

$$\Rightarrow 0 < f(x) < f(2x) < f(3x)$$

$$\Rightarrow 0 < 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\Rightarrow \lim_{x\to\infty} 1 \le \lim_{x\to\infty} \frac{f(2x)}{f(x)} \le \lim_{x\to\infty} \frac{f(3x)}{f(x)}$$

By sandwich theorem.

$$\Rightarrow \lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$$

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NEHA MA'AM'S ARMY

JEE Main 2010

Q. Let f: R → R be a positive increasing function with

$$\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1. \text{ Then } \lim_{x\to\infty} \frac{f(2x)}{f(x)} =$$

A 2/3

B 3/2

C 3

D 1

@nehamamsarmy

NEHA MA'AM'S ARMY

JEE(Main): 31 Jan 2024 -II

$x < 5x < 7x$

$\Rightarrow f(x) < f(5x) < f(7x)$

$\Rightarrow \lim_{x\to\infty} 1 < \lim_{x\to\infty} \frac{f(5x)}{f(x)} < \lim_{x\to\infty} \frac{f(7x)}{f(x)}$

$1 < \lim_{x\to\infty} \frac{f(5x)}{f(x)} < 1$

$\lim_{x\to\infty} \frac{f(5x)}{f(x)} = 1$

Q. Let f: R→(0, ∞) be strictly increasing function such that $\lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1.$ Then, the value of $\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right]$ is equal to

$[1-1] = 0$

A. 0

B. $\frac{7}{5}$

C. 1

D. 4

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NEHA MA'AM'S ARMY

Solution:

$$f : R \to (0, \infty)$$

$$\lim_{x \to \infty} \frac{f(7x)}{f(x)} = 1$$

$\because$ f is increasing

$$f(x) \le f(5x) \le f(7x), \ \forall \ x > 0$$

$$1 \le \frac{f(5x)}{f(x)} \le \frac{f(7x)}{f(x)}$$

$$\text{As } x \to \infty \quad \frac{f(5x)}{f(x)} \to 1^{+}$$

$$\lim_{x \to \infty} \left[\frac{f(5x)}{f(x)} - 1 \right] = 0$$

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NEHA MA'AM'S ARMY

JEE(Main): 31 Jan 2024 -II

Q. Let f: R→(0, ∞) be strictly increasing function such that $\lim_{x\to\infty}\frac{f(7x)}{f(x)}=1.$ Then, the value of $\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)}-1\right]$ is equal to

A 0

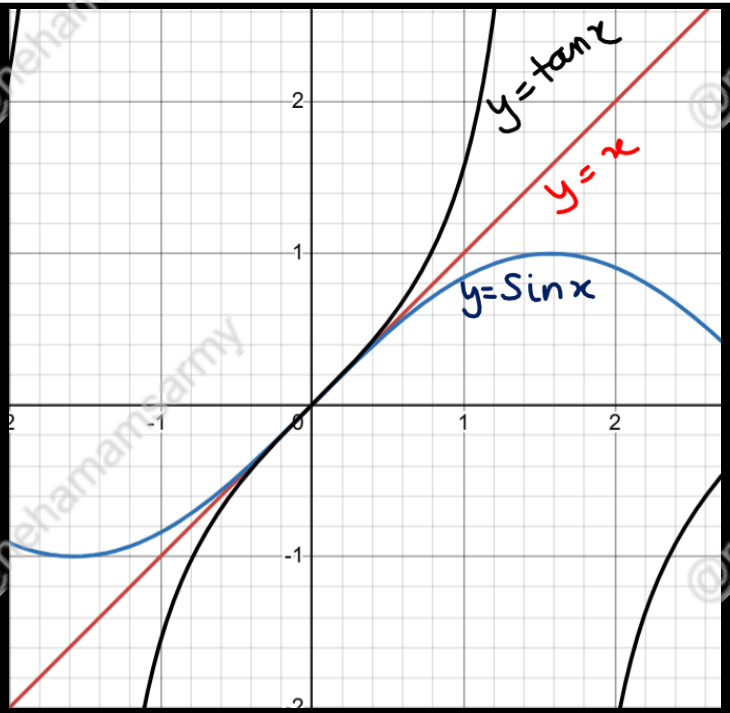
B $\frac{7}{5}$

C 1

D 4

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NEHA MA'AM'S ARMY



LIMITS involving GIF

① $\lim_{x \to 0^+} \left[\frac{\sin x}{x}\right] = 0$ $\quad \rightarrow < 1$

② $\lim_{x \to 0^+} \left[\frac{x}{\sin x}\right] = 1$ $\quad \rightarrow > 1$

→ tanx mein ulta ∴

$\frac{\sin x}{x} < 1$

$\frac{x}{\sin x} > 1$

As x—> 0+ tanx > x > sinx

Click



NEHA MA'AM'S ARMY

Q. The value of $\lim_{x\to 0}\left(\left[\frac{11x}{\sin x}\right]+\left[\frac{21\sin x}{x}\right]\right)$, where [x] is the greatest integer less than or equal x is

$\frac{x}{\sin x} > 1$

$11\frac{x}{\sin x} > 11$

$\left[11\frac{x}{\sin x}\right] = 11$

$\frac{\sin x}{x} < 1$

$21\left(\frac{\sin x}{x}\right) < 21$

$\left[21\frac{\sin x}{x}\right] = 20$

31

A 32

B 31

C 11

D 21

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NEHA MA'AM'S ARMY

Solution:

Since $x > \sin x$, for $x > 0$ and $\lim_{x \to 0+} \frac{x}{\sin x} = 1$

so $\frac{11x}{\sin x} \to 11$ as $x \to 0+$ but $\frac{11x}{\sin x} > 11$. Thus

$\left[\frac{11x}{\sin x}\right] = 11$ for value $x \to 0+$. Similarly

$\frac{21 \sin x}{x} \to 21$ as $x \to 0+$ but $20 \le \frac{21 \sin x}{x} < 21$

$\left[\frac{21 \sin x}{x}\right] = 20$. Hence

$$\lim_{x \to 0+} \left(\left[\frac{11x}{\sin x}\right] + \left[\frac{21 \sin x}{x}\right]\right) = 31$$

Similarly $x < \sin x$ for $x < 0$, so $\left[\frac{11x}{\sin x}\right] = 11$

as $x \to 0-$ and $\left[21 \frac{\sin x}{x}\right] = 20$ as $x \to 0-$. Thus

$$\lim_{x \to 0-} \left(\left[\frac{11x}{\sin x}\right] + \left[\frac{21 \sin x}{x}\right]\right) = 31$$

NEHA MA'AM'S ARMY

Q. The value of $\lim_{x\to 0}\left(\left[\frac{11x}{\sin x}\right]+\left[\frac{21\sin x}{x}\right]\right)$, where [x] is the greatest integer less than or equal x is

A 32

B 31

C 11

D 21

@nehamamsarmy

NEHA MA'AM'S ARMY

Q. The value of $\lim_{x\to 0}\left\{\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right\}$, where [.] represents the greatest integer function is

$100\frac{x}{\sin x} > 100$

$\left[\ \right] = 100$

$99\frac{\sin x}{x} < 99$

$\left[\ \right] = 98$

A. 199

B. 198

C. 0

D. 197

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NEHA MA'AM'S ARMY

Solution:

We know , $x > \sin x \ \forall x > 0$ and $x < \sin x \ \forall x < 0$

$$So, x \to 0 \Rightarrow \frac{\sin x}{x} < 1 and \frac{x}{\sin x} > 1$$

$$= \lim_{x \to 0} \left(\left[\frac{100x}{\sin x} \right] + \left[\frac{99 \sin x}{x} \right] \right)$$

$$= 100 + 98$$

$$= 198$$

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NEHA MA'AM'S ARMY

Q. The value of $\lim_{x\to 0}\left\{\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right\}$, where [.] represents the greatest integer function is

A 199

B 198

C 0

D 197

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NEHA MA'AM'S ARMY

CONTINUITY

At a point

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

or

$$\lim_{x \to a} f(x) = f(a) \checkmark$$

$a \in \text{Dom } f$

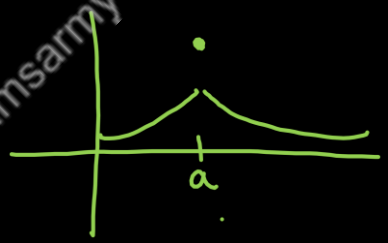
At an interval

on (a, b) — on $[a, b]$

Cts. at every pt. on ....

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REASONS of DISCONTINUITY

(1) $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

(2) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

BUT $\neq f(a)$

(3) $f(a)$ is NOT defined

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CONTINUITY

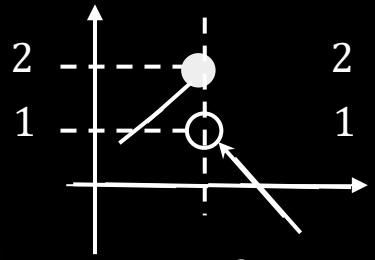


$LHL = 2$

$RHL = 2$

$f(a) = 2$

Cont

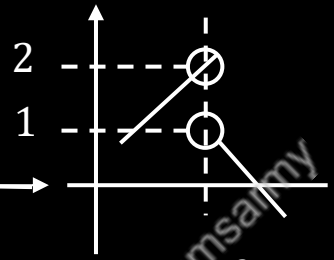


$LHL = 2$

$RHL = 1$

$f(a) = 2$

discont.



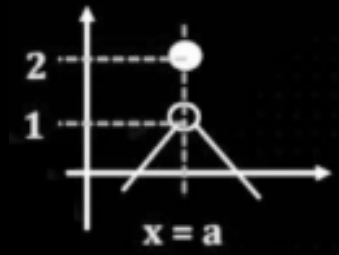
discont

$LHL = 2$

$RHL = 1$

$f(a)$

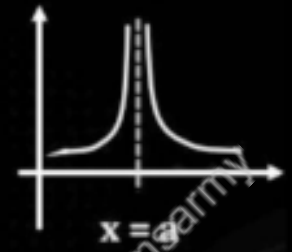
discont.



$LHL = RHL = 1$

$f(a) = 2$

discont.



$LHL \to +\infty$

$RHL \to +\infty$

$f(a) \to n.d$

discont.

NEHA MA'AM'S ARMY

Q. Let $f(x) = \begin{cases} ax - b & , \quad x \leq 1 \\ 3x & , \quad 1 < x < 2 \\ bx^2 - a & , \quad x \geq 2 \end{cases}$

If f is continuous function then (a, b) is equal to

At $x=1$

$LHL = \lim_{x \to 1^-} (ax - b) = a - b$

$RHL = \lim_{x \to 1^+} (3x) = 3$

$f(1) = 3$

$a - b = 3$

At $x=2$

$LHL = \lim_{x \to 2^-} 3(x) = 6$

$RHL = \lim_{x \to 2^+} (bx^2 - a)$

$4b - a = f(2)$

$4b - a = 6$

A. (1, 2)

B. (1, 3)

C. (3, 6)

D. (6, 3)

NEHA MA'AM'S ARMY

Solution:

$f(1) = a - b$ and $\lim_{x \to 1} 3x = 3$ so for f to be continuous $a - b = 3$. Also $f(2) = 4b - a$, so $\lim_{x \to 2-} f(x) = \lim_{x \to 2-} 3x = 6$. Thus $4b - a = 6$ solving, we have $a = 6$, $b = 3$.

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NEHA MA'AM'S ARMY

Q. Let $f(x) = \begin{cases} ax - b & , \quad x \leq 1 \\ 3x & , \quad 1 < x < 2 \\ bx^2 - a & , \quad x \geq 2 \end{cases}$

If f is continuous function then (a, b) is equal to

A (1, 2)

B (1, 3)

C (3, 6)

D (6, 3)

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NEHA MA'AM'S ARMY

Q. If $f(x) = \begin{cases} x + \lambda, & x < 3 \\ 4, & x = 3 \\ 3x - 5, & x > 3 \end{cases}$ is continuous at $x = 3$, then $\lambda =$

$3 + \lambda = 4$

$\Rightarrow \lambda = 1$

$4 = 4$

A 4

B 3

C 2

D 1 ✓

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NEHA MA'AM'S ARMY

Solution:

L.H.L. at $x = 3$, $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (x+\lambda) = \lim\limits_{h\to 0} (3-h+\lambda) = 3+\lambda$ .....(i)

R.H.L. at $x = 3$, $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (3x-5) = \lim\limits_{h\to 0} \{3(3+h)-5\} = 4$ .....(ii)

Value of function $f(3) = 4$ .....(iii)

For continuity at $x = 3$

Limit of function = value of function $3 + \lambda = 4 \Rightarrow \lambda = 1$.

NEHA MA'AM'S ARMY

Q. If $f(x) = \begin{cases} x + \lambda, & x < 3 \\ 4, & x = 3 \\ 3x - 5, & x > 3 \end{cases}$ is continuous at $x = 3$, then $\lambda =$

A 4

B 3

C 2

D 1

@nehamamsarmy

NEHA MA'AM'S ARMY

Q. If $f(x) = \begin{cases} x \sin x, & when\ 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x), & when\ \frac{\pi}{2} < x < \pi \end{cases}$

then

Check Point(s)
↳ "Split"

LHL = $\frac{\pi}{2}$

RHL = $\frac{\pi}{2}(-1)$

A. f(x) is discontinuous at $x = \frac{\pi}{2}$

B. f(x) is continuous at $x = \frac{\pi}{2}$

C. f(x) is continuous at x=0

D. None of these

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NEHA MA'AM'S ARMY

Solution:

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \frac{\pi}{2}, \lim_{x \to \frac{\pi}{2}^+} f(x) = -\frac{\pi}{2} \text{ and } f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}.$$

Since $\lim_{x+\frac{\pi}{2}^-} \neq \lim_{x+\frac{\pi}{2}^+} f(x)$, $\therefore$ Function is discontinuous at $x = \frac{\pi}{2}$

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NEHA MA'AM'S ARMY

Q. If $f(x) = \begin{cases} x \sin x, & when\ 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x), & when\ \frac{\pi}{2} < x < \pi \end{cases}$

then

A. f(x) is discontinuous at $x = \frac{\pi}{2}$

B. f(x) is continuous at $x = \frac{\pi}{2}$

C. f(x) is continuous at x=0

D. None of these

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NEHA MA'AM'S ARMY

JEE Main 2020: 07 Jan - II

$$\lim_{x\to 0} \frac{\ln(1+3x)-\ln(1-2x)}{x}$$

$$= \lim_{x\to 0} \left(3\frac{\ln(1+3x)}{3x} + 2\frac{\ln(1-2x)}{(-2)x} \right)$$

$\to 1$, $\to 1$

$$= 5 = f(0) = K \Rightarrow \boxed{K=5}$$

Q. If the function f defined on

$$\left(-\frac{1}{3}, \frac{1}{3}\right) \text{ by } f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$$

, is continuous, then k is equal to.

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NEHA MA'AM'S ARMY

Solution:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{\ln(1+3x)}{x} - \frac{\ln(1-2x)}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{3\ln(1+3x)}{3x} + \frac{2\ln(1-2x)}{-2x}\right) = 3 + 2 = 5$$

$$\left[\lim_{f(x)\to 0} \frac{\ln\{1+f(x)\}}{f(x)} = 1\right]$$

$\therefore$ f(x), will be continuous if $f(0) = \lim_{x\to 0} f(x)$

Thus, $k = 5$

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NEHA MA'AM'S ARMY

JEE MAIN 2022 29th July - II

Q . Let the function $f(x) = \begin{cases} \frac{\ln(1+5x)-\ln(1+\alpha x)}{x}, & x \neq 0 \\ 10, & x = 0 \end{cases}$ be continuous at $x = 0$, then α is equal to :

$$\lim_{x\to 0}\left(5\frac{\ln(1+5x)}{5x} - \alpha\frac{\ln(1+\alpha x)}{\alpha x}\right)$$

$5 - \alpha = 10$

$\Rightarrow \alpha = -5$

A 10

B -10

C 5

D -5

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NEHA MA'AM'S ARMY

Solution:

$$f(x) = \begin{cases} \dfrac{\ln(1+5x) - \ln(1+\alpha x)}{x}, & x \neq 0 \\ 10, & x = 0 \end{cases}$$

$$\lim_{x \to 0} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x}$$

$$= \lim_{x \to 0} \frac{\ln(1+5x)}{x} - \frac{\ln(1+\alpha x)}{x}$$

$$= 5 - \alpha$$

$$\Rightarrow 5 - \alpha = 10 \Rightarrow \alpha = -5$$

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JEE MAIN 2022 29th July - II

Q . Let the function $f(x) = \begin{cases} \frac{\ln(1+5x)-\ln(1+\alpha x)}{x}, & x \neq 0 \\ 10, & x = 0 \end{cases}$ be continuous at $x = 0$, then α is equal to :

A 10

B -10

C 5

D -5

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NEHA MA'AM'S ARMY

JEE Main 2021: 18 March - II

Q. Let f:R→R be a function defined as

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin 2x}{2x}, & If\ x<0\\ b, & If\ x=0\\ \frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{\frac{5}{2}}}, & if\ x>0\end{cases}$$

If f is continuous on x=0, then (a+b) equals

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

$$\frac{1}{2}\lim_{x\to 0^-}\left(\frac{(a+1)\sin(a+1)x}{(a+1)x}+\frac{\sin 2x}{2x}2\right)$$

$$\frac{1}{2}\left((a+1)+2\right)=b \Rightarrow \boxed{a+3=2b}$$

$$\boxed{a=-2}$$

$$\lim_{x\to 0^+}\frac{(x+bx^3)^{1/2}-x^{1/2}}{bx^{5/2}}$$

$$=\lim_{x\to 0}\frac{x+bx^3-x}{bx^{5/2}\left((x+bx^3)^{1/2}+x^{1/2}\right)}$$

$$\lim_{x\to 0}\frac{x^{1/2}}{(x+bx^3)^{1/2}+x^{1/2}}$$

$$\lim_{x\to 0}\frac{1}{(1+bx^2)^{1/2}+1}$$

$$\boxed{b=\frac{1}{2}}$$

$$a+b=-2+\frac{1}{2}=-\frac{3}{2}$$

A. $-\frac{5}{2}$

B. -3

C. -2

D. $-\frac{3}{2}$

NEHA MA'AM'S ARMY

Solution:

$f(x)$ is continuous at $x = 0$

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x) \quad \ldots(1)$$

$$f(0) = b \quad \ldots(2)$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$$

$$= \frac{a+1}{2} + 1 \quad \ldots(3)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to 0^+} \frac{(x+bx^3-x)}{bx^{5/2}\left(\sqrt{x+bx^3}+\sqrt{x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+bx^2}+1\right)} = \frac{1}{2} \quad \ldots(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow b = \frac{1}{2}, a = -2$$

$$a + b = \frac{-3}{2}$$

JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

NEHA MA'AM'S ARMY

JEE Main 2021: 18 March - II

Q. Let f:R→R be a function defined as

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin 2x}{2x}, If\ x<0\\ b,\ If\ x=0\\ \frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{\frac{5}{2}}},\ if\ x>0\end{cases}$$

If f is continuous on x=0, then a+b equals

A $-\frac{5}{2}$

B -3

C -2

D $-\frac{3}{2}$

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NEHA MA'AM'S ARMY

VIDEO SOLUTION

JEE(Main): 9 April 2024 -I

Q. Lef f : $(0, \pi) \to R$ be a function given by

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$$

where a, b $\in$ Z. If f is continuous at $x = \frac{\pi}{2}$, then $a^2 + b^2$ is equal to _____.

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Solution:

$$\text{LHL at } x = \frac{\pi}{2}$$

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = \left(\frac{8}{7}\right)^{0} = 1$$

$$\text{RHL at } x = \frac{\pi}{2}$$

$$\lim_{x \to \frac{\pi}{2}} (1 + |\cot x|)^{\frac{b}{a}|\tan x|}$$

$$= e^{\lim\limits_{x \to \frac{\pi}{2}} |\cot x| \frac{b}{a} |\tan x|} = e^{\frac{b}{a}}$$

As f(x) is continuous at $x = \frac{\pi}{2}$

$$\Rightarrow Lf\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = Rf\left(\frac{\pi}{2}\right)$$

$$\Rightarrow 1 = a - 8 = e^{\frac{b}{a}}$$

$$\Rightarrow a = 9, b = 0$$

$$\Rightarrow a^2 + b^2 = 81$$

NEHA MA'AM'S ARMY

JEE MAIN 2021: 20th July- I

Q. Let a function f:R→R be defined as, $f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$

Where [x] is the greatest integer less than or equal to x. If f is continuous on R, then (a+b) is equal to:

- A 4
- B 3
- C 2
- D 1

cts. at $x=0$

$\lim_{x\to 0^-} (\sin x - e^x) = -1 = f(0)$

$\lim_{x\to 0^+} (a+[-x]) = a-1$

$\Rightarrow a-1=-1$

$\Rightarrow \boxed{a=0}$

$[-0.001]$

cts. at $x=1$

$\lim_{x\to 1^-} (a+[-x]) = a-1$

$\lim_{x\to 1^+} (2x-b) = 2-b$

$2-b = a-1 \Rightarrow \boxed{b=3}$

0.999

$[-0.999] = -1$

$a+b = 0+3 = 3$

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Solution:

We have,

$$f(x)=\begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a+[-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$$

If $f(x)$ is continuous at $x = a$ then $\lim\limits_{x \to a^-} f(x) = f(a) = \lim\limits_{x \to a^+} f(x)$.

Continuous at $x = 0$

$f(0^+) = f(0^-)$

$\Rightarrow \; a - 1 = 0 - e^0$

$\Rightarrow \; a = 0$

Continuous at $x = 1$

$f(1^+) = f(1^-)$

$\Rightarrow \; 2(1) - b = a + (-1)$

$\Rightarrow \; b = 2 - a + 1$

$\Rightarrow \; b = 3$

$\therefore \; a + b = 3$

NEHA MA'AM'S ARMY

JEE MAIN 2021: 20th July- I

Q. Let a function f:R→R be defined as, $f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$

Where [x] is the greatest integer less than or equal to x. If f is continuous on R, then (a+b) is equal to:

A 4

B 3

C 2

D 1

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NEHA MA'AM'S ARMY

PROPERTIES OF CONTINUOUS FUNCTIONS

Let $f(x)$ and $g(x)$ be continuous functions at $x=a$.

1) $cf(x)$ is cts. at $x=a$, c: constant

2) $f(x) \pm g(x)$ is cts. at $x=a$

3) $f(x) \cdot g(x)$ is cts. at $x=a$

4) $\frac{f(x)}{g(x)}$ is cts. at $x=a$ $g(a) \neq 0$

5) fog and gof are cts. at $x=a$.

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LiST OF CONTINUOUS FUNCTIONS

1. CONSTANT
2. POLYNOMIAL
3. MODULUS
4. RATIONAL
5. ALL TRIGONOMETRIC FUNCTiONS
6. ALL INVERSE TRIG. FUNCTIONS
7. e^x
8. $\log_e x$

All these functions are continuous on their DOMAIN

9. y = [x] and {x} are continuous $\forall X \in R - Z$
10. y = tan x is continuous everywhere except at $x = (2n+1)\frac{\pi}{2}; n \in Z$

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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

JEE MAIN 2022

Q. Let f(x)=[2x²+1] and $g(x) = \begin{cases} 2x-3, & x<0 \\ 2x+3, & x \geq 0 \end{cases}$,

where [t] is the greatest integer ≤ t. Then, in the open interval (-1, 1), the number of points where fog is discontinuous is equal to 62.

$fog(x) = f(g(x)) = [2(g(x))^2+1]$

$= [2(g(x))^2] + 1$

$$f(g(x)) = \begin{cases} [2(2x-3)^2]+1 & ; \ x<0 \\ [2(2x+3)^2]+1 & ; \ x \geq 0 \end{cases}$$

$-1 < x < 0$

$\lim_{x\to 0^-} f(g(x)) = \lim_{x\to 0^-} \left([2(2x-3)^2]+1\right)$

$-2 < 2x < 0$

$-5 < 2x-3 < -3$

$25 > (2x-3)^2 > 9$

$\Rightarrow 18 < 2(2x-3)^2 < 50$

31 integers dlscts

Similarly, for $0<x<1$, f(g(x)) will have 31 pts. of disctty

$[x+n] = [x] + n$, $n \in Z$

[x] is dlscts when $x \in Z$

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$f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\geq 0\end{cases}$

Now, $f(g(x))=[2g^2(x)]+1$

$$=\begin{cases}[2(2x-3)^2]+1; & x<0\\ [2(2x+3)^2]+1; & x\geq 0\end{cases}$$

If $x<0$ then $-1<x<0 \Rightarrow -2<x<0$

$\Rightarrow -5<2x-3<-3 \Rightarrow\Rightarrow 9<(2x-3)^2<25$

$\Rightarrow 18<2(2x-3)^2<50$, So there will be 31 values where $2(2x-3)^2$ is integer, Similarly for $2(2x+3)^2$ there will be 31, so total 62 points of discontinuity

NEHA MA'AM'S ARMY

Q. Find the points of discontinuity of

$$y = \frac{1}{u^2 + u - 2}, \text{ where } u = \frac{1}{x-1}$$

$$y = \frac{1}{\frac{1}{(x-1)^2} + \frac{1}{(x-1)} - 2}$$

$$= \frac{(x-1)^2}{1 + (x-1) - 2(x-1)^2}$$

$$= \frac{(x-1)^2}{x - 2(x^2 + 1 - 2x)}$$

$$= \frac{(x-1)^2}{-2x^2 + 5x - 2}$$

$$= \frac{-(x-1)^2}{(2x-1)(x-2)}$$

$x = \frac{1}{2}$ $\quad$ $x = 2$

Pts. of discty $\frac{1}{2}, 1, 2$

u: discts at $x = 1$

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The function $u = f(x) = \frac{1}{x-1}$ is discontinuous at the point

$x = 1.$ ...(i)

The function $y = g(x) = \frac{1}{u^2 + u - 2} = \frac{1}{(u+2)(u-1)}$ is

discontinuous at $u = -2$ and $u = 1$.

when $u = -2, \frac{1}{x-1} = u = -2$

$\Rightarrow \quad x - 1 = -\frac{1}{2}$

$\Rightarrow \quad x = 1/2$ ...(ii)

when $u = 1, \frac{1}{x-1} = u = 1$

$\Rightarrow \quad x - 1 = 1$

$\Rightarrow \quad x = 2$ ...(iii)

Hence, the composite function $y = g(f(x))$ is

discontinuous at three points $x = \frac{1}{2}$, 1 and 2.

NEHA MA'AM'S ARMY

Q. If $f(x) = \frac{1}{1-x}$ then the points of discontinuity of the function $f^{30}(x)$ where $f^n(x) =$ fof ..... Of (n times) are

$x = 1$

$f^2(x) = f(f(x))$

$= \frac{1}{1-\left(\frac{1}{1-x}\right)}$

$= \frac{1-x}{-x}$

$f(f(x)) = \frac{x-1}{x}$ → discts at $x = 0$

$f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1-\left(\frac{x-1}{x}\right)} = \frac{x}{1} = x = f^3(x)$

$f^4(x) = \frac{1}{1-x}$

⋮

A $x = 2, 1$

B $x = 0, 1$

C $x = 2, 3$

D No points of discontinuity

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Solution:

Clearly, $x = 1$ is a point of discontinuity of the function $f(x)= \frac{1}{1-x}$.

if $x \neq 1$, then

$(fof)(x)= f[f(x)]= f(\frac{1}{1-x})= \frac{x-1}{x}$, which is discontinuous at $x = 0$.

If $x \neq 0$ and $x \neq 1$, then

$(fo\ fof)(x)= f[(fof)(x)]= f(\frac{x-1}{x})= x$

Which is continuous everywhere.

Hence, $f^{30}(x)= x$, which is continuous everywhere.

So, the only points of discontinuity are $x = 0$ and $x = 1$

NEHA MA'AM'S ARMY

Q. If $f(x) = \frac{1}{1-x}$ then the points of discontinuity of the function $f^{30}(x) where\ f^n(x) =$ fof …….. Of (n times) are

A $x = 2, 1$

B $x = 0, 1$

C x= 2, 3

D No points of discontinuity

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NEHA MA'AM'S ARMY

Algebra of Continuous functions

$g(x) \neq 0$

f(x)	g(x)	f(x) ± g(x)	f(x).g(x) and f(x)/g(x)
Continuous	Continuous	Continuous	Continuous
Continuous	Not Continuous	Not Continuous	May be Continuous
Not Continuous	Not Continuous	May be Continuous	May be Continuous

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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

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f(x)	g(x)	f(x) ± g(x)	f(x).g(x) and f(x)/g(x)
Continuous	Not Continuous	Not Continuous	May be Continuous

☀ $f(x) = x^3$ cts

$g(x) = \text{Sgn}(x) = \begin{cases} -1 & x<0 \\ 0 & x=0 \\ 1 & x>0 \end{cases}$ discts

$f(x)\cdot g(x) = \begin{cases} -x^3 & x<0 \\ 0 & x=0 \\ x^3 & x>0 \end{cases}$

$x \to 0^-$ 0
$x \to 0^+$ 0
$x = 0$ 0

} cts at $x=0$

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$f(x) = x^3$ $\quad$ $g(x) = \text{Sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

$f(x)$ cts. at $x = 0$ $\quad$ $g(x)$ discts. at $x = 0$

$$f(x) \cdot g(x) = \begin{cases} -x^3 & x < 0 \\ 0 & x = 0 \\ x^3 & x > 0 \end{cases}$$

cts. at $x = 0$

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f(x)	g(x)	f(x) ± g(x)	f(x).g(x) and f(x)/g(x)
Continuous	Not Continuous	Not Continuous	May be Continuous

☀ $f(x) = x$ cts. at R $g(x) = [x]$ discts at Z

$f(x) \cdot g(x) = x[x]$

$\lim_{x \to 1^-} = 1^-[1^-] = 0$

$\lim_{x \to 1^+} = 1^+[1^+] = 1$

$\neq$ discts at $x = 1$

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$f(x) = x$ ⟶ cts. at $x \in R$

$g(x) = [x]$ ⟶ discts at $x \in Z$

$f(x) \cdot g(x) = x[x]$

$$\lim_{x \to 1^-} x[x] = 1[1^-] = 1 \times 0 = 0$$

$$\lim_{x \to 1^+} x[x] = 1[1^+] = 1 \times 1 = 1$$

discts at $x = 1$ but will be cts. at 0

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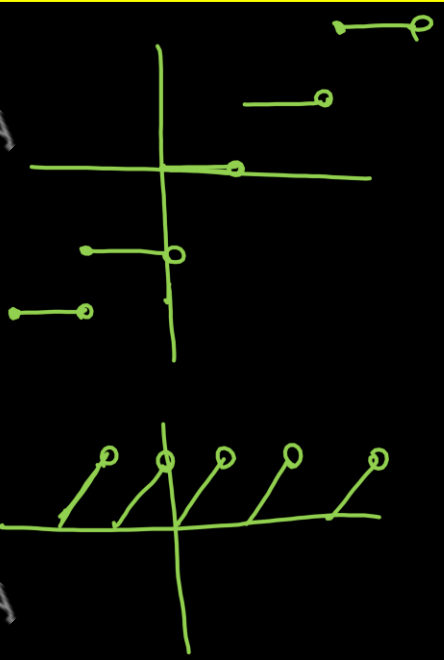
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f(x)	g(x)	f(x) ± g(x)	f(x).g(x) and f(x)/g(x)
Not Continuous	Not Continuous	May be Continuous	May be Continuous

☀ $f(x) = [x]$ discts $\quad g(x) = \{x\}$ discts

$f(x)+g(x) = [x]+\{x\}$

$= x$ cts at $x \in R$



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f(x)	g(x)	f(x) ± g(x)	f(x).g(x) and f(x)/g(x)
Not Continuous	Not Continuous	May be Continuous	May be Continuous

☼ $f(x) = \begin{cases} -1, & x \le 0 \\ 1, & x > 0 \end{cases}$ Discts

$g(x) = \begin{cases} 1, & x \le 0 \\ -1, & x > 0 \end{cases}$ Discts

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$f(x) = \begin{cases} -1, & x \le 0 \\ 1, & x > 0 \end{cases}$ $\quad g(x) = \begin{cases} 1, & x \le 0 \\ -1, & x > 0 \end{cases}$

$f(x) = \begin{cases} -1, & x \le 0 \\ 1, & x > 0 \end{cases}$ $\quad g(x) = \begin{cases} 1, & x \le 0 \\ -1, & x > 0 \end{cases}$

$f(x) + g(x) = 0 \ \forall \ x \in R$ ✓ cts.

$f(x) - g(x) = \begin{cases} -2 & x \le 0 \\ 2 & x > 0 \end{cases}$ ✓ ✓ disct at 0

$f(x) \cdot g(x) = -1 \ \forall x \in R$ cts. & $\frac{f(x)}{g(x)} = -1 \ \forall x \in R$ ✓ cts.

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Continuity of Composite Functions

NOTE:

- If $g(x)$ is continuous at $x = a$ and $f(x)$ is continuous at $g(a)$ then $f(g(x))$ is continuous at $x = a$.
- If $g(x)$ is discontinuous at $x = a$ then $f(g(x))$ may or may not be discontinuous at $x = a$.
- If $g(x)$ is undefined at $x = a$ then $f(g(x))$ is discontinuous at $x = a$.

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TYPES OF DISCONTINUITY



CLICK

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TYPES OF DISCONTINUITY

- **Removable** — $\lim_{x \to a} f(x)$ = exist
  - **Missing Point** — f(a) not defined
  - **Isolated Point** — f(a) ≠ Limit
- **Non-Removable** — Limit do not exist
  - **Finite Type** — LHL/RHL → Finite
  - **Infinite Type** — LHL/RHL → Infinite
  - **Oscillatory** — No fixed value

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Differentiability & Geometrical Meaning of Differentiability

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Condition for Differentiability :

If $\lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0^-} \frac{f(a-h)-f(a)}{-h}$ = a finite value.

(R.H.D) . (L.H.D)

$f'(a)$

Geometrically implies that there is a unique tangent at $x = a$

If LHD = RHD = finite value

Graph is smooth

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)$$

$x \to a+h$

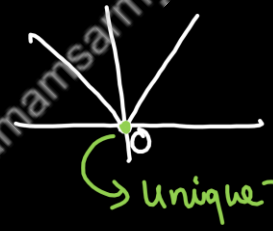
$x \to a-h$

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Relation between Continuity and Differentiability

If function is differentiable, then it is continuous



Converse need NOT be true

If function is not continuous then function is not differentiable

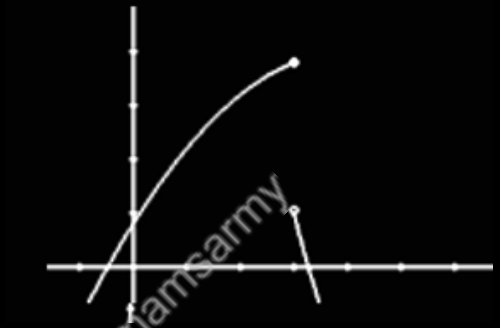
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Reasons of Non-differentiability

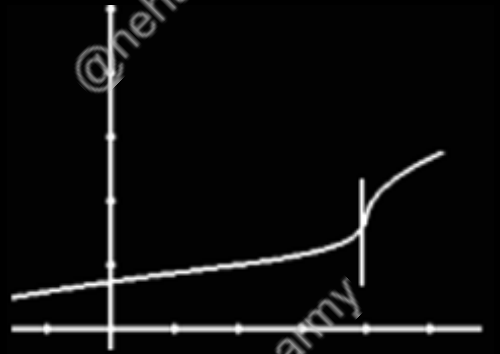
Discontinuity

A Function is not differentiable at 'a' if there is any types of discontinuity at 'a'.



Vertical Tangent Line

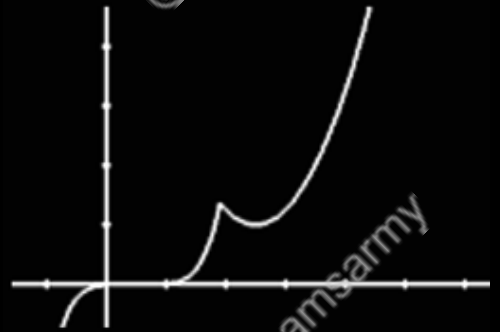
A Function is not differentiable at 'a' if its graph has a vertical line at 'a'.



Corner

for PYQs

A Function is not differentiable at 'a' if its graph has a corner or kink at 'a'.



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Differentiability TABLE

f(x)	g(x)	$f(x) \pm g(x)$	f(x).g(x) and f(x)/g(x)
Differentiable	Differentiable	Differentiable	Differentiable
Differentiable	Not Differentiable	Not Differentiable	May be Differentiable
Not Differentiable	Not Differentiable	May be Differentiable	May be Differentiable

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f(x)	g(x)	f(x).g(x) and f(x)/g(x)
Differentiable	Not Differentiable	May be Differentiable

$f(x) = x^3$ $g(x) = \text{Sgn}(x)$

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$f(x) = x^3$ Diff

$g(x) = \text{Sgn}(x)$ non-diff.

$h(x) = f(x) \cdot g(x) = \begin{cases} -x^3 & x < 0 \\ 0 & x = 0 \\ x^3 & x > 0 \end{cases}$

$h'(x) = \begin{cases} -3x^2 & x < 0 \\ 0 & x = 0 \\ 3x^2 & x > 0 \end{cases}$

Diff. at $x = 0$

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NOTE :

$(x - a)|x - a|$ is differentiable at $x = a$

$$f(x) = (x-a)|x-a| = \begin{cases} -(x-a)^2 & x < a \\ (x-a)^2 & x \geq a \end{cases}$$

$$f'(x) = \begin{cases} -2(x-a) & x < a \\ 2(x-a) & x \geq a \end{cases}$$

$0 \quad 0$

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f(x)	g(x)	f(x) ± g(x)
Not Differentiable	Not Differentiable	May be Differentiable

$f(x) = |x|$ $\quad g(x) = \sin|x|$

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f(x)	g(x)	f(x) ± g(x)
Not Differentiable	Not Differentiable	May be Differentiable

$f(x) = |x|$ $\quad g(x) = \sin|x|$

$$f(x) - g(x) = h(x) = \begin{cases} -x + \sin x & x < 0 \\ x - \sin x & x \geq 0 \end{cases}$$

$h'(x)$ at 0

$$\Rightarrow \begin{cases} -1 + \cos x, & x < 0 \\ 1 - \cos x, & x \geq 0 \end{cases}$$

0

0

$h(x)$ is diff at $x = 0$

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f(x)	g(x)	f(x).g(x) and f(x)/g(x)
Differentiable	Not Differentiable	May be Differentiable

$f(x) = |x|$

$g(x) = sgn(x)$

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f(x)	g(x)	f(x).g(x) and f(x)/g(x)
Differentiable	Not Differentiable	May be Differentiable

$f(x) = |x|$ $\quad g(x) = sgn(x)$

$f(x) = |x|$ not diff at 0 $\quad g(x) = sgn(x)$ not diff at 0

$$f(x)\cdot g(x) = \begin{cases} (-x)(-1) & x < 0 \\ 0 & x = 0 \\ (x)(1) & x > 0 \end{cases}$$

$\therefore f(x).g(x)$ is diff at 0

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$x \to 2^-$ $\quad 1+2=3$

$x \to 2^+$ $\quad 5-2=3$

Q. A function $f(x) = \begin{cases} 1+x, & x \le 2 \\ 5-x, & x > 2 \end{cases}$ is

$$f'(x) = \begin{cases} 1 & x \le 2 \\ -1 & x > 2 \end{cases}$$

A. Not continuous at x = 2

B. Differentiable at x=2

C. Continuous but not differentiable at x = 2

D. None of these

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Solution:

$$\lim_{h\to 0^-} 1+(2-h)=3\,,\quad \lim_{h\to 0^+} 5-(2+h)=3\,,\quad f(2)=3$$

Hence, f is continuous at $x=2$

$$\text{Now}\quad Rf'(x)=\lim_{h\to 0}\frac{5-(2+h)-3}{h}=-1$$

$$Lf'(x)=\lim_{h\to 0}\frac{1+(2-h)-3}{-h}=1$$

$\because\ Rf'(x)\neq Lf'(x)$

$\therefore$ f is not differentiable at $x=2$.

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Q. A function $f(x) = \begin{cases} 1+x, & x \leq 2 \\ 5-x, & x > 2 \end{cases}$ is

A. Not continuous at x = 2

B. Differentiable at x=2

C. Continuous but not differentiable at x = 2

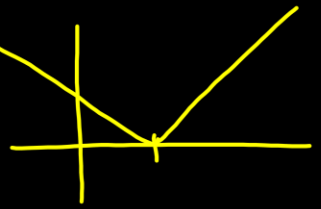
D. None of these

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$x|x| = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

$(x|x|)' = \begin{cases} 2x & x \geq 0 \\ -2x & x < 0 \end{cases}$



$|x-1|$ =

$|x-2|^2 = (x-2)^2$ diff at 2

Q. Let $f : (-\infty, \infty) \to (-\infty, \infty)$ be given by $f(x) = x|x| + |x-1| - |x-2|^2 + |x-3|^3$ If A denotes the set of points where $f(\cdot)$ is not differentiable, then A =

x0 1 x2 x3

$|x-3|^3 = \begin{cases} -(x-3)^3, & x < 3 \\ (x-3)^3 & x \geq 3 \end{cases}$

$(|x-3|^3)' = \begin{cases} -3(x-3)^2, & x < 3 \\ 3(x-3)^2, & x \geq 3 \end{cases}$

diff at $x = 3$

A {3}

B {1}

C {0}

D {2}

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Solution:

. $h(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ is a differentiable function

$u(x) = |x - 2|^2 = (x - 2)^2$ is a differentiable function

$v(x) = |x - 3|^3 = \begin{cases} (x-3)^3, & x \geq 3 \\ -(x-3)^3, & x < 3 \end{cases}$ is a differentiable function

$w(x) = |x - 1|$ is not differentiable at $x = 1$.

f is differentiable everywhere except at $x = 1$.

Hence $A = \{1\}$

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Q. Let $f:(-\infty,\infty)\to(-\infty,\infty)$ be given by $f(x)=x|x|+|x-1|-|x-2|^2+|x-3|^3$ If A denotes the set of points where $f(\cdot)$ is not differentiable, then A =

A {3}

B {1}

C {0}

D {2}

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Q. The value of the derivative $|x-1|+|x-3|$ at $x=2$ is :



$$f(x)=\begin{cases}1-x+3-x, & x\le 1\\ (x-1)+(3-x), & 1<x<3\\ x-1+x-3, & x\ge 3\end{cases}$$

$$=\begin{cases}4-2x, & x\le 1\\ 2, & 1<x<3\\ 2x-4, & x\ge 3\end{cases}$$

for $x=2 \Rightarrow f(x)=2$

$f'(x)=0$

A. -2

B. 0

C. 2

D. Not defined

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Solution:

Let $f(x) = |x-1| + |x-3| = \begin{cases} -(x-1)-(x-3), & x<1 \\ (x-1)-(x-3), & 1 \le x < 3 \\ (x-1)+(x-3), & x \ge 3 \end{cases} = \begin{cases} -2x+4, & x<1 \\ 2, & 1 \le x < 3 \\ 2x-4, & x \ge 3 \end{cases}$

Since, $f(x) = 2$ for $1 \le x < 3$. Therefore $f'(x) = 0$ for all $x \in (1,3)$.

Hence, $f'(x) = 0$ at $x = 2$.

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Q. The value of the derivative $|x-1|+|x-3| at\ x=2$ is :

A -2

B 0

C 2

D Not defined

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JEE Main 2021

Q. If $f(x) = \begin{cases} \frac{1}{|x|}; & |x| \geq 1 \\ ax^2 + b; & |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:

(A) $\frac{1}{2}, \frac{1}{2}$

(B) $\frac{1}{2}, -\frac{3}{2}$

(C) $\frac{5}{2}, -\frac{3}{2}$

(D) $-\frac{1}{2}, \frac{3}{2}$

$f(x) = \begin{cases} -\frac{1}{x} & x \leq -1 \\ ax^2 + b & -1 < x < 1 \\ \frac{1}{x} & x \geq 1 \end{cases}$

Diff. $\Rightarrow$ cty

at -1: $-\frac{1}{(-1)} = a(-1)^2 + b \Rightarrow \boxed{a + b = 1}$

$f'(x) = \begin{cases} \frac{1}{x^2} & x \leq -1 \\ 2ax & -1 < x < 1 \\ -\frac{1}{x^2} & x \geq 1 \end{cases}$

$2a(-1) = 1 \Rightarrow a = -\frac{1}{2}$

$\Rightarrow b = \frac{3}{2}$

$|x| \geq 1$

$x \geq 1$ or $x \leq -1$

$|x| < 1 \Rightarrow -1 < x < 1$

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Solution:

$$f(x)=\begin{cases}\frac{1}{|x|} & ;\ |x|\geq 1\\ ax^2+b; & |x|<1\end{cases}$$

$$f(x)=\begin{cases}\frac{-1}{x}\ ; & x\leq -1\\ \frac{1}{x}; & x\geq 1\\ ax^2+b\ ; & -1<x<1\end{cases}$$

$f(x)$ *is continuous at* $x=1,\ LHL=RHL$

$f(x)$ *is differentiable at* $x=1 \ \Rightarrow\ 2ax+0=\frac{-1}{x^2}$

$a+b=1,\ \ 2a=-1$

$\Rightarrow a=-\frac{1}{2}\ ,\ b=1+\frac{1}{2}=\frac{3}{2}$

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JEE Main 2021

Q. If $f(x) = \begin{cases} \frac{1}{|x|}; & |x| \geq 1 \\ ax^2 + b; & |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:

A $\frac{1}{2}, \frac{1}{2}$

B $\frac{1}{2}, -\frac{3}{2}$

C $\frac{5}{2}, -\frac{3}{2}$

D $-\frac{1}{2}, \frac{3}{2}$

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JEE(Main): 30 Jan 2024 -I

TRY

Q. If the function

$$f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$$

is differentiable on R, then 48(a+b) is equal to

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Solution:

$$f(x)=\begin{cases}\frac{1}{x} & x\geq 2\\ -\frac{1}{x} & x<-2\\ ax^2+2b, & -2<x<2\end{cases}$$

$$f'(x)=\begin{cases}-\frac{1}{x^2} & x\geq 2\\ \frac{1}{x^2} & x<-2\\ 2ax & -2<x<2\end{cases}$$

$$\text{at } x=2$$

$$-\frac{1}{2^2}=2a\times 2$$

$$a=-\frac{1}{16}$$

& continuous also

[Any differentiable function is always continuous]

$$4a+2b=\frac{1}{2}$$

$$-\frac{1}{4}+2b=\frac{1}{2} \qquad b=\frac{3}{8}$$

$$48\,(a+b)$$

$$48\left(\frac{-1}{16}+\frac{3}{8}\right)$$

$$48\left(\frac{-1+6}{16}\right)$$

$$=15$$

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JEE Main 2016

Q. For $x \in R, f(x) = |\log 2 - \sin x| and\, g(x) = f(f(x))$, is

A $g'(0) = -\cos(\log 2)$

B g is differentiable at $x = 0\, and\, g'(0) = -\sin(\log 2)$

C g is not differentiable at $x = 0$

D $g'(0) = \cos(\log 2)$

As $x \to 0 \Rightarrow \log 2 > \sin x$

$\Rightarrow f(x) = \log 2 - \sin x$

$g(x) = f(f(x))$

$= \log 2 - \sin(\log 2 - \sin x)$

$g'(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$\Rightarrow g'(0) = -\cos(\log 2 - 0)(-1)$

$= \cos(\log 2)$

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Solution:

In the neighborhood of $x = 0$, $f(x) = \log 2 - \sin x$

$\therefore g(x) = f(f(x)) = \log 2 - \sin(f(x))$

$= \log 2 - \sin(\log 2 - \sin x)$

$\therefore g'(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$\therefore g'(0) = \cos(\log 2)$

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JEE Main 2016

Q. For $x \in R, f(x) = |\log 2 - \sin x| and\, g(x) = f(f(x))$, is

A $g'(0) = -\cos(\log 2)$

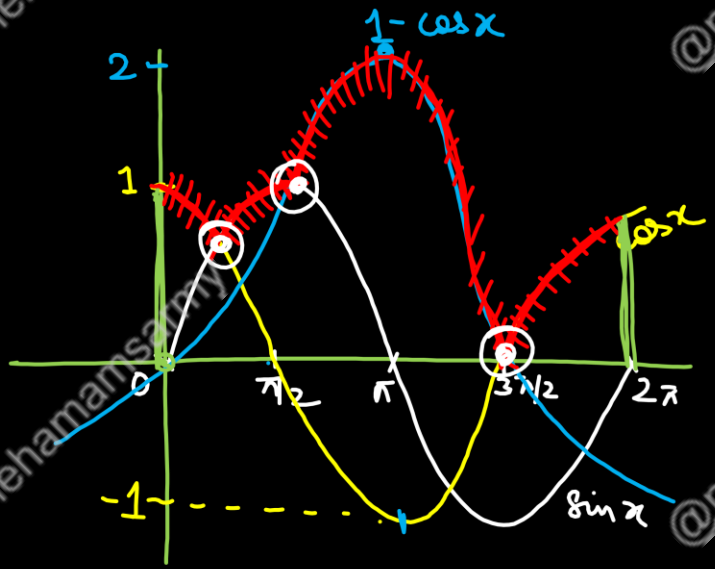
B g is differentiable at $x = 0\, and\, g'(0) = -\sin(\log 2)$

C g is not differentiable at $x = 0$

D $g'(0) = \cos(\log 2)$

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National Testing Agency
Excellence in Assessment

Q. In $(0, 2\pi)$, the total number of points where $f(x) = \max.\{\sin x, \cos x, 1 - \cos x\}$

is not differentiable are equal to

$y = 1 - \cos x$

$\Rightarrow y - 1 = \ominus \cos x$

A 3

B 4

C 5

D 6

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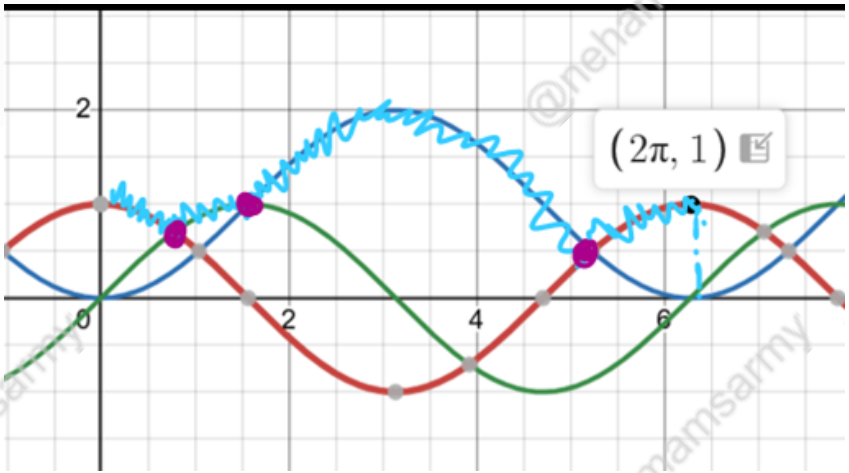
NEHA MA'AM'S ARMY

Solution:

The graph of $f(x) = \max.\{\sin x, \cos x, 1-\cos x\}$ is

$\Rightarrow$ f(x) is not differentiable at $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{3}$.

$\Rightarrow$ f(x) is not differentiable at 3 points



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Excellence in Assessment

Q. In $(0, 2\pi)$, the total number of points where $f(x) = \max. \{\sin x \ \cos x, \ 1 - \ \cos x\}$ is not differentiable are equal to

A 3

B 4

C 5

D 6

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JEE MAIN 2022 29th July - II

VIDEO SOLN.

Q If $[t]$ denotes the greatest integer $\leq t$, then the number of points at which the function $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval $(-20,20)$ is _____.

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Solution:

$$f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$$

$$\Rightarrow f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x] - 240$$

$f(x)$ is non differentiable at $x = -\frac{3}{2}$

$f(x)$ is discontinuous at $\{-19, -18, \cdots, 18, 19\}$

as well as $\left\{-\frac{39}{2}, -\frac{37}{2}, \cdots, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \cdots, \frac{39}{2}\right\}$

At same points, they are also non differentiable.

$\therefore$ Total number of points of non differentiability $= 39 + 40 = 79$

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JEE Main 2021 : 25 Feb - II

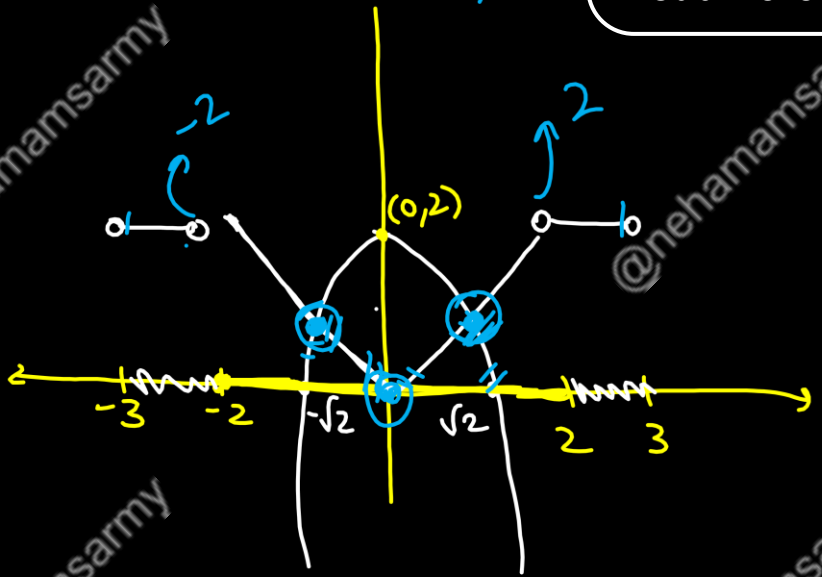
$x = 2 - x^2$

$x^2 + x - 2 = 0$

$(x+2)(x-1)$

Q. A function f is defined on [-3, 3] as

$f(x) = \min\{|x|, 2 - x^2\}, \; -2 \le x \le 2 \quad [|x|], 2 < |x| \le 3$ where [x] denotes the greatest $\le x$. The number of points, where f is Not differentiable in (-3,3) is 5.



$y = 2 - x^2$

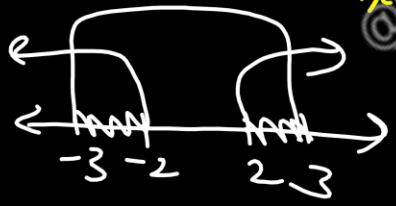
$\Rightarrow y - 2 = -x^2$

$[|x|] \quad |x| \le 3$

$|x| > 2 \quad -3 < x < 3$

$x > +2$

$x < -2$



$-3 < x < -2$

$2 < x < 3$

$2 < |x| < 3$

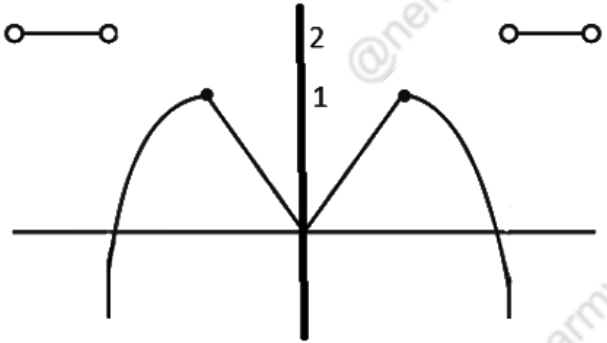
$[|x|] = 2$

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Solution:

$$f(x)=\begin{cases}\min\{|x|, 2-x^2\}, & -2 \le x \le 2 \\ [|x|], & 2 < |x| \le 3\end{cases}$$

$$\Rightarrow x \in [-3,-2) \cup (2,3]$$



Thus the function $f(x)$ is not continuous at $(-2,-1,0,1,2)$.

Hence, the number of points, where f is not differentiable in $(-3,3)$ is 5 points.

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NOTE :

Let

$$f(x) = \begin{cases} x^p \sin\left(\frac{1}{x}\right) & ; \quad x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$$

- f(x) is continuous at x = 0 for $p > 0$
- f(x) is differentiable at x = 0 for $p > 1$
- f′(x) is cont at x = 0 for $p > 2$

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NEHA MA'AM'S ARMY

JEE(Main): 24th Jan 2023 -I

Q. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$; Then at x=0 is

$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

$0 \times (-1 < \text{se} < 1)$

$f(0)=0$

$f'(x) = \begin{cases} 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cos\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right) & x \neq 0 \\ \text{?} & x=0 \end{cases}$

$x \to 0$

$f'(x) = \boxed{2x \sin\left(\frac{1}{x}\right)} - \cos\left(\frac{1}{x}\right)$; $0 \to 0$; $\not\exists$

A. f is continuous but not differentiable

B. f is continuous but f' is not continuous

C. f and f' both are continuous

D. f' is continuous but not differentiable

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Solution:

Continuity of $f(x)$: $f(0^+) = h^2 \cdot \sin\frac{1}{h} = 0$

$f(0^-) = (-h)^2 \cdot \sin\left(\frac{-1}{h}\right) = 0$

$f(0) = 0$

$f(x)$ is continuous

$$f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \frac{h^2 \cdot \sin\left(\frac{1}{h}\right) - 0}{h} = 0$$

$$f'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \frac{h^2 \cdot \sin\left(\frac{1}{-h}\right) - 0}{-h} = 0$$

f(x) is differentiable.

$$f'(x) = 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}$$

$$f'(x) = \begin{cases} 2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

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Now,

$$\lim_{x\to 0} f'(x) = \lim_{x\to 0}\left[2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)\right]$$

$$\Rightarrow \lim_{x\to 0} f'(x) = \lim_{x\to 0}\left[2x\left\{\frac{1}{x} - \frac{1}{3!}\frac{1}{x^3} + \frac{1}{5!}\frac{1}{x^5} - \ldots.\right\} - \left\{1 - \frac{1}{2!}\frac{1}{x^2} + \frac{1}{4!}\frac{1}{x^4} - \ldots.\right\}\right]$$

$$\Rightarrow \lim_{x\to 0} f'(x) = \lim_{x\to 0}\left[\left\{1 - \frac{2}{3!}\frac{1}{x^2} + \frac{2}{5!}\frac{1}{x^4} - \ldots.\right\} - \left\{-\frac{1}{2!}\frac{1}{x^2} + \frac{1}{4!}\frac{1}{x^4} - \ldots.\right\}\right]$$

This limit does not exists finitely, hence $f'(x)$ is discontinuous at $x = 0$.

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JEE(Main): 24th Jan 2023 -I

Q. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$; Then at x=0 is

A f is continuous but not differentiable

B f is continuous but f' is not continuous

C f and f' both are continuous

D f' is continuous but not differentiable

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Derivative Using First Principle

The derivative of function y= f(x) is defined as the Instantaneous rate of change of y {or f(x)} with respect to the change in the independent variable x.

Derivative = $\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

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JEE(Main): 29 Jan 2024 -I

Q. Suppose $f(x) = \dfrac{(2^x + 2^{-x})\tan x \sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$.

Then the value of f'(0) is equal to

$f'(0) = \lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$

$\lim_{h \to 0} \dfrac{(2^h + 2^{-h}) \tanh \sqrt{\tan^{-1}(h^2 - h + 1)} - 0}{h(7h^2 + 3h + 1)^3}$ (h → 1)

$= \dfrac{(1+1)\,1\,\sqrt{\tan^{-1}(1)}}{(0+0+1)^3} = 2 \times \dfrac{\sqrt{\pi}}{4} = \sqrt{\pi} = f'(0)$

A $\dfrac{\pi}{2}$

B 0

C $\sqrt{\pi}$

D π

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Solution:

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(2^h + 2^{-h}) \tan h \sqrt{\tan^{-1}(h^2 - h + 1)} - 0}{(7h^2 + 3h + 1)^3 h}$$

$$= (2^h + 2^{-h}) \frac{\tanh}{h} \frac{\sqrt{\tan^{-1}(h^2 - h + 1)}}{(7h^2 + 3h + 1)^3}$$

$$= \frac{2 \times 1 \times \sqrt{\tan^{-1} 1}}{1} = \frac{2.\sqrt{\pi}}{2} = \sqrt{\pi}$$

NEHA MA'AM'S ARMY

JEE(Main): 29 Jan 2024 -I

Q. Suppose $f(x) = \dfrac{(2^x + 2^{-x}) \tan x \sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$.

Then the value of $f'(0)$ is equal to

A $\dfrac{\pi}{2}$

B 0

C $\sqrt{\pi}$

D π

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Standard Formulae of Differentiation

$(i)\quad \frac{d}{dx}(x^n) = nx^{n-1}$

$(ii)\quad \frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1}\cdot a$

$(iii)\quad \frac{d}{dx}(\sin x) = \cos x$

$(iv)\quad \frac{d}{dx}(\cos x) = -\sin x$

$(v)\quad \frac{d}{dx}(\tan x) = \sec^2 x$

$(vi)\quad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$(vii)\quad \frac{d}{dx}(\sec x) = \sec x \tan x$

$(viii)\quad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$(ix)\quad \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$(x)\quad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$(xi)\quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$(xii)\quad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$(xiii)\quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$(xiv)\quad \frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

$(xv)\quad \frac{d}{dx}(a^x) = a^x \log_e a$

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$$(\log_5(x))'$$

$$= \left(\frac{\log_e x}{\log_e(5)}\right)'$$

$$= \frac{1}{\ln 5}\left(\frac{1}{x}\right)$$

Standard Formulae of Differentiation

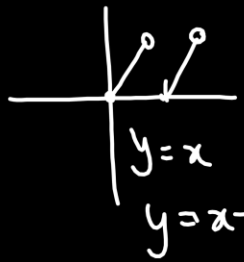
$(xvi)\ \frac{d}{dx}(e^x) = e^x$

$(xvii)\ \frac{d}{dx}(\log_e a) = \frac{1}{x\log_e a} \Rightarrow \frac{d}{dx}(\log_e x) = \frac{1}{x}$

$(xviii)\ \frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0,$

$y = |x|$ is not differentiable at $x = 0$

$(xix)\ \frac{d}{dx}([x]) = \begin{cases} 0 \text{ for } x \in R - I \\ \text{does not exist for } x \in I \end{cases}$ gives a constant



$(xx)\ \frac{d}{dx}(\{x\}) = \begin{cases} 1 \text{ if } x \in R - I \\ \text{does not exist if } x \in I \end{cases}$

NEHA MA'AM'S ARMY

SOME RULES FOR DIFFERENTIATION

1. The derivative of a constant function is zero i.e. $\frac{d}{dx}(c) = 0$
1. The derivative of constant times a function is constant times the derivative of the function, i.e. $\frac{d}{dx}\{c.f(x)\} = c.\frac{d}{dx}\{f(x)\}$
1. The derivative of the sum or difference of two function is the sum or difference of their derivatives, i.e.,

$$\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}\{f(x)\} \pm \frac{d}{dx}\{g(x)\}$$

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NEHA MA'AM'S ARMY

NTA PAPER 110

Q. If $f(x) = \tan^{-1}\left(\frac{\log(ex^{-2})}{\log ex^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$ $x > 0$,

then, find f'(5)?

①

$\tan^{-1}\left(\frac{\log(ex^{-2})}{\log(ex^2)}\right)$

$= \tan^{-1}\left(\frac{\log e + \log(x^{-2})}{\log e + \log(x^2)}\right)$

$= \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right)$

$= \tan^{-1}(1) - \tan^{-1}(2\log x)$

②

$\tan^{-1}\left(\frac{3+2\log x}{1-(3)(2\log x)}\right)$

$= \tan^{-1}(3) + \tan^{-1}(2\log x)$



① + ② ⇒ $f(x) = \tan^{-1}(1) + \tan^{-1}(3)$

$f'(x) = 0$

$\Rightarrow f'(5) = 0$.

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Solution:

$$f(x) = \tan^{-1}\left(\frac{\log(ex^{-2})}{\log ex^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$= \tan^{-1}\left(\frac{\log e+\log x^{-2}}{\log e+\log x^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-3(2\log x)}\right)$$

$$= \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-3(2\log x)}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}(2\log x) + \tan^{-1}(3) + \tan^{-1}(2\log x)$$

$$\Rightarrow f(x) = \tan^{-1}(1) + \tan^{-1}(3)$$

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow f'(5) = 0$$

NEHA MA'AM'S ARMY

PRODUCT RULE OF DIFFERENTIATION

$$\frac{d}{dx}\{f(x).g(x)\} = f(x).\frac{d}{dx}\{g(x)\} + g(x).\frac{d}{dx}\{f(x)\}$$

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NEHA MA'AM'S ARMY

JEE(Main): 6 April 2024 -II

Q. Suppose for a differentiable function h, $h(0) = 0$, $h(1) = 1$ and $h'(0)=h'(1) = 2$. If $g(x) = h(e^x)\, e^{h(x)}$, then $g'(0)$ is equal to:

$$g'(x) = \left(h'(e^x)\cdot e^x\right)e^{h(x)} + h(e^x)\left(e^{h(x)}\cdot h'(x)\right)$$

$$g'(0) = \left(h'(1)\cdot 1\right)e^{h(0)} + h(1)\left(e^{h(0)}\cdot h'(0)\right)$$

$$= 2\times 1 + 1\left(e^{1}\times 2\right)$$

A 5

B 3

C 8

D 4

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Solution:

$$g(x) = h(e^x) \cdot e^{h(x)}$$

$$g'(x) = h(e^x) \cdot e^{h(x)} \cdot h'(x) + e^{h(x)} h'(e^x) \cdot e^x$$

$$g'(0) = h(1)e^{h(0)}h'(0) + e^{h(0)}h'(1)$$

$$= 2 + 2 = 4$$

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NEHA MA'AM'S ARMY

JEE(Main): 6 April 2024 -II

Q. Suppose for a differentiable function h, $h(0) = 0$, $h(1) = 1$ and $h'(0)=h'(1) = 2$. If $g(x) = h\left(e^x\right) e^{h(x)}$, then $g'(0)$ is equal to:

A 5

B 3

C 8

D 4

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QUOTIENT RULE OF DIFFERENTIATION

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x).\frac{d}{dx}\{f(x)\} - f(x).\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$$

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NEHA MA'AM'S ARMY

DABBE mein DABBA!

DERIVATIVE OF A FUNCTION OF A FUNCTION (CHAIN RULE)

If y is a differentiable function of t and t is a differentiable function of x

$y = f(t)$ and $t = g(x)$, then $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

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NEHA MA'AM'S ARMY

$x = e^{\sin^{-1} y}$ then $\left.\frac{dy}{dx}\right|_{x=1}$

$\ln x = \sin^{-1}(y)$

Diff wrt x

$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$

When $x=1 \Rightarrow \sin^{-1} y = 0$

$y = 0$

$\frac{dy}{dx} = \frac{1}{1} = 1$

$y = \sin\left(e^{x^2+3x-2}\right)$ then $\left.\frac{dy}{dx}\right|_{x=1}$

$y' = \cos(e^{x^2+3x-2}) \times e^{x^2+3x-2} \times (2x+3)$

at $x=1$

$\cos(e^2) \times e^2 \times 5$

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Solution:

$$x = e^{\sin^{-1} y} \Rightarrow \log x = \sin^{-1} y$$

$$\Rightarrow \quad y = \sin(\log x)$$

$$\frac{dy}{dx} = \cos(\log x)\frac{1}{x},$$

so $$\left.\frac{dy}{dx}\right|_{x=1} = \cos(\log 1) = \cos 0 = 1$$

$$\frac{dy}{dx} = \cos\left(e^{x^2+3x-2}\right)\frac{d}{dx}\left(e^{x^2+3x-2}\right)$$

$$= \cos\left(e^{x^2+3x-2}\right)e^{x^2+3x-2}\frac{d}{dx}(x^2+3x-2)$$

$$= \cos\left(e^{x^2+3x-2}\right)e^{x^2+3x-2}(2x+3)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 5e^2\cos(e^2)$$

NEHA MA'AM'S ARMY

JEE Main 2012

Q. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$ then $\frac{dy}{dx}$ equals

$$\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right) \times \left(\frac{2(3-2x)+(2x+3)(+2)}{(3-2x)^2}\right)$$

$$= \sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right) \frac{12}{(3-2x)^2}$$

A $\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

B $\frac{12}{(3-2x^2)}$

C $\frac{12}{(3-2x^2)}\sin\left[\left(\frac{2x+3}{3-2x}\right)\right]$

D $\frac{12}{(3-2x^2)}\cos\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

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Solution:

Let $f'(x) = \sin[\log x]$ and $y = f\left(\frac{2x+3}{3-2x}\right)$

Now, $\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right) \cdot \frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$

$$= \sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right] \frac{[(6-4x-)-4x-6]}{(3-2x^2)}$$

$$= \frac{12}{(3-2x^2} \cdot \sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$$

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NEHA MA'AM'S ARMY

JEE Main 2012

Q. If $f'(x) = \sin(\log x) and\, y = f\left(\frac{2x+3}{3-2x}\right) then\, \frac{dy}{dx}$ equals

A $\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

B $\frac{12}{(3-2x^2)}$

C $\frac{12}{(3-2x^2)}\sin\left[\left(\frac{2x+3}{3-2x}\right)\right]$ log

D $\frac{12}{(3-2x^2}\cos\left[\log\left(\frac{2x+3}{3-2x}\right)\right.$

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NEHA MA'AM'S ARMY

JEE(Main): 31 Jan 2023 -I

Q. Let $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)\right)$ then , at x = 1

$y = \sin^3\left(\frac{\pi}{3}\cos g(x)\right)$

$y' = 3\sin^2\left(\frac{\pi}{3}\cos g(x)\right) \times \cos\left(\frac{\pi}{3}\cos g(x)\right) \times \frac{\pi}{3}(-\sin g(x)) \times g'(x)$

$y'|_{x=1} = 3\sin^2\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right) \times \cos\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right) \times -\frac{\pi}{3}\sin\frac{2\pi}{3} \cdot g'(1)$

$\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $g'(1) = -\pi$

$g'(x) = \frac{\pi}{3\sqrt{2}} \times \frac{3}{2}(-4x^3+5x^2+1)^{1/2} \times (-12x^2+10x)$

$g'(1) = \frac{\pi}{2\sqrt{2}}(\sqrt{2})(-12+10) = -\pi$

$\frac{\pi}{3\sqrt{2}}(-4+5+1)^{3/2} = \frac{\pi \times 2\sqrt{2}}{3\sqrt{2}}$

(A) $2y' + \sqrt{3}\pi^2 y = 0$

(B) $2y' + 3\pi^2 y = 0$

(C) $\sqrt{2}y' - 3\pi^2 y = 0$

(D) $y' + 3\pi^2 y = 0$

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NEHA MA'AM'S ARMY

Solution: The given equation can be written as $y = \sin^3\left(\frac{\pi}{3}\cos g(x)\right)$

where, $g(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}$

$$\Rightarrow g'(x) = \frac{\pi}{2\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{1}{2}}\left(-12x^2 + 10x\right)$$

$$\Rightarrow g'(1) = \frac{\pi\sqrt{2}}{2\sqrt{2}}(-2) = -\pi$$

and, $g(1) = \frac{2\pi}{3} = \left(\pi - \frac{\pi}{3}\right)$

Now, $y' = 3\sin^2\left(\frac{\pi}{3}\cos g(x)\right) \times \cos\left(\frac{\pi}{3}\cos g(x)\right) \times \frac{\pi}{3}(-\sin g(x))g'(x)$

$$y'(1) = 3\sin^2\left(-\frac{\pi}{6}\right) \cdot \cos\left(\frac{\pi}{6}\right) \cdot \frac{\pi}{3}\left(-\sin\frac{2\pi}{3}\right)g'(1)$$

$$y'(1) = \frac{3}{4} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\pi}{3}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$$

$$y(1) = \sin^3\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right) = -\frac{1}{8}$$

Therefore, $2y'(1) + 3\pi^2 y(1) = 0$

NEHA MA'AM'S ARMY

JEE(Main): 31 Jan 2023 -I

Q. Let $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)\right)$ then , at x = 1

A $2y' + \sqrt{3}\pi^2 y = 0$

B $2y' + 3\pi^2 y = 0$

C $\sqrt{2}y' - 3\pi^2 y = 0$

D $y' + 3\pi^2 y = 0$

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NEHA MA'AM'S ARMY

JEE Main 2019: 12 Jan - I

Q. For $x > 1$, *if* $(2x)^{2y} = 4e^{2x-2y}$, *then* $(1+\log_e 2x)^2 \frac{dy}{dx}$ is equal to

$2y \ln(2x) = \ln(4e^{2x-2y})$

$= \ln 4 + \ln(e^{2x-2y})$

$2y \ln(2x) = \ln 4 + (2x - 2y)$

$2y(1+\ln(2x)) = \ln 4 + 2x$

$\Rightarrow 2y = \frac{\ln 4 + 2x}{1+\ln(2x)}$

Diff wrt x ⇒

$2y' = \frac{2(1+\ln 2x) - (\ln 4 + 2x) \times \frac{2}{2x}}{(1+\ln(2x))^2}$

$(1+\ln(2x))^2 y' = 2x + \frac{2x\ln 2x - \ln 4 - 2x}{x}$

$= \frac{2x\ln 2x - 2\ln 2}{2x}$

A $\log_e 2x$

B $\frac{x \log_e 2x - \log_e 2}{x}$

C $x \log_e 2x$

D $\frac{x \log_e 2x + \log_e 2}{x}$

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Given, $(2x)^{2y} = 4.\, e^{2x-2y}$

Taking natural logarithm on both sides, we get

$2y \log_e 2x = \log_e 4 + (2x - 2y)$

$\Rightarrow 2y = \left(\frac{\log_e 4 + 2x}{1 + \log_e 2x}\right)$

Differentiating both sides with respect to x, we get

$\frac{2dy}{dx} = \frac{(1+\log_e 2x).2 - (\log_e 4 + 2x)\frac{1}{x}}{(1+\log_e 2x)^2}$ (Using quotient rule)

$\Rightarrow (1 + \log_e 2x)^2 \frac{dy}{dx} = \left(\frac{x.\log_e 2x - \log_e 2}{x}\right).$

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JEE Main 2019: 12 Jan - I

Q. For $x > 1$, *if* $(2x)^{2y} = 4e^{2x-2y}$, *then* $(1+\log_e 2x)^2 \frac{dy}{dx}$ is equal to

A $\log_e 2x$

B $\frac{x \log_e 2x - \log_e 2}{x}$

C $x \log_e 2x$

D $\frac{x \log_e 2x + \log_e 2}{x}$

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KHICHDI

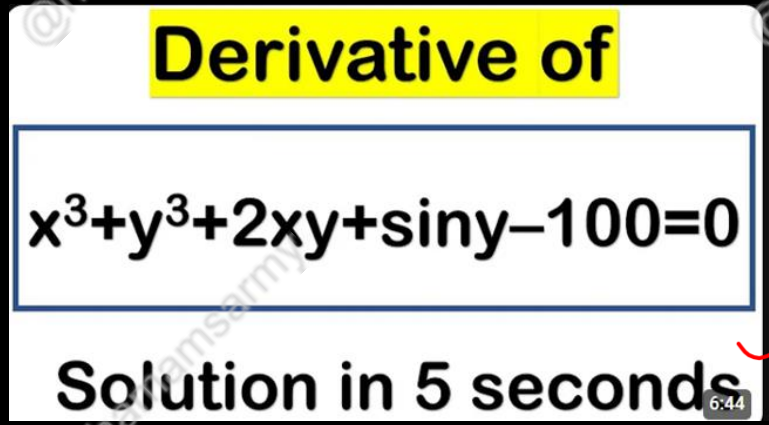
DIFFERENTIATION OF IMPLICIT FUNCTIONS



If in an equation, x and y both occurs together $i.e.$ $f(x, y) = 0$ and this equation can not be solved either for y or x, then y (or x) is called the implicit function of x (or y).

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JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES



DIFFERENTIATION SHORTCUT-2/ IMPLICIT DIFFERENTIATION/DERIVATIVES TRICK

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Neha Agrawal Mathematically Inclined

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LINK

$f(x, y)$

$$\frac{dy}{dx} = -\frac{f'(x)}{f'(y)}$$

[y is a constant]

[x is a constant]

NEHA MA'AM'S ARMY

Q. If $x^2 + 2xy + y^3 = 4$, find $\frac{dy}{dx}$

Diff wrt x

$2x + 2(y + xy') + 3y^2 y' = 0$

$\Rightarrow y'(2x + 3y^2) = -2(x + y)$

$y' = \frac{-2(x+y)}{2x+3y^2}$

TRICK

$\frac{dy}{dx} = \frac{-(2x + 2y)}{0 + (2x) + 3y^2}$

Q. Differentiate $y^2 + x^3 - xy + \cos y = 0$

TRICK

$\frac{dy}{dx} = \frac{-(0 + 3x^2 - y + 0)}{(2y + 0 - x - \sin y)}$

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NEHA MA'AM'S ARMY

Solution:

1

We have

$x^2+2xy+y^3=4$

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}(x^2)+2\frac{d}{dx}(xy)+\frac{d}{dx}(y^3)=\frac{d}{dx}(4)$$

$$\Rightarrow 2x+2\left(x\frac{dy}{dx}+y.1\right)+3y^2\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{2(x+y)}{(2x+3y^2)}$$

2

$$2y\frac{dy}{dx}+3x^2-\frac{d}{dx}(xy)-\sin y\frac{dy}{dx} = 0$$

$$(2y-\sin y)\frac{dy}{dx}+3x^2-\left\{x\frac{dy}{dx}+y.1\right\} = 0$$

$$(2y-\sin y-x)\frac{dy}{dx}+3x^2-y = 0$$

$$(2y-\sin y-x)\frac{dy}{dx} = y-3x^2$$

$$\frac{dy}{dx}=\frac{y-3x^2}{2y-\sin y-x}$$

JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

JEE Main 2020: 07 Jan - II

Q. Let y=y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and y(1/2) = -¼. Then dy/dx at x= ½ , is equal to

Trick

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = k$$

$$\frac{dy}{dx} = -\frac{\left(\frac{y(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-y^2}\right)}{\left(\sqrt{1-x^2} + \frac{x(-2y)}{2\sqrt{1-y^2}}\right)}$$

$x = 1/2$

$$= -\frac{\left(\frac{+\frac{1}{4}\times\frac{1}{2}}{\sqrt{1-\frac{1}{4}}} + \sqrt{1-\frac{1}{16}}\right)}{\left(\sqrt{1-\frac{1}{4}} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)}{\sqrt{1-\frac{1}{16}}}\right)}$$

A. $-\sqrt{5}/4$

B. $-\sqrt{5}/2$

C. $2/\sqrt{5}$

D. $\sqrt{5}/2$

NEHA MA'AM'S ARMY

Solution:

$$x = \frac{1}{2}, y = \frac{-1}{4} \Rightarrow xy = \frac{-1}{8}$$

$$y.\frac{(-2x)}{2\sqrt{1-x^2}} + y'.\sqrt{1-x^2} = 0 - \{1.\sqrt{1-y^2} + \frac{x.(-2y)}{2\sqrt{1-y^2}}y'\}$$

$$-\frac{xy}{\sqrt{1-x^2}} + y'\sqrt{1-x^2} = -\sqrt{1-y^2} + \frac{xy.y'}{\sqrt{1-y^2}}$$

$$y'(\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}) = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

$$y'(\frac{\sqrt{3}}{2} + \frac{1}{8.\frac{\sqrt{15}}{4}}) = \frac{-1}{8.\sqrt{\frac{3}{4}}} - \frac{\sqrt{15}}{4}$$

$$y'(\frac{\sqrt{45}+1}{2\sqrt{15}}) = -\frac{(1+\sqrt{45})}{4\sqrt{3}}$$

$$y' = -\frac{\sqrt{15}}{2\sqrt{3}} = -\frac{\sqrt{5}}{2}$$

NEHA MA'AM'S ARMY

JEE Main 2020: 07 Jan - II

Q. Let y=y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and y(1/2) = -¼. Then dy/dx at x= ½ , is equal to

A -√5/4

B -√5/2

C 2/√5

D √5/2

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NEHA MA'AM'S ARMY

COMMON FRIEND

DERIVATIVE OF PARAMETRIC FUNCTIONS

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{f'(t)}{g'(t)}$$

$$\frac{d^2y}{d^2x} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx} = \frac{d}{dt}\frac{\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}.$$

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NEHA MA'AM'S ARMY

Q. If $x = 2t + 3t^2$, $y = t^2 + 2t^3$ then $y'^2 + 2y'^3$ $\left(y' = \frac{dy}{dx}\right)$ is equal to

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 6t^2}{2 + 6t} = t$$

$$(y')^2 + 2y'^3$$

$$t^2 + 2t^3 = y$$

A. $2y$

B. y^2

C. y

D. $3y + 2$

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NEHA MA'AM'S ARMY

Solution:

$$\frac{dx}{dt} = 2 + 6t, \quad \frac{dy}{dt} = 2t + 6t^2$$

So that $$y' = \frac{dy}{dx} = \frac{2t + 6t^2}{2 + 6t} = t$$

$$y'^2 + 2\,y'^3 = t^2 + 2t^3 = y$$

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Q. If $x = 2t + 3t^2$, $y = t^2 + 2t^3$ then $y'^2 + 2y'^3$ $\left(y' = \frac{dy}{dx}\right)$ is equal to

A. $2y$

B. y^2

C. y

D. $3y + 2$

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Q. $x=3\cos t$ and $y=5\sin t$, where t is a parameter, then then $9\frac{d^2y}{dx^2}$ at $t=-\frac{\pi}{6}$ is equal to

$$\frac{dy}{dx}=\frac{5\cos t}{-3\sin t}=-\frac{5}{3}\cot(t)$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}=+\frac{5}{3}\operatorname{cosec}^2(t)\times\frac{dt}{dx}$$

$$\frac{d^2y/dt^2}{dx^2/dt^2}$$ ✗

$$=\frac{5}{3}\operatorname{cosec}^2(t)\times\frac{1}{-3\sin t}$$

$$9\frac{d^2y}{dx^2}=-5\operatorname{cosec}^3(t)$$

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Solution:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5\cos t}{-3\sin t} = \frac{-5}{3}\cot(t)$$

$$\frac{d^2y}{dx^2} = \frac{-5}{3}\cdot(-\text{cosec}^2 t)\cdot\frac{dt}{dx} = \frac{5}{3}\left(\frac{1}{\sin^2 t}\right)\left(\frac{1}{-3\sin t}\right)$$

$$= \frac{5}{9}(8)$$

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JEE MAIN 2022 28th July - II

Q Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to :

Soln.

$$x'(t) = 2\sqrt{2}\left(-\sin t\sqrt{\sin 2t} + \cos t \times \frac{2\cos 2t}{2\sqrt{\sin 2t}}\right)$$

$$= 2\sqrt{2}\left(\frac{-\sin t(\sin 2t) + \cos t\cos 2t}{\sqrt{\sin 2t}}\right)$$

$$x'(t) = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}} \quad -(1)$$

$$\text{Also, } y'(t) = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\Rightarrow y' = \frac{2\sqrt{2}\sin 3t}{2\sqrt{2}\cos 3t} = \tan 3t$$

$$y'' = 3\sec^2(3t)\cdot\frac{dt}{dx}$$

$$= 3\sec^2(3t) \times \frac{\sqrt{\sin 2t}}{2\sqrt{2}\cos 3t} \quad (\text{using } (1))$$

A $-\frac{2\sqrt{2}}{3}$

B $\frac{2}{3}$

C $\frac{1}{3}$

D $\frac{-2}{3}$

JEE MAINS: LIMITS, CONTINUITY & DERIVATIVES

JEE MAIN 2022 28th July - II

Q Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\ \sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to :

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Solution:

$$x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t} \text{ and } y(t) = 2\sqrt{2}\sin t\ \sqrt{\sin 2t}\ , t \in \left(0, \frac{\pi}{2}\right)$$

$$\boxed{\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}} \quad \boxed{\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}}$$

$$\therefore \frac{dy}{dx} = \tan 3t \Rightarrow \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = -1$$

$$\frac{d^2y}{dx^2} = 3\sec^2 3t \cdot \frac{dt}{dx} = \frac{3\sec^2 3t \cdot \sqrt{\sin 2t}}{2\sqrt{2}\cos 3t} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = -3$$

$$\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{2}{-3} = -\frac{2}{3}$$

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JEE MAIN 2022 28th July - II

Q Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\ \sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to :

A $-\frac{2\sqrt{2}}{3}$

B $\frac{2}{3}$

C $\frac{1}{3}$

D $\frac{2}{3}$

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DIFFERENTIATION OF LOGARITHMIC FUNCTIONS

When base and power both are the functions of x $i.e.$, the function is of the form $[f(x)]^{g(x)}$.

$$y = [f(x)]^{g(x)}$$

$$\log y = g(x) \log [f(x)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx} g(x) \cdot \log [f(x)]$$

$$\frac{dy}{dx} = [f(x)]^{g(x)} \cdot \left\{ \frac{d}{dx} [g(x) \log f(x)] \right\}.$$

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Q. If $x^y \cdot y^x = 16$, *then the value of* $\frac{dy}{dx}$ at (2, 2) is

$\ln(x^y \cdot y^x) = \ln(16)$

$\Rightarrow \ln(x^y) + \ln(y^x) = \ln(16)$

$\Rightarrow y\ln x + x\ln y = \ln 16$

Diff wrt x

$\Rightarrow \left(y'\ln x + y\cdot\frac{1}{x}\right) + \left(1\ln y + x\times\frac{1}{y}y'\right) = 0$

Trick!!

$$\frac{dy}{dx} = -\frac{\left(\frac{y}{x} + \ln y\right)}{\left(\ln x + \frac{x}{y}\right)} = -\frac{(1+\ln 2)}{(\ln 2 + 1)}$$

A. -1

B. 0

C. 1

D. None of these

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Solution:

Since, $x^y . y^x = 16$

$\therefore \log_e x^y + \log_e y^x = \log_e 16$

$\Rightarrow y\log_e x + x\log_e y = 4\log_e 2$

Now, on differentiating both sides w.r.t. x, we get

$$\frac{y}{x} + \log_e x \frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log_e y . 1 = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(\log_e y + \frac{y}{x})}{(\log_e x + \frac{x}{y})}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{(\log_e 2 + 1)}{(\log_e 2 + 1)} = -1$$

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Q. If $x^y . y^x = 16$, *then the value of* $\frac{dy}{dx}$ at (2, 2) is

A -1

B 0

C 1

D None of these

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JEE(Main): 01 Feb 2023 -II

Q. If $y(x)=x^x$, $x>0$, then $y''(2)-2y'(2)$ is equal to:

$\ln y = x \ln x$

$\frac{1}{y} y' = 1 \ln x + x \cdot \frac{1}{x}$

$y' = x^x (\ln x + 1)$

$y'' = x^x (\ln x + 1)^2 + x^x \cdot \frac{1}{x}$

A. $8 \log_e 2 - 2$

B. $4 \log_e 2 + 2$

C. $(4 \log_e 2)^2 - 2$

D. $(4 \log_e 2)^2 + 2$

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Solution:

Given, $y = x^x$

$\Rightarrow \ln(y) = x\ln(x)$

Differentiate both sides w.r.t. x

$$\frac{1}{y}\cdot y' = 1 + \ln(x)$$

$$\Rightarrow y' = y(1+\ln x) \ldots\ldots\ldots (1)$$

$$\Rightarrow y' = x^x(1+\ln x)$$

at $x = 2 \Rightarrow y = 4$

So, $y'(2) = 4(1+\ln 2) \ldots\ldots\ldots (2)$

Now differentiate the equation (1),

$$y'' = y'(1+\ln x) + \frac{y}{x}$$

$$\Rightarrow y''(2) = y'(1+\ln 2) + 2$$

$$\Rightarrow y''(2) - y'(2) = y'(\ln 2) + 2$$

$$\Rightarrow y''(2) - 2y'(2) = (\ln 2 - 1)y'(2) + 2$$

$$= 4(\ln 2 - 1)(\ln 2 + 1) + 2$$

$$= 4(\ln 2)^2 - 2$$

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JEE(Main): 01 Feb 2023 -II

Q. If $y(x)=x^x$, $x>0$, then $y''(2)-2y'(2)$ is equal to:

A $8\log_e 2-2$

B $4\log_e 2+2$

C $(4\log_e 2)^2-2$

D $(4\log_e 2)^2+2$

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CLICK

JEE(Main): 6th April 2023 -I

Q. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to:

A $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

B $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

C $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

D $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

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Solution:

$$2x^y + 3y^x = 20$$

$$v_1^{v_2}\left(v_2\frac{1}{v_1} + \ln v_1 \cdot v_2^1\right)$$

$$2x^y\left(y\cdot\frac{1}{x} + \ln x\frac{dy}{dx}\right) + 3y^x\left(x\frac{1}{y}\cdot\frac{dy}{dx} + \ln y\cdot 1\right) = 0$$

Put $(2, 2)$

$$2.4\left(1 + \ln 2\frac{dy}{dx}\right) + 3.4\left(1\cdot\frac{dy}{dx} + \ln 2\right) = 0$$

$$\frac{dy}{dx}[8\ln 2 + 12] + 8 + 12\ln 2 = 0$$

$$\frac{dy}{dx} = -\left[\frac{2 + 3\ln 2}{3 + 2\ln 2}\right] = -\left[\frac{2 + \ln 8}{3 + \ln 4}\right]$$

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JEE(Main): 6th April 2023 -I

Q. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to:

A $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

B $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

C $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

D $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

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DIFFERENTIATION BY TRIGONOMETRIC SUBSTITUTIONS

	Function	Substitution	
(i)	$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $a\cos\theta$	where $\theta \in \left[0, \frac{\pi}{2}\right]$
(ii)	$\sqrt{x^2 + a^2}$	$x = a\tan\theta$ or $a\cot\theta$	where $\theta \in \left(0, \frac{\pi}{2}\right)$
(iii)	$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $a\operatorname{cosec}\theta$	where $\theta \in \left(0, \frac{\pi}{2}\right)$
(iv)	$\sqrt{\frac{a-x}{a+x}}$	$x = a\cos 2\theta$	where $\theta \in \left[0, \frac{\pi}{2}\right]$

$1 - \cos 2\theta = 2\sin^2\theta$

$1 + \cos 2\theta = 2\cos^2\theta$

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Q. If $x = \dfrac{1-t^2}{1+t^2}$ and $y = \dfrac{2t}{1+t^2}$, then $\dfrac{dy}{dx}$ is equal to

$t = \tan\theta$

$\Rightarrow x = \cos 2\theta$

$\Rightarrow y = \sin 2\theta$

$\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{2\cos 2\theta}{-2\sin 2\theta} = -\dfrac{x}{y}$

A $-\dfrac{y}{x}$

B $\dfrac{y}{x}$

C $-\dfrac{x}{y}$

D $\dfrac{x}{y}$

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Solution:

$\because x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan\theta$ in both the equations, we get

$$x = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta \ldots(i)$$

$$\text{and } y = \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta \ldots(ii)$$

On differentiating both the eqs.(i) and (ii), we get

$$\frac{dx}{d\theta} = -2\sin 2\theta \text{ and } \frac{dy}{d\theta} = 2\cos 2\theta$$

$$\text{Therefore, } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$$

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Q. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

A $-\frac{y}{x}$

B $\frac{y}{x}$

C $-\frac{x}{y}$

D $\frac{x}{y}$

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Q. The value of $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]$ is, if $x \in \mathbf{R} - \left\{\frac{1}{3}\right\}$

$y = 3\tan^{-1}\sqrt{x}$

$\Rightarrow y' = \frac{3}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}}$

$\sqrt{x} = \tan\theta$

$\tan^{-1}\left(\frac{\tan\theta(3-\tan^2\theta)}{1-3\tan^2\theta}\right)$

$\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}(\tan 3\theta) = 3\theta$

A $\frac{1}{2(1+x)\sqrt{x}}$

B $\frac{3}{(1+x)\sqrt{x}}$

C $\frac{2}{(1+x)\sqrt{x}}$

D $\frac{3}{2(1+x)\sqrt{x}}$

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Solution:

If $x \in \mathbf{R} - \left\{\frac{1}{3}\right\}$, then

$$\frac{d}{dx}\left(\tan^{-1}\frac{(\sqrt{x}(3-x))}{1-3x}\right)$$

$$= \frac{d}{dx}\left(\tan^{-1}\frac{\tan\theta(3-\tan^2\theta)}{1-3\tan^2\theta}\right)$$

(putting $\sqrt{x} = \tan\theta$ or $\theta = \tan^{-1}\sqrt{x}$)

$$= \frac{d}{dx}\left(\tan^{-1}\frac{(3\tan\theta-\tan^2\theta)}{1-3\tan^2\theta}\right)$$

$$= \frac{d}{dx}\left[\tan^{-1}(\tan 3\theta)\right] = \frac{d}{dx}(3\theta)$$

$$= \frac{d}{dx}\left[3\tan^{-1}\sqrt{x}\right] = \frac{3}{2\sqrt{x}(1+x)}$$

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Q. The value of $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]$ is, if $x \in \mathbf{R} - \left\{\frac{1}{3}\right\}$

A $\frac{1}{2(1+x)\sqrt{x}}$

B $\frac{3}{(1+x)\sqrt{x}}$

C $\frac{2}{(1+x)\sqrt{x}}$

D $\frac{3}{2(1+x)\sqrt{x}}$

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Differentiation of Infinite Series

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Q. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$, then $\frac{dy}{dx} =$

$y = \sqrt{x + y}$

$y^2 = x + y$

$2yy' = 1 + y'$

$\Rightarrow y' = \frac{1}{2y - 1}$

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Given, $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$

$\Rightarrow y = \sqrt{x + y}$

$\Rightarrow y^2 = x + y$

$\Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow (2y - 1)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y - 1}$

Hence, if $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$, then $\frac{dy}{dx} = \frac{1}{2y - 1}$

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Q. If $y = \sqrt{\sin x + \underbrace{\sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}_{y}}$, then $\frac{dy}{dx} =$

$y = \sqrt{\sin x + y}$

$y^2 = \sin x + y$

$2y\,y' = \cos x + y'$

$\Rightarrow y' = \frac{\cos x}{2y - 1}$

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Given, $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$

$\Rightarrow y = \sqrt{\sin x + y}$

Squaring both side in above the equation,

$\Rightarrow y^2 = \sin x + y$

$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$\Rightarrow (2y - 1)\frac{dy}{dx} = \cos x$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

Hence, if $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$ then $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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Q. Let $y = x^{x^{x^{..\infty}}}$, *then* $\frac{dy}{dx}$ is equal to (given x>0)

$y = x^y$

$\ln y = y \ln x$

$\Rightarrow \frac{1}{y} y' = y' \ln x + \frac{y}{x}$

$\Rightarrow y\left(\frac{1}{y} - \ln x\right) = \frac{y}{x}$

A yx^{y-1}

B $\frac{y^2}{x(1-y\ln x)}$

C $\frac{y}{x(1+y\ln x)}$

D None of these

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Solution:

Since by deleting a single term from an infinite series, it remains same.

Therefore, the given function may be written as

$y = x^y$

$\Rightarrow \log y = y \log x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx} \times \log x + y\frac{1}{x} \qquad \text{[Diff. both sides w.r.t. } x\text{]}$$

$$\Rightarrow \frac{dy}{dx}\frac{\{1 - y\log x\}}{y} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$$

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Q. Let $y = x^{x^{x..\infty}}$, *then* $\frac{dy}{dx}$ is equal to (given x>0)

A yx^{y-1}

B $\frac{y^2}{x(1-y\ln x)}$

C $\frac{y}{x(1+y\ln x)}$

D None of these

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Higher order Derivatives

Let y = f(x), then the higher order differentiation of this function is given by

First Derivative: $\frac{dy}{dx} = \frac{d}{dx}(y) = \frac{d}{dx} f(x) = f'(x) = y'$

Second Derivative: $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x) = y''$

Third Derivative: $\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = f'''(x) = y'''$

Fourth Derivative: $\frac{d^4y}{dx^4} = \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right) = f^{(4)}(x) = y^{(4)}$

nth Derivative: $\frac{d^ny}{dx^n}, n = 1, 2, 3, 4, 5.....$

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Note :

$$\frac{d^2y}{dx^2} \neq \frac{f''(t)}{g''(t)} \neq \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$$

$\frac{d^2y}{dx^2}$ → $\frac{d}{dx}\left(\frac{dy}{dx}\right)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\quad) = (\quad)\cdot\frac{dt}{dx}$$

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NTA Paper 15

Q. If $y = a\cos(\log x) + b\sin(\log x)$, where a & b are parameters, then $x^2y'' + xy'$ is equal to

$$y' = -a\sin(\log x)\times\frac{1}{x} + b\cos(\log x)\times\frac{1}{x}$$

$$\Rightarrow xy' = -a\sin(\log x) + b\cos(\log x)$$

Diff wrt x

$$\Rightarrow 1y' + xy'' = -\frac{a\cos(\log x)}{x} - \frac{b\sin(\log x)}{x}$$

$$\Rightarrow xy' + x^2y'' = -(y)$$

A. y

B. -y

C. 2y

D. -2y

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$$y = a\cos(\log x) + b\sin(\log x)$$

On differentiating w.r.t. x, we get

$$y' = \frac{-a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$$

$$\Rightarrow xy' = -a\sin(\log x) + b\cos(\log x)$$

Again, on differentiating w.r.t. x, we get

$$xy'' + y' = -a\cos(\log x)\frac{1}{x} - b\sin(\log x)\frac{1}{x}$$

$$\Rightarrow x^2y'' + y'x = -[a\cos(\log x) + b\sin(\log x)]$$

$$\Rightarrow x^2y'' + xy' = -y$$

NEHA MA'AM'S ARMY

NTA Paper 15

Q. If $y = a\cos(\log x) + b\sin(\log x)$, where a & b are parameters, then $x^2y'' + xy'$ is equal to

A. y

B. -y

C. 2y

D. -2y

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NEHA MA'AM'S ARMY

Q. $f(x) = e^x - e^{-x} - 2\sin x - \frac{2}{3}x^3.$

Then the least value of n for which $\left.\frac{d^n}{dx^n} f(x)\right|_{x=0}$ is nonzero is

$f^{VII}(x) = e^x + e^{-x} + 2\cos x$

$x = 0$

$1 + 1 + 2 = 4$

$\neq 0$

$f'(x) = e^x + e^{-x} - 2\cos x - \frac{2}{3}(3x^2)$ → 0

$f''(x) = e^x - e^{-x} + 2\sin x - 4x$ → 0

$f'''(x) = e^x + e^{-x} + 2\cos x - 4$ → 0

$f^{IV}(x) = e^x - e^{-x} - 2\sin x$ → 0

$f^{V}(x) = e^x + e^{-x} - 2\cos x$ → 0

$f^{VI}(x) = e^x - e^{-x} + 2\sin x$ → 0

A 5

B 6

C 7

D 8

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Q.

$$f(x) = e^x - e^{-x} - 2\sin x - \frac{2}{3}x^3.$$

Then the least value of n for which $\left.\frac{d^n}{dx^n} f(x)\right|_{x=0}$ is nonzero is

A 5

B 6

C 7

D 8

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