GATE

Previous Questions with solutions

Subject wise & Chapter wise

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Preface

Dear Reader,

First and foremost let me discuss about GATE exam. GATE is basically an objective type examination, conducts IITs and IISc in the month of February every year. Now days GATE examination gained lot of importance because, not only for M.Tech admission but also for Job in PSU. These PSUs are providing fascinating career to young engineering graduates with excellent packages.

So now question is all about how to crack this exam? For this exam one need to prepare according to syllabus provided in notification. In GATE exam basically examiners test your basics and concepts in each and every subject according to their weightages. So, one need to know clearly what to prepare for secure good rank, for this Vani Institute is providing solution with this book.

In this book we are providing Mathematics previous years questions with solutions. One can use this book for practice and quick revision. student will understand clearly what to focus in each topic.

We developed to the best of our knowledge, in case any mistake and suggestions please feel free to inform us.

DIRECTOR,

VANI INSTITUTE

SYLLABUS

COMMON MATHS FOR ALL BRANCHES

1. **Linear Algebra:**
   Linear Algebra: Finite dimensional vector spaces; Linear transformations and their matrix representations, rank; systems of linear equations, eigen values and eigen vectors, minimal polynomial, Cayley-Hamilton Theroem, diagonalisation, Hermitian, Skew-Hermitian and unitary matrices; Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, self-adjoint operators.
2. **Fourier Series :** Fourier series
3. **Probability and statistics:**
   Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis
4. **Calculus:**
   Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial derivatives, Maxima and Minima, Multiple integrals.
5. **Numerical Methods:**
   Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.
6. **Differential equations:**
   First order equation (linear and non-linear), Higher order linear differential equations with constant coefficients, methods of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial Differential Equations and Variable separable method.
7. **Complex variables:**
   Analytic functions, Cauchy's integral theorem and integral formula, Taylors and Laurent's series, Residue theorem, Solution integrals.
8. **Vector Calculus:** Vector identities, directional derivatives, line, surface and volume integrals, Stokes, Gauss and Green's theorems.
9. **Laplace Transforms:** Linear Property, First shifting theorem , change of scale property, second shifting theorem, multiplication by 't', division by 't', Laplace transform of integral, inverse Laplace transform, Convolution theorem.

Previous GATE Questions & Solutions to Engineering Mathematics topic wise

CONTENTS

CHAPTER- 1
LINEAR ALGEBRA

01. The eigen vector(s) of the matrix $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \alpha \neq 0$ is (are) **(GATE-93)**

(a) $(0,0,\alpha)$ (b) $(\alpha,0,0)$ (c) $(0,0,1)$ (d) $(0,\alpha,0)$

02. If $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{bmatrix}$ the matrix A^4, Calculated by the use of Cayley-Hamilton theorem (or) otherwise is **(GATE-93)**

03. If A and B are real symmetric matrices of order n then which of the following is true. **(GATE-94[CS])**

(a) $AA^T = I$ (b) $A = A^{-1}$ (c) $AB = BA$ (d) $(AB)^T = B^T A^T$

04. The inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ is **(GATE-94)**

05. A 5×7 matrix has all its entries equal to -1. Then the rank of a matrix is **(GATE-94[EE])**

(a) 7 (b) 5 (c) 1 (d) zero

06. The eigen values of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are **(GATE-94[EE])**

(a) (a+1), 0 (b) a, 0 (c) (a-1), 0 (d) 0, 0

07. The number of linearly independent solutions of the system of equations

$\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ is equal to **(GATE-94[EE])**

(a) 1 (b) 2 (c) 3 (d) 0

08. The rank of $(m \times n)$ matrix $(m<n)$ cannot be more than **(GATE-94[EC])**

(a) m (b) n (c) mn (d) none

09. Solve the following system **(GATE-94[EC])**

$$x_1 + x_2 + x_3 = 3$$
$$x_1 - x_2 = 0$$
$$x_1 - x_2 + x_3 = 1$$

(a) unique solution (b) No solution

(c) Infinite number of solutions (d) Only one solution

10. The rank of matrix $\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$ is **(GATE-94[CS])**

11. The matrix $\begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$ is an inverse of the matrix $\begin{bmatrix} 5 & -4 \\ 1 & -1 \end{bmatrix}$ **(GATE-94[PI])**

(a) True (b) False

12. If for a matrix, rank equals both the number of rows and number of columns, then the matrix is called **(GATE-94[PI])**

(a) Non-singular (b) Singular (c) Transpose (d) Minor

13. The value of the following determinant $\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$ is **(GATE-94[PI])**

(a) 8 (b) 12 (c) -12 (d) -8

14. For the following matrix $\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$ the number of positive roots is **(GATE-94[PI])**

(a) one (b) two (c) four (d) cannot be found

15. Rank of the matrix $\begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix}$ is 3 **(GATE-94[ME])**

(a) True (b) False

16. Find out the eigen value of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$ for any one of the eigen values, find out the corresponding eigen vector? **(GATE-94[ME])**

17. Given matrix $L = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$ and $M = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ then L×M is **(GATE-95[PI])**

(a) $\begin{bmatrix} 8 & 1 \\ 13 & 2 \\ 22 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 8 \\ 2 & 13 \\ 5 & 22 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 2 \\ 9 & 4 \\ 0 & 5 \end{bmatrix}$

18. Solve the system $2x + 3y + z = 9, 4x + y = 7, x - 3y - 7z = 6$ **(GATE-95[ME])**

19. Among the following, the pair of the vector orthogonal to each other is **(GATE-95[ME])**

(a) $[3,4,7],[3,4,7]$ (b) $[1,0,0],[1,1,0]$ (c) $[1,0,2],[0,5,0]$ (d) $[1,1,1],[-1,-1,-1]$

20. The inverse of the matrix $S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ **(GATE-95[EE])**

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$

21. Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$ its eigen values are **(GATE-95[EE])**

22. The matrix of the following (n+1) × (n+1) matrix, where 'a' is a real number is

$$\begin{bmatrix} 1 & a & a^2 \cdots\cdots a^n \\ 1 & a & a^2 \cdots\cdots a^n \\ \vdots & & \\ 1 & a & a^2 \cdots\cdots a^n \end{bmatrix}$$

(GATE-95[EE])

(a) 1 (b) 2 (c) n (d) depends on the value of a

23. Let AX=B be a system of linear equations where A is an m×n matrix B is an n×1 column Matrix which of the following is false? **(GATE-96[CS])**

(a) The system has a solution, if $\rho(A) = \rho(A/B)$

(b) If m = n and B is a non-zero vector then the system has a unique solution

(c) If m<n and B is a zero vector then the system has infinitely many solutions.

(d) The system will have a trivial solution when m=n, B is the zero vectors and rank of A is n.

24. The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication.

(GATE-96[CS])

(a) If a=b (or) $\theta = n\pi$, n is an integer (b) always

(c) never (d) If $a\cos\theta \neq b\sin\theta$

25. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be two matrices such that AB = I. Let C= A$\begin{bmatrix} 1\,0 \\ 1\,1 \end{bmatrix}$ and CD = I. Express the elements of D in terms of the elements of B.

(GATE-96[CS])

26. The eigen values of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are **(GATE- 96[ME])**

(a) 0,0,0 (b) 0,0,1 (c) 0,0,3 (d) 1,1,1

27. In the Gauss – elimination for a solving system of liner algebraic equations, triangularization leads to **(GATE- 96[ME])**

(a) Diagonal matrix (b) lower triangular matrix

(c) upper triangular matrix (d) angular matrix

28. Let $[A]_{n\times n}$ be matrix of order n and I_{12} be the matrix obtained by inter changing the first and second rows of I_n. then A I_{12} is such that its first. **(GATE- 97[CS])**

(a) Row is the same as its second row (b) row is the same as second row of a

(c) column is the same as the second column of A (d) row is a zero row.

29. Express the given matrix A = $\begin{bmatrix} 2 & 1 & 5 \\ 4 & 8 & 13 \\ 6 & 27 & 31 \end{bmatrix}$ as a product of triangular matrices L and U where the diagonal elements of the lower triangular matrices L are unity and U is an upper triangular matrix. **(GATE- 97[EE])**

30. For the following set of simultaneous equations

$1.5x - 0.5y + z = 2$

$4x + 2y + 3z = 9$

$7x + y + 5z = 10$

(a) The system is unique (b) infinitely many solutions exist

(c) the equations are incompatible (d) finite many solutions exist

31. If the determinant of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ is 26, then the determinant of the matrix $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$ is **(GATE- 97[CE])**

(a) -26 (b) 26 (c) 0 (d) 52

32. If A and B are two matrices and AB exists then BA exists , **(GATE- 97[CE])**

(a) only if A has as many rows as B has columns

(b) only if both A and B are square matrices

(c) only if A and B are skew matrices (d) Only if both A and B are symmetric

33. Inverse of matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is **(GATE- 97[CE])**

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

34. The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ is **(GATE- 97[CS])**

(a) 11 (b) -48 (c) 0 (d) -24

35. If $\Delta = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$ then which of the following is a factor ofΔ. **(GATE- 98[CS])**

(a) a+b (b) a-b (c) abc (d) a+b+c

36. Consider the following set of equations x+2y=5, 4x+8y=12, 3x+6y+3z=15.
This set **(GATE- 98[CS])**

(a) Has unique solution (b) has no solutions

(c) has infinite number of solutions (d) has 3 solutions

37. The rank of matrix $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$ is **(GATE- 98[CS])**

(a) 3 (b) 1 (c) 2 (d) 4

38. If A is a real square matrix then $\boldsymbol{AA^T}$ is **(GATE- 98[CE])**

(a) un symmetric (b) always symmetric

(c) skew – symmetric (d) some times symmetric

39. In matrix algebra AS = AT (A, S, T, are matrices of appropriate order) implies S=T only if **(GATE- 98[CE])**

(a) A is symmetric (b) A is singular

(c) A is non- singular (d) A is skew- symmetric

40. The real symmetric matrix C corresponding to the quadratic form Q=4 $\boldsymbol{x_1 x_2}$-5$\boldsymbol{x_2 x_2}$ is **(GATE- 98[CE])**

(a) $\begin{bmatrix} 1 & 2 \\ 2 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2 \\ 2 & -5 \end{bmatrix}$

41. Obtain the eigen values and eigen vectors of A= $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ **(GATE- 98[CE])**

42. The eigen values of the matrix A= $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ are **(GATE- 98[CE])**

(a) 1,1 (b) -1,-1 (c) j,-j (d) 1,-1

43. If the vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then one of the eigen value of A is **(GATE- 98[EE])**

(a) 1 (b) 2 (c) 5 (d) -1

44. A= $\begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$ the sum of the eigen values of the matrix A is **(GATE- 98[EE])**

(a) 10 (b) -10 (c) 24 (d) 22

45. If A= $\begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ then $\boldsymbol{A^{-1}}$ = **(GATE- 98[EE])**

(a) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1/3 & 0 \\ -2 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & 2 \\ 0 & -1/3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1/5 & 0 & 1/2 \\ 0 & 1/3 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1/5 & 0 & -1/2 \\ 0 & 1/3 & 0 \\ -1/2 & 0 & 1 \end{bmatrix}$

46. A set of linear equation is represented by the matrix equations Ax = b. The necessary condition for the existence of a solution for this system is **(GATE- 98[EE])**

(a) A must be invertible

(b) b must be linearly dependent on the columns of A

(c) b must be linearly independent on the rows of A (d) None

47. If A= $\begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$ and adj (A)= $\begin{bmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & k & 7 \end{bmatrix}$ then k= **(GATE- 99)**

(a) -5 (b) 3 (c) -3 (d) 5

48. Find the eigen values and eigen vectors of The matrix $\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ **(GATE- 99)**

49. If A is any nxn matrix and k is a scalar then $|kA| = \alpha\, |A|$ where α is **(GATE- 99[CE])**

(a) kn (b) $\boldsymbol{n^k}$ (c) $\boldsymbol{k^n}$ (d) $\boldsymbol{\frac{k}{n}}$

50. The number of terms in the expansion of general determinant of order n is

(GATE- 99[CE])

$(\boldsymbol{a})\boldsymbol{n^2}$ (b) n! (c) n (d) $(\boldsymbol{n+1})^{\,\boldsymbol{2}}$

51. The equation $\begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{-1} \\ \boldsymbol{y} & \boldsymbol{x^2} & \boldsymbol{x} \end{bmatrix}$ = 0 reperents a parabola passing the points.

(GATE- 99[CE])

(a) (0, 1), (0,2), (0,-1) (b) (0,0),(-1,1),(1,2)

(c) (1,1), (0,0),(2,2) (d) (1,2),(2,1),(0,0)

52. An n × n array V is defined as follows v [i,j] = i-j for all i,j, 1≤ i,j ≤ n then the sum of the elements of the array V is **(GATE- 2000[CS])**

(a) $\boldsymbol{O_2}$ (b) n-1 (c) $\boldsymbol{n^2}$-3n+2 (d) n (n+1)

53. The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is **(GATE- 2000[CS])**

(a) 4 (b) 0 (c) 15 (d) 20

54. If A, B, C are square matrices of the same order then $(\boldsymbol{ABC})^{-\mathbf{1}}$ is equal be

(GATE- 2000[CS])

(a) $C^{-1}A^{-1}B^{-1}$ (b) $C^{-1}B^{-1}A^{-1}$ (c) $A^{-1}B^{-1}C^{-1}$ (d) $A^{-1}C^{-1}B^{-1}$

55. Consider the following two statements. **(GATE- 2000[CS])**

(I) The maximum number of linearly independent column vectors of a matrix A is called the Rank of A

(II) If A is n×n square matrix then it will be non-singular if rank of A=n

(a) Both the statements are false (b)Both the statements are true

(c) (I) is true but (II) is false (d) (I) is false but (II) is true

56. The eigen valus of the matrix $\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 4 \end{bmatrix}$ are **(GATE- 2000[EC])**

(a) 2,-2,1,-1 (b) 2,3,-2,4 (c) 2,3,1,4 (d) None

57. The rank of matrix A = $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{3} & \mathbf{4} & \mathbf{5} \\ \mathbf{4} & \mathbf{6} & \mathbf{8} \end{bmatrix}$ is **(GATE- 2000[IN])**

(a) 0 (b) 1 (c) 2 (d) 3

58. Consider the following statements

S_1 : The sum of two singular matrices may be singular.

S_2 : The sum of two non- singular may be non- singular.

Which of the following statements is true? **(GATE- 01[CS])**

(a) S_1 & S_2 are both true (b) S_1 & $S_2$are both false

(c) S_1 is true and S_2 is false (d) S_1 is false and S_2 is true

59. The necessary condition to diagonalize a matrix is that **(GATE- 01[CS])**

(a) all its eigen value should be distinct (b) its eigen vectors should be independent

(c) its eigen values should be real (d) the matrix is non- singular

60. The determinant of the following matrix $\begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{bmatrix}$ **(GATE- 01[CE])**

(a) -76 (b) -28 (c) 28 (d) 72

61. The eigen values of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 9 \end{bmatrix}$ are **(GATE- 01[CE])**

(a) (5. 13,9.42) (b) (3.85,2.93) (c) (9.00,5.00) (d) (10.16,3.84)

62. The product [P] $[Q]^T$ of the following two matrics [P] and [Q] **(GATE- 01[CE])**

Where [P] = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, [Q] = $\begin{bmatrix} 4 & 8 \\ 9 & 2 \end{bmatrix}$

(a) $\begin{bmatrix} 32 & 24 \\ 56 & 46 \end{bmatrix}$ (b) $\begin{bmatrix} 46 & 56 \\ 24 & 32 \end{bmatrix}$ (c) $\begin{bmatrix} 35 & 22 \\ 61 & 42 \end{bmatrix}$ (d) $\begin{bmatrix} 32 & 56 \\ 24 & 46 \end{bmatrix}$

63. The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is **(GATE- 02[CS])**

(a) 4 (b) 2 (c) 1 (d) 0

64. Obtain the eigen values of the matrix A = $\begin{bmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ **(GATE- 02[CS])**

(a) 1,2,-2,-1 (b) -1,-2,-1,-2 (c) 1,2,2,1 (d) None

65. The determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix}$ is **(GATE- 02[EE])**

(a) 100 (b) 200 (c) 1 (d) 300

66. Eigen values of the following matrix are $\begin{bmatrix} -1 & 4 \\ 4 & -1 \end{bmatrix}$ **(GATE- 02[CE])**

(a) 3,-5 (b) -3,5 (c) -3,-5 (d) 3,5

67. Consider the following system of linear equations $\begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$

(GATE- 03[CS])

Notice that the 2^{nd} and 3^{rd} column of the coefficient matrix are linearly dependent. For how many value of α, does systems of equations have infinitely many solutions.

(a) 0 (b) 1 (c) 2 (d) infinitel many.

68. Given matrix [A] = $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$, the rank of the matrix is **(GATE- 03[CE])**

(a) 4 (b) 3 (c) 2 (d) 1

69. A system of equations represented by AX = 0 where X is a column vector of unknown and A is a matrix containing coefficient has a non- trivial solution when A is.

(GATE- 03)

(a) non-singular (b) singular (c) symmetric (d) hermitian

70. What values of X, Y, Z satisfy the following system of linear equations **(GATE- 04)**

(a) X= 6, Y= 3, Z= 2 (b) X= 12, Y= 3, Z= -4

(c) X= 6, Y= 6, Z= -4 (d) X= 12, Y= -3, Z= 4

71. If matrix X=$\begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$ and X^2-X+1=0 then the inverse of X is **(GATE- 04)**

(a) $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$ (b) $\begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$

(c) $\begin{bmatrix} -a & 1 \\ -a^2+a-1 & a-1 \end{bmatrix}$ (d) $\begin{bmatrix} a^2-a+1 & a \\ 1 & 1-a \end{bmatrix}$

72. The number of different n×n symmetric matices with each elements being either 0 or 1 is **(GATE- 04[CS])**

(a) 2^n (b) 2^{n^2} (c) $2^{\frac{n^2+n}{2}}$ (d) $2^{\frac{n^2-n}{2}}$

73. Let A, B, C, D be n×n matrices, each with non- zer5o determinant ABCD= I then B^{-1} = **(GATE- 04[CS])**

(a) $D^{-1}C^{-1}A^{-1}$ (b) CDA (c) ABC (d) does not exist

74. How many solutions does the following system of linear equations have

-X+5Y= -1 X-Y=2 X+3Y=3 **(GATE- 04[CS])**

(a) Infinitely many (b) two distinct solutions (c) unique (d) none

75. The some of the eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is **(GATE- 04[ME])**

(a) 5 (b) 7 (c) 9 (d) 18

76. For what value of X will the matrix given below become singular? $\begin{bmatrix} 8 & X & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$

(GATE- 04[ME])

77. Real matrices$[A]_{3\times1}$, $[B]_{3\times3}$, $[C]_{3\times5}$, $[D]_{5\times3}$, $[E]_{5\times5}$, $[F]_{5\times1}$ are given. Matrices [B] and [E] are symmetric. Following statements are made with respect to their matrices.

(I) Matric product $[F]^T$ $[C]^T$ [B] [C] [F] is a scalar.

(II) Matric product $[D]^T$ [F] [D] is always symmetric.

With reference to above statements which of the following applies? **(GATE- 04[CE])**

(a) Statement (I) is true but (II) is false (b) Statement (I) is false but (II) is true

(c) Both the statements are true (d) both statements are false

78. The eigen values of the matrix $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ are **(GATE- 04[CE])**

(a) 1,4 (b) -1,2 (c) 0,5 (d) cannot be determined

79. Considered the following system of equations in three real variable X_1, X_2 and X_3 :

$2X_1 - X_2 + 3X_3 = 1$ $3X_1 + 2X_2 + 5X_3 = 2$ $-X_1 + 4X_2 + X_3 = 3$

This system of equations has **(GATE- 05[CE])**

(a) No solution (b) a unique solution

(c) More than one but a finite number of solutions (d) an infinite number of solutions

80. Consider a non- homogeneous system of linear equations represents mathematically an over determined system. Such a system will be **(GATE- 05[CE])**

(a) Consistent having a unique solution (b) Consistent having many solutions.

(c) Inconsistent having a unique solution (d) Inconsistent having no solution

81. What are the given values of the following 2×2 matrix $\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ **(GATE- 05[CS])**

(a) -1,1 (b) 1, 6 (c) 2, 5 (d) 4, -1

82. Consider the matrices $X_{4\times3}$, $Y_{4\times3}$, and $P_{4\times3}$. The order of $[P(X^T \; Y)^{-1} P^T]^T$ will be **(GATE- 05[CE])**

(a) 2×2 (b) 3×3 (c) 4×3 (d) 3×4

83. The deteminant of the matrix given below is $\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$ **(GATE- 05[CE])**

(a) -1 (b) 0 (c) 1 (d) 2

84. Consider the system of equations, $X_{n\times1} = \lambda X_{n\times1}$ where λ is a scalar. Let (λ_i, X_i) be an eigen value and its corresponding eigen vector for real matrix A. Let $I_{n\times n}$ be unti matrix. Which one of the following statement is not correct? **(GATE- 05[CE])**

For a homogeneous n×n system of linear equations (A- λI) X = 0, having a non

(a) Trivial solution, the rank of (A- λI) is less than n.

(b) For matrix(λ_i^m , X_i^m) will be eigen pair for all i.

(c) If $A^T = A^{-1}$ THEN $|\lambda_i|$=1 for all i. (d) If $A^T = A$ then λ_i are real for all i.

85. In the matrix equation PX= Q which of the following is a necessary condition for one solution for the existence of atleast one solution for the unknown vector X. Argumented matrix [P|Q] must have the same rank as matrix P . **(GATE- 05[EE])**

Vector Q must have only non-zero elements.

Matrix P must be singular Matrix P must be square

86. For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is -2. Which of the following is an eigen Vector? **(GATE- 05[EE])**

(a) $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

87. If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$ then the top row of R^{-1} is **(GATE- 05[EE])**

(a) [5 6 7] (b) [5 -3 1] (c) [2 0 -1] (d) [2 -1 0]

88. The eigen values of the matrix M given below are 15, 3 and 0. $M = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, the value of the determinant of a matrix is **(GATE- 05[PI])**

(a) 20 (b) 10 (c) 0 (d) -10

89. Identify which one of the following is an eigen vector of the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$ **(GATE- 05[IN])**

(a) $[-1 \quad 1]^T$ (b) $[3 \quad -1]^T$ (c) $[1 \quad -1]^T$ (d) $[-2 \quad 1]^T$

90. A is a 3 × 4 matrix and AX= B is an inconsistent system of equations. The highest Possible rank of A is **(GATE- 05[ME])**

(a) 1 (b) 2 (c) 3 (d) 4

91. If $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$ then a+b = **(GATE- 05[EE])**

(a) $\frac{7}{20}$ (b) $\frac{3}{20}$ (c) $\frac{19}{60}$ (d) $\frac{11}{20}$

92. Which one of the following is an eigen vector of the matrix $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$ is **(GATE- 05[IN])**

(a) $[1 \quad -2 \quad 0 \quad 0]^T$ (b) $[0 \quad 0 \quad 1 \quad 0]^T$ (c) $[1 \quad 0 \quad 0 \quad -2]^T$ (d) $[1 \quad -1 \quad 2 \quad 1]^T$

93. Let A be 3×3 matrix with rank 2. Then AX= O has **(GATE- 05[IN])**

(a) only the trivial solution X = O (b) one independent solution

(c) two independent solutions (d) there independent solutions

94. Given an orthogonal matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $(A \quad A^T)^{-1}$ is **(GATE- 05[EC])**

(a) $\frac{1}{4}I_4$ (b) $\frac{1}{2}I_4$ (c) I (d) $\frac{1}{3}I_4$

95. Given the matrix $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$ the eigen vector is **(GATE- 05[EC])**

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

96. Eigen values of a matrix S = $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigen values of the matrix S^2 = SS ? **(GATE- 05[EC])**

(a) 1 and 25 (b) 6, 5 (c) 5, 1 (d) 2, 10

97. Multiplication of matrices E and F is G. Matrices E and G are

E = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and G = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ what is the matrix F? **(GATE- 06[ME])**

(a) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

98. For a given 2×2 matrix A, it is observed that A $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ = -1 $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and A $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ = -2 $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ then the matrix A is **(GATE- 06[IN])**

(a) A = $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ (b) A = $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(c) A = $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ (d) A = $\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

99. A system of linear simultaneous equations is given as AX = b

Where A = $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ & b = $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ then the rank of matrix A is **(GATE- 06[IN])**

(a) 1 (b) 2 (c) 3 (d) 4

100. A system of linear simultaneous equations is given as Ax = b. Where A = $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and b = $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ which of the following statement is true? **(GATE- 06[IN])**

(a) x is a null vector (b) x is unique

(c) x does not exist (d) x has infinitely many values

101. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is **(GATE- 06[EC])**

(a) 0 (b) 1 (c) 2 (d) 3

102. The eigen values and the corresponding eigen vectors of a 2×2 matrix are given by

Eigen Value	Eigen Vector
$\lambda_1 = 8$	$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 4$	$V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is **(GATE- 06[EC])**

(a) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

103. For the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$. The eigen value corresponding to the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is

(GATE- 06[EC])

(a) 2 (b) 4 (c) 6 (d) 8

104. Solution for the system defined by the set of equations 4y + 3z = 8, 2x – z = 2 & 3x + 2y = 5 is **(GATE- 06[EC])**

(a) X = 0 , y = 1, z = 4/5 (b) X = 0 , y = 1/2, z = 2

(c) X = 1 , y = 1/2, z = 2 (d) non existent

105. For a given matrix A = $\begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen value is 3. The other two eigen values are **(GATE- 06[CE])**

(a) 2, -5 (b) 3, -5 (c) 2, 5 (d) 3, 5

106. Let A be an n×n real matrix such that A^2 = I and Y be an n-dimensional vector. Than the linear system of equations Ax = Y has **(GATE- 07[IN])**

(a) No solution (b) unique solution

(c) more than only but infinitely many dependent solutions.

(d) infinitely many dependent solutions.

107. Let A = [a_{ij}], 1≤ i, j≤ n with n ≥ 3 and a_{ij} = i.j. Then the rank of A is

(GATE- 07[IN])

(a) 0 (b) 1 (c) n-1 (d) n

108. The minimum and maximum eigen values of matrix $\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ are -2 and 6 respectively. What is the other eigen value? **(GATE- 07[CE])**

(a) 5 (b) 3 (c) 1 (d) -1

109. For what values of α and β the following simultaneous equations have an infinite Number of solutions x + y + z = 5, x + 3y + 3z = 9, x + 2y + αz = β

(GATE- 07[CE])

(a) 2, 7 (b) 3, 8 (c) 8, 3 (d) 7, 2

110. The inverse of 2 × 2 matrix $\begin{bmatrix}1 & 2\\ 5 & 7\end{bmatrix}$ is **(GATE- 07[CE])**

(a) $\frac{1}{3}\begin{bmatrix}-7 & 2\\ 5 & -1\end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix}7 & 2\\ 5 & 1\end{bmatrix}$ (c) $\frac{1}{3}\begin{bmatrix}7 & -2\\ -5 & 1\end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix}-7 & -2\\ -5 & -1\end{bmatrix}$

111. If a square matrix A is real and symmetric then the eigen values **(GATE- 07[ME])**

(a) are always real (b) are always real and positive

(c) are always real and non-negative (d) occur in complex conjugate pairs

112. The number of linearly independent eigen vectors of $\begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}$ is **(GATE- 07[ME])**

(a) 0 (b) 1 (c) 2 (d) infinite

113. The determinant $\begin{bmatrix}1+b & b & 1\\ b & 1+b & 1\\ 1 & 2b & 1\end{bmatrix}$ equals to **(GATE- 07[PI])**

(a) 0 (b) 2b(b-1) (c) 2(1-b)(1+2b) (d) 3b(1+b)

114. If A is square symmetric real valued matrix of dimension 2n, then the eigen values of A are **(GATE- 07[PI])**

(a) 2n distinct real values (b) 2n real values not necessarily distinct

(c) n distinct pairs of complex conjugate numbers

(d) n pairs of complex conjugate numbers, not necessarily distinct

The number of eigen values of A is n & eigen values of real symmetric matrix are always real. .'. The number of eigen values of real symmetric matrix A of order 2n (or dimension 2n) are 2n real values which may or rrr-a] not be repeated.

115. $q_1, q_{2\,,}\, q_3$, -------q_m are n-dimensional vectors with m < n. This set of vectors is linearly dependent . Q is the matrix with . $q_1, q_{2\,,}\, q_3$,----- q_m as the columms. The rank of Q is

(GATE- 07[EE])

(a) Less than m (b) m (c) between m and n (d) n

116. X = [X_1 X_2.................x_n $]^T$ is an n – tuple non-zero vector. The n × n matrix V=XX^T

(GATE- 07[CE])

(a) Has rank zero (b) has rank 1 (c) is orthogonal (d) has rank n

117. If A = $\begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A satisfies the relation **(GATE- 07[EE])**

(a) A + 31 + 2 A^{-1} = O (b) A^2 + 2A + 21 = O

(c) (A + I) (A + 2I) = O (d) e^A = O

118. If A = $\begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A^9 equals **(GATE- 07[EE])**

(a) 511 A + 510 I (b) 309 A + 104 I (c) 154 A + 155 I (d) e^{9A}

119. Let x and y be two vectors in a 3 – dimensional space and < x, y > denote their dot product. Then the determinant det $\begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix}$ = **(GATE- 07[EE])**

(a) is zero when x and y are linearly independent

(b) is positive when x and y are linearly independent

(c) is non-zero for all non-zero x and y

(d) Is zero only when either x (or) y is zero

120. The characteristic equation of a 3 × 3 matrix P is defined as α(λ) = │λI-P│= λ^3 + 2λ + λ^2 + 1 = 0. If I denotes identity matrix then the inverse of P will be **(GATE- 08[EE])**

(a) P^2 + P + 2I (b) P^2 + P + I (c) - (P^2 + P + I) (d) -(P^2 + P + 2I)

121. If the rank of a 5 × 6 matrix Q is 4 then which one of the following statements is correct?

(GATE- 08[EE])

(a) Q will have four linearly independent rows and four linearly independent columns

(b) Q will have five linearly independent rows and four linearly independent columns

(c) QQ^T will be invertible (d) Q^TQ will be invertible

122. A is m × n full rank matrix with m > n and I is an identity matrix . Let matrix $A^+ = (A^T A)^T A^T$. Then which one of the following statement is false? **(GATE- 08[EE])**

(a) $AA^+A = A$ (b) $(AA^+)^2 = AA^+$ (c) $A^+A = I$ (d) $AA^+A = A^+$

123. All the four entries of 2 × 2 matrix P = $\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ are non – zero and one of the eigen values is zero. Which of the following statement is true? **(GATE- 08[EC])**

(a) $P_{11} P_{22} - P_{12} P_{21} = 1$ (b) $P_{11} P_{22} - P_{12} P_{21} = -1$

(c) $P_{11} P_{22} - P_{21} P_{12} = 0$ (d) $P_{11} P_{22} - P_{12} P_{21} = 0$

124. The system of linear equations $\left.\begin{matrix} 4x + 2y = 7 \\ 2x + y = 6 \end{matrix}\right\}$ has **(GATE- 08[EC])**

(a) unique solution (b) no solution

(c) an infinite no. of solutions (d) exactly two distinct solution.

125. The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & P \end{bmatrix}$ has one eigen value to 3. The sum of the other two eigen values is **(GATE- 08[ME])**

(a) P (b) p -1 (c) p -2 (d) p -3

126. The eigen vectors vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is a + b ? **(GATE- 08[ME])**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

127. For what values of 'a' if any will the following system of equations in x, y and z. Have a solution? 2x + 3y = 4, x + y + z = 4, x + 2y – z = a **(GATE- 08[ME])**

(a) Any real number (b) 0 (c) 1 (d) there is no such value

128. The eigen vector pair of the matrix $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ is **(GATE- 08[PI])**

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

129. The Inverse of matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is **(GATE- 08[PI])**

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

130. Let P be 2×2 real orthogonal matrix and $\bar{x}$ is a real vector $[X_1 \quad X_2]^T$ with length $\|\bar{x}\|$ = $(x_1^2 + x_2^2)^{1/2}$. Then which one of the following statement is correct?

(GATE- 08[EE])

$\|\overline{Px}\| \leq \|\bar{x}\|$ where at least one vector satisfies $\|\overline{Px}\| < \|\bar{x}\|$

$\|\overline{Px}\| = \|\bar{x}\|$ for all vectors $\bar{x}$

$\|\overline{Px}\| \geq \|\bar{x}\|$ where atleast one vector satisfies $\|\overline{Px}\| > \|\bar{x}\|$

No relationship can be established between $\|\bar{x}\|$ and $\|\overline{Px}\|$

131. The following system of equations $x_1 + x_2 + 2x_3 = 1$, $x_1 + 2x_2 + 3x_3 = 2$, $x_1 + 4x_2 + \alpha x_3 = 4$, has a unique solution. The only possible value (s) for α is/are

(GATE- 08[CS])

(a) 0 (b) either 0s (or) 1

(c) one of 0, 1 (or) -1 (d) any real number

132. How many of the following matrices have an eigen value 1?

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ & $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **(GATE- 08[CS])**

(a) One (b) two (c) three (d) four

133. The product of marices $(PQ)^{-1}$ P is **(GATE- 08[CE])**

(a) P^{-1} (b) Q^{-1} (c) $P^{-1}\, Q^{-1}$ P (d) $P\, Q\, P^{-1}$

134. The eigen values of the matrix $[P] = \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$ are **(GATE- 08[CE])**

(a) -7 and 8 (b) -6 and 5 (c) 3 and 4 (d) 1 and 2

135. The following system of equations x + y + z =3, x + 2y + 3z =4, x + 4y + kz = 6 will not have a unique solution for k equal to **(GATE- 09[CE])**

(a) 0 (b) 5 (c) 6 (d) 7

136. A square matrix B is symmetric if _ **(GATE- 09[CE])**

(a) 10 and -2 (b) 10 and 2 (c) 5 and 4 (d) 5 and -4

137. In the solution of the following set of linear equations by Gauss-elimination using partial pivoting 5x + y + 2z = 34.

For elimination of x and y are **(GATE- 09[CE])**

(a) 10 and 4 (b) 10 and 2 (c) 5 and 4 (d) 5 and -4

138. The eigen values of the following matrix $\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$ are **(GATE- 09[EC])**

(a) 3, 3+5j, 6-j (b) -6 + 5j, 3+j, 3-j (c) 3+j, 3-j, 5+j (d) 3, -1+3j, -1-3j

139. The eigen values of a 2×2 matrix X are -2 and -3. The eigen values of matrix $(X+I)^{-1}$ (X+5I) are **(GATE- 09[IN])**

(a) -3, -4 (b) -1, -2 (c) -1,-3 (d) -2, -4

140. For a matrix [M] = $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$. The transpose of the matrix is equal to inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by **(GATE- 09[ME])**

(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

141. The trace and determinant of a 2×2 matrix are shown to be -2 and -35 respectively . Its eigen values are **(GATE- 09[EE])**

(a) -30, -5 (b) -37, -1 (c) -7, 5 (d) 17.5, -2

142. The value of the determinant $\begin{bmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ is **(GATE- 09[PI])**

(a) -28 (b) -24 (c) 32 (d) 36

143. The value of X_3 obtained by solving the following system of linear equations is

$X_1 + 2X_2 - 2X_3 = 4$ $2X_1 + X_2 + X_3 = -2$ $-X_1 + X_2 - X_3 = 2$ **(GATE- 09[PI])**

(a) -12 (b) -2 (c) 0 (d) 36

144. For the set of equations

$X_1 + 2X_2 + X_3 + 4X_4 = 2,$

$3X_1 + 6X_2 + 3X_3 + 12X_4 = 6.$

The following statement is true **(GATE- 09[EE])**

(a) only the trivial solution $X_1 + 2X_2 + X_3 + 4X_4 = 0$ exist

(b) there are no solutions

(c) a unique non – trivial solutions exist (d) multiple non – trivial solutions exist

145. An eigen vector of P = $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is **(GATE- 10[EE])**

(a) $[-1\ 1\ 1]^T$ (b) $[1\ 2\ 1]^T$ (c) $[1\ -1\ 2]^T$ (d) $[2\ 1\ -1]^T$

146. The eigen values of a skew – symmetric matrix are **(GATE- 10[EC])**

(a) always zero (b) always purre imaginary

(c) either zeao (or) pure imaginary (d) always real

147. One of the eigen vector of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is **(GATE- 10[ME])**

(a) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

148. A real n×n matrix $A = [a_{ij}]$ is defined as follows $\begin{cases} a_{ij} = i, \ \forall\ i = j \\ = 0, \ \text{otherwise} \end{cases}$

The sum of all n eigen values of A is **(GATE- 10[IN])**

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n(n+1)(2n+1)}{2}$ (d) n^2

149. X and y are n×n – zero square matrices of size n n.

If $XY = O_{n\times n}$ Then **(GATE- 10[IN])**

(a) $|X| = 0$ and $|Y| \neq 0$ (b) $|X| \neq 0$ and $|Y| = 0$

(c) $|X| = 0$ and $|Y| = 0$ (d) $|X| \neq 0$ and $|Y| \neq 0$

150. Consider the following matrix $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$. If the eigen values of A are 4 and 8 then

(GATE- 10[CS])

(a) X = 4, y = 10 (b) x = 5, y = 8 (c) x = -3, y = 9 (d) x = -4, y = 10

151. The inverse of the matrix $\begin{bmatrix} 3+ & 2i & i \\ -i & 3 & -2i \end{bmatrix}$ is **(GATE- 10[CS])**

(a) $\frac{1}{2}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$ (b) $\frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

152. The value of q for which the following set of linear equations 2x + 3y = 0, 6x + qy = 0 can have non-trivial solution is **(GATE- 10[PI])**

(a) 2 (b) 7 (c) 9 (d) 11

153. If $\{1, 0, -1\}^T$ is an eigen vector of the following matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ then the corresponding eigen value is **(GATE-10)[PI]**

154. The two vectors [1 1 1] and $[1\ a\ a^2]$ where $a = -1/2 + j\,\sqrt{3}/2$ and $j = \sqrt{-1}$ are

(GATE- 11[EE])

(a) Orthonormal (b) orthogonal (c) parallel (d) collinear

155. The matrix [A] = $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of lower triangular matrix [L] And an upper triangular [U]. The property decomposed [L] and [U] matrices Respectively are **(GATE- 11[EE])**

(a) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

156. The system of equations x + y + z = 6, x + 4y + 6z = 20, x + 4y + $\lambda z = \mu$ has no solution for values of λ and μ given by **(GATE- 11[EC])**

(a) $\lambda = 6$, $\mu = 20$ (b) $\lambda = 6$, $\mu \neq 20$ (c)) $\lambda \neq 6$, $\mu = 20$ (d)) $\lambda \neq 6$, $\mu \neq 20$

157. The matrix M = $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$ has eigen values -3, -3, 5. An eigen vector corresponding to the eigen values 5 is $[1\ 2\ -1]^T$. One of the eigen vector of the matrix M^3 is **(GATE- 11[IN])**

(a) $[1\ 8\ -1]^T$. (b) $[1\ 2\ -1]^T$ (c) $[1\ \sqrt[3]{2}\ -1]^T$ (d) $[1\ 1\ -1]^T$.

158. The eigen values of the following matrix $\begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$ are **(GATE- 11[PI])**

(a) 4, 9 (b) 6, -8 (c) 4, 8 (d) -6, 8

159. If a matrix A = $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and matrix B = $\begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix}$ the transpose of product of these two matrices i.e., $(AB)^T$ is equal to **(GATE- 11[PI])**

(a) $\begin{bmatrix} 28 & 19 \\ 34 & 47 \end{bmatrix}$ (b) $\begin{bmatrix} 19 & 34 \\ 47 & 28 \end{bmatrix}$ (c) $\begin{bmatrix} 48 & 33 \\ 28 & 19 \end{bmatrix}$ (d) $\begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}$

160. Eigen values of a real symmetric matrix are always **(GATE- 11[ME])**

(a) positive (b) negative (c) real (d) 162. [A] is a square

161. [A] is a square matrix which is neither symmetric nor skew – symmetric And differences of these matrices are defined as [S] = [A] + $[A]^T$ and [D] = [A] - $[A]^T$ respectively. Which of the following Statements is true? **(GATE- 11[CS])**

(a) Both [S] and [D] are symmetric
(b) Both [S] and [D] are skew – symmetric
(c) [S] is skew- symmetric and [D] is symmetric
(d) [S] is skew- symmetric and [D] is skew-symmetric

162. Consider the following system of equations $2x_1 + x_2 + x_3 = 0, \quad x_2 - x_3 = 0 \ \ and \ \ x_1 + x_2 = 0$. This system has **(GATE- 11[ME])**

(a) a unique solution (b) no solution

(c) Infinite number of solutions (d) five solutions

163. Consider the matrix as given below $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$. Which one of the following options provides the correct values of the eigen values of the matrix? **(GATE- 11[CS])**

(a) 1, 4, 3 (b) 3, 7, 3 (c) 7, 3, 2 (d) 1, 2, 3

164. Given that A = $\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ $and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

(GATE- 11[EC,EE,IN])

(a) 15 A + 12 I (b) 19 A + 30 I (c) 17 A + 15 I (d) 17 A + 21 I

165. For the matrix A = $\begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors is given as

(GATE- 11[ME, PI])

(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{5} \\ \frac{2}{\sqrt{3}} \end{pmatrix}$

166. x + 2y + z = 4, 2x + y + 2z = 5, x – y + x = 1

The system of algebraic equations given above has **(GATE- 11[ME, PI])**

(a) A unique solution of x = 1, y = 1 and z = 1

(b) Only the two solutions of (x = 1, y = 1, z = 1) and (x = 2, y = 1, z = 0)

(c) Infinite number of solutions (d) No feasible solution.

167. The eigen values of matrix $\begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$ are **(GATE- 12[CE])**

(a) 0 (b) 1 (c) 2 (d) 3

168. The equation $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has **(GATE- 11[EE])**

(a) no solution (b) only one solution

(c) non-zero unique solution (d) multiple solutions

169. A matrix has eigen values -1 and -2. The corresponding eigenvectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ respectively. The matrix is **(GATE- 13[EE])**

(a) $\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\ -2 & -4\end{bmatrix}$ (c) $\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$

170. The minimum eigenvalue of the following matrix is $\begin{bmatrix}3 & 5 & 2\\ 5 & 12 & 7\\ 2 & 7 & 5\end{bmatrix}$ **(GATE- 13[EC])**

(a) 0 (b) 1 (c) 2 (d) 3

171. Let A be an m × n matrix and B an n×m matrix. It is given that determinant $(I_m + AB)$ = determinant $(I_m + AB)$, where I_k is the $k \times k$ identity

Matrix. Using the above property, the determinant of the matrix given below

Is $\begin{bmatrix}2 & 1 & 1 & 1\\ 1 & 2 & 1 & 1\\ 1 & 1 & 2 & 1\\ 1 & 1 & 1 & 2\end{bmatrix}$

(GATE- 13[EC])

(a) 2 (b) 5 (c) 8 (d) 16

172. The dimension of the null space of the matrix $\begin{bmatrix}0 & 1 & 1\\ 1 & -1 & 0\\ -1 & 0 & -1\end{bmatrix}$ is **(GATE- 13[IN])**

(a) 0 (b) 1 (c) 2 (d) 3

173. One pair of eigenvectors corresponding to the two eigenvalues of the matrix $\begin{bmatrix}0 & -1\\ 1 & 0 -\end{bmatrix}$ is **(GATE- 13[IN])**

(a) $\begin{bmatrix}1\\ -j\end{bmatrix}, \begin{bmatrix}j\\ -1\end{bmatrix}$ (b) $\begin{bmatrix}0\\ 1\end{bmatrix}, \begin{bmatrix}-1\\ 0\end{bmatrix}$ (c) $\begin{bmatrix}1\\ j\end{bmatrix}, \begin{bmatrix}0\\ 1\end{bmatrix}$ (d) $\begin{bmatrix}1\\ j\end{bmatrix}, \begin{bmatrix}j\\ 1\end{bmatrix}$

174. The eigen values of a symmetric matrix are all **(GATE- 13[ME])**

(a) Complex with non-zero positive imaginary part.

(b) 9 Complex with non-zero negative imaginary part.

(c) real (d) Pure imaginary

175. Choose the CORRECT set of functions, which are linearly dependent (**GATE- 13[ME]**)

(a) $\sin x, sin^2x \ and \ cos^2x$ (b) cos x, sin x and tan x

(c) cos 2x, $sin^2x \ and \ cos^2x$ (d) cos 2x, sin x and cos x

176. What is the minimum number of multiplications involved in computing the matrix product PQR ? Matrix P has 4 rows and 2 columns, matrix Q has 2 rows and 4 columns, matrix Q has 2 rows and 4 columns and matrix R has 4 rows and 1 column

(GATE- 13[CE])

177. Which one of the following does NOT equal $\begin{bmatrix}1 & x & x^2\\1 & y & y^2\\1 & z & z^2\end{bmatrix}$?

(a) $\begin{bmatrix}1 & x(x+1) & x+1\\1 & y(y+1) & y+1\\1 & z(z+1) & z+1\end{bmatrix}$ (b) $\begin{bmatrix}1 & x+1 & x^2+1\\1 & y+1 & y^2+1\\1 & z+1 & z^2+1\end{bmatrix}$

(c) $\begin{bmatrix}0 & x-y & x^2-y^2\\0 & y-z & y^2-z^2\\1 & z & z^2\end{bmatrix}$ (d) $\begin{bmatrix}2 & x+y & x^2+y^2\\2 & y+z & y^2+z^2\\1 & z & z^2\end{bmatrix}$

178. For matrices of same dimension M, N and scalar c, which one of these properties DOES NOT ALWAYS hold? **(GATE-14-EC-Set1)**

(a) $(M^T)^T = M$ (b) $(cM)^T = c(M)^T$

(c) $(M+N)^T = M^T + N^T$ (d) MN = NM

179. A real (4 × 4) matrix A Satisfies the equation $A^2 = I$, where I is the (4 × 4) identity matrix. The positive eigen value of A is_ **(GATE-14-EC-Set 1)**

180. Consider the matrix

$$j_6 = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 1 & 0 & 0\\0 & 0 & 1 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$ which is obtained by reversing the order of the columns of the identity matrix I_6. Let $= I_6 + \alpha j_6$, Where α is a non-negative real number.

The value of α for which det (P) = 0 is. **(GATE-14-EC-Set 1)**

181. The determinant of matrix A is 5 and the determinant of matrix B is 40. The determinant of matrix AB is _. **(GATE-14-EC-Set 2)**

182. The ystem of linear equations $\begin{pmatrix}2 & 1 & 3\\3 & 0 & 1\\1 & 2 & 5\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}5\\-4\\14\end{pmatrix}$ has **(GATE- 14-EC-Set 2)**

(a) a unique solution (b) infinitely many solutions

(c) no solution (d) exactly two solutions.

183. The maximum value of the determinant among all 2 × 2 real symmetric matrices with trace 14 is _. **(GATE-14-EC-Set 2)**

184. Which one of the following statements is NOT true for a square matrix A?

(GATE-14-EC-Set 2)

(a) If A is upper triangular , the eigenvalues of A are the diagonal elements of it

(b) If A is real symmetric, the eigenvalues of A are always real and positive

(c) If A is real, the eigenvalues of A and A^T are always the same

(d) If all the principal minors of A are positive, all the eigenvalues of A are also positive

185. Given a system of equations

X + 2y + 2z = b_1 5x + y + 3z = b_2

Which of the following is true its solutions **(GATE-14-EE-Set 2)**

(a) The system has a unique solution for any given b_1 *and* b_2

(b) The system will have infinitely many solutions for any given b_1 *and* b_2

(c) Whether or not a solution exists depends on the given b_1 *and* b_2

(d) The system would have no solution for any values of b_1 *and* b_2

186. A system matrix is given as follows A = $\begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}$. The absolute value of the ratio of the maximum eigen value to the minimum eigen value is **(GATE-14-EE-Set 1)**

187. Which one of the following statements is true for all real symmetric matrices? **(GATE-14-EE-Set 2)**

(a) All the eigen values are real (b) All the eigen values are positive

(c) All the eigen values are distinct (d) sum of all the eigen values is zero

188. A = $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$; B = $\begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$. If the rank of matrix A is N, then the rank of matrix B is **(GATE-14-EE-Set 3)**

(a) N/2 (b) N-1 (c) N (d) 2N

189. A scalar valued function is defined as f(x) = $x^T Ax + b^T x + c$, where A is a symmetric positive definite matrix with dimension n×n ; b and x are vectors of dimension n×1. The minimum value of f(x) will occur when x equals. **(GATE-14-IN-Set 1)**

(a) $(A^T A)^{-1}$ B (b) $-(A^T A)^{-1} B$ (c) $-\left(\frac{A^T B}{2}\right)$ (d) $\frac{A^T B}{2}$

190. For the matrix A satisfying the equation given below, the eigen values are

$[A]\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ **(GATE-14-IN-Set 1)**

(a) (1, -j, j) (b) (1, 1, 0) (c) (1, 1, -1) (d) (1, 0, 0)

191. Given that the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is -12,

The determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is **(GATE-14-ME-Set 1)**

(a) -96 (b) -24 (c) 24 (d) 96

192. Which one of the following describes the relationship among the there vectors, $\hat{\imath} + \hat{\jmath} + \hat{k}$, $\widehat{2\imath} + \widehat{3\jmath} + \hat{k}$ *and* $\widehat{5\imath} + \widehat{6\jmath} + \widehat{4k}$? **(GATE- 14- ME-Set 1)**

(a) The vectors are mutually perpendicular (b) The vectors are linearly dependent
(c) The vectors are linearly independent (d) The vectors are unit vectors

193. One of the eigen vectors of the matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is **(GATE-14-ME-Set 2)**

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$ (b) $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$ (c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

194. Consider a 3 × 3 real symmetric matrix S such that two of its eigen values are

A ≠ 0, b ≠ 0 with respective eigen vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

if a ≠ b then $x_1y_1 + x_2y_2 + x_3y_3$ equals **(GATE-14-ME-Set 3)**

(a) A (b) b (c) ab (d) 0

195. Which one of the following equations is a correct identity for arbitrary 3 × 3 real matrices P,Q and R? **(GATE-14-ME-Set 4)**

(a) P(Q + R) = PQ + RP (b) $(P - Q)^2 = P^2 - 2PQ + Q^2$
(c) det (P + Q) = det P + det Q (d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$

196. Given the matrices J = $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and

K = $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, the product K^TJK is_ **(GATE- 14- CE-Set 1)**

197. The sum of Eigen values of the matrix , [M] is where[M] = $\begin{bmatrix} 215 & 650 & 795 \\ 655 & 150 & 835 \\ 485 & 355 & 550 \end{bmatrix}$

(GATE- 14- CE-Set 1)

(a) 915 (b) 1355 (c) 1640 (d) 2180

198. The determinant of matrix $\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$ is _ **(GATE- 14- CE-Set 2)**

199. The rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is_ **(GATE- 14- CE-Set 2)**

200. The system of equations, given below, has **(GATE-14-PI-Set 1)**

$X + 2y + 4z = 2$

$4x + 3y + z = 5$

$3x + 2y + 3z = 1$

(a) a unique solution (b) Two solution

(c) no solution (d) more than two solutions

201. Consider the following system of equations : **(GATE-14-CS-Set 1)**

$3x + 2y = 1$

$4x + 7z = 1$

$X + y + z = 3$

$X - 2y + 7z = 0$

The number of solutions for this system is _

202. The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of a 4×4 symmetric positive definite matrix is. **(GATE-14-CS-Set 1)**

203. If the matrix A is such that

$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$ then determinant of A is equal to _. **(GATE- 14- CS-Set 2)**

204. The product of the non-zero eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ is .

(GATE-14-CS-Set 2)

205. Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigen values? Option (a) **(GATE-14-CS-Set 3)**

(a) If the trace of the matrix is positive and the determinant is negative, at Least one of its eigen values is negative.

(b) If the trace of the matrix is positive, all its eigen values are positive.

(c) If the determinant of the matrix is positive, all its eiven values are positive.

206. If V_1 *and* V_2 are 4-dimensionl subspaces of a 6-dimensional vector space V, Then the smallest possible dimension of $V_1 \cap V_2$ is _. **(GATE-14-CS-Set 3)**

207. Consider a system of linear equations **(GATE-EC-15)**
$x-2y+3z=-1, x-3y+4z=1 \ \ and \ -2x+4y-6z=k$ the value of k for which the system has infinitely many solutions.

208. The value of p such that the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigen vector of the matrix $\begin{pmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{pmatrix}$ is

(GATE-EC-15)

209. The value of x for which all eigen values of matrix given below are real is

$$\begin{pmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{pmatrix}$$ **(GATE-EC-15)**

A) 5 + j B) 5 – j C) 1-5j D) 1+5j

210. For $A=\begin{pmatrix} 1 & \tan x \\ \tan x & 1 \end{pmatrix}$ det of $A^T.A^{-1}$ is **(GATE-EC-15)**

A) $\sec^2 x$ B) cos4x C) 1 D) 0

211. The max value of A such that the matrix $\begin{pmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{pmatrix}$ has three linearly independent real eigen vectors is **(GATE- -15)**

A) $\frac{2}{3\sqrt{3}}$ B) $\frac{1}{3\sqrt{3}}$ C) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$ D) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

212. If any two columns of a det $p=\begin{pmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{pmatrix}$ are interchanged which of the following is true for det? **(GATE-ME-15)**

A) Absolute value remains unchanged, but sign will change

B) Both value and sign changes

C) Value change, but sign will not change

D) Both obsolute value and sign will not change

213. The least eigen value of a singular matrix is **(GATE-ME-15)**

A) positive B) Zero C) Negative D) Imaginary

214. The lowest eigen value 2×2 matrix $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ is **(GATE-ME-15)**

215. For a given matrix $p=\begin{pmatrix} 4-3i & i \\ -i & 4+3i \end{pmatrix}$ where i = $\sqrt{-1}$, the inverse of p is

(GATE-ME-15)

A) $\frac{1}{24}\begin{pmatrix} 4-3i & i \\ -i & 4+3i \end{pmatrix}$ B) $\frac{1}{25}\begin{pmatrix} i & 4-3i \\ 4+3i & -i \end{pmatrix}$

C) $\frac{1}{24}\begin{pmatrix} 4+3i & -i \\ i & 4-3i \end{pmatrix}$ D) $\frac{1}{25}\begin{pmatrix} 4+3i & -i \\ i & 4-3i \end{pmatrix}$

216. For what value of p, the following set of equations will have no solution 2x+3y=5, 3x+py=10 **(GATE-CE-15)**

217. The smallest and largest eigen values of a matrix $\begin{pmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{pmatrix}$ are **(GATE-CE-15)**

A) 1.5 and 2.5 B) 0.5 and 2.5 C) 1.0 and 3.0 D) 1.0 and 2.0

218. Let A = $A=[i,j]$, $1\le i,j\le n$ with $n\ge 3$ and $a_{ij}=i.j$, the rank of A is **(GATE-CE-15)**

A) 0 B) 1 C) n – 1 D) n

219. The Eigen values of matrix $\begin{pmatrix} 2 & 1 \\ 1 & p \end{pmatrix}$ have a ratio 3:1 for p = 2, what is the other value of p for which the eigen values have same ratio 3:1 **(GATE-CE-15)**

A) -2 B) 1 C) $7/3$ D) $14/3$

220. In 20 decomposition of a matrix $\begin{pmatrix} 2 & 2 \\ 4 & 9 \end{pmatrix}$ if the diagonal elements of 0 are both 1 then the lower diagonal entry l_{22} is **(GATE-CS-15)**

221. Perform the following operations to the matrix $\begin{pmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{pmatrix}$ **(GATE-CS-15)**

(i) Add third row to secong row

(ii) Subtract 3rd column to first column, then the det of the resulting matrix is

222. The following set of three vectors $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} x \\ 6 \\ x \end{pmatrix} and \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$ is linearly dependent then x is

(GATE-CH-15)

A) 0 B) 1 C) 2 D) 3

223. For the matrix $\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} if \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is eigen vector, the corresponding eigen value is

(GATE-CH-15)

224. The solution to the system of equation is $\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$ is **(GATE-ME-16)**

(A) 6, 2 (B) −6, 2 (C) −6, −2 (D) 6, −2 (1M)

225.The condition for which the eigen values of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$

are positive, is **(GATE-ME-16)**

(A) $k > 1/2$ (B) $k > -2$ (C) $k > 0$ (D) $k < -1/2$

226. A real square matrix A is called skew-symmetric if **(GATE-ME-16)**

(A) $A^T = A$ (B) $A^T = A^{-1}$ (C) $A^T = -A$ (D) $A^T = A + A^{-1}$

227.The number of linearly independent eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

is _________;;;; Ans: (2) **(GATE-ME-16)**

228. Let $M^4 = I$ (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals: **(GATE-EC-16)**

(A) M^{4k+1} (B) M^{4k+2} (C) M^{4k+3} (D) M^{4k}

229. The value of x for which the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$ has zero as an eigen value is

______________ **(GATE-EC-16)**

230. The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det (A) = 100 and trace (A) = 14. The value of $|a-b|$ is

(GATE-EC-16)

231. Consider a 2×2 square matrix

$$A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$$

Where x is unknown. If the eigen values of the matrix A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$, then x is equal to **(GATE-EC-16)**

(A) $+j\omega$ (B) $-j\omega$ (C) $+\omega$ (D) $-\omega$

232. If the vectors $e_1 = (1,0,2)$, $e_2 = (0,1,0)$ and $e_3 = (-2,0,1)$ form an orthogonal basis of the three dimensional real space R^3, then the vector $u = (4,3,-3) \in R^3$ can be expressed as

(GATE-EC-16)

(A) $u = -\frac{2}{5}e_1 - 3e_2 - \frac{11}{5}e_3$ (B) $u = -\frac{2}{5}e_1 - 3e_2 + \frac{11}{5}e_3$

(C) $u = -\frac{2}{5}e_1 + 3e_2 + \frac{11}{5}e_3$ (D) $u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$

233. A, 3×3 matrix P is such that, $P^3 = P$. Then the eigen values of P are **(GATE-EE-16)**

a) 1,1, -1 b) 1, 0.5 + j0.866, 0.5 - j0.866
c) 1, - 0.5 + j0.866, -0.5 -j0.866 d) 0,1,-1

234.Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$ Then S is **(GATE-EE-16)**

a) A circle of radius $\sqrt{10}$

b) a circle of radius $\frac{1}{\sqrt{10}}$

c) an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d) an ellipse with minor axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

235. Consider 3×3 matrix with every element being equal to 1. Its only non-zero eigenvalue is ________;; **(GATE-EE-16)**

236. Let the Eigen values of a 2 × 2 matrix A be 1, –2 with eigenvectors x_1 and x_2 respectively. Then the Eigen values and eigenvectors of the matrix A^2–3A+4I would respectively, be **(GATE-EE-16)**

a) 02, 14; x_1, x_2

b) 2, 14; x_1+x_2: x_1-x_2

c) 2, 0; x_1, x_2

d) 2, 0; x_1+x_2, x_1-x_2

237. Let A be a 4×3 real matrix which rank2. Wich one of the following statement is TRUE?

a) Rank of A^T is less than 2

b) Rank of A^TA is equal to 2

c) Rank of A^TA is greater than 2

d) Rank of A^TA can be any number between 1 and 3

(GATE-EE-16)

238. If the entries in each column of a square matrix M add up to 1, then an eigenvalue of M is

(A) 4 (B) 3 (C) 2 (D) 1

(GATE-CE-16)

239. Consider the following linear system.

$x + 2y - 3z = a$

$2x + 3y + 3z = b$

$5x + 9y - 6z = c$

This system is consistent if a, b and c satisfy the equation **(GATE-CE-16)**

(A) $7a - b - c = 0$ (B) $3a + b - c = 0$ (C) $3a - b + c = 0$ (D) $7a - b + c = 0$

240. Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is ____

(GATE-CS-16)

241. Two eigen values of a 3x3 real matrix P are $(2+\sqrt{-1})$ and 3.The determinant of P is_____

(GATE-CS-16)

SOLUTIONS

01. Ans : (b&d)

Sol.

Given matrix is A = $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(i) Eigen values:-

Given matrix is an upper triangular matrix.

Therefore diagonal elements of A are eigen values of A.

i.e. λ = 0,0,0.

(ii) Eigen vectors for $\lambda = 0$

Consider $(A - \lambda I) X = O$

$$= \begin{bmatrix} 0-\lambda & 0 & \alpha \\ 0 & 0-\lambda & 0 \\ 0 & 0 & 0-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ ---- (1)}$$

Put λ = 0 in (1)

$$\text{i.e. } \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ ---- (2)}$$

from (2), we have

$\alpha x_3 = 0$

$x_3 = 0$ $(\because \propto \neq 0)$,

r = 1, n = 2, r = 2

put $x_2 = k_1$

$x_1 = k_2$

$$x_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k_2 \\ k_1 \\ 0 \end{bmatrix}$$

$$= k_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ From the given options (b) and (d) are correct.

02.

Sol. Given A = $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{bmatrix}$

$\Rightarrow$ eigen values of A are λ = 1, -1, i, -i

$\Rightarrow$ the characteristic equation of A is

$(\lambda - 1)\ (\lambda + 1)\ (\lambda - i)\ (\lambda + i) = 0$

$\Rightarrow$ $(\lambda^2 - 1)\ (\lambda^2 + 1) = 0$

$\Rightarrow$ $\lambda^4 - 1 = 0$

By, cayley - Hamilton theorem we have

A^4 - I = 0

A^4 = I

03. Ans: (d)

Sol. By the properties of transpose of matrices option (d) is correct.

ie., By the reversal law of the transpose of the product matrices, we have

$(AB)^T = B^T\ A^T$

04. Sol.

$A^{-1} = \frac{adj(A)}{|A|}$

$|A| = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ = 1(0-1)+0+1 (-1 -0) = -1-1=-2

Adj (A) = $\begin{bmatrix} -1 & 1 & -1 \\ 0 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$

$$\therefore A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{(-2)}\begin{bmatrix}-1 & 1 & -1\\ 0 & 0 & -2\\ -1 & -1 & 1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & -1 & 1\\ 0 & 0 & 2\\ 1 & 1 & -1\end{bmatrix}$$

05. **Ans: (c)**

Sol. Let $A = \begin{bmatrix}1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\end{bmatrix}$ be the given matrix

Here all rows / all columns are same. Applying

$R_2 - R_1$, $R_3 - R_1$, $R_4 - R_1$, $R_5 - R_1$, We get the echelon form of A as

$$\sim\begin{bmatrix}1&1&1&1&1&1&1\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix}$$

$\therefore$ P (A) = 1 = number of non-zero rows.

06. Sol. Given $A = \begin{bmatrix}a & 1\\ a & 1\end{bmatrix} \Rightarrow |A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix}a - \lambda I & 1\\ a & 1-\lambda\end{vmatrix} = 0 \qquad \Rightarrow \lambda^2 - (a + 1)\lambda + 0 = 0$

$\lambda = 0$, a + 1 are the eigen values of A.

Ans : (a)

07. Sol. Given $\begin{bmatrix}1 & 0 & 2\\ 1 & -1 & -2\\ 2 & -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix}1 & 0 & 2\\ 0 & -1 & -2\\ 0 & -2 & -4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$\begin{bmatrix}1 & 0 & -2\\ 0 & -1 & -2\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ r = ρ(A) = 2, n = number of variables = 3

Number of independent solutions = n – r = 3 – 2 = 1

Ans: (a)

08. Sol. We know that, ρ $(A_{m\times n}) \leq$ min {m, n} But it is given that m < n

∴ $\rho(A_{m\times n}) \leq$ m

Hence ρ $(A_{m\times n})$ cannot be more than 'm'.

Ans: a

09. Sol. Given matrix can be written in a form

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}3\\ 0\\ 1\end{bmatrix}$$ Consider the Augmented matrix [A | B]

$$[A|B] = \left[\begin{array}{ccc|c}1 & 1 & 1 & 3\\ 1 & 0 & 1 & 0\\ 1 & -1 & 1 & 1\end{array}\right] \quad R_2 \to R_2 - R_1\ ,\ R_3 \to R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c}1 & 1 & 1 & 3\\ 0 & 0 & 1 & -3\\ 0 & -2 & 1 & -2\end{array}\right] \quad R_3 \to R_3 - 2R_2$$

ρ(A) = 2, ρ(A | B) = 3

Here ρ(A) ≠ ρ(A | B)

∴ Solution does not exist

Ans: (b)

10. Sol. Given $A = \begin{bmatrix}0 & 0 & -3\\ 9 & 3 & 5\\ 3 & 1 & 1\end{bmatrix}$

$$R_1 \leftrightarrow R_3 \quad \sim \begin{bmatrix}3 & 1 & 1\\ 9 & 3 & 5\\ 0 & 0 & -3\end{bmatrix}$$

$$R_2 \to R_2 - 3R_1 \quad \sim \begin{bmatrix}3 & 1 & 1\\ 0 & 0 & 2\\ 0 & 0 & 0\end{bmatrix}$$

∴ ρ(A) = 2

11. Sol. Given $A = \begin{bmatrix}5 & -4\\ 1 & -1\end{bmatrix}$

$$\Rightarrow \quad A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{-5+4}\begin{bmatrix}-1 & 4\\ -1 & 5\end{bmatrix} = \begin{bmatrix}1 & -4\\ 1 & -5\end{bmatrix}$$

Ans: (a)

12. Sol. Given that both rows and columns are equal.

By the definition of rank option (a) is correct

Ans: (a)

13. Sol. $|A| = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$ = 1 (225 – 256) -4 (100 – 144) + 9(64 – 81) = -8

Ans : (d)

14. Sol. Given A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Characteristic equation is 1A – λ I1 = 0

$\Rightarrow \quad \begin{vmatrix} 1-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix}$ => λ^2 - 4 λ + 5 = 0

λ – 2 $\pm$ I , There are two positive roots

These two are positive roots

Ans : (b)

15. Sol. Given A = $\begin{bmatrix} 7 & 4 & 8 \\ 0 & 2 & 2 \\ -7 & 0 & -4 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_1$ $\qquad$ $R_3 \rightarrow R_3 - 2R_1$

$\sim \begin{bmatrix} 7 & 4 & 8 \\ 0 & 2 & 2 \\ -7 & 0 & -4 \end{bmatrix}$ $\qquad$ $\sim \begin{bmatrix} 7 & 4 & 8 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

.'. ρ(A) = 2

Ans : (b)

16. Sol. Given A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$

Eigen values :

$\Rightarrow |A - \lambda I| = 0 \quad \Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & 3-\lambda & 1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda) \quad [(3-\lambda) \quad (4-\lambda) \quad -2] = 0$

$\Rightarrow (1-\lambda) \quad [(12 - 7\lambda \quad + \lambda^2 - 2] = 0$

$\Rightarrow (1-\lambda) \quad [\lambda^2 \quad -7\lambda \quad +10] = 0$

$\Rightarrow (1-\lambda) \quad [(\lambda-2) \quad (\lambda-5)] = 0$

$\therefore \quad \lambda = 1,2,5$ are the eigen values of A.

Eigen vector:

Consider $[A - \lambda I]\, X = 0$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 2 & 3-\lambda & 1 \\ 0 & 2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - (1)$$

Case (i) Put $\lambda = 1$ in (1)

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_2 + 2x_2 + x_3 = 0$

$2x_2 + 3x_3 = 0$

Let $x_3 = K$

Then $x_2 = \frac{-3K}{2}$ and $x_1 = K$

$$\therefore \; x_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3K/2 \\ K \end{bmatrix} = K \begin{bmatrix} 1 \\ -3/2 \\ 1 \end{bmatrix}$$ is an eigen vector corresponding to $\lambda = 1$

17. Sol. $\quad L\,M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}$

Ans : (b)

18. Sol. Given system is in a form AX = B

$$\Rightarrow \begin{bmatrix} 1 & -3 & -7 \\ 4 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 9 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$; $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -3 & -7 \\ 0 & 13 & 28 \\ 0 & 9 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -17 \\ -3 \end{bmatrix}$$

$R_3 \to 13R_3 - 9R_2$

$$\begin{bmatrix} 1 & -3 & -7 \\ 0 & 13 & 28 \\ 0 & 0 & -57 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -17 \\ 114 \end{bmatrix}$$

-57z = 114, 13y + 28z = -17 and x – 3y – 7z = 6

z = $\frac{114}{-57}$ = -2,

y = $\frac{-17-28(z)}{13}$ = $\frac{-17-28(-2)}{13}$ = 3

and x = 3y + 7z + 6 = 3 $\frac{(39)}{13}$ + 7(-2) + 6 = 1.

∴ x = 1, y = 3, z = -2

19. Sol. A) 3x3 + 4x4+7x7 = 9+16+49 = 74 ≠ 0

B) 1 x1 + 0x 1+0 x 0 = 1 ≠ 0

C) 1 x 0 + 0 x 5 + 2x 0 = 0

Ans : (C)

20. Sol. Given S = $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ ⇒ $|S|$ = 1 (1 + 1) = 2 ≠ 0

adj (S) = $\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

∴ $S^{-1} = \frac{adj(S)}{|s|} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$

Ans: (d)

21. Sol. The characteristic equation of A is $|A - \lambda I| = 0$

$\begin{vmatrix} 0-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ -6 & -11 & -6-\lambda \end{vmatrix} = 0$

$(-\lambda)$ $[6\lambda + \lambda^2 + 11]$ - [0+6] = 0

$[\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0]$

∴ The eigen values of A are $\lambda = -1, -2, -3$

22. Sol. **Ans: (a)**

23. **Ans: (b)**

Sol. Given that $A_{m\times n} \; X_{n\times 1} = B_{m\times 1}$

⇒ $A_{n\times n} \; X_{n\times 1} = B_{n\times 1}$ (m = n)

In this case, the given system may (or) may not have unique solution. If A is singular then unique solution does not exist and if A is non- singular then unique solution exisi.

.'. Option (b) is wrong statement

24. **Ans: (a)**

Sol. Given A = $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ & B = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$\Rightarrow$ AB = $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

= $\begin{bmatrix} a\,cos\theta & -b\,sin\theta \\ a\,sin\theta & b\,cos\theta \end{bmatrix}$

BA = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ = $\begin{bmatrix} a\,cos\theta & -a\,sin\theta \\ b\,sin\theta & b\,cos\theta \end{bmatrix}$

If a = b (or) θ = nЛ, for an integer n then AB = BA.

i.e. A and B commute when a = b (or) θ = nЛ, n is an integer.

25. Sol. Given A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and B = $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

Also given CD = I --------------------(1)

From (I) , we have CD = I

D = C^{-1}

D = $\left(A \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right)^{-1}$

D = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} \quad A^{-1}$

D = $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ B $\quad (\therefore A^{-1} = B)$

.'. D = $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ = $\begin{bmatrix} b_{11} & b_{12} \\ -b_{11} + b_{21} & -b_{12} + b_{22} \end{bmatrix}$

26. Sol. The characteristic equation of a given matrix A is $|A - \lambda I| = 0$

$\Rightarrow \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = 0$

$\Rightarrow (1-\lambda) \quad [(1-\lambda)^2 - 1] - 1[1-\lambda-1] + 1[1-1+\lambda] = 0$

$\Rightarrow (1-\lambda) \quad [(1-\lambda)^2 - 1] + 2\lambda = 0$

$\Rightarrow \lambda^2 (\lambda - 3) = 0$.'. The eigen values of A are λ = 0, 0, 3.

Ans: (C)

27. Sol. In the Gauss - elimination method, the coefficient matrix of a given system reduces to Upper-triangular matrix.

∴ option (c) is correct.

28. Sol. Post multiplication of A with the elementary matrix A (Theorem).

For example consider some matrix

A = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ and also unit matrix I = $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

From the data $I_{12} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (ie. $R_1 \leftrightarrow R_2$)

Now A I_{12} = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}2 & 1\\4 & 3\end{bmatrix}$

Here the first column of A is same as 2^{nd} column of A I_{12}

∴ Option (c) is correct.

29. Sol. Then $\begin{bmatrix}2 & 1 & 5\\4 & 8 & 13\\6 & 27 & 31\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\1_{21} & 1 & 0\\1_{31} & 1_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\0 & u_{21} & u_{23}\\0 & 0 & u_{33}\end{bmatrix}\begin{bmatrix}2 & 1 & 5\\4 & 8 & 13\\6 & 27 & 31\end{bmatrix}$

$$= \begin{bmatrix}u_{11} & u_{12} & u_{13}\\l_{21}u_{11} & l_{21}u_{12}+u_{22} & u_{13}l_{21}+u_{23}\\l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{bmatrix}$$

Equating the corresponding elements on both sides

$u_{11} = 2$ $\quad u_{12} = 1$ $\quad u_{13} = 5$

$l_{21}u_{11} = 4 \Rightarrow l_{21} = 4/u_{11} = \frac{4}{2} = 2$

$l_{21}u_{12} + u_{22} = 8 \Rightarrow l_{21}u_{12} = -(2)(1) = 6$

$l_{21}u_{13} + u_{23} = 13 \Rightarrow u_{23} = 13 - l_{21}u_{13} = 13 - (2)(5) = 3$

$l_{31}u_{11} = 6 \Rightarrow l_{31} = \frac{6}{u_{11}} = \frac{6}{2} = 3$

$l_{31}u_{12} + l_{32}u_{22} = 27 \Rightarrow l_{32} = (27 - l_{31}u_{12})\frac{1}{u_{12}} = \frac{(27-(3)(1))}{6} = \frac{24}{6} = 4$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 31$

$\Rightarrow u_{33} = 31 - l_{31}u_{13} - l_{32}u_{23} = 31 - (3)(5) - (4)(3) = 4$

∴ L = $\begin{bmatrix}1 & 0 & 0\\l_{21} & 1 & 0\\l_{31} & l_{32} & l_{33}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 4 & 1\end{bmatrix}$ and U = $\begin{bmatrix}u_{11} & u_{12} & u_{13}\\0 & u_{12} & u_{23}\\0 & 0 & u_{33}\end{bmatrix} = \begin{bmatrix}2 & 1 & 5\\0 & 6 & 3\\0 & 0 & 4\end{bmatrix}$

30. Sol. $\begin{bmatrix} 3/2 & -1/2 & 1 \\ 4 & 2 & 3 \\ 7 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \\ 10 \end{bmatrix}$

Consider augmented matrix $[A|B]$

$$[A|B] = \left[\begin{array}{ccc|c} 3/2 & -1/2 & 1 & 2 \\ 4 & 2 & 3 & 9 \\ 7 & 1 & 5 & 10 \end{array}\right]$$

$R_2 \to \frac{3}{8} R_2 - R_1, \quad R_3 \to \frac{3}{14} R_3 - R_1$

$$\left[\begin{array}{ccc|c} 3/2 & -1/2 & 1 & 2 \\ 0 & 5/4 & 1/8 & -1/2 \\ 0 & 5/7 & 1/14 & 1/7 \end{array}\right]$$

$R_3 \to \frac{1}{4} R_3 - \frac{1}{2} R_2$

$$\left[\begin{array}{ccc|c} 3/2 & -1/2 & 1 & 2 \\ 0 & 5/4 & 1/8 & -1/2 \\ 0 & 0 & 0 & 3/28 \end{array}\right]$$

$\rho(A) = 2, \quad \rho(A|B) = 3$

But $\rho(A) \neq \rho(A|B)$

.'. No solution and the equations are inconsistent.

31. Sol. By the properties of determinant of the matrices, if two rows are interchanged in a determinant then the value of the determinant does not change but sign will change

32. Sol: Option (b) is correct since remaining options are not correct.

Ans : (b)

33. Sol: Given A = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\Rightarrow |A| = 1 \neq \text{and} \;\; adj\,(A) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\therefore \; A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\therefore$ (a)

34. Sol. Ans : (b)

Given A = $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ $\Longrightarrow$ It is an upper triangular matrix

$\Longrightarrow$ The determinant of an upper triangular matrix is equal to product of its principal diagonal elements.

$\therefore$ $|A|$ = (6) (2) (4) (-1) = -48

35. Sol. $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$R_1 \to R_1 - R_2$

$\Delta = \begin{vmatrix} 1 & a-b & bc-ca \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ If we simply this 'det' (a-b) is a factor

36. Sol. Given AX = B

$\Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 4 & 8 & 0 \\ 3 & 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 15 \end{bmatrix}$

Consider $[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 4 & 8 & 0 & 12 \\ 3 & 6 & 3 & 15 \end{array}\right]$

$R_2 \to R_2 - 4R_1$; $R_3 \to R_3 - 3R_1$

$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 3 & 0 \end{array}\right]$

$R_2 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -8 \end{array}\right]$

$\rho(A) = 2, \quad \rho(A|B) \Rightarrow P(A) \neq P(A/B)$

$\therefore$ The system has no solution

Ans : (b)

37. Sol .

$$\text{Given } A = \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$$

$$R_3 \to R_3 \to 4R_1 : R_4 \to R_4 \to 3R_1$$

$$\sim \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 0 & -14 & -29 & -27 \\ 0 & 0 & 0 & -19 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & -14 & -29 & -27 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -19 \end{bmatrix}$$

$$\therefore \rho(A) = 4$$

38.Sol Ans : (b)

Given that A is real square matrix

$$\text{Consider} \left(AA^T\right)^T = \left(A^T\right)^T A^T \left(\because (AB)^T = B^T A^T\right)$$

$$= AA^T \left(\because \left(A^T\right)^T = A\right)$$

$\therefore$ A is symmetric matrix

39. Ans: (c)

By left cancellation law of matrix multiplication, we have

$$AS = AT \Rightarrow S = T \text{ only if } A \text{ is non} - \text{singular}$$

40. Ans : (d)

The given coefficient is in 2- variables $x_1 - x_2$.

$$a_{11} = \text{The coefficient of } x_1 x_1 \left(\text{or}\right) x_1^2 = -5$$

$$a_{12} = a_{21} = \frac{1}{2}\left[\text{The Coefficient of } x_1 x_2\right]$$

$$= \frac{1}{2}(4) = 2$$

a_{22} = The coefficient of $x_2.x_2$ (or) $x_2^2 = -5$

$\therefore$ The Matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & -5 \end{bmatrix}$

41. Sol:

Given $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

(1) <u>Eigen Values:-</u>

Let λ be an eigen valued determinant of A_{2x2}.

Then the characteristic equation of $A_{2\times2}$ is $|A - \lambda I| = 0$

$\Rightarrow \begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix} = 0$

$\Rightarrow \lambda^2 - 10\lambda + 24 = 0$

$\Rightarrow \lambda = 4, 6.$

which are the eigen values of A.

(2) <u>Eigen Vectors:-</u>

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be an eigen vector of A_{2x2} corresponding to an eigen values λ. Then X is given by $(A - \lambda I)X = 0$

i.e, $\begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$...........(1)

<u>Case-(i)</u> Put λ = 4 in (1)

i.e. $\begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 4x_1 - 4x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2$

Let $x_2 = k$ Then $x_1 = k$

Hence $x_1 = k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ where k is any non zero number

Case -(ii) putting λ=6 in (1) we have

i.e. $\begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_1 - 4x_2 = 0$

$\Rightarrow x_1 - 2x_2 = 0$

$\Rightarrow x_1 = 2x_2$

Let $x_2 = k_1$ Then $x_1 = 2k_1$

$\therefore X_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2k_1 \\ k_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} k_1$

Where k_1 is any non zero number

44. Sol. Sum of the eigen values of A = Trace of A = 10

45. Sol. Given A = $\begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $\Rightarrow |A| = 3$ *and* $adj(A) = \begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{bmatrix}$

$\therefore A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{3}\begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1/3 & 0 \\ -2 & 0 & 5 \end{bmatrix}$

46. Sol. If the system is inconsistent then $\rho(A) = \rho(A\ B)$ = number of linearly independent columns of A.

∴ The column B must be linearly dependent on the columns of A.

47. Sol. Let P be the probability of getting head

∴ P(E) = p + $q^2p + q^4p$ + - - - -

= $P[1 + q^2 + q^4 + - - - - - \propto]$ $= \frac{p}{1-q^2} = 2/3$

48. Sol. Given $\sum x = 6, \sum y = 21, \sum x^2 = 14,$

$\sum xy = 46$ and there values of x and y to fit the straight line

Let y = a + bx

Then the normal equations are

$\sum y = na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

Where n = number of points

$\therefore 21 = 3a + 6b$

$46 + 6b + 14b \Rightarrow 23 = 3a + 7b$

$-21 = -3a + (-)6b$

$2 = b$

$\therefore a = 3$

49. Sol.

By the properties of determinant of matrices, we have $|kA_{nxn}| = k^n |A_{nxn}|$

$\therefore \alpha = k^n$ where k is a scalar

50. Sol. Probability for first two tosses to yield heads is $(1/2)^2$, so remaining tosses must be tails. Therefore the probability for remaining tosses to be tail is $(1/2)^8$ Hence required probability = $(1/2)^2 \cdot (1/2)^8 = (1/2)^{10}$

51. Sol. $P\left(\frac{X+Y=2}{X-Y=0}\right) = \frac{P[(X+Y=2)(X-Y=0)]}{P(X-Y=0}$

$= \frac{P(X=1,Y=1)}{P(X-Y=0)} = \frac{1/4 \times 1/4}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}} = 1/6$

52. Ans : (a)

Sol: Given that $v[i,j] = i - j$

$$V = \begin{bmatrix} 0 & -1 & -2 & -3 & \dots & 1-n \\ 1 & 0 & -1 & -2 & \dots & 2-n \\ 2 & 1 & 0 & -1 & \dots & 3-n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ n-1 & n-2 & n-3 & n-4 & \dots & 0 \end{bmatrix}_{n\times n}$$

Here the given matrix is skew symmetric matrix

$\therefore$ Sum of all the elements of V = 0

53. Ans : (a)

Sol: $$\Delta = \begin{vmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{vmatrix} = 2(2-0) = 4$$

54. Ans : (b)

Sol: By the property of reversal law of inverse of product of three matrices A,B,C wer have $(ABC)^{-1} = C^{-1}B^{-1}A^{-1} \left(\because (AB)^{-1} = B^{-1}A^{-1}\right)$

Where A,B,C are square matrices of same order.

55. Ans: (b)

Sol: By the definition of rant of a matrix A, the first statement is correct (i.e. true).

If A is nxn matrix and $|A_{nxn}| \neq 0$ then $\rho(A_{nxn}) = n$

Therefore second statement is also correct i.e. true).

Hence option (b) is correct.

56.Ans : (b)

If λ is an eigen value(s) of matrix A_{4x4} then the eigen value(s) are given by $|A-\lambda I|=0$

$$\text{i.e.}\begin{vmatrix} 2-\lambda & -1 & 0 & 0 \\ 0 & 3-\lambda & 0 & 0 \\ 0 & 0 & -2-\lambda & 0 \\ 0 & 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\begin{vmatrix} 3-\lambda & 0 & 0 \\ 0 & -2-\lambda & 0 \\ 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(3-\lambda)\left[-(2+\lambda)\right](4-\lambda) = 0$$

$$\Rightarrow (2-\lambda)(3-\lambda)(2+\lambda)(4-\lambda) = 0$$

$\therefore$ The Eigen values are given by

$\lambda = 2, -2, 3, 4$

57.Ans : (c)

Sol: $\text{Given } A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

$R_2 \rightarrow R_2 \rightarrow 3R_1; R_3 \rightarrow R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -2 & -4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$\therefore \rho(A) = 2$

58. Ans : (a)

Sol : S_1 is true

Ex : $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

where $|A| = 0, |B| = 0$

$\Rightarrow A + B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow |A + B| = 0$

S_2 is true

Ex :

$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

where $|A| \neq 0, |B| \neq 0$

$\Rightarrow A + B = \begin{bmatrix} 2 & 3 \\ 2 & 6 \end{bmatrix} \Rightarrow |A + B| \neq 0$

59. Ans : (b)

Sol : By at theorem, A square matrix A of order n is diagonalizable if and only if it has n linearly independent eigen vectors.

60. Ans : (b)

Sol: Given $A = \begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{vmatrix} = -28$

61. Ans : (d)

Sol: Given $A = \begin{bmatrix} 5 & 3 \\ 2 & 9 \end{bmatrix}$

Consider |A-λI|=0 where λ is an eigen value of A

$\Rightarrow \begin{vmatrix} 5-\lambda & 3 \\ 2 & 9-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 14\lambda + 39 = 0$

$\Rightarrow \lambda = 7 + \sqrt{10}, 7 - \sqrt{10}$

$\Rightarrow \lambda = 10.16, 3.84$

62.Ans : (a)

Sol :

$$\text{Given}\left[P\right]=\begin{bmatrix}2&3\\4&5\end{bmatrix}\text{and}\left[Q\right]=\begin{bmatrix}4&8\\9&2\end{bmatrix}$$

$$\Rightarrow\left[P\right]\left[Q\right]^T=\begin{bmatrix}2&3\\4&5\end{bmatrix}\begin{bmatrix}4&9\\8&2\end{bmatrix}=\begin{bmatrix}32&24\\56&46\end{bmatrix}$$

63.Ans : (c)

Sol : The given matrix is in an echelon form

$\therefore \rho(A)=$ no of non-zero rows in the echelon form = 1

64. Ans : (a)

Sol: Eigen values of an upper triangular matrix are just its diagonal elements. The eigen values of A are 1,2,-2,-1

65. Ans : (C)

$$\text{Sol : Given A=}\begin{bmatrix}1&0&0&0\\100&1&0&0\\100&200&1&0\\100&200&300&1\end{bmatrix}_{4\times4}$$

66. Ans : (c)

$$\text{Sol : Given A}=\begin{bmatrix}-1&4\\4&-1\end{bmatrix}$$

$$\Rightarrow|A-\lambda I|=0$$

$$\Rightarrow\begin{vmatrix}-1-\lambda&0\\4&-1-\lambda\end{vmatrix}=0$$

$$\Rightarrow\lambda^2+2\lambda-15=0$$

$$\Rightarrow\lambda=3,-5.$$

67. Ans : (b)

$$\text{Sol: }(A|B)=\left[\begin{array}{ccc|c}1&2&-8&7\\4&3&-12&5\\2&1&-4&\alpha\end{array}\right]$$

$$R_2\to R_2-4R_1;R_3\to R_3-2R_1$$

$$\left[\begin{array}{ccc|c}1&2&-8&7\\0&-5&20&-23\\0&-3&12&\alpha-14\end{array}\right]$$

$R_2 \to 3R_2 - 5R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -8 & 7 \\ 0 & -5 & 20 & -23 \\ 0 & 0 & 0 & 1-5\alpha \end{array}\right]$$

For infinitely many solutions, last row must be zero row.

$\text{i.e} 1-5\alpha = 0 \Rightarrow \alpha 1/5$

Hence for only one value of $\alpha = 1/5$, the system will have infinitely many solutions.

68. Ans : (c)

Sol : $\text{Given } A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

$R_2 \to 4R_2 \to 6R_1; R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$R_3 \to (10)R_3 + R_2$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2.$

69. Ans : (b)

Sol: Given system is Homogeneous system

AX = 0. This Homogeneous system will have trivial and non-trivial solution.

If the coefficient matrix A is non-singular in AX = 0 then AX = 0 will have trivial solution.

If the coefficient matrix A is singular matrix in AX = 0 then AX =0 will have non-trivial solution.

Hence option (b) is correct.

70. Ans : (c)

Sol : $[A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 1 & 3 & 4 & 8 \\ 2 & 2 & 3 & 12 \end{array}\right]$

$R_2 \to R_2 \to R_1, R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & -3 & 0 \end{array}\right]$$

$R_3 \to R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & 4 \end{array}\right] --------(1)$$

$\rho(A) = 3, \rho(A|B) = 3,$

$n = \text{no.of unkowns} = 3$

$\Rightarrow \rho(A) = \rho(A|B) = n$

$\therefore$ Unique solution exist.

From Eq.(1), we have

$x + 2y + 3z = 6, y + z = 2 \text{ and } -z = 4$

$\Rightarrow z = -4, y = 6 \text{ and } x = 6$

71. Ans : (b)

Sol : Given $X^2 - X + I = 0$

$\Rightarrow X^{-1}(X^2 - X + I) = 0$

$\Rightarrow X - I + X^{-1} = O$

$\Rightarrow X^{-1} = I - X$

$\therefore X^{-1} = I - X$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & 1 \\ -a^2 + a - 1 & 1 - a \end{bmatrix} = \begin{bmatrix} 1-a & -1 \\ a^2 - a + 1 & a \end{bmatrix}$$

72. Ans : (c)

Sol : Let $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ : & : & : & : \\ : & : & : & : \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$

Be the symmetric matrix.

The total no. of different elements in A_{nxn} is $\frac{n^2+n}{2}$ and each element can be filled in 2 ways using either 0 or 1.

$\therefore$ The total no.of different $n \times n$ different

Symmetric matrices is $2^{\frac{n^2+n}{2}}$

73. Ans : (b)

Sol :

$$ABCD = I$$
$$\Rightarrow (ABCD)^{-1} = I^{-1}$$
$$\Rightarrow D\left(D^{-1}C^{-1}B^{-1}A^{-1}\right)A = DIA$$
$$\Rightarrow C\left(C^{-1}B^{-1}\right) = CDA$$
$$\therefore B^{-1} = CDA$$

74.Ans : (c)

Sol : $(A\,|\,B) = \left[\begin{array}{cc|c} 1 & -5 & 1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -5 & 1 \\ 0 & 4 & 1 \\ 0 & 8 & 2 \end{array}\right]$

$$\sim \left[\begin{array}{cc|c} 1 & 5 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2 = \rho(A\,|\,B) = 2$ & number of unkowns $= n = 2$

Here $\rho(A) = \rho(A\,|\,B) = n = 2$

$\therefore$ Unique solution exists

75. Ans : (b)

Sol : Sum of the eigen values = sum of diagnonal elements

= 1+5+1=7

76.

Sol : For singular matrix A, we have |A| = 0

$$\Rightarrow \begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0 \Rightarrow 8(0-12) - x(0-24) = 0$$

$\Rightarrow -96+24x=0 \Rightarrow x=4$

77. Ans : (a)

Sol :

Given the $[A]_{3x1}, [B]_{3x3}, [C]_{3x5}\ [A]_{3x1}, [E]_{5x5}$, and $[F]_{5x1}$ are real matrices. And also given that $[B]$ & $[E]$ are symmetric.

In the first statement the product

$[F]^T_{1\times5}[C]^T_{5\times3}[B]_{3\times3}[C]_{3\times5}[F]_{5\times1}$ is a first order matrix,

$\therefore$ First statement is correct

In the second statement, the product $[D]^T_{3\times5}[F]_{5\times1}[D]_{5\times3}$ is not defined.

$\therefore$ Second statement is wrong Hence option (a) is correct

78.Ans : (c)

Sol : Given $A=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}$

$\Rightarrow |A-\lambda I|=0 \Rightarrow \begin{vmatrix}4-\lambda & -2\\ -2 & 1-\lambda\end{vmatrix}=0$

$\Rightarrow \lambda^2-5\lambda-0=0 \Rightarrow \lambda=0,5$

which are the eigen values of A.

79. Ans : (b)

Sol: $(A|B)=\left[\begin{array}{ccc|c}1 & -4 & -1 & -3\\ 2 & -1 & 3 & 1\\ 3 & 2 & & 2\end{array}\right]$

$R_2 \to R_2-2R_1; R_3 \to R_3-3R_1$

$\left[\begin{array}{ccc|c}1 & -4 & -1 & -3\\ 0 & 7 & 5 & 7\\ 0 & 14 & 8 & 11\end{array}\right]$

$\rho(A)=3=\rho(A|B)=3=$ no.of unknowns.

$\therefore$ Unique solution exists.

80. Ans :

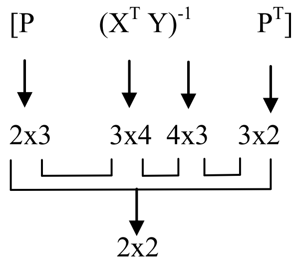
Sol : An over determined system may or may not solution.

$\therefore$ No option is correct

81. Ans : (b)

Sol : Similar to Example 78

82. Ans : (a)



83. Ans : (a)

Sol :

$$|A| = \begin{vmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{vmatrix} = (-1)\begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix} = (-1)(-1)(0-1) = -1$$

84. Ans ;(b)

Sol: Eigen values of A, A^2,A^3,………..A^m are different

But Eigen vectors of A,A^2,A^3,……….A^m are same

$AX = \lambda X$

$A^2X = \lambda^2X$

:

:

$A^mX= \lambda^mX$

λ^m is an eigen value ofA^m and X is an eigen vectors of A^m.

i.e; X^m is not an eigen vector of A^m. Hence the statement in option (b) is wrong.

85. Ans : (a)

Sol : If $\rho(P) = \rho(P|Q)$ then $PX = Q$ is consistent system and has atleast one solution.

86. Ans : (d)

$$(P-\lambda I)X=0,$$

Sol : For $\lambda=-2$

$$\Rightarrow\begin{pmatrix}3-(-2) & -2 & 2\\ 0 & -2-(-2) & 1\\ 0 & 0 & 1-(-2)\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\Rightarrow 5x_1-2x_2+2x_3=0$ and $x_3=0$

$\Rightarrow 5x_1-2x_2=0$

Let $x_2=k_1$, where k_1 is arbitary constant

$$X\begin{pmatrix}2k_1/5\\k_1\\0\end{pmatrix}\Rightarrow X_1=\begin{pmatrix}2\\5\\0\end{pmatrix}\text{for } k_1=5$$

87. Ans : (b)

Sol:

$$|R|=\begin{vmatrix}1 & 0 & -1\\ 2 & 1 & -1\\ 2 & 3 & 2\end{vmatrix}=1(2+3)+0-1(6-2)=1$$

$$\text{Adj}(R)=\begin{bmatrix}5 & -3 & 1\\ -6 & 4 & 1\\ 4 & -3 & 1\end{bmatrix}$$

$$R^{-1}=\frac{\text{adj}(R)}{|R|}=\begin{bmatrix}5 & -3 & 1\\ -6 & 4 & -1\\ 4 & -3 & 1\end{bmatrix}$$

$\therefore$ Top row of R^{-1} is $[5\ \ -3\ \ \ 1]$

88. Ans : (b)

Sol: |M| = The perfect of the eigen

Values = 15 x3 x0 =0

89.

Sol : Eigen values are 1,-2

For λ =1, eigen vectors are given by

$(A-I)X=0$

$$\begin{bmatrix}0 & 0\\ -1 & -3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$x+3y=0$

$$X_1=\begin{bmatrix}3\\ -1\end{bmatrix}$$

90. Ans : (b)

Sol: Given that A is 3x4 matrix and AX = B is inconsistent

$AX=B\rightarrow$

$\downarrow \qquad \downarrow$

$A_{3x4} \quad (A/B)_{3x5}$

If the system is inconsistent then $\rho(A)<3$ and $\rho(A)<\rho(A|B)$

$\therefore$ The highest possible rank of A is 2.

91. Ans: (a)

Sol :

Given $A=\begin{bmatrix}2 & -0.1\\ 0 & 3\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1/2 & a\\ 0 & b\end{bmatrix}$

$\Rightarrow A^{-1}=\begin{bmatrix}1/2 & a\\ 0 & b\end{bmatrix}=\begin{bmatrix}3 & 0.1\\ 0 & 2\end{bmatrix}\frac{1}{6}$

$a=\frac{1}{60}$ & $b=\frac{20}{60}$

$\therefore a+b=\frac{21}{60}=7/20$

92. Ans : (a)

Sol: Consider |A-λI|=0

$$\Rightarrow\begin{vmatrix}5-\lambda & 0 & 0 & 0\\ 0 & 5-\lambda & 0 & 0\\ 0 & 0 & 2-\lambda & 1\\ 0 & 0 & 3 & 1-\lambda\end{vmatrix}=0$$

$\Rightarrow (5-\lambda)(5-\lambda)((2-\lambda)(1-\lambda)-3)=0$

$\Rightarrow (5-\lambda)(5-\lambda)\left[2-3\lambda+\lambda^2-3\right]=0$

$\Rightarrow (5-\lambda)(5-\lambda)\left(\lambda^2-3\lambda-1\right)=0$

$\Rightarrow$ Real eigen values are 5, 5.

Consider $(A-\lambda)X = O$ for $\lambda = 5$

$$\begin{bmatrix} 5-5 & 0 & 0 & 0 \\ 0 & 5-5 & 0 & 0 \\ 0 & 0 & 2-5 & 1 \\ 0 & 0 & 3 & 1-5 \end{bmatrix} X = 0$$

$\Rightarrow -3x_3 + x_4 = 0 \; \& \; -3x_4 = 0$

$\Rightarrow x_4 = 0$ and $x_3 = 0$

Let $x_1 = k_1$ and $x_2 = k_2$

$\therefore X = \begin{bmatrix} k_1 & k_2 & 0 & 0 \end{bmatrix}^T$

$\therefore X = \begin{bmatrix} 1 & -2 & 0 & 0 \end{bmatrix}^T$

for $k_1 = 1$ and $k_2 = -2$

93. Ans :(b)

Sol: $\rho(A_{3x3}) = 2 < n = 3 =$ no.of variables

$\therefore$ number of independent solutions $= 3 - 2 = 1$

94. Ans : (c)

Sol: Given that A is an orthogonal matrix.

By the definition of orthogonal matrix,

We have

$AA^T = I = A^T A$

$\therefore \left(AA^T\right) = (I)^{-1} = I$

95.Ans : (c) & (d)

Sol: Given $A = \begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$

Eigen values are $\lambda = 4, -5$

For $\lambda = 5$, eigen vectors are given by

$[A + 5I]X = 0$

$\Rightarrow x + 2y = 0$

Vectors given in options (c) & (d) satisfy this equation

96. Ans : (a)

Sol: Eigen values of a matrix 'S' are 1 and 5, we know that, the eigen values of matrix S^2 are 1^2 and 5^2 i.e., 1 & 25.

97. Ans : (C)

$$\text{Sol : Given that } EF = G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow F \text{ is the inverse of } E^{-1} \left(\text{Since } AA^{-1} = 1\right)$$

$$\therefore E^{-1} = F = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

98. Ans : (c)

$$\text{Given } A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{and } A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = (-2)\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\text{Sol : } \Rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix} \& \begin{bmatrix} 1 \\ -2 \end{bmatrix} \text{are eigen vectors of A corresponding to}$$

an eigen values $\lambda = -1$ and $\lambda = -2$ respectively. Now we have to find 2x2 matrix A.

A square matrix A_{nxn} is said to be diagonalizable if their exist a non-singular matrix P such that P^{-1} AP=D. Where D is a diagonal matrix whose diagonal elements are eigen values of A and columns of P are eigen vectors of A.

$$\therefore P^{-1}AP = D \Leftrightarrow A = PDP^{-1}$$

$$\therefore A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

Hence option (c) is correct

99. Ans : (d)

$$\text{Sol : Given } A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore \rho(A) = 4.$

100. Ans : (b)

Sol : From the previous example.

$\rho(A) = 4, \rho(A|b) = 4$ and $n = 4$

$=$ number of variables

$\therefore$ **x** is unique

101. Ans : (c)

Sol: Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

102. Ans : (a)

Sol : Given $\lambda_1 = 8$, $\lambda_2 = 4$ and

$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} V$

$A = PDP^{-1}$

where $P = [V_1 V_2]$ and $D = \begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}$$

$$\therefore A = \begin{bmatrix} 8 & 4 \\ 8 & -4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\frac{1}{2}$$

$$= \frac{1}{2}\begin{bmatrix} 12 & 4 \\ 4 & 12 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

103. Ans : (c)

Given that $X = \begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is an eigen vector of $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

Corresponding to some an eigen value λ. Then by definition of an eigen vecot of A, we have, $AX = \lambda X$.

$$AX = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 101 \\ 101 \end{bmatrix} = \begin{bmatrix} 606 \\ 606 \end{bmatrix} = 6\begin{bmatrix} 101 \\ 101 \end{bmatrix} = \lambda X$$

$\therefore X = \begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is an eigen vector of A corresponding to an eigen value $\lambda = 6$.

104. Ans :(d)

Sol : Given AX = B

Consider $[A \mid B] = \left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 0 & 4 & 3 & 8 \\ 2 & 0 & -1 & 2 \end{array}\right]$

$R_3 \to 3R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 0 & 4 & 3 & 8 \\ 0 & -4 & -3 & -4 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 0 & 4 & 3 & 8 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

Here $\rho(A) = 2, \rho(A \mid B) = 3$

But $\rho(A) \neq \rho(A \setminus B)$

$\therefore$ No solution i.e. inconsistent

105.Ans : (b)

Sol : Given $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$ and also that one of the eigen value is 3.

We know that the sum of the eigen values is equal to the sum of the diagonal elements of A.

tr(A) = 2-1+0 =1

From the option (b) the eigen values are 3,-5 sum of the given eigen values in option (b) is1. Which is same as trace of A.

Option is (b) is correct

106.Ans : (b)

Sol : Given $A_{nxn}X_{nxn} = Y_{nx1}$ and $A^2 = I$

$\Rightarrow AA = I$

$\Rightarrow$ A is invertable i.e., A^{-1} exists.

$\Rightarrow$ A is non – singular

$\therefore$ Unique solutin exists.

107. Ans : (b)

$$\text{Sol : } A \begin{bmatrix} 1 & 2 & 3 & ---- & n \\ 2 & 4 & 6 & ---- & 2n \\ 3 & 6 & 9 & ---- & 3n \\ | & & & & \\ | & & & & \\ n & 2n & 3n & ---- & 3n \end{bmatrix}_{nxn}$$

$\therefore \rho(A) = 1 (\because \text{all the rows are proportional})$

108. Ans :(b)

Sol: Two eigen values are given -2&6 Sum of eigen values of A= tr(A)

$\lambda + (-2) + 6 = 1 + 5 + 1 \Rightarrow 3$

109.Ans: (a)

Sol : Given AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 2 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ \beta \end{bmatrix}$$

$$\left(A/B\right)=\left[\begin{array}{ccc|c}1&1&1&5\\1&3&3&9\\1&2&\alpha&\beta\end{array}\right]$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\sim\left[\begin{array}{ccc|c}1&1&1&5\\0&2&2&4\\0&1&\alpha-1&\beta-5\end{array}\right]$$

$R_3 \to 2R_3 - R_2$

$$\sim\left[\begin{array}{ccc|c}1&1&1&5\\0&2&2&4\\0&0&2\alpha-4&2\beta-14\end{array}\right]$$

If $2\alpha - 4 = 0$ and $2\beta - 14 = 0$ then the system will have infinite number of solutions.

$\therefore$ For $\alpha = 2$ and $\beta = 7$ the system has infinite number of solutions

110. Ans :(a)

Sol: $A^{-1} = \dfrac{1}{ab-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \dfrac{1}{-3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix} = \dfrac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$

111. Ans : (a)

Sol: By the property, the eigen values of a real symmetric matrix are always real.

112. Ans: (b)

Sol : Eigen Values of A are 2,2

Consider $\left(A - \lambda I\right) = \begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}$

For $\lambda = 2, \left(A - 2I\right) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\therefore$ The number of linearly independent eigen vectors of matrix A

$= \left(\text{number of variables}\right) - \rho\left(A - \lambda I\right)$

113. Ans : (b)

Sol:

$$\int_{-\alpha}^{\alpha} f(t)\delta(t-a) = f(a) \text{ where } a > 0$$

$$\therefore \int_{-\alpha}^{\alpha} 6\delta\left(t-\frac{\pi}{6}\right)\text{Sin}(t)dt = 6\sin\frac{\pi}{6} = 3$$

114. Sol. (b)

Sol: the number of eigen values of A_{nxn} is n & eigen values of real symmetric matrix are always real.

$\therefore$ the number of eigen values of real symmetric matrix A of order 2n (or dimension 2n) are 2n real values which may or may not be repeated.

115. Ans: (a)

Sol: $Q = [q_1\ q_2\ q_3 ------q_m]_{n\times n}$ and m<n

We knows that ρ(Q) = no. of independent vectors (rows/column)

i.e., ρ(Q) ≤m

But given that q_1, q_2, q_3 ------ q_m are dependent vectors.

$\therefore \rho(Q) < m(\text{dependent vectors})$

116. Ans : (b)

$$\text{Sol}: V = \begin{bmatrix} x_1 \\ x_2 \\ : \\ : \\ x_n \end{bmatrix}_{n\times 1} \begin{bmatrix} x_1 & x_2 & \ldots & \ldots & x_n \end{bmatrix}_{1\times n} = XX^T$$

$\rho(X_{n\times 1}) = 1$ and $\rho(X^T{}_{1\times n}) = 1$

$\therefore \rho(V) = 1$

117. Ans : (c)

Sol: Characteristics equation of A is $|A-\lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -3-\lambda & 2 \\ -1 & 0-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda^2 - 1(-3)\lambda) + 2 = 0$$

$$\Rightarrow \lambda^2 + 3\lambda + 2 = 0$$

$$A^2 + 3A + 2I = (\text{or})(A+I)(A+2I) = 0$$

118. Ans: (a)

Sol : Characteristics equation of $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ is $\begin{bmatrix} -3-\lambda & 2 \\ -1 & 0-\lambda \end{bmatrix} = 0$

$\Rightarrow A^2 + 3A + 2I = 0$

$\Rightarrow A^3 + 2A^2 + 2A = 0$

$\therefore\ A^2 = -3A - 2I$

$A^3 = -3A^2 - 2A = -3(-3A - 2I) - 2A$

$\Rightarrow A^3 = 9A + 6I - 2A$

$\therefore A^3 = 7A + 6I$

$A^4 = 7A^2 + 6A = 7(-3A - 2I) + 6A$

$\Rightarrow A^4 = -21A - 14I + 6A$

$\therefore A^4 = -15A - 14I$

$A^5 = -15A^2 - 14A = -15(-3A - 2I) - 14A$

$\Rightarrow A^5 = 45A + 30I - 14A$

$\therefore A^5 = 45A + 30I$

Similarly $A^9 = 511A + 5101$

119. Ans : (b)

Sol: Given that <x,y> denote their dot product of vector x and y

$\therefore$ By the definition of dot product of vector x and y,

We have

$< x, y >= x, y < x, y >= x^2 \,\&\, x.y = y.x$

$\det \begin{bmatrix} < x,x > & < x,y > \\ < y,x > & < y,y > \end{bmatrix}$

But dot product is scaler.

$\therefore (1)$ becomes positive when x and y are linearly independent

120.Ans : (d)

Sol: $|\lambda I - P| = 0$

$\Rightarrow \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$

$\Rightarrow P^3 + P^2 + 2P + I = 0$

$$P^{-1}\left(P^3+P^2+I\right)=P^{-1}0$$

$$\therefore P^{-1}=-P^2-2P-2I$$

121. Ans : (a)

Sol: Given $\rho\left(Q_{5\times6}\right)=4$

But, we know that $\rho(Q)=$ number of linearly independent row/columns

$\therefore Q_{5\times6}$ will have four linearly independent rows & columns.

122. Ans : (d)

Sol: Given $A^+=\left(A^TA\right)^{-1}A^T$ and $\rho(A)=n$

LHS of option $(d)= AA^+A$

$$\begin{aligned}&=A\left[\left(A^TA\right)^{-1}A^T\right]A\\&=A\left[\left(A^TA\right)^{-1}\left(A^TA\right)\right]\\&=AI\\&=A\neq \text{R.H.S of }(d)\end{aligned}$$

123. Ans : (c)

Sol: Given $P=\begin{bmatrix}P_{11}&P_{12}\\P_{21}&P_{22}\end{bmatrix}$ where $P_{ii}\neq\forall$ i and one of the sign value is zero

$\Rightarrow$ P is sin gular matrix

$\therefore |P|=0(\text{or})P_{11}P_{22}-P_{12}P_{21}=0$

124. Ans : (b)

Sol :

Given

$$\left.\begin{aligned}4x+2y&=7\\2x+y&=6\end{aligned}\right\}$$

$$\Rightarrow\left.\begin{aligned}2x+y&=7/2\\2x+y&=6\end{aligned}\right\}$$

$\Rightarrow$ Solution does not exist

$\therefore$ The System has no solution.

125. Ans : (c)

Sol : One of the eigen value of A is 3 and other two eigen value are λ_1,λ_2.

$\Rightarrow 3+\lambda_1+\lambda_2 = 1+0+P$

$\therefore \lambda_1+\lambda_2 = p+1-3 = p-2$

126. Ans : (b)

Sol: $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ and eigen values are 1 and 2

Eigen vectors

Consider $\begin{bmatrix} 1-\lambda & 2 \\ 0 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ - - - - - (1)

Case-(i): λ= (1)

$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x_2 = 0 \Rightarrow x_2 = 0$

Let $x_1 = k_1$

$\therefore X = \begin{bmatrix} k_1 \\ 0 \end{bmatrix}$ where k_1 is an arbitrary constant

Case-(ii): λ=2 in(1)

$\begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - x_1 + 2x_2$

$= 0 \Rightarrow x_1 = 2x_2$

Let $x_2 = k_2$

$\therefore x_1 = 2k_2$

$X = \begin{bmatrix} 2k_2 \\ k_2 \end{bmatrix}$ where k_2 is an arbitrary constant

$X_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ corresponding to $\lambda = 1$ for $k_1 = 1$

$X_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (or) $X_2 = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$ corresponding to

$\lambda = 2$ for $k_2 = 1$ (or) $1/2$

$a = 0$ and $b = \frac{1}{2}$

$\therefore a+b = \frac{1}{2}$

127. Ans (b)

Sol . Consider $[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 3 & 0 & 4 \\ 1 & 2 & -1 & a \end{array}\right]$

$R_2 \to R_2 - 2R_1; R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & -4 \\ 0 & 1 & -2 & a-4 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & -4 \\ 0 & 1 & -2 & a \end{array}\right]$$

If a = 0 then the system will have system

128. Sol. (a) Similar to example 41

129. Sol. (a)

The given matrix A can be obtained from the unit matrix with elementary operation R_1 <) R2. The inverse matrix corresponding to the elementary matrix A is A itself.

130. Sol. Let us consider an orthogonal matrix P of order 2×2

$$P = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix} \text{ (or) } \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Given $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Consider $P\bar{x} = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\Rightarrow \|\overline{Px}\| = \sqrt{\left[\frac{3x_1}{5} - \frac{4x_2}{5}\right]^2 + \left[-\frac{4x_1}{5} - \frac{3x_2}{5}\right]^2} = \sqrt{x_1^2 + x_2^2} = \|x\|$$

$\therefore \|\overline{Px}\| = \|x\|$ for every vector.

131. Sol. Given AX = B

To have unique solution the determinant of a coefficient matrix must be Non-singular.

i.e. $\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & \alpha \end{vmatrix} \neq 0 \Rightarrow \alpha - 5 \neq 0 \quad \Rightarrow \alpha \neq 5$.

132. Sol. *Let* A = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, D = $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

The eigen values of A are 1, 0 The eigen values of B are 0, 0

The eigen values of C are 1 + I, 1 – i

And the eigen values of D are -1, -1

∴ only one of the matrix A has an eigen value 1.

133. Sol. Consider $(PQ)^{-1}\,P = (Q^{-1}P^{-1})P$

$= Q^{-1}(P^{-1}P) \qquad = Q^{-1}I \qquad = Q^{-1}$

134. Sol. Given [P] = $\begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$

Consider $|P - \lambda I| = 0$

Where λ is an eigen value of P.

$\Rightarrow \begin{vmatrix} 4-\lambda & 5 \\ 2 & -5-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 + \lambda - 30 = 0$

$\Rightarrow \lambda = 5, -6$

135. Sol. Given AX = B

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & k-1 & 3 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k-7 & 0 \end{array}\right]$

If $k - 7 \neq 0$ then system will have unique solution.

.'. For k = 7 the system will not have unique solution.

136. Sol. By the definition of symmetric matrix if B^T = B for a square matrix B then B is called symmetric matrix.

137. Sol. (a)

Sol. According to partial pivoting Gauss- elimination method, the pivot for elimination of x is the numerically largest coefficient of x in the given 3 equations. And the pivot for elimination of y is the numerically largest coefficient of y in the remaining equations of the given system. .'.The pivots for elimination of x and y are 10 and4.

138. Sol. Trace of A = sum of the eigen values =1

Therefore options (d) is correct.

139. Sol. Eigen values of X are -2, -3

Eigen values of I are 1, 1

Eigen values of X + I are -2 +1, -3 + 1 i.e. , -1, -2

Given $(X+I)^{-1}(X+5I) = (X+I)^{-1}(X+I+4I)$

$= (X+I)^{-1}(X+I) + 4(X+I)^{-1}$

$= I + 4(X+I)^{-1}$

∴ The eigen values of $I + 4(X+I)^{-1}$ $are -3, -1$

140. Sol. Given $M^T = M^T \Rightarrow MM^T = I$

$$\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements on both sides, we have x = -$^4/_5$.

141. Sol. Given trace = -2 : Det = 35

$\Rightarrow \lambda_1 + \lambda_2 = -2 \; and \; \lambda_1\lambda_2 = -35$

$\therefore \lambda_1 = -7 \; and \; \lambda_2 = 5$

142. Sol. $|A| = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ = 1(3-1)-3(12-2)+2(4-2)= -24

143. Sol. Consider $(A|B) = \left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 2 & 1 & 1 & -2 \\ -1 & 1 & -1 & 2 \end{array}\right]$

$R_2 \rightarrow R_2 - 2R_1$; $R_3 \rightarrow R_3 + R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 0 & -3 & 5 & -10 \\ 0 & 3 & -3 & 6 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 0 & -3 & 5 & -10 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

$\Rightarrow\ x_1 + 2x_2 + 2x_3 = 4$

$\Rightarrow\ 3x_2 + 5x_3 = -10$

$\Rightarrow\ 2x_3 = -4$

$\therefore\ x_3 = -2$

144. Sol. Given $x_1 + 2x_2 + x_3 - 4x_4 = 2$ $3x_1 + 6x_2 + 3x_3 + 12x_4 = 6$

There are two equations with 4 unknowns

One equation in 3 unknowns

$\Rightarrow \therefore$ the system will have infinite number of solutions.

146. Sol. (c)

By a property that the eigen values of a skew symmetric matrix are

Always either zero (or) purely imaginary.

147. Sol. Similar to example 41

148. Sol. $\begin{bmatrix} 1\ 0\ 0 - - - 0 \\ 0\ 2\ 0 - - - 0 \\ 0\ 0\ 3 - - - 0 \end{bmatrix}$

$\Rightarrow$ eigen values of A are 1, 2, 3, -----n

$\therefore$ Sum of all eigen values = 1 + 2 + 3 + ….+ n = $\frac{n(n+1)}{2}$.

149. Sol. If product of two non zero square matrices is zero matrix,

then both the matrices are singular.

150. Sol. Eiegn values of A are 4 and 8

A = $\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$ sum of the eigen values = Tr(A)

2 + y = 4 + 8

$\therefore$ y = 10 Product of eigen values = det(A)

2y – 3x = (4)(8) = 32

3x = 20 – 32

$3x = 20 - 32 = -12$

$\therefore \quad x = -4$

151. Ans: (b)

$|A| = (9+4) - 1 = 12$

$\therefore A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

152. Ans: (c)

Given $2x + 3y = 0$

$6x + qy = 0$

$\Rightarrow AX = 0$

$\begin{bmatrix} 2 & 3 \\ 6 & q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

For non-trival solution, $|A| = 0$

i.e., $\begin{vmatrix} 2 & 3 \\ 6 & q \end{vmatrix} = 0$

$\Rightarrow 2q - 18 = 0 \Rightarrow q = 0$

153. Ans: (a)

$AX = \lambda X$

$AX = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0+0 \\ -1+0+1 \\ 0+0-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \lambda X$

$\therefore$ Eigen value is $\lambda = 1$

154. Ans: (b)

$X = [1 \quad 1 \quad 1], \; Y = [1 \quad a \quad a^2]$

$XY^T = [1 \quad 1 \quad 1]\begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$

$= 1 + a + a^2 = 0$

Where $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ and

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$XX^T \neq 0, \qquad YY^T \neq 0$

$\therefore$ X, Y are orthogonal vectors but not orthonormal.

155. Ans: (b)

A = LU

$$LU = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

156. Ans: (b)

Given AX = B

$$\text{Consider } [A\,|\,B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda-1 & \mu-6 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & \lambda-6 & \mu-20 \end{array}\right]$$

For no solution, $\lambda = 6$ and $\mu \neq 20$

157. Ans: (b)

$AX = \lambda X$

$A^2X = \lambda^2 X$

| |

| |

$A^mX = \lambda^m X$

A and A^m have same eigen vectors

$\therefore [1 \quad 2 \quad -1]^T$ is one of the eigen vector of M^3.

158. Ans: (b)

Consider $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 10-\lambda & -14 \\ 18 & -12-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (-2)\lambda + (-120+72) = 0$$

$\Rightarrow \lambda^2 + 2\lambda - 48 = 0$

$\Rightarrow \lambda = 6, -8$

159. Ans: (d)

$(AB)^T = B^T A^T$

$= \begin{bmatrix} 4 & 5 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}$

160. Sol. (c)

By a property, the eigen values of a real symmetric matrix are always real

161. Ans: (d)

By a property, if A is any square matrix then

(i) $A + A^T$ is always symmetric and

(ii) $A - A^T$ is always skew-symmetric

$\therefore$ option (d) is correct.

162. Ans: (c)

$2x_1 + x_2 + x_3 = 0$

$x_2 - x_3 = 0$

$x_1 + x_2 = 0$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

$R_3 \to R_3 - 2R_1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

$R_3 \to R_3 + R_2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2 < n = 3$

$\therefore$ Infinite3 no.of solutions exist

From (1), we have $x_1 + x_2 = 0$

And $x_2 - x_3 = 0$

Let $x_3 = k$, then $x_2 = k$ and $x_1 = k$

$\therefore$ The solution is $X = \begin{bmatrix} k \\ k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

163. Sol. (a)

The diagonal elements of an upper triangular matrix A are eigen values of A .'. 1,4,3 are the eigen values of given matrix A

164. Ans: (b)

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

Characteristic Equation is $\lambda^2 + 5\lambda + 6 = 0$

$\Rightarrow A^2 + 5A + 6I = 0, \quad A^2 = -5A - 6I$

$\Rightarrow A^3 = -5A^2 - 6A = -5(-5A - 6I) - 6A$

$\therefore A^3 = 19A + 30I$

165. Ans: (b)

Normalised eigen vector of

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{(1)^2 + (-1)^2}} \\ \dfrac{1}{\sqrt{(1)^2 + (-1)^2}} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

166. Ans: (c)

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 1 & 2 & 5 \\ 1 & -1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & -3 & 0 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \rho(A/B) = \rho(A) = 2 < 3$

$\therefore$ Infinite number of solutions exists.

167. Ans: (b)

$A = \begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$

Sum of the eigen values $= x + y = 17$

Product of the eigen values $= x \times y = 47$

From options, $3.48 + 13.53 = 17$

$(3.48)(13.53) = 47$

168. Ans: (d)

Given $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_1 - 2x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

Solving above two equations, we get $x_1 = x_2$.

$\therefore$ The given system has infinite number of solutions.

169. Ans: (d)

Given $\therefore X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ corresponding to $\lambda_1 = -1$

And $X_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ corresponding to $\lambda_2 = -2$

Now (i) $AX = \lambda X$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\Rightarrow a - b = -1$ ----------------- (1)

$\& \, 2c - d = 1$ ----------------- (2)

(ii) $AX = \lambda X$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = (-2)\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\Rightarrow a - 2b = -2$ ----------------- (3)

$\& \, c - 2d = 4$ ----------------- (4)

Solving equations (1), (2), (3) & (4), we get

$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

170. Sol. (a)

Since the determinant of matrix

A is '0'. .'0' will be the minimum eigen value of matrix A

171. Ans: (b)

$$|A| = \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3 + C_4$

$$= \begin{vmatrix} 5 & 1 & 1 & 1 \\ 5 & 2 & 1 & 1 \\ 5 & 1 & 2 & 1 \\ 5 & 1 & 1 & 2 \end{vmatrix}$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1$

$$= \begin{vmatrix} 5 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 5$$

172. Sol. (b)

Dimension of the null space= (number of variables - rank)

= (3 - 2) = 1,

173. Sol : Marks to All

174. Sol. (c)

By the property of eigen values we have the following statement :

The eigen values of a real symmetric matrix are real.

175. Ans: (c)

$\cos 2x = \cos^2 x - \sin^2 x \qquad (\because \cos 2\theta = \cos^2\theta - \sin^2\theta)$

$\Rightarrow \cos 2x = (1)\cos^2 x + (-1)\sin^2 x$

Here one of the function is written as a linear combination of other two functions.

$\therefore$ $\cos 2x$, $\sin^2$ & $\cos^2 x$ are linearly dependent.

176. Sol. 16

The number of multiplication in PQR computing the matrix product (PQ)R is 48 & the matrix product P(QR) is 16.

.'. The minimum number of multiplicatimis 15.

177. Ans: (a)

$$|A| = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$C_2 \to C_2 + C_1,\ C_3 \to C_3 + C_1$

$$|A| = \begin{vmatrix} 1 & x+1 & x^2+x \\ 1 & y+1 & y^2+y \\ 1 & z+1 & z^2+z \end{vmatrix} = (-1)\begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix}$$

$$\therefore \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \neq \begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix}$$

178. Ans (d)

Sol ; Since matrix multiplication is not cumulative MN $\neq$ NM

179. Ans :1

Sol : Let 'λ' be an eigen value of 'A' then λ^2 is an eigen value of A^2. But A^2. But A^2=1 and 1 is only eigen value of 'I'

$\therefore \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

$\therefore$ The + ve eigen value = 1

180. Ans: 1

Sol : P = I_6+α J_6

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_1 = R_6$ if $\alpha = 1$ to get $|P| = 0$

$\therefore$ Required value = 1

181. Ans: 200

Det (A.B) = Det (A). Det(B)

$= 5 \times 40 = 200$

182. Ans: (b)

The augmented matrix (AB)

$$= \begin{pmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & -4 \\ 1 & 2 & 5 & 14 \end{pmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 2 & 5 & 14 \\ 3 & 0 & 1 & -4 \\ 2 & 1 & 3 & 5 \end{pmatrix}$$

$R_2 - 3R_1,\ R_3 - 2R_1$

$$\sim \begin{pmatrix} 1 & 2 & 5 & 14 \\ 0 & -6 & -14 & -46 \\ 0 & -3 & -7 & -23 \end{pmatrix}$$

$2R_3 - R_2$

$$(AB) \sim \begin{pmatrix} 1 & 2 & 5 & 14 \\ 0 & -6 & -14 & -46 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore \rho(A) = \rho(AB) = 2 < 3$ (no. Of variables)

$\therefore$ The given system has infinitely many solutions.

183. Ans: 49

Let $A = \begin{pmatrix} x & y \\ y & 14-x \end{pmatrix}$

$\text{Det}\,A = x(14-x) - y^2$

For maximum value of Det A, $y = 0$

$\therefore A = \begin{pmatrix} x & 0 \\ 0 & 14-x \end{pmatrix}$

$|A| = x(14-x) = (14x - x^2) = f(x)(\text{say})$

$\Rightarrow f'(x) = 14 - 2x$

$\Rightarrow f'(x) = 0 \Rightarrow x = 7$

$f''(x) = -2 < 0$

$\therefore$ At x = 7, we get f(x) as maximum and is equal to 49.

184. Sol. (b)

For an upper triangular, lower triangular or diagonal matrix, the eigen values are the diagonal elements of the matrix. Hence option (A) is true option (B) is not true. For example: $\begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$ is a real symmetric ' (.0 -3) but eigen values are -ve options (C) and (D) are standard theorems.

185. Ans: (b)

Let the given system of equations be AX = B

The augmented matrix

$$(AB) = \begin{pmatrix} 1 & 2 & 2 & b_1 \\ 5 & 1 & 3 & b_2 \end{pmatrix}$$

$R_2 - 5R_1$

$$\sim \begin{pmatrix} 1 & 2 & 2 & b_1 \\ 0 & -9 & -7 & b_2 - 5b_1 \end{pmatrix}$$

$\therefore$ Rank of A = 2 = Rank of (AB) is less than the number of variables '3'

$\therefore$ The system has infinite number of solutions.

186. Ans: 3

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & -1 \\ -6 & -11-\lambda & 6 \\ -6 & -11 & 5-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$

$\Rightarrow \lambda = -1, +2, +3$

$\therefore$ The required ratio $= \dfrac{3}{1} = 3$

187. Ans: (a)

By properties of eigen values, only (A) is true.

188. Ans: (c)

$$B = \begin{pmatrix} p^2 + q^2 & pr + qs \\ pr + qs & r^2 + s^2 \end{pmatrix} = AA^T$$

$\Rightarrow |B| = |AA^T|$

$\Rightarrow |B| = |A||A^T|$

$\Rightarrow |B| = |A||A|$

$\Rightarrow |B| = |A|^2$

$\therefore p(B) = p(A) = N$

189. Ans: (c)

Let n = 2 and $A = \begin{pmatrix} p & q \\ q & r \end{pmatrix}$

$X = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

Then $f(x) = X^T AX + B^T X + C$

$= (px^2 + 2qxy + ry^2) + (b_1 x + b_2 y) + C$

$\dfrac{\partial f}{\partial x} = 0$ gives $2px + 2qy + b_1 = 0$ ------------------ (1)

$\frac{\partial f}{\partial y} = 0$ gives $2qx + 2ry + b_2 = 0$ ------------------ (2)

(1) & (2) can be written as

$$2\begin{pmatrix} p & q \\ p & r \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

i.e., $2AX = -B$

$$AX = -\frac{B}{2}$$

190. Sol. (c)

The given matrix equation can be written as AB = C

The matrix 'C' can be obtained from 'B' by interchanging R2 and R3

.'. The above elementary operation on 'B' is equivalent to multiplying 'B' by the elementary matrix which can be obtained from R1 by interchanging R2 and R3.

$$\therefore \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Now the eigen values of 'A' are obtained by solving $\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = 0$

$$\Rightarrow (1-\lambda)(\lambda^2 - 1) = 0$$

$$\Rightarrow \lambda = 1, 1, -1$$

191. Sol. Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$

Then $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix} = 2A$

$$\therefore \quad |2A| = 2^3\,|A| = 8\,(-12) = -96$$

192. Sol. Let $\bar{a} = (\bar{\imath} + \bar{\jmath} + \bar{k})$

$\bar{b} = (\overline{2\imath} + \overline{3\jmath} + \bar{k})$

$\bar{c} = (\overline{5\imath} + \overline{6\jmath} + \overline{4k})$

$\bar{a}\,\bar{b} = (2 + 3 + 1) = 6 \neq 0$

$|\bar{a}| = \sqrt{3}$

∴ options (A), (C) and (D) are false To check linear dependency, let us consider the given vectors as rows of A is 3 matrix (A). If rank of A is 3 then the vectors are linearly independent otherwise dependent

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{bmatrix}$$

$R_2 - 2R_1,\ R_3 - 5R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} R_3 + R_2 \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ Rank of 'A' = 2

Hence the given vectors are linearly dependent.

193. Sol. Let $A = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -5-\lambda & 2 \\ -9 & 6-\lambda \end{vmatrix} = 0$$

$\lambda^2 - \lambda - 12 = 0$

$(\lambda - 4)(\lambda + 3) = 0$

$\lambda = 4, -3$ are eigen values

At $\lambda = 4$ → The eigen vectors are given by (A – 4I) X = 0

i.e., $\begin{pmatrix} -9 & 2 \\ -9 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

∴ -9x + 2y = 0 …………………………..(1)

∴ At $\lambda = -3 \rightarrow (A - 3I)X = 0$

$$\begin{pmatrix} -2 & 2 \\ -9 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

∴ -2x + 2y = 0 …………………………..(2)

∴ (2) satisfied by (D)

194. Sol. Since the eigen vectors of a real symmetric matrix are pair-wise Orthogonal. (i.e., dot product = 0) i.e., $x_1y_1 + x_2y_2 + x_3y_3 = 0$.

195. Sol. From option (A)

P(Q + R) = (PQ + PR) ≠ (PQ + RP)

From option (B)

$(P-Q)^2 = (P-Q)\,(P-Q)$

$(P^2 - PQ - QP + Q^2) \neq (P^2 - 2PQ + Q^2)$

From option (C)

Det (P + Q) ≠ (det P + det Q)

$$\begin{bmatrix} Take\ p = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\ Q = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} \\ \Rightarrow\ Det\ P = 0 = Det\ Q \\ Det\ (P+Q) = 6 \end{bmatrix}$$

From option (D)

$(P+Q)^2 = (P+Q)\,(P+Q)$

$(P^2 + PQ + QP + Q^2)$ is only correct

Matrix multiplication need not be commutative in options (A) and (B).

196. Sol. $K^T J K = (1\ 2\ -1)\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ = (6 8 -1) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ = (23)

197. Sol. The sum of Eigen values of M = Trace of 'M'

= 215 + 150 + 550 = 915

198. Sol. The required determinant = $\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$

$R_3 - 3R_1$

$$= \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & -6 & -8 \\ 3 & 0 & 1 & 2 \end{vmatrix} = (-1)\begin{vmatrix} 1 & 3 & 0 \\ 2 & -6 & -8 \\ 3 & 1 & 2 \end{vmatrix}$$

= (-1) [1(-12+8) -3 (4+24)] = (-1) (-4 – 84) = 88.

199. Sol. $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$$\sim \begin{bmatrix} -2 & 14 & 8 & 18 \\ 6 & 0 & 4 & 4 \\ 14 & -14 & 0 & -10 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 3R_1;\ R_3 \rightarrow R_3 + 7R_1$

$$\sim \begin{bmatrix} -2 & 14 & 8 & 18 \\ 0 & 42 & 28 & 58 \\ 0 & -84 & 0 & -116 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 2R_2$

$$\sim \begin{bmatrix} -2 & 14 & 8 & 18 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

∴ Required rank = 2

200. Sol. The given system can be written as AX = B

$$(AB) = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 4 & 3 & 1 & 5 \\ 3 & 2 & 3 & 1 \end{bmatrix}$$

$R_4 - 4R_1,\ R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & -5 & -15 & -3 \\ 0 & -4 & -9 & -5 \end{bmatrix}$$

$5R_3 - 4R_2$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & -5 & -15 & -3 \\ 0 & -4 & 15 & -13 \end{bmatrix}$$

∴ Rank of A = Rank of (A/B) = 3 = number of variables

Hence unique solution exists.

201. Sol. $(A/B) = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{pmatrix}$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 4 & 0 & 7 & 1 \\ 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \end{pmatrix}$$

$R_4 - 4R_1,\ R_3 - 3R_1,\ R_4 - R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & -1 & -3 & -8 \\ 0 & -3 & 6 & -3 \end{pmatrix}$$

$R_2 \leftrightarrow R_3$

$$\sim\begin{pmatrix}1 & 1 & 1 & 3\\ 0 & -1 & -3 & -8\\ 0 & -4 & 3 & -11\\ 0 & -3 & 6 & -3\end{pmatrix}$$

$R_3 - 4R_2,\ R_4 - 3R_2$

$$\sim\begin{pmatrix}1 & 1 & 1 & 3\\ 0 & -1 & -3 & -8\\ 0 & 0 & 15 & 21\\ 0 & 0 & 15 & 21\end{pmatrix}$$

$R_4 - R_3$

$$\sim\begin{pmatrix}1 & 1 & 1 & 3\\ 0 & -1 & -3 & -8\\ 0 & 0 & 15 & 21\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$\therefore\ \rho(A) = \rho(A/B) = 3 = no.$ of variables

Hence there exists only one solution.

202. Sol. The eigen vectors corresponding to a symmetric positive definite are orthogonal.

$\therefore$ Dot product between any two eigen vectors corresponding to distinct eigen values is zero.

203. Sol. A is a 3×3 matrix, for this matrix remaining two rows are identical with first row

$\Rightarrow \text{rank}(A) = \rho(A) = 0$

204. Sol. Let the given matrix be 'A'

Characteristic equation is $|A-\lambda I| = 0$

$$\begin{vmatrix}1-\lambda & 0 & 0 & 0 & 1\\ 0 & 1-\lambda & 1 & 1 & 0\\ 0 & 1 & 1-\lambda & 1 & 0\\ 0 & 1 & 1 & 1-\lambda & 0\\ 1 & 0 & 0 & 0 & 1-\lambda\end{vmatrix} = 0$$

$$(1-\lambda)\begin{vmatrix}1-\lambda & 1 & 1 & 0\\ 1 & 1-\lambda & 1 & 0\\ 1 & 1 & 1-\lambda & 0\\ 0 & 0 & 0 & 1-\lambda\end{vmatrix}$$

$$+1\begin{vmatrix}0 & 1-\lambda & 1 & 1\\ 0 & 1 & 1-\lambda & 1\\ 0 & 1 & 1 & 1-\lambda\\ 1 & 0 & 0 & 0\end{vmatrix} = 0$$

$$= (1-\lambda)^2 \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} [(1-\lambda)^2 - 1] = 0$$

$$\lambda^3(\lambda-2)(\lambda-3) = 0$$

$\lambda = 0, 0, 0, 2, 3$ ∴ The required value $= 2 \times 3 = 6$

205. Sol. Determinant of a matrix = Product of its eigen values.

∴ Determinant is negative ⇒ there exists atleast one eigen value, which is negative.

206. Sol. dim $(V_1 + V_2)$ = dim (V_1) + dim (V_2)-dim $(V_1 \cap V_2)$

Minimum value of dim $(V_1 \cap V_2)$

= dim (V_1) + dim (V_2) - max of dim $(V_1 + V_2)$ = 4 + 4 – 6 = 2

207. Sol. Augmented matrix for given system is

$$[A/B] = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & k \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 + 2R_2$

$$\begin{pmatrix} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & k-2 \end{pmatrix}$$

If a system has infinitely many solutions, we have k-2=0$\Rightarrow$k=2

208 Sol. Let X = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigen vector $\Rightarrow AX = \lambda X$

$$\Rightarrow \begin{pmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 12 \\ p+7 \\ 36 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda \\ 3\lambda \end{pmatrix} \Rightarrow \lambda = 12$$

Now $2\lambda = p+7 \Rightarrow p+7 = 24 \Rightarrow p = 17$

209. Sol. All eigen values are real $\Rightarrow$ A is Hermitian matrix $\Rightarrow \left(\overline{A}\right)^T = A$

$$\Rightarrow \begin{pmatrix} 10 & \overline{x} & 4 \\ 5-j & 20 & 2 \\ 4 & 2 & -10 \end{pmatrix} = \begin{pmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{pmatrix} \Rightarrow x = 5-j$$

210. Sol. $A = \begin{pmatrix} 1 & \tan x \\ \tan x & 1 \end{pmatrix}$, $A^T = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$

$$|A| = 1 + \tan^2 x = \sec^2 x$$

$$A^{-1} = \frac{1}{\sec^2 x}\begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

Now, $A^T.A^{-1} = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix} \frac{1}{\sec^2 x}\begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$

$$= \frac{1}{\sec^2 x}\begin{vmatrix} 1-\tan^2 x & -2\tan^2 x \\ 2\tan x & 1-\tan^2 x \end{vmatrix}$$

$$\left|A^T.A^{-1}\right| = \frac{1}{\sec^2 x}\left[\left(1+\tan^2 x\right)^2\right] = \sec^2 x$$

211. Sol. The characteristic Equation is $|A - XI| = 0$

$$\Rightarrow x^3 + 6x^2 + 11x + 6 + 2a = 0$$

Let f(x) = $x^3 + 6x^2 + 11x + 6 + 2a = 0$

$= (x+1)(x+2)(x+3) + 2a = 0$

*f(x) cannot have 3 real roots, if k is a root then $f(x) = (x-k)^3$, by comparing 6=-3k;

$3k^2 = 11$

No such 'k' exists

Case I : If f(x)=0 has 2 repeated real roots say α, α, β

Case ii: f(x)=0 has 3 real distinct roots α, β, f

Now $f'(x)=0 \Rightarrow x_1 = \frac{-6-\sqrt{3}}{3},\ x_2 = \frac{-6+\sqrt{3}}{3}$

At x_1 f(x) has a relative maxima

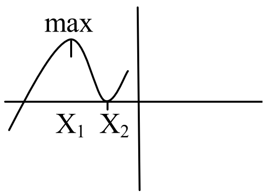
At x_2 f(x) has a relative minima

The graph of f(x) will be

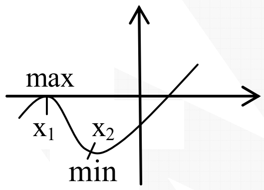
Case i: Fig

Case ii: The graph for distinct roots

Case I :

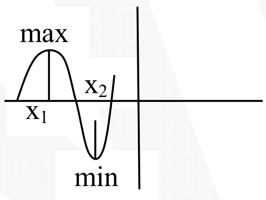


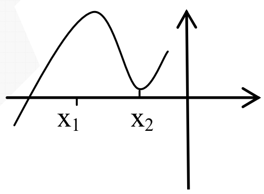
X_2 repeated proof



X_1 repeated proof

Case II: The graph for distinct roots





$\therefore$ In all cases $f(x_2) \le 0 \Rightarrow 2\left(a - \frac{\sqrt{3}}{9}\right) \le 0 \Rightarrow a \le \frac{1}{3\sqrt{3}}$

212. Sol. Option A, from the properties of determinant

213. Sol:option b, from the properties of eigen values

214. Sol. Let A = $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$, ch. Equation is $|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 7\lambda + 10 = 0 \Rightarrow \lambda = 2.5$

Least Eigen value is 2

215. Sol. $|p| = 24$

$$adjp = \begin{pmatrix} 4-3i & -i \\ i & 4+3i \end{pmatrix} : P^{-1} = \frac{1}{24}\begin{pmatrix} 4-3i & i \\ -i & 4+3i \end{pmatrix}$$

216. Sol. $AX = B \Rightarrow \begin{pmatrix} 2 & 3 \\ 3 & p \end{pmatrix}\begin{pmatrix} x \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

$$R_2 \to 2R_2 - 3R_1 \begin{pmatrix} 2 & 3 & 5 \\ 0 & 2p-9 & 5 \end{pmatrix}$$

System has no solution if $\rho(A/B) \neq \rho(A)$

$\Rightarrow 2p - 9 = 0 \Rightarrow p = 9/2 = 4.5$

217. Sol. Ch. Equation is $\begin{vmatrix} 3-\lambda & -2 & 2 \\ 4 & -4-\lambda & 6 \\ 2 & -3 & 5-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$

$\Rightarrow (\lambda - 1)(\lambda^2 - 3\lambda + 2) = 0$

$\Rightarrow (\lambda - 1)(\lambda - 1)(\lambda - 2) = 0 \Rightarrow \lambda = 1, 1, 2$

Smallest eigen value is 1, largest eigen value is 2.

218. Sol. Given $A = \{a_{ij}\}$, $1 \le i, j \le n, n \ge 3$

Consider 3×3 matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$ r(A) = 1

Option (B) is correct

219. Sol. Given eigen values are in the ratio 3:1. For p = 2,

Ch. Equation is $\lambda^2 - 4\lambda + 3 = 0 \Rightarrow \lambda = 1, 3$

From the options if we take $p = 14/3$ then $A = \begin{pmatrix} 2 & 1 \\ 1 & 14/3 \end{pmatrix}$

$$\Rightarrow \lambda^2 - (2 + \frac{14}{3})\lambda + \left(\frac{28}{3} - 1\right) = 0$$

$$\Rightarrow \lambda^2 - \frac{20}{3}\lambda + \frac{25}{3} = 0 \Rightarrow 3\lambda^2 - 20\lambda + 25 = 0 \Rightarrow \lambda = 5, 5/3 \text{ having the ratio 3:1}$$

220. sol: A = LU $\Rightarrow \begin{pmatrix} 2 & 2 \\ 4 & 9 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{pmatrix}\begin{pmatrix} 1 & u_{12} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} l_{11} & l_{11}u_{12} \\ l_{21} & l_{21}u_{12} + l_{22} \end{pmatrix}$

$$\Rightarrow l_{11} = 2; l_{11}u_{12} = 2 \Rightarrow u_{12} = 1$$

$$l_{21} = 9 : l_{21}u_{12} + l_{22} = 9 \Rightarrow l_{22} = 5$$

221. Sol. Let A = $\begin{pmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{pmatrix}$

By performing the given operations, matrix is converted as B= $\begin{pmatrix} -42 & 4 & 3 \\ 280 & 11 & 300 \\ -182 & 2 & 195 \end{pmatrix}$

$$|B| = 0$$

222. Sol. For linearly dependent vectors $|A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & x & 3 \\ 2 & 6 & 4 \\ 1 & x & 2 \end{vmatrix} = 0 \Rightarrow 2x - 6 = 0 \Rightarrow x = 3$$

223. Sol. AX = λ X$\Rightarrow \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 7 \\ 7 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow 7\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \lambda = 7$$

224.**Ans: (D)**

Sol: x = 6, y = –2 is the solution of equation

2x+5y = 2 and –4x + 3y = –30

225.**Ans: (A)**

Sol: A = $\begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$

If λ_1, λ_2 are eigen values of A_{2x2}

then $|A| = \lambda_1\lambda_2$

$$|A| = 2k - 1$$

$$\lambda_1.\lambda_2 > 0$$

$$2k - 1 > 0$$

$$\therefore k > \frac{1}{2}$$

226. **Ans: (C)**

Sol: By using definition

A real square matrix A is said to be skew-symmetric matrix if

$$A^T = -A \text{ (or)} a_{ij} = -a_{ji} \ \forall \ ij$$

227.

Sol: $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow \lambda = 2,2,3$

For $\lambda = 2$. The eigen vector is (A-2I) x = 0

$$(A - 2I) = \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$(A - \lambda I) = 2, n = 3$

P = n- r = 3 – 2 = 1

The no. of Linearly independent eigen vectors corresponding to an eigen value λ= 2 is one &

corresponding to an eigen value λ = 3 is one

$\therefore$ The number of linearly independent eigen vectors of A is 2.

228.**Ans: (C)**

$\Rightarrow M^8 = M^4 = I \Rightarrow M^7 = M^{-1}$

$\Rightarrow M^{12} = M^8 = I \Rightarrow M^{11} = M^{-1}$

$\Rightarrow M^{16} = M^{12} = I \Rightarrow M^{15} = M^{-1}$

…………………………

$\therefore M^{-1} = M^{4K+3}$, K is a natural number.

229. **Ans:** x = 1

Sol: For eigen value of A is to be zero, det (A) = 0

$3\{(-63 + 7x) + 52\} - 2\{(-81 + 9x) + 78\} + 4\{-36 + 42\} = 0$

$\therefore x = 1$

230.**Ans:** 3

Sol: trace (A) = 14

a +b + 7 = 14

a +b = 7

det (A) = 100

$$5\begin{vmatrix} a & 3 & 7 \\ 0 & 2 & 4 \\ 0 & 0 & b \end{vmatrix} = 100$$

$10\,ab = 100 \Rightarrow ab = 10$

$\therefore$ a = 5, b = 2 (or) a = 2, b = 5

$\Rightarrow |a - b| = 3$

231.**Ans: (D); Sol:** det(A) = $\sigma^2 - \omega x$

$= \sigma^2 + \omega^2 = \sigma^2 - \omega x$

$\Rightarrow = \omega^2 = -\omega x$

$\Rightarrow x = -\omega$

232.**Ans: (D)**

Sol: $u = x_1e_1 + x_2e_2 + x_3e_3$

$(4,3,-3) = x_1(1,0,2) + x_2(0,1,0) + x_3(-2,0,1)$

$x_1 - 2x_3 = 4 \rightarrow (3),\ x_2 = 3, 2x_1 + x_3 = -3$

On solving these equations, we get

$x_1 = -\dfrac{2}{5},\quad x_2 = 3,\quad x_3 = \dfrac{-11}{5}$

$\therefore\quad u = -\dfrac{2e_1}{5} + 3e_2 - \dfrac{11}{5}e_3$

233.Ans: (d)

Sol: Given $P^3 = P$

Let λ be an eigen value of P

Then $\lambda^3 = \lambda \Rightarrow \lambda$=0,1,-1

234.Ans: (d)

Sol: $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \qquad \left(\because \begin{bmatrix} a \\ b \end{bmatrix} = P\begin{bmatrix} x \\ y \end{bmatrix}\right)$

$3x + y = a$ ……..(1)

$x + 3y = b$ ……..(2)

$a^2 + b^2 = 9x^2 + y^2 + 6xy + x^2 + 9y^2 + 6xy$

$\Rightarrow 10x^2 + 10y^2 + 12xy = 1 \qquad (\because a^2 + b^2 = 1)$

$a = 10, b = 10, h = 6$

$h^2 - ab < 0$

It represents ellipse

The lengths of semi-axes are $(AB - H^2)r^4 - (A + B)r^2 + 1 = 0$

$64r^4 - 20r^2 + 1 = 0$

$r^2 = \frac{1}{4} (or) r^2 = \frac{1}{16}$

Both r^2 values are positive, so it represents ellipse.

$r = \frac{1}{2} (or) r = \frac{1}{4}$

Length of Major axis = 2r = 1

Length of Minor axis = 2r = $\frac{1}{2}$

Equation of the major axis is $\left(a - \frac{1}{r_1^2}\right)x + hy = 0$

$(10 - 4)x + 6y = 0$

$\Rightarrow x + y = 0$

Equation of the minor axis is $\left(a - \frac{1}{r_2^2}\right)x + hy = 0$

$(10 - 16)x + 6y = 0$

$\Rightarrow y - x = 0$

Major axis exists along y = – x and minor axis exists along y = x

The vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ lies on the line y = x

$\therefore$ Option (d) is correct.

235.Ans : 3

Sol: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Char equation is $|A - \lambda I| = 0 \Rightarrow -\lambda^3 + 3\lambda^2 = 0$

$\Rightarrow \lambda = 3, 0, 0$

236.Ans:(a)

Sol: A $\rightarrow$ 1, –2

$A^2 \rightarrow$ 1, 4

$-3A \rightarrow$ –3, 6

$4I \rightarrow$ 4, 4

$A^2 - 3A + 4I \rightarrow$ 2, 14

$\therefore$ eigen values 2,14

Eigen vectors do not change.

237.Ans: (b)

Sol: $\rho(A_{4\times3}) = 2; \quad \rho(A^T_{3\times4}) = 2$

$\rho(A\times B) \le \min\{\rho(A), \rho(B)\}$

AA^T of order 4×4 whose rank ≤ 2

A^TA is of order 3×3 whose rank ≤ 2

238.Ans: All options are correct

239.**ANS. (B)**

Soln:

$$(AB) = \begin{pmatrix} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{pmatrix}$$

$(R_2 - 2R_1) : (R_3 - 5R_1)$

$$\approx \begin{pmatrix} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{pmatrix}$$

$(R_3 - R_2)$

$$\approx \begin{pmatrix} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & 0 & 0 & (c-b-3a) \end{pmatrix}$$

$(c-b-3a) = 0$

$3a + b - c = 0$

240.Ans: 0.125

Sol: $\lambda = 1, 2, 4$;; $|A| = 1 \times 2 \times 4 = 8 \Rightarrow |A^{-1}| = \frac{1}{|A|} = \frac{1}{8}$

$$\therefore |(A^{-1})^T| = |A^{-1}| = \frac{1}{8}$$

(OR)

Eigen values of A are 1, 2, 4

Eigen values of A^{-1} are $1, \frac{1}{2}, \frac{1}{4}$;;;$|A^{-1}| = 1\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$

$$|(A^{-1})^T| = |A^{-1}| = \frac{1}{8} = 0.125$$

241.**Ans**: 15 ;;;

Soln: If $\lambda = 2 + \sqrt{-1} = 2 + i$ is an Eigen value then 2- i is also Eigen value

$\therefore |p| = (2+i)(2-i)3 = (4+1)3 = 15$

(Or)

Complex roots of a polynomial equation always occur in pairs.

If (2+i) is an eigen vale then (2-i) is also an eigen value of P.

Determinant of P = (2+i) (2-i) 3=15

CHAPTER- 2
Fourier Series

01. Let g: $[0, \infty) \to [0, \infty)$ be a function defined by g(x) = x –[x], where [x] represents The integer part of x. (That is, it is the largest integer which is less than or equal to x). The value of the constant term in the Fourier series expansion of g(x) is ----------------

(GATE – 14 – EE –Set 1)

02. A function with a period 2Л is shown below. The Fourier series for this function is given by

(GATE –2000 [CE])

(a) $f(x) = \frac{1}{2} + \sum_{n-1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos nx$

(b) $f(x) = \sum_{n-1}^{\infty} \frac{2}{n\pi} \left(Sin\frac{n\pi}{2}\right) \cos nx$

(c) $f(x) = {}^{1}/_{2} + \sum_{n-1}^{\infty} \frac{2}{n\pi}\ Sin\ \frac{n\pi}{2} \sin nx$

(d) $f(x) = \sum_{n-1}^{\infty} \frac{2}{n\pi}\ Sin\ \frac{n\pi}{2} \sin nx$

03. Let f(x) be a real, periodic function satisfying f(–x) = – f(x). The general form of its Fourier series representation would be **(GATE –EE-16)**

a) $f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx)$

b) $f(x) = \sum_{k=1}^{\infty} b_k \sin(kx)$

c) $f(x) = a_0 + \sum_{k=1}^{\infty} a_{2k} \cos(kx)$

d) $f(x) = \sum_{k=0}^{\infty} a_{2k+1} \sin(2k+1)x$

04.The Fourier series of the function, **(GATE –CE-16)**

$f(x) = 0, -\pi < x \le 0$

$= \pi - x, 0 < x < \pi$

in the interval $[-\pi, \pi]$ is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{cosx}{1^2} + \frac{cos3x}{3^2} + ----\right] + \left[\frac{sinx}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + ---\right]$$

The convergence of the above Fourier series at $x = 0$ gives

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

$(c) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ $(d) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$

SOLUTIONS
FOURIER SERIES

01. Sol. g(x) = x – [x] is a periodic function with period '1'.

$\therefore\ a_0 = \frac{1}{2L}\int_0^{2L} g(x)dx$

(if '2L' is the period of g(x)

$= \int_0^1 X\,dx \quad (\because x - [x] = x \text{ in } (0,1)) \quad = \frac{1}{2}$.

02. Sol. $f(x)\begin{cases} 0 & if -\pi < X < \frac{-\pi}{2} \\ 1 & if \frac{-\pi}{2} < X < \frac{\pi}{2} \\ 0 & if \frac{\pi}{2} < X < \pi \end{cases}$

$a_o = \frac{2}{\pi}\int_o^{\pi/2} 1\,dx = 1$

$a_o = \frac{2}{\pi}\int_o^{\pi/2} \cos nx\,dx = \frac{2}{n\pi}\sin\frac{n\pi}{2},$

$b_n = 0$

03.Ans; (b)

Sol: Given f(x) is a odd periodic function so, cosine terms will be zero in trigonometric fourier series.

$\therefore f(x) = \sum_{k=1}^{\infty} b_k \sin(kx)$ __

04. (C)

Soln: $f(0) = \frac{\pi}{4} + \frac{2}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\right)$

$\frac{f(0^-) + f(0^+)}{2} = \frac{\pi}{4} + \frac{2}{\pi}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\right)$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{2}{\pi}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots..\right)$

$1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

CHAPTER - 3

PROBABILITY AND STATISTICS QUESTIONS

01. The probability that a number selected at randombetween100 and 999 (both inclusive) will not contain the digit 7 is **(GATE – 95)**

(a) $\frac{16}{25}$ (b) $\left(\frac{9}{10}\right)^3$ (c) $\frac{27}{75}$ (d) $\frac{18}{25}$

02. The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6.The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow? **(GATE – 97)**

(a) 0.3 (b) 0.25 (c) 0.35 (d) 0.4

03. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is **(GATE - 98)**

(a) $\frac{1}{6}$ (b) $\frac{3}{8}$ (c) $\frac{10}{8}$ (d) $\frac{1}{2}$

04. The probability that two friends share the same birth-month is **(GATE – 98)**

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) $\frac{1}{144}$ (d) $\frac{1}{24}$

05. Suppose that the expectation of a random variable X is 5. Which of the following statement is true? **(GATE – 99)**

(a) There is a sample point at which X has the value = 5

(b) There is a sample point at which X has the value > 5

(c) There is a sample point at which X has the value ≥ 5 (d) None of the above

06. Consider two events E_1 and E_2 such that $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$ and $(E_1 I\ E_2) = \frac{1}{5}$.

Which of the following statement is true? **(GATE – 99)**

(a) $P(E_1 Y E_2) = \frac{2}{3}$ (b) $E_1$and E_2 are independent

(c) E_1 and E_2 are not independent (d) $P\left(\frac{E_1}{E_2}\right) = \frac{4}{5}$

07. Four arbitrary points (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4) are given in the xy-plane using the method of least squares, if regressing y upon x gives the fitted line y = ax + b; and regressingx upon y gives the fitted line x = cy + d, then **(GATE – 99)**

(a) The two fitted lines must coincide (b) The two lines need not coincide

(c) It is possible that ac = 0 (d) a must be $\frac{1}{c}$

08. E_1 and E_2 are events in a probability space satisfying the following constraints $P(E_1)=P(E_2)$; $P(E_1 \text{ Y } E_2) = 1$; E_1& E_2 are independent then $P(E_1) =$ **(GATE-2000)**

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

09. In a manufacturing plant, the probability of making a defective bolt is 0.1, the mean and Standard deviation of defective bolts in a total of 900 bolts is respectively. **(GATE–2000)**

(a) 90 & 9 (b) 9 & 90 (c) 81 & 9 (d) 9 & 81

10. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day? **(GATE-2001)**

(a) $\frac{1}{7^7}$ (b) $\frac{1}{7^6}$ (c) $\frac{1}{2^7}$ (d) $\frac{7}{2^7}$

11. Four fair coins are tossed simultaneously. The probability that at least one heads and at least one tails turn up is? **(GATE-2002)**

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{15}{16}$

12. A regression model is used to express a variable Y as a function of another variable X. This implies that **(GATE-2002)**

(a) There is a causal relationship between Y & X
(b) A value of X may be used to estimate a value of Y
(c) Values of X exactly determine values of Y
(d) There is no causal relationship between Y & X

13. Let P(E) denote probability of an event E. given P(A) = 1, P(B) = ½ the values of P(A/B) & P(B/A) respectively are **(GATE-2003)**

(a) $\frac{1}{4}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{4}$ (c) $\frac{1}{2}$, 1 (d) 1, $\frac{1}{2}$

14. A box contains 10 screws, 3 of which are defective two screws are drawn at random with replacement the probability that none of the 2 screws is defective will be **(GATE-2003)**

(a) 100% (b) 50% (c) 49% (d) none of these

15. In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a Randomly picked child belongs to a family with two children? **(GATE-2004[IT])**

(a) $\frac{3}{23}$ (b) $\frac{6}{23}$ (c) $\frac{3}{10}$ (d) $\frac{3}{5}$

16. If a fair coin is tossed 4 times, what is the probability that two heads and two tails will result? **(GATE-2004[CS])**

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

17. An exam paper has 150 multiple choice questions of 1 mark each, with each question having four choices. Each incorrect answer fetches -0.25 marks. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all the students is **(GATE-2004[CS])**

(a) 0 (b) 2550 (c) 7525 (d) 9375

18. In a class of 200 students, 125 have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization, 30 students havetaken both data structures and computer organization, 15 students have taken all the threecourses. How many students have not taken any of the three courses?

(GATE-2004[IT])

(a) 15 (b) 20 (c) 25 (d) 35

19. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. given that gate 1 has failed, the probability that both gates 2 and 3 will fail is **(GATE-2004[IT])**

(a) 0.240 (b) 0.200 (c) 0.040 (d) 0.008

20. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the card is NOT replaced? **(GATE-2004[IT])**

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$ (c) $\frac{1}{169}$ (d) $\frac{1}{221}$

21. The following data about the flow of liquid was observed in a continuous chemical process plant

Flow rate(litres/sec)	7.5 to 7.7	7.7 to 7.9	7.9 to 8.1	8.1 to 8.3	8.3 to 8.5	8.5 to 8.7
Frequency	1	5	35	17	12	10

Mean flow rate of liquid is **(GATE-2004)**

(a) 8.00 liters/sec (b) 8.06 liters/sec (c) 8.16 liters/sec (d) 8.26 liters/sec

22. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3times. The probability that no 2 of the marbles drawn have the same color is **(GATE-2005[IT])**

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

23. If P and Q are two random events, then the following is true **(GATE-2005[EE])**

(a) Independence of P and Q implies that probability $(P \cap Q) = 0$

(b) Probability $(P \cap Q) \geq$ probability (P) + probability (Q)

(c) If P and Q are mutually exclusive then they must be independent

(d) Probability $(P \cap Q) \leq$ probability (P)

24. A fair coin is tossed three times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is **(GATE-2005[EE])**

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

25. Two dice are thrown simultaneously. The probability that the sum of numbers on both exceeds 8 is **(GATE-2005[PI])**

(a) $\frac{4}{36}$ (b) $\frac{7}{36}$ (c) $\frac{9}{36}$ (d) $\frac{10}{36}$

26. A lot had 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly two of the chosen items are defective is **(GATE-2005[ME])**

(a) 0.0036 (b) 0.1937 (c) 0.2234 (d) 0.3874

27. A single die is thrown two times. What is the probability that the sum is neither 8 nor 9? **(GATE-2005[ME])**

(a) $\frac{1}{9}$ (b) $\frac{5}{36}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

28. The probability that there are 53 Sundays in a randomly chosen leap year is **(GATE-2005[IN])**

(a) $\frac{1}{7}$ (b) $\frac{1}{14}$ (c) $\frac{1}{28}$ (d) $\frac{2}{7}$

29. A fair dice is rolled twice. The probability that an odd number will follow an even number is **(GATE-2005[EC])**

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

30. Lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is **(GATE-2005)**

(a) 0.0036 (b) 0.1937 (c) 0.2234 (d) 0.3874

31. The life of a bulb (in hours) is random variable with an exponential distribution $f(t) = \alpha e^{-\alpha t}$, $0 \leq t \leq \infty$. the probability that its value lies between 100 & 200 hours is **(GATE-2005[PI])**

(a) $e^{-100\alpha} - e^{-200\alpha}$ (b) $e^{-100} - e^{-200}$ (c) $e^{-100\alpha} + e^{-200\alpha}$ (d) $e^{-200\alpha} - e^{-100\alpha}$

32. Using given data points tabulated below, a straight line passing through the origin is fitted using least squares method. The slope of the line is **(GATE-2005)**

X	1	2	3
Y	1.5	2.2	2.7

(a) 0.9 (b) 1 (c) 1.1 (d) 1.5

33. Assume that the duration in minutes of a telephone conversation follows the exponentialdistribution f(x) = $\frac{1}{5}e^{-x/5}$, $x \geq 0$. The probability that the conversation will exceed fiveminutes is **(GATE-2007[IN])**

(a) $\frac{1}{e}$ (b) $1-\frac{1}{e}$ (c) $\frac{1}{e^2}$ (d) $1-\frac{1}{e^2}$

34. If the standard deviation of the spot speed of vehicles in a high way is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the co-efficient of variation in speed is **(GATE-2007[CE])**

(a) 0.1517 (b) 0.1867 (c) 0.2666 (d) 0.3646

35. Let X &Y be two independent random variables. Which one of the relations between expectation (E), Variance (Var) & Co variance (Cov) given below is FALSE? **(GATE-2007[ME])**

(a) E(XY) = E(X) E(Y) (b) Cov(X,Y) = 0

(c) Var(X+Y) = Var(X) + Var(Y) (d) $E(X^2Y^2) = (E(X))^2 (E(Y))^2$

36. Two cards are drawn at random in succession with replacement from a deck of 52 well shuffled cards. Probability of getting both 'Aces' is **(GATE-2007[PI])**

(a) $\frac{1}{169}$ (b) $\frac{2}{169}$ (c) $\frac{1}{13}$ (d) $\frac{2}{13}$

37. The random variable X taken on the values 1, 2 or 3 with probabilities $\frac{2+5P}{5}$, $\frac{1+3P}{5}$, $\frac{1.5+2P}{5}$ respectively. The values of P and E(X) are respectively **(GATE-2007[PI])**

(a) 0.05, 1.87 (b) 1.90, 5.87 (c) 0.05, 1.10 (d) 0.25, 1.40

38. If X is a continuous random variable whose probability density function is given by f(x) = $\begin{cases} k(5x-2x^2), 0 \leq x \leq 2 \\ 0, otherwise \end{cases}$ then P(X>1) is **(GATE-2007[PI])**

(a) $\frac{3}{14}$ (b) $\frac{4}{5}$ (c) $\frac{14}{7}$ (d) $\frac{17}{28}$

39. If E denotes expectation, the variance of a random variable X is given by **(GATE-2007[EC])**

(a) $E(X^2)-E^2(X)$ (b) $E(X^2)+E^2(X)$ (c) $E(X^2)$ (d) $E^2(X)$

40. An examination consists of two papers, paper 1 & paper 2. The probability of failing in Paper1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2. The Probabilityof failing in paper 1 is 0.6. The probability of a student failing in both the papers is **(GATE-2007[EC])**

(a) 0.5 (b) 0.18 (c) 0.12 (d) 0.06

41. X is uniformly distributed random variable that takes values between zero and one.The value of E(X^3) will be **(GATE-2008[EE])**

(a) 0 (b) 1/8 (c) ¼ (d) 1/2

42. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be **(GATE= 2008[IN])**

(a) $\frac{16}{3}$ (b) 6 (c) $\frac{256}{9}$ (d) 36

43. Consider a Gaussian distributed random variable with zero mean & standard deviation σ the value of its cumulative distribution function at the origin will be **(GATE-2008[IN])**

(a) 0 (b) 0.5 (c) 1 (d) 10 σ

44. P_X (X) = $Me^{(-2|X|)}$ +$Ne^{(-3|X|)}$ is the probability density function for the real random variableX, over the entire X-axis, M and N are both positive real numbers. The equation relating M and N is **(GATE-2008[IN])**

(a) M+$\frac{2}{3}$ N = 1 (b) 2M+$\frac{1}{3}$N = 1 (c) M+N = 1 (d) M+N = 3

45. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times? **(GATE-2008[ME])**

(a)$\frac{1}{4}$ (b)$\frac{3}{8}$ (c)$\frac{1}{2}$ (d) $\frac{3}{4}$

46. For a random variable X ($-\infty < X < \infty$) following normal distribution, the mean is μ =100. If the probability is P = α for X ≥110. Then the probability of X lying between 90 & 110, i.e.,P(90 ≤ X ≤ 110) & equal to **(GATE-2008[PI])**

(a)1-2α (b) 1-α (c)1-α/2 (d) 2α

47. In a game, two players X and Y toss a coin alternately. Whosever gets a 'head' first wins the game and the game is terminated. Assuming that player X starts the game. The probability of player X winning the game is **(GATE-2008[PI])**

(a) 1/3 (b) 1/2 (c) 2/3 (d) 3/4

48. Three values of x and y are to be fitted in a straight line in the form y = a + bx by the method of least squares. Given $\sum x$= 6, $\sum y$=21, $\sum x^2$ =14, $\sum xy$= 46. The valuesof a& b are respectively **(GATE-2008)**

(a) 2, 3 (b) 1, 2 (c)2, 1 (d) 3, 2

49. The standard normal probability function can be approximated asF $\left(X_N\right)$ = $\frac{1}{1+\exp(-1.7255X_N|X_N|^{0.12})}$ where X_N = Standard Normal Deviate. If Mean &Standard

Deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between90cm and 102cm is

(GATE-2009[CE])

(a) 66.7% (b) 50.0% (c) 33.3% (d) 16.7%

50. A fair coin is tossed 10 times. What is the probability that only the first two tosses willyield heads? **(GATE-2009[EC])**

(a) $\left(\frac{1}{2}\right)^2$ (b) $10c_2\left(\frac{1}{2}\right)^2$ (c) $\left(\frac{1}{2}\right)^{10}$ (d) $10c_2\left(\frac{1}{2}\right)^{10}$

51. Consider two independent random variables X and Y with identical distributions. The variables X and Y take values 0,1 and 2 with probability 1/2 , 1/4 and 1/4 respectively.What is the conditional probability P(X+Y = 2 / X-Y = 0)?

(GATE-2009[EC])

(a) 0 (b) 1/16 (c) 1/6 (d) 1

52. A discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates statements is true? **(GATE-2009[EC])**

K	1	2	3	4	5
P(X = K)	0.1	0.2	0.4	0.2	0.1

(a) Both the student and the teacher are right

(b) Both the student and the teacher are wrong

(c) The student is wrong but the teacher is right

(d) The student is right but the teacher is wrong

53. A screening test is carried out to detect a certain disease. It is found that 12% of thepositive reports and 15% of the negative reports are incorrect. Assuming that the probability ofa person getting positive report is 0.01, the probability that a person tested gets an incorrect report is **(GATE-2009[IN])**

(a) 0.0027 (b) 0.0173 (c) 0.1497 (d) 0.2100

54. If three coins are tossed simultaneously. The probability of getting at least one head is

(GATE-2009[ME])

(a) 1/8 (b) 3/8 (c)1/2 (d) 7/8

55. The standard deviation of a uniformly distributed random variable between 0 and 1 is

(GATE-2009[ME])

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

56. Assume for simplicity that N people, all born in April (a month of 30 days) are collected in aroom, consider the event of at least two people in the room being born on the same date of the month even if in different years. Ex. 1980 & 1985. What is the smallest N so that the Probability of this exceeds 0.5 is? **(GATE-2009[EE])**

(a) 20 (b) 7 (c) 15 (d) 16

57. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that first removed ball is white. The probability that the second removed ball is **(GATE-2010[EE])**

(a) 1/3 (b) 3/7 (c) 1/2 (d) 4/7

58. A fair coin is tossed independently four times. The probability of the event "The number of times heads show up is more than the number of times tails show up" **(GATE-2010[EC])**

59. What is the probability that a divisor 1099 is a multiple of 1096 **(GATE-2010[CS])**

60. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company there for subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q.What is the probability of a computer being declared faulty ? **(GATE-2010[CS])**

(a) pq + (1-p) (1-q) (b) (1-q) p (c) (1-p)q (d) pq

61. A box contains 2 washers, 3 nuts & 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is **(GATE-2010[ME])**

(a) 2/315 (b) 1/630 (c) 1/1260 (d) 1/2520

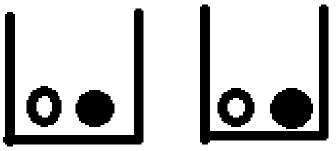
62. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing **(GATE-2010[CE])**

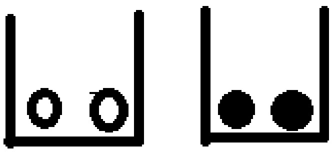
(a) 1/8 (b) 1/6 (c) 1/4 (d) 1/2

63. If a random variable X satisfies the Poisson's distribution with a mean value of 2. Then the probability that X > 2 is **(GATE-2010[PI])**

(a) $2e^{-2}$ (b) $1-2e^{-2}$ (c) $3e^{-2}$ (d) $1-3e^{-2}$

64. Two white and two black balls kept in two bins, are arranged in four ways as shown below in each arrangement, bin has to chosen randomly and only one ball needs to be picked randomly from the chosen bin. Which one of the following arrangement has the highest probability for getting a white ball picked? **(GATE-2010[PI])**

(a) 

(b) 

(c) 

(d) 

65. A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss **(GATE-2011[EC])**

(a) $\frac{2}{36}$ (b) $\frac{2}{6}$ (c) $\frac{5}{12}$ (d) $\frac{1}{2}$

66. The box 1 contains chips numbered 3,6,9,12 and 15. The box 2 contains chips numbered 6,11, 16, 21 and 26. Two chips, one from each box are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is **(GATE-2011[IN])**

(a) $\frac{6}{25}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{19}{25}$

67. It is estimated that the average number of events during a year is 3. What is the probability ofoccurance of not more than two more events over a two year duration? Assume that the no. of events follow a poisson distribution **(GATE-2011[PI])**

(a) 0.052 (b) 0.062 (c) 0.072 (d) 0.082

68. An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail.Probability of getting at least one head is **(GATE-2011[ME])**

(a) $\frac{1}{32}$ (b) $\frac{13}{32}$ (c) $\frac{16}{32}$ (d) $\frac{31}{32}$

69. There are two containers with one containing 4 red and 3 green balls and the other containing3blue balls and 4 green balls. one ball is drawn at random from each container. The probabilities that one of the ball is red and the other is blue will be (GATE-2011[CE])

(a) $\frac{1}{7}$ (b) $\frac{9}{49}$ (c) $\frac{12}{49}$ (d) $\frac{3}{7}$

70. If two fair coins are flipped and at least one of the outcomes is known to be a head. What is the probability that both outcomes are heads? **(GATE-2011[CS])**

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

71. If the difference between the expectation of the square of a random variable $[E(X^2)]$ and the square of the expectation of the random variable $[E(X)]^2$ is denoted by R, then **(GATE-2011[CS])**
(a) R = 0 (b) R < 0 (c) R ≥ 0 (d) R> 0

72. Consider a finite sequence of random values X = { x_1, x_2, x_3,……x_n }. Let μ_x be the mean and σ_x be the standard deviation of X. let another finite sequence Y of the equal length be derived from this $y_i = ax_i + b$, where a and b are positive constants. Let μ_y be the mean σ_y be the standard deviation of this sequence. Which one of the following statements isincorrect? **(GATE-2011[CS])**
(a) Index position of mode of X in X is the same as the index position of mode of Y in Y
(b) Index position of median of X in X is the same as the index position of median of Yin Y.
(c) $\mu_y = a\mu_x + b$ (d) $\sigma_y = a\sigma_x + b$

73. Two independent random variables X and Y are uniformly distributed in the interval [-1, 1]. The probability that max [X, Y] is less than ½ is **(GATE-2012[EC, EE, IN])**
(a) ¾ (b) 9/16 (c) ¼ (d) 2/4

74. A fair coin is tossed till a head appears for the first time. The probability thatthe number of required tosses is odd, is **(GATE-2012[EC, EE, IN])**
(a) 1/3 (b) ½ (c) 2/3 (d) ¾

75. A box contains 4 red balls and 6 black balls. There balls are selected randomly from the box one after another, without replacement. The probability that the selected setcontains set contains one red ball and two black balls is **(GATE-2012[ME, PI])**
(a) 1/20 (b) 1/12 (c) 3/10 (d) ½

76. An automobile plant contracted to buy shock absorbers from two suppliers X and Y.X supplies 60% and Y supplies40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable of X' s shock absorbers, 96% are reliable. Of Y' s shock absorbers, 72% are reliable the probability that a randomly choosen shock absorber, which is found to reliable, is made by Y is **(GATE-2012[ME, PI])**
(a) 0.288 (b) 0.334 (c) 0.667 (d) 0.720

77. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000mm and 200mm, respectively. The probabilitythat the annual precipitation will be more than 1200 mm is **(GATE-2012[CE])**
(a) <50% (b) 50% (c) 75% (d) 100%

78. In an experiment, positive and negative values are equally likely to occur.The probability of obtaining at most one negative value in five trials is **(GATE-2012[CE])**

(a) $\frac{1}{32}$ (b) $\frac{2}{32}$ (c) $\frac{3}{32}$ (d) $\frac{6}{32}$

79. A continuous random variable X has a probability density functionF(X) = e^{-x} , 0 < x <∞. Then P { X> 1 } is **(GATE-2013[EE, IN])**

(a) 0.368 (b) 0.5 (c) 0.632 (d) 1.0

80. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability P(3V ≥ 2U) IS **(GATE-2013[EC])**

(a) 4/9 (b) ½ (c) 2/3 (d) 5/9

81. Consider two identically distributed zero – mean random variables U and V.Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of X **(GATE-2013[EC])**

(a) F(x) – G(x) ≤ 0 (b) F(x) – G(x) ≥ 0

(c) (F(x) – G(x)).x ≥ 0 (d) (F(x) – G(x)).x ≥ 0

82. Let X be a normal random variable with mean 1 and variance 4.The probability P {X < 0} is **(GATE-2013[ME])**

(a) 0.5 (b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0 (d) 1.0

83. The probability that a student knows the correct answer to a multiple choice question is $\frac{2}{3}$.If the student does not know the answer, then the student guesses the answer. The probability of guessing the answer being correct is $\frac{1}{4}$. Given that the student has answeredthe question correctly, the conditional probability that the student knows the correct answer is **(GATE-2013[ME])**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{8}{9}$

84. Find the value of λ such that the function f(x) is a valid probability density function___________. **(GATE-2013[CE])**

F(x) = λ(x-1)(2-x) for 1 ≤ x ≤ 2 otherwise

(a) (b) (c) (d)

85. Suppose P is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval? **(GATE-2013[CS])**

(a) $8/(2e^3)$ (b) $9/((2e^3)$ (c) $17/(2e^3)$ (d) $26/(2e^3)$

86. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _________ **(GATE-2014 [EC-Set 1])**

87. Let X_1, X_2 and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability p{X_1 is the largest} is____________ **(GATE-2014 [EC-Set1])**

88. Let X be a real – valued random variable with E[X] and E[X^2] denoting the mean values of X and X^2, respectively. The relation which always holds true is **(GATE-2014 [EC-Set1])**

(a) $(E[X])^2 > E[X^2]$ (b) $E[X^2] \geq (E[X])^2$
(c) $E[X^2] = (E[X])^2$ (d) $E[X^2] > (E[X])^2$

89. Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, E[X], is **(GATE-2014 [EC-Set 2])**

90. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is **(GATE-2014 [EC-Set 3])**

(a) 0.067 (b) 0.073 (c) 0.082 (d) 0.091

91. A fair coin is tossed repeatedly till both head and tail appear at least once.The average number of tosses required is__________. **(GATE-2014 [EC-Set 3])**

92. Let X_1, X_2, and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{X_1 + X_2 \leq X_3\}$ is __________. **(GATE-2014 [EC-Set 3])**

93. Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to ____________. **(GATE-2014 [EC-Set 4])**

94. If calls arrive at a telephone exchange such that the time of arrival of any call is independent of the time of arrival of earlier or future calls, the probability distribution function of the total number of calls in a fixed time interval will be **(GATE-2014 [EC-Set 4])**

(a) Poisson (b) Gaussian (c) Exponential (d) Gamma

95. Parcels from sender S to receiver R pass sequentially through two post – offices. Each post – office has a probability $\frac{1}{5}$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post – office is __________. **(GATE-2014 [EC-Set 4])**

(a) 1/4 (b) 1/5 (c) 1/3 (d) 1/6

96. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is (n – 3) is **(GATE-2014 [EE-Set 1])**

(a) 2^{-n} (b) 0 (c) ${}^{n}C_{n-3}2^{-n}$ (d) 2^{-n+3}

97. Consider a die with the property that the probability of a face with 'n' dots showing up is proportional to 'n'.The probability of the face with three dots showing up is_________.

(GATE-2014 [EE-Set 2])

98. Let X be a random variable with probability density function

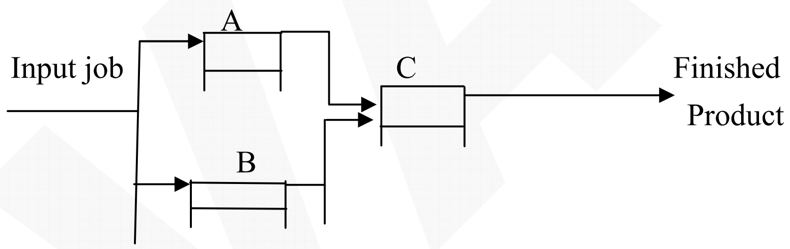
$$f(x) = \begin{cases} 0.2 \ \ for|x| \le 1 \\ 0.1 \quad for\ 1 < |x| \le 4 \\ 0 \qquad otherwise \end{cases}$$

The probability P(0.5 < x < 5) is_____________ **(GATE-2014 [EE-Set 2])**

99. Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years respectively, then the value of k is__________. **(GATE-2014 [EE-Set 3])**

100. Given that x is a random variable in the range [0, ∞] with a probability density function $\frac{e^{-\frac{x}{2}}}{K}$, the constant K is **(GATE-2014 IN -Set 1)**

101. The figure shown the schematic of a production process with machines A, B and C. An input job needs to be pre-processed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as:$P_A = 0.15, P_B = 0.05$ &$P_c = 0.1$



Assuming independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place)is **(GATE-2014 IN -Set 1)**

102. In the following table, x is a discrete random variable and p(x) is the probability density. The standard deviation of x is

X	1	2	3
P(X)	0.3	0.6	0.1

(GATE-2014ME -Set1)

(a) 0.18 (b) 0.36 (c) 0.56 (d) 0.6

103. A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is **(GATE-2014 ME -Set 2)**

(a) $\frac{7}{20}$ (b) $\frac{42}{125}$ (c) $\frac{25}{29}$ (d) $\frac{5}{9}$

104. Consider an unbiased cubic die with opposite faces coloured red, blue or green such that each colour appears only two times on the die. If the die is thrown thrice, the probability of obtaining red colour on top face of the die at least twice is **(GATE-2014 ME -Set 2)**

105. A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is **(GATE-2014 ME-Set 3)**

106. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of $\frac{1}{6}$, $\frac{2}{3}$ and $\frac{1}{6}$, respectively. Then mean value and the variance of the number of defective pieces produced by **(GATE-2014 ME -Set 3)**

(a) 1 and 1/3 (b) 1/3 and 1 (c) 1 and 4/3 (d) 1/3 and 4/3

107. A nationalized bank has found that the daily balance available in itssavings accounts follows a normal distribution with a mean of Rs. 500 and a standard deviation of Rs. 50. The percentage of savings account holders, who maintain an average daily balance more than Rs.500 is **(GATE-2014 ME-Set 4)**

108. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2 The probability of occurrence of less than2 accidents in the plant during a randomly selected month is **(GATE-2014 ME-Set 4)**

(a) 0.029 (b) 0.034 (c) 0.039 (d) 0.044

109. The probability density function of evaporation E on any day during a year in a watershed is given by f(E) = $\begin{cases} \frac{1}{5} & 0 \le E \le 5\ mm/day \\ 0 & otherwise \end{cases}$ the probability that E lies in between 2 and 4 mm/day in the watershed is (in decimal) ___ **(GATE-2014 CE -Set1)**

110. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is_____. **(GATE-2014 CE-Set 1)**

111. A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes; (i) Head, (ii) Head, (iii) Head, (iv) Head. The probability of obtaining a "Tail" when the coin is tossed again is **(GATE-2014 CE -Set 1)**

(a) 0 (b) ½ (c) 4/5 (d) 1/5

112. If {x} is a continuous, real valued random variable defined over the interval$(-\infty, +\infty)$ and its occurrence is defined by the density function given as:F(x) = $\frac{1}{\sqrt{2\pi^{*}b}}e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where 'a' and 'b' are the statistical attributes of the random variable {x}. the value of the integral $\int_{-\infty}^{a}\frac{1}{\sqrt{2\pi^{*}b}}e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ dx is **(GATE-2014 CE -Set 2)**

(a) 1 (2) 0.5 (c) π (d) $\frac{\pi}{2}$

113. An observer counts 240veh/h at a specific highway location. Assume that the vehicle arrival at the location is poisson distributed, the probability of having one vehicle arriving over a 30- second time interval is______________. **(GATE-2014 CE -Set 2)**

114. A simple random sample of 100 observations was taken form a large population the sample mean and the standard deviation were determined to be 80 and 12, respectively. The standard error of mean is____________. **(GATE-14- PI -Set1)**

115. Marks obtained by 100 students in an examination are given in the table

SI.NO	Marked Obtained	Number of Students
1	25	20
2	30	20
3	35	40
4	40	20

What would be the mean, median, and mode of the marks obtained by the students?

(GATE-14- PI -Set 1)

(a) Mean 33; Median 35; Mode 40. (b) Mean 35; Median 32.5; Mode 40

(c) Mean 33; Median 35; Mode 35 (d) Mean 35; Median 32.5; Mode 35

116. In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair will make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only10%. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day?

(GATE-14- PI-Set 1)

(a) 3/10 (b) 9/11 (c) 14/17 (d) 27/41

117. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is____________. **(GATE-14- CS -Set 1)**

118. Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is X/1296. The value of X is______________. **(GATE-14- CS-Set1)**

119. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p. Then 100p = ____. **(GATE-14- CS-Set2)**

120. Each of the nine words in the sentence "The Quick brown fox jumps over the lazy dog" is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is ______.(The answer should be rounded to one decimal place.) **(GATE-14- CS -Set 2)**

121. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is__________. **(GATE-14- CS -Set 2)**

122. Let S be a sample space and two mutually exclusive events A and B be such that A∪B = S. If P(.) denotes the probability of the event, the maximum value of P(A) P(B) is_____________. **(GATE-14-CS-Set 3)**

123. Suppose A and B are two independent events which probabilities $P(A) \neq 0 : P(B) \neq 0$. Let $\overline{A}, \overline{B}$ are their complements which of the following statements is false **(GATE-EC-15)**

(a) $P(A \cap B) = P(A).P(B)$ (b) $P(A/B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$ (d) $P(\overline{A} \cap \overline{B}) = P(\overline{A}).P(\overline{B})$

124. The input X to the binary symmetric channel (BSC) shown in figure is '1' with the probability 0.8. The cross over probability is $1/7$. If the received bit Y = 0, the conditional probability that '1' was transmitted is **(GATE-EC-15)**

125. A source omits bit '0' with probability $\frac{1}{3}$ and bit '1' with probability $\frac{2}{3}$. The emitted bits are communicated to the receiver. The receiver decides for either 0 or 1 based on the received value R. It is given that conditional density function of R is **(GATE-EC-15)**

$$f_{R/0}(r) = \begin{cases} \frac{1}{4} & ; -3 \leq x \leq 1 \\ 0 & ; \ otherwise \end{cases} \quad and \quad f_{R/1}(r) = \begin{cases} \frac{1}{6} & ; -1 \leq x \leq 5 \\ 0 & ; \ otherwise \end{cases}$$

The minimum decision error probability is

A) 0 B) $1/12$ C) $1/9$ D) $1/6$

126. Ram and Ramesh appeared in an interview for two vacancies in same dept. The probability that Ram's selection is $\frac{1}{6}$ and Ramesh is $\frac{1}{8}$. What is the probability that only one of them is selected? **(GATE-EC-15)**

A) $47/48$ B) $1/4$ C) $13/48$ D) $35/48$

127. Let X = {0, 1} and Y = {0, 1} be two independent binary random variables. If P (X=0) = p and P(Y=0) = q; then $P(X+Y) \geq 1$ is equal to **(GATE-EC-15)**

A) pq +(1-p)(1-q) B) pq< 0 C) p(1-q) D) 1-pq

128. A random variable X represents number of times a fair coin needs to be tossed till two consecutive heads appear for first time. The expectation of X is **(GATE-EC-15)**

129. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown. The expected value of X is **(GATE-EC-15)**

130. The variance of a random variable X with p.d.f $f(x) = \frac{1}{2}|x|e^{-|x|}$ is **(GATE-EC-15)**

131. Given set A = {2, 3, 4, 5} and B = {11, 12, 13, 14, 15} two numbers are selected randomly one from each set. What is the probability that sum of two number is equal to 1 to 10 **(GATE-EE-15)**

(A) 0.20 (B) 0.25 (C) 0.30 (D) 0.33

132. The probability that a student pass in maths, physics, chemistry are m, p, c respectively of these subjects students has 75% chance of passing atleast one, 50 % chance of passing atleast two and 40% chance of pasiing exactly two : Following relations are drawn in m, p, c

I : $p + m + c = 27/20$ II : $p + m + c = 13/20$ III: $p \times m \times c = 1/10$ **(GATE-EE-15)**

(A) only I is true (B) only II is true (C) II & III are true (D) I & III are true

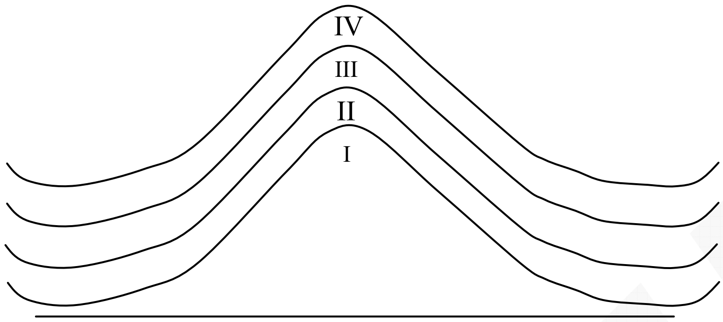
133. A random variable X has a p.d.f $f(x) = \begin{cases} a+bx : & 0<x<1 \\ 0 & otherwise \end{cases}$ If the expected value E(X) = $2/3$ then $P(X < 0.5)$ is **(GATE-EE-15)**

134. Two players A, B alternatively keep rolling a fair dice. The person to get six first wins the game. If A starts the game, the chance of winning of B is **(GATE-ME-15)**

(A) $5/11$ (B) $1/2$ (C) $7/13$ (D) $6/11$

135. Among the four normal distribution with probability density function as shown below, which one has the lowest varience? **(GATE-ME-15)**

(A) I (B)II (C) III (D) IV



136. The probability of obtaining at least two sixes in throwing fair dice, 4 times is

(GATE-ME-15)

(A) $^{425}/_{432}$ (B) $^{19}/_{144}$ (C) $^{13}/_{144}$ (D) $^{125}/_{432}$

137. The vendors are asked to supply very high precision component. The respective probabilities of their meeting the strict design specifications are 0.8, 0.7 and 0.5. Each vendor supplies one component. The probability that out of total three components supplied by the vendors at least one will meet the design specification is

(GATE-ME-15)

138. The chance of a student passing an exam is 20%, the chance of a student passing an exam and getting above 90% marks in it is 5%. Given that a student pass the examination the probability that student gets 90% marks is **(GATE-ME-15)**

(A) $\frac{1}{18}$ (B) $\frac{1}{4}$ (C) $\frac{2}{9}$ (D) $\frac{5}{18}$

139. A coin is tossed thrice. Let X be an event has head occurs in each of the first two tosses. Let Y be an event that tail occurs on third toss. Let Z be an event that two tails occurs in 3 tosses based on the above information which of the following statement is true.

(GATE-ME-15)

(A) X and Y are not independent (B) Y and Z are dependent

(C) Y and Z are independent (D) X and Z are independent

140. If P(X) = $\frac{1}{4}$; P(Y) = $\frac{1}{3}$ and $P(X \cap Y) = \frac{1}{2}$, the value of $P\left(\frac{Y}{X}\right)$ is **(GATE-ME-15)**

(A) $\frac{1}{4}$ (B) $\frac{4}{25}$ (C) $\frac{1}{3}$ (D) $\frac{29}{50}$

141. Consider the following probability mass function of a random variable X

$$p(x,q) = \begin{cases} q & \text{if } X = 0 \\ 1-q & \text{if } X = 1 \\ 0 & \text{otherwise} \end{cases}$$ **(GATE-CE-15)**

If $\varepsilon = 0.4$, the variance of X is

142. Four cards are randomly selected from a pack of 52 cards. If first two cards are kings what is the probability that third card is king **(GATE-CE-15)**

(A) $4/52$ (B) $2/50$ (C) $1/52 \times 1/52$ (D) $1/52 \times 1/52 \times 1/52$

143. The probability density function of a random variable X is **(GATE-CE-15)**

$$f(x) = \frac{x}{4}(4 - x^2) \quad for\ 0 \le x \le 2$$
$$= 0 \quad otherwise$$

The mean μ_z of a random variable is

144. X and Y denote the sets containing 2 and 20 distinct objects respectively and F denotes the set of all possible function defined from X to Y. Let f be randomly chosen function in F. The probability of f being one to one is **(GATE-CE-15)**

145. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by **(GATE-ME-16)**

(A) μ (B) μ^2 (C) μ (D) $1/\mu$

146. The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is __________ **(GATE-ME-16)**

147. Students taking an exam are divided into two groups, **P** and **Q** such that each group has the same number of students. The performance of each of the students in a test was evaluated out of 200 marks. It was observed that the mean of group **P** was 105, while that of group **Q** was 85. The standard deviation of group **P** was 25, while that of group **Q** was 5.

Assuming that the marks were distributed on a normal distribution, which of the following statements will have the highest probability of being **TRUE**?

(A) No student in group **Q** scored less marks than any student in group **P**.

(B) No student in group **P** scored less marks than any student in group **Q**.

(C) Most students of group **Q** scored marks in a narrower range than students in group **P**.

(D) The median of the marks of group **P** is 100.

148.Three cards were drawn from a pack 52 cards. The probability that they are a king, a queen, and a jack is **(GATE-ME-16)**

(A) $\frac{16}{5525}$ (B) $\frac{64}{2197}$ (C) $\frac{3}{13}$ (D) $\frac{8}{16575}$

149. The area (in percentage) under standard normal distribution curve of random variable Z within limits from −3 to +3 is __________ **(GATE-ME-16)**

150. The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is _______. **(GATE-EC-16)**

151. Two random variables X and Y are distributed according to **(GATE-EC-16)**

$$f_{X,Y}(x,y) = \begin{cases} (x+y), & 0 \le x \le 1, \quad 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability P(X +Y ≤ 1) is _________

152. The probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a "head" is obtained. If the tosses are independent, then the probability of getting "head" for the first time in the fifth toss is _____ **(GATE-EC-16)**

153. Let the probability density function of a random variable X, be given as:

$f_x(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$ where u(x) is the unit step function. **(GATE-EE-16)**

Then the value of 'a' and Prob {X ≤0}, respectively, are

a. $2, \frac{1}{2}$ b) $4, \frac{1}{2}$ c) $2, \frac{1}{4}$ d) $4, \frac{1}{4}$

154.Probability density function of a random variable X is given below **(GATE-CE-16)**

$$f(x) = \begin{cases} 0.25 & \text{if } 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$P(X \le 4)$ is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

155. X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which one of the following is the value of $P(X \cup Y)$? **(GATE-CE-16)**

(A) 0.7 (B) 0.5 (C) 0.4 (D) 0.3

156. If f(x) and g(x) are two probability density function. **(GATE-CE-16)**

$$f(x) = \begin{cases} \frac{x}{a} + 1 & : -a \le x < 0 \\ -\frac{x}{a} + 1 & : \quad 0 \le x \le a \\ 0 & \quad otherwise \end{cases}$$

$$g(x) = \begin{cases} -\frac{x}{a} & : -a \le x < 0 \\ \frac{x}{a} & : \quad 0 \le x \le a \\ 0 & \quad otherwise \end{cases}$$

Which one of the following statements is true?

(A) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are same

(B) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are different

(C) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are same

(D) Mean of f(x) and g(x) are different; Variance of f(x) and g(x)are different

157. In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is ____.

(GATE-CE-16)

(A) 40.00 (B) 46.02 (C) 60.01 (D) 92.02

158.Type II error in hypothesis testing is **(GATE-CE-16)**

(A) acceptance of the null hypothesis when it is false and should be rejected

(B) rejection of the null hypothesis when it is true and should be accepted

(C) rejection of the null hypothesis when it is false and should be rejected

(D) acceptance of the null hypothesis when it is true and should be accepted

159. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100hours is________ **(GATE-CS-16)**

160. A probability density function on the interval [a, 1] is givenby$1/x^2$ and outside this interval the value of the function is zero. The value of a is __________ **(GATE-CS-16)**

161. A sequence x[n] is specified as $\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, for $n \geq 2$

The initial conditions are x[0] = 1, x[1] = 1 and x[n] = 0 for n<0. The value of x[12] is ____ **(GATE-EC-16)**

PROBABILITY AND STATISTICS SOLUTIONS

01. Sol: (d)

Total number of elements in a sample space $n(S) = {}^{900}C_1 = 900$:

Number of favorable cases= $8 \times 9 \times 9$:

Required probability = $\frac{8 \times 9 \times 9}{900} = \frac{18}{25}$

02. Sol: (d)

From the given data P (E_1) = 0.5, P (E_2) = 0.6, P (E_1 Y E_2) = 0.7

Required probability = P ($E_1 \cap E_2$) = P (E_1) + P (E_2) – P (E_1 Y E_2)= 0.4

03. Sol: (b)

Probability of getting an odd number when a die is rolled = $\frac{3}{6} = \frac{1}{2}$

here number of trials $n = 3$

By Binomial distribution required probability = $3_{c_1} \times \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{3}{8}$

04. Sol: (b)

Number of elements in a sample space = 12×12

Number of favorable cases = ${}^{12}C_1$

Required probability $\frac{{}^{12}C_1}{12 \times 12} = \frac{1}{12}$

05. Sol: (c)

Since all sample points are within the limits wich are less than 5, so expected value can not exceed 5

06. Sol: (c)

Since P (E_1 Y E_2) = $\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = \frac{19}{30}$ and P (E1 / E2) = $\frac{3}{5} \neq \frac{4}{5}$

Remaining options are not possible so option 'C' is correct

07. Sol: option (b) is only possible, remaining cases are not possible

08. Sol: (b)

given P (E_1) = 1,since $E_1$and E_2 are independent

$P(E_1 U E_2) = P(E_1) + P(E_2) – P(E_1)P(E2)$

= 1+1-1 =1

09. Sol: (a)

Given p = 0.1, n = 900, q=1-p = 0.9

Mean of a binomial distribution = n × p = 90

Standard deviation S.D = σ = $\sqrt{n \times p \times q}$ = 9

10. Sol: (b)

The probability that an accident can occurred on any day of the week = $\frac{1}{7}$

By Binomial distribution, required probability = $7_{c_1} \times \frac{1}{7^7} = \frac{1}{7^6}$

11. Sol: (c)

n(s) = 16

since atleast one head and atleast one tail will turn up, number of favourable cases= 14

∴the required probability = $\frac{14}{16} = \frac{7}{8}$

12. Sol: (b)

Y = ax + b

represent a line of regression, which estimate Y using a known parameter $'x'$

13. Sol: (d)

Given P(A) = 1, P(B) = 1/2, since $B \le A:\ A \cap B = B$

$$\Rightarrow P(A \cap B) = P(B) = \frac{1}{2}$$

Now $P\left(\frac{A}{B}\right) = \frac{P(AIB)}{P(B)} = 1$,

$$P\left(\frac{B}{A}\right) = \frac{P(BIA)}{P(A)} = \frac{1}{2}$$

14. Sol: (d)

The probability that first screw drawn is defective is 3/10.

The probability that second screw drawn is also defective is 3/10. (because first one is replaced).

Required probability = $\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$

15. Sol: (b)

Let x be the number of families

Number of children belonging to families with 3 children = $\frac{x}{2}\times 3$ (since 50% of the families belongs to 3 children)

Number of children belonging to families with 2 children = $\frac{3x}{10}\times 2$ (since 30%of the families belongs to 2 children)

Number of children belonging to families with 1 child = $\frac{2x}{5}$ (since 20% of the families belongs to 1 child)

∴probability of the family having 2 children = $\dfrac{\frac{3x}{10}\times 2}{\frac{3x}{2}+\frac{3x}{5}+\frac{2x}{5}}=\frac{6}{23}$

16. Sol: (a)

By binomial distribution required probability = P (X = 2) = $4_{c_2}\times\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^2=\frac{3}{8}$

17. Sol: (d)

Let X denote the marks obtained for each question. The probability distribution for X is given below.

X	1	-0.25
P(X)	0.25	0.75

Expected marks for one question = E(X) = $\sum X.P(X)$

= 1. (0.25) + (- 0.25) (0.75)

= 0.0625

Expected marks for 150 questions = 150. (0.0625)

Total expected marks for 1000 students = 1000.(150).(0.0625)

= 9375

18. Sol: (c)

Given $P(P) = \frac{125}{200}, P(D) = \frac{85}{200}, P(C) = \frac{65}{200}$

$$P(PID) = \frac{50}{200}, P(PIC) = \frac{35}{200}, P(DIC) = \frac{30}{200}, P(PIDIC) = \frac{15}{200}$$

$$P(PYDYC) = P(P) + P(D) + P(C) - P(PID) - P(DIC) - P(PIC) + P(PIDIC) = \frac{7}{8}$$

$$\Rightarrow P(\overline{P}I\overline{D}I\overline{C}) = \frac{1}{8}$$

Number of students so have not taken any of the three courses = $\frac{1}{8} \times 200 = 25$

19. Sol: (c)

Given P (failure of each gate) = 0.2

Since all gates are operating independently,we have $P(G_2 \cap G_3) = P(G_2).P(G_3)$

= 0.2 ×0.2 = 0.04

20. Sol: (d)

Probability of drawing two king cards without replacement = $\frac{4_{C_2}}{52_{C_2}} = \frac{1}{221}$

21. Sol: (c) for the values of x we have to take mid values for the given data

Mid value x	7.6	7.8	8.0	8.2	8.4	8.6
Freq	1	5	35	17	12	10

Mid value: 7.6 7.8 8.0 8.2 8.4 8.6

Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{65.8}{80} = 8.16$

22. Sol: (b)

3 balls of different colors can be drawn in 6 ways

Required probability = $6\left(\frac{10}{60} \times \frac{20}{60} \times \frac{30}{60}\right) = \frac{1}{6}$

23. By observing all options (d) is correct answer

24. Sol: (b)

Since first outcome is head,Sample space = {HHT, HTT, HTH, HHH}

Required probability= 2/4 = ½

25. Sol: (d)

n(s) = 6×6 = 36

sum exceeds 8 means it may be 9 or 10 or 11 or 12

$\Rightarrow$ E = { (3,6), (4,5), (5,4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6)}

Required probability = $\frac{n(E)}{n(s)} = \frac{10}{36}$

26. Sol: (b)

Given p = 0.1, q=0.9, n=10

Required probability = P (X = 2)

= $10c_2$ × (0.1)2 × (0.9)8 = 0.1937

27. Sol: (d)

Let E be the event of getting the sum 8 or 9

$\Rightarrow$ n(E) = 9

$\Rightarrow$ P(E) = $\frac{9}{36} = \frac{1}{4}$

$\Rightarrow$ Required probability = 1-P(E) = $\frac{3}{4}$

28. Sol: (d)

leap year = 366 days = $\frac{364 days + 1 + 1}{(52 \times 7)}$

Sample space = {SM,MT,TW,WTH,THF,FSAT,SATS} (remaining two days)

Required probability = $\frac{2}{7}$

29. Sol: (d)

Probability of getting an odd number when a fair dice is rolled = $\frac{3}{6} = \frac{1}{2}$

Probability of getting an even number when a fair dice is rolled = $\frac{3}{6} = \frac{1}{2}$ Required probability = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (Because both are independent since one after other case)

30. Sol: (b)

p=0.1, n=10

Mean $\mu = nP = 10 \times 0.1 = 1$

P(X =2) = $e^{-\mu}\frac{\mu^2}{2} = \frac{1}{2e} = 0.18$

31. Sol: (a)

P(100 < X < 200) = $\int_{100}^{200} f(t)dt = \int_{100}^{200} \alpha e^{-\alpha t} dt = e^{-100\alpha} - e^{-200\alpha}$

32. Sol: (b)

Let the required line be $y = bx \cdots (1)$

Then the normal of above equation is given by

$\sum xy = b\sum x^2 \Rightarrow b = \frac{\sum xy}{\sum x^2} = \frac{14}{14} = 1$

x	y	xy	x2
1 2 3	1.5 2.2 2.7	1.5 4.4 8.1	1 4 9
$\sum x = 6$	$\sum y = 6.4$	$\sum xy$=14	$\sum x^2 = 14$

33. Sol: (a)

$P(5 < X < \infty) = \int_5^\infty f(x)dx = \int_5^\infty \frac{1}{5}e^{-x/5}dx = \frac{1}{e}$

34. Sol: (c)

σ = 8.8μ = 33

Co-efficient of variation = $\frac{\sigma}{\mu} = \frac{8.8}{33} = 0.266$

35. Sol: (d)

All (Other cases are true for independent random variables)

36. Sol: (a)

Probability of drawing an Ace card from a pack of cards =$\frac{4_{c_1}}{52_{c_1}}$

Required probability = $\frac{4_{c_1}}{52_{c_1}} \times \frac{4_{c_1}}{52_{c_1}} = \frac{1}{169}$ (since cards are drawn one after other wish replacement)

37. Sol: (a)

Sum of probabilities = 1

$$\frac{2+5P}{5}+\frac{1+3P}{5}+\frac{1.5+2P}{5}=1 \Rightarrow 4.5+10P=5 \Rightarrow P=0.05$$

$$E(X)=xP(X=x)=1\left(\frac{2+5P}{5}\right)+2\left(\frac{1+3P}{5}\right)+3\left(\frac{1.5+2P}{5}\right)$$

$$=\frac{8.5+0.85}{5}=1.87$$

38. Sol: (d)

We have $\int_{-\infty}^{\infty} f(x)dx=1 \Rightarrow \int_0^2 k\left(5x-2x^2\right)dx=1 \Rightarrow k=\frac{3}{14}$

Now $P\left(X>1\right)=\int_1^{\infty} f(x)dx=\int_1^2 \frac{3}{14}\left(5x-2x^2\right)dx=\frac{17}{28}$

39. Sol: (a)

We have $V_{ar}\left(X\right)=E(X^2)-\left[E(X)\right]^2$

40. Sol: (c)

Given that P (1) = 0.3, P(2) = 0.2, P(1/2) = 0.6

Using conditional probability, the probability that a student is failed in paper1 given that He is failed in paper2 is given by

$$P\left(\frac{1}{2}\right)=\frac{P(1\cap 2)}{P(2)}$$

$$0.6=\frac{P(1\cap 2)}{0.2}$$

Required probability = $P(1\cap 2)=0.12$

41. Sol: (c)

The p.d.f of uniform distribution is

f(x) = $\frac{1}{b-a}$ for a < x < b

Here f(x) = $\frac{1}{1-0}=1$

$$E\left(X^3\right)=\int_0^1 x^3 f(x)dx=\frac{1}{4}$$

42. Sol: (a)

For uniform distribution Var (X) $=\frac{(b-a)^2}{12}$ for a < x < b

$$=\frac{16}{3}$$

43. Sol: (b)

Since the curve is symmetric about $x = 0$ is i.e at $z = 0$

44. Sol: (a)

$$\int_{-\infty}^{\infty} P_x(X)dx = 1$$

$$\int_{-\infty}^{\infty} \left[Me^{-2|x|} + Ne^{-3|x|} \right] dx = 1$$

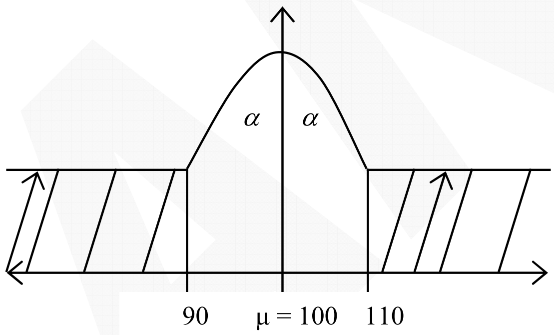
$$\Rightarrow 2M\int_0^{\infty} e^{-2x}dx + 2N\int_0^{\infty} e^{-3x}dx = 1 \Rightarrow M + 2\frac{N}{3} = 1$$

45. Sol: (a)

n=4, p=1/2, q =1/2

$$P(X = 3) = 4_{c_3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{4}{16} = \frac{1}{4}$$

46. Sol: (a)



P (X ≥ 110) = α

P (X ≤ 90) = α

P (90 ≤ X ≤ 110) = 1-2α   90 $\mu = 100$ 110

47. Sol: (c)

Let P be the probability of getting head by any of the three persons

$$P(E) = p + q^2 p + q^4 p + \ldots\ldots = P(1 + q^2 + q^4 + \ldots\ldots\infty) = \frac{p}{1-q^2} = \frac{2}{3}$$

48. Sol:

Given $\sum x = 6, \sum y = 21, \sum x^2 = 14, \sum xy = 46$ and three values of x and y to fit the straight line

Let y = a + bx

Then the normal equations are

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

21 = 3a + 6b

46 = 6a + 14b

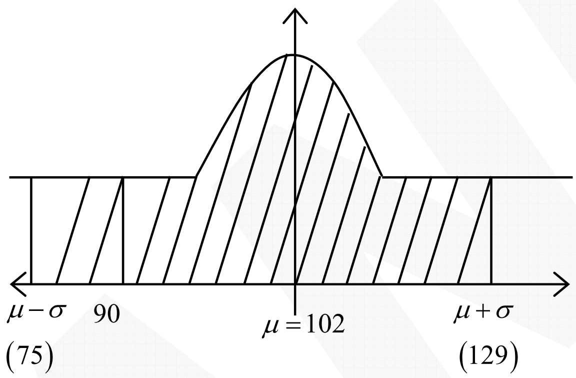
$\Rightarrow$ 23 = 3a + 7b

-21 = -3a + (-) 6b

2 = b

$\therefore$ a = 3

49. Sol: (d)



μ= 102, σ = 27

We know that P (μ−σ < X <μ+σ) = 66%

$\Rightarrow$ P(75 < X < 129) = 66%

$\Rightarrow$ P(75 < X < 102) = 33%

Therefore P (90 < X < 102) = 16.7%

50. Sol: (c)

Probability for first two tosses to yield heads is $\left(\frac{1}{2}\right)^2$, so remaining tosses must be tails.

Therefore the probability for remaining tosses to be tails is $\left(\frac{1}{2}\right)^8$

Hence required probability = $\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{10}$

51. Sol: (c)

$$P\left(\frac{X+Y=2}{X-Y=0}\right)=\frac{P\left[\left(X+Y=2\right)\cap\left(X-Y=0\right)\right]}{P(X-Y=0)}$$

$$P=\frac{P(X=1,Y=1)}{P(X-Y=0)}=\frac{\frac{1}{4}\times\frac{1}{4}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}}=\frac{1}{6}$$

52. Sol: (b)

Given $\mu=3.5,\sigma^2=1.5$

$calculated,\mu=\sum XP(X=x)=3.3$

$\sigma^2=\sum x^2P(X=x)-\left[\sum xP(X=x)^2\right]^2$

$10.6-9=1.6$

53. Sol: (c)

Given probability of getting positive report = 0.01

And probability of getting negative report = 0.99

Required probability = probability of getting incorrect report when it is test positive or Negative

= (0.01) (0.12) + (0.99) (0.15)

$=\frac{12}{10000}+\frac{99\times15}{10000}=0.1497$

54. Sol: (d)

Probability of getting at least one head = 1- probability of getting none heads

$=1-P(X=0)$

$==1-3_{c_0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$

$=\frac{7}{8}$

55. Sol: (a)

For uniform distribution, $VarX=\frac{(b-a)^2}{12}$ in $[0,1]$

$=\frac{(1-0)^2}{12}=\frac{1}{12}$ for a < X < b

$S..D=\sigma=\frac{1}{\sqrt{12}}$

56. Sol:(b)

Let N = 2,

Probability that none of them born on the same day = $1-\frac{1}{30}=\frac{29}{30}$

Required probability $=1-\frac{29}{30}=\frac{1}{30}$ is not greater than 0.5

Let N = 3, required probability = $1-\frac{29}{30}\times\frac{28}{30}=0.098$

Let N = 4, required probability = $1-\frac{29}{30}\times\frac{28}{30}\times\frac{27}{30}=0.188$

Similarly N = 7,

Required probability = $1-\frac{29}{30}\times\frac{28}{30}\times\frac{27}{30}\times\frac{26}{30}\times\frac{25}{30}\times\frac{24}{30}=0.53>0.5$

57. Sol: (c)

Given that first removed ball is white.

Then the balls left in box are 3 white and 3 red balls.

Probability of second removed ball is red $=\frac{3_{c_1}}{6_{c_1}}=\frac{1}{2}$

58. Sol: (d)

let X be the random variable which denote number of heads.

Given n = 4

Required probability is minimum 3 heads

$$\Rightarrow P(x\geq 3)=4_{c_3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)+4_{c_4}\left(\frac{1}{2}\right)^4\left(\because P(x=3)+P(x=4)\right)=\frac{5}{16}$$

59. Sol: (a)

Number of divisors of $10^n=(n+1)^2$

Number of divisors of $10^{99}=10{,}000$

Number of divisors of 10^{99} which are multiples of 10^{96}

= Number of divisors of $10^3=(3+1)^2=16$

Required probability = 16/10000 = 1/625

60. Sol: (a)

Probability of faulty assembly of any computer = p

Probability that testing process gives the correct result = q

Required probability = probability of faulty assemble when it is tested correct (or)

Probability of right assemble when it is tested incorrect

61. Sol: (c)

w-2, n-3, b-4

Required probability = $\frac{2}{9}\times\frac{1}{8}\times\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}\times\frac{4}{5}\times\frac{3}{3}$

62. Sol: (c)

Sample space = {HH, HT, TH, TT}

Required probability, P (E) = 1/4

63. Sol: (d)

Given mean of Poisson distribution is 2. i.e. $\lambda=2$

Required probability P $(X \geq 2)$ = 1- P(X = 0) + P(X = 1)

$= 1- [\lambda^0.e^{-2} + e^{-2}.\lambda]$

$= 1- e^{-2}[1+2] = 1-3e^{-2}$

64. Sol: (c)

probability of picking a white ball randomly is

(a) $\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)=\frac{1}{2}$ (b) $\left(\frac{1}{2}\times\frac{2}{2}\right)+\left(\frac{1}{2}\times 0\right)=\frac{1}{2}$

(c) $\left(\frac{1}{2}\times\frac{1}{1}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)=\frac{2}{3}$ (d) $\left(\frac{1}{2}\times 0\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)=\frac{1}{6}$

65. Sol: (c)

E = { (1,2) ,(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5),(4,6), (5,6) }

N (E) =15, n(s) = 36

Required probability = $\frac{n(E)}{n(s)}=\frac{15}{36}=\frac{5}{12}$

66. Sol: (d)

n(s) = $5_{c_1}\times 5_{c_1}=25$

Let E be the event of picking one chip from each box such that product of numbers

On chips is even number.

$\therefore$ n(E) = (2×5) + (3×3) = 19

Required probability = $\frac{19}{25}$

67. Sol: (b)

Apply poisson distribution

Given n=2, p=3

$\therefore \lambda = np = 6$

Required probability is given by,

$= P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2}\right] = 0.0619$

68. Sol: (d)

$n = 5, p = \frac{1}{2}, q = \frac{1}{2}$

By binomial distribution $P(X \ge 1) = 1 - P(X=0) = 1 - 5_{c_0}\left(\frac{1}{2}\right)^5 = \frac{31}{32}$

69. Sol: (c)

Required probability $= \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$

70. Sol: (a)

n(s) = 4

Let E_1 be the event of at least one of the outcomes is head.

$\therefore P(E_1) = \frac{3}{4}$

Let E_2 be the event that both outcomes are heads.

$\therefore P(E_2) = \frac{1}{4}$

$\therefore P(E_1 I E_2) = \frac{1}{4}$

Required probability = $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_2 I E_1)}{P(E_1)} = \frac{1}{3}$

71. Sol: (d)

$\left(R = Var(X) = E(X^2) - [E(X)]^2 > 0\right)$ ($\because$ variance is positive)

72. Sol: (d)

Since Variance of constant is zero.

73. Sol: (b)

P[max(X, Y)]

= P [X ≤ x, Y ≤ y]

= P[X ≤ x] .P[Y ≤ y] as X & Y are independent

$= \int_{-1}^{1/2} \frac{1}{2}\, dx \int_{-1}^{1/2} \frac{1}{2}\, dy = \frac{9}{16}$

74. Sol:- (c)

Req. Prob = $\frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots\ldots\ldots.\infty$

$= \frac{\frac{1}{2}}{-1\frac{1}{4}} = \frac{2}{3}$

75. Sol:- (d)

Given 4R and 6B

required probability = $\frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{1}{2}$

76. Sol:- (b)

P(X) = 0.6 P(Y) = 0.4

P($\frac{R}{X}$) =0.96 P($\frac{R}{Y}$) = 0.72

P($\frac{Y}{R}$) = $\frac{P(Y \cap R)}{P(R)}$

P($\frac{Y}{R}$) = $\frac{P(Y)P(R/_Y)}{P(X)P(R/_X) + P(Y)P(R/_Y)}$

$= \frac{(0.4)(0.72)}{(0.6)(0.96) + (0.4)(0.72)} = 0.334$

77. Ans :- (a)

Given μ = 1000 $\sigma = 200$

We know that Z $= \frac{X-\mu}{\sigma}$

When X = 1200, Z = $\frac{1200-1000}{200} = 1$

Req. Prob = P(x > 1200)

= P(Z > 1) = 0.5 – P(0 < Z < 1)

< 0.5

78. Ans (d)

P(X ≤ 1) = P(X = 0) + P(X = 1)

$= 5_{C_0} \times \left(\frac{1}{2}\right)^5 + 5_{C_1} \times \left(\frac{1}{2}\right)^5 = \frac{1+5}{32} = \frac{6}{32}$

79. Sol:- (a)

$P(X > 1) = \int_1^\infty f(x).\,dx = \int_1^\infty e^{-x}dx = \left.\frac{e^{-x}}{-1}\right|_1^\infty = e^{-1} \text{ or } \frac{1}{e} = 0.368$

80 Sol:- (b)

Difference between the two normal random variables is also normal

Random variable.

$\therefore$P(3V ≥ 2U) = P(3V- 2U ≥ 0)

P(Z ≥ 0) = ½

81. Sol:- (d)

Difference between the two identically distributed zero mean random variable

Cumulative distribution functions is also normal random variable.

82. Sol: (b)

$\mu = 1;\ \sigma^2 = 4 \Rightarrow \lambda = 2$

P(X < 0) = P$\left(Z < \frac{-u}{\sigma}\right) = P\ \left(Z <\ -\frac{1}{2}\right)$

= P(Z < -0.5)

P(X < 0) = 0.5-P(0<Z<5)

Greater than zero & less than 0.5

83. Sol:- (d)

The probability that the student known the answer and answered the question correctly

$= \frac{2}{3} \cdot 1$

The probability of answering correctly by guessing $= \frac{1}{3} \cdot \frac{1}{4}$

$\therefore$ The required probability $= \frac{\frac{2}{3}}{\frac{2}{3}+\frac{1}{3}\times\frac{1}{4}} = \frac{8}{9}$

84. Sol: 6

We have $\int_1^2 f(x)dx - 1$

$\Rightarrow \int_1^2 \lambda(x-1)(2-x)dx = 1$

$\Rightarrow \lambda \int_1^2 (-X^2 + 3X - 2)dx = 1$

$\Rightarrow \lambda \left[\frac{-x^3}{3} + 3\frac{x^2}{2} - 2X\right]_1^2 = 1$

$\Rightarrow \lambda \left[\frac{27-26}{6}\right] = 1 \Rightarrow \frac{\lambda}{6} = 1$

85. Ans (c)

P(X < 3) = P(x=0) + P(x = 1) + P(x = 2)

$= e^{-3} + 3\ e^{-3} + 9\ e^{-3}/2 = 17/2e^3$

86. Sol: - 0.66

Let N = the number of families

Total No. of children

$= \left(\frac{N}{2} \times 1\right) + \left(\frac{N}{2} \times 2\right) = \frac{3N}{2}$

Therefore required Probability = $\frac{\left(\frac{N}{2}\times 2\right)}{\frac{3N}{2}} = \frac{2}{3} = 0.66$

87. As X_1, X_2&$X_3$are independent and identically distributed

P(X_1 is largest) = P($X_2$is largest) = P(X_3 is largest) $= \frac{1}{3} = 0.33$

88. Sol: (B)

We know that

V(X) = E(X^2) – $[E(X)]^2 \geq 0$

$\Rightarrow$ E(X^2) $\geq [E(X)]^2$

89. Sol:- 50

X	1	3	5	…..	97	99
P(X)	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{50}$	…..	$\frac{1}{50}$	$\frac{1}{50}$

$\therefore \; E(X) = \sum X$ P(X)

$= \sum \frac{1}{50}(1 + 3 + \cdots + 99)$

$= \frac{1}{50}(2500) = 50$ ($\therefore$ Sun of first 'n' odd umbers = n^2)

90. Sol:- (C)

Let p = $\frac{1}{2}$ = probability of getting a head any time

q = $\frac{1}{2}$; n = 9

We have to get '3' heads and '6' tails in first '9' tosses, before getting a head

In the 10th toss $\Rightarrow$ 3heads in 9 tosses

$\therefore$ The required probability

$= \left[9C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^6\right] \times \frac{1}{2} = \frac{9C_3}{2^{10}} = \frac{64}{1024} = 0.082$

91. Sol:- (3)

Let 'X' be a random variable which denotes the number of tosses required for the First head to appear.

Similarly 'Y' be another random variable which denotes the number of tosses required For the first tail to appear.

Then P(X=2) = T.H = $\frac{1}{2}.\frac{1}{2}$

P(X=3) = T.T.H = $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$ ... ...

$\therefore$ E(X) = $\left(2 \times \frac{1}{2^2}\right) + \left(3 \times \frac{1}{2^3}\right) + \left(4 \times \frac{1}{2^4}\right) + \cdots$

$= \frac{1}{2}\left[2.\frac{1}{2} + 3.\frac{1}{2^2} + 4.\frac{1}{2^3} + \ldots\right]$

$= \frac{1}{2}\left[\left(1 + 2.\frac{1}{2} + 3.\frac{1}{2^2} + \cdots\right) - 1\right]$

$= \frac{1}{2}\left[\left(1 - \frac{1}{2}\right)^{-2} - 1\right]$

$= \frac{1}{2}(4 - 1) = \frac{3}{2}$

Similarly E(Y) $= \frac{3}{2}$

$\therefore$ E(X+Y) = E(X) + E(Y) = $\frac{3}{2} + \frac{3}{2} = 3$

92. Sol:- 0.16

The probability density function of a uniform distribution function on [a, b] is

F(x) = $\frac{1}{(b-a)}$.

In the given interval f(x) = $\frac{1}{(1-0)} = 1$

$E(X_1) = \frac{a+b}{2} = E(X_2) = E(X_3) = \frac{1}{2}$

$V(X_1) = \frac{(b-a)^2}{12} = \frac{1}{12} = V(X_2) = V(X_3)$

$\therefore P(X_1 + X_2 \leq X_3) = P(X_1 + X_2 - X_3 \leq 0)$

$= P(Y \leq 0)(where\ Y = X_1 + X_2 - X_3)$

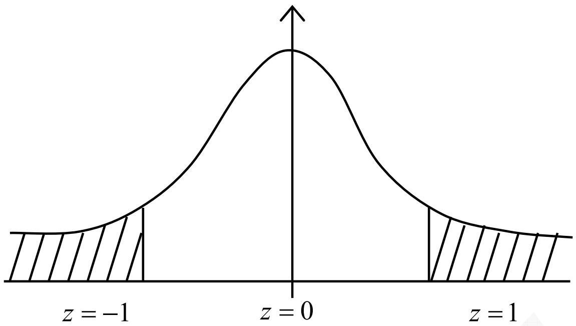
E(Y) $= \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$

V(Y) $V(X_1) + V(X_2) + V(X_3) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$

$\Rightarrow S.D = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore P(Y \leq 0) = P\left(\frac{Y-\mu}{\sigma} \leq \frac{0-\mu}{\sigma}\right) = P\left(z \leq \frac{0-\frac{1}{2}}{\frac{1}{2}}\right)$

(where z = $\frac{u-\mu}{\sigma}$ is a standard normal variable)



$= P(z \leq -1) \qquad = P(z \geq 1)$ (by symmetry of normal curve)

$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

93. Sol:- 0.8

- Given that $\mu = 0,\ \sigma = 1$

Then $f_x(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$E[|X|] = \int_{-\infty}^{\infty} |X| f_X(X) dx$

$= 2\int_0^{\infty} \frac{Xe^{-\frac{X^2}{2}}}{\sqrt{2\pi}} dx = \frac{2}{\sqrt{2\pi}} \int_0^{\infty} Xe^{-\frac{X^2}{2}} dx$

$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} Xe^{-\frac{X^2}{2}} dx = \sqrt{\frac{2}{\pi}} \left(\frac{e^{-\frac{X^2}{2}}}{-1} \right)_0^{\infty}$

$\left(Put\ \frac{X^2}{2} = t \Rightarrow xdx = dt \right)$

$= \sqrt{\frac{2}{\pi}} \left(\frac{0-1}{-1} \right) = \sqrt{\frac{2}{\pi}} = 0.8$

94. The number of phone calls in a fixed time interval is discrete random variable.

∴ Option (a) follows discrete probability distribution and remaining option

Option (b), (c) & (d) are continuous probability distributions.

95. Sol:- 0.44

Probability that a parcel is lost by first Post-office $= \frac{1}{5}$

Probability that it is lost by second post-office $= \frac{4}{5} \times \frac{1}{5}$

The Probability for losing the parcel $= \frac{1}{5} + \left(\frac{4}{5} \times \frac{1}{5} \right)$

By Baye's theorem

Required Probability = $\frac{\frac{4}{5}\times\frac{1}{5}}{\frac{1}{5}+\left(\frac{4}{5}\times\frac{1}{5}\right)} = \frac{4}{9} = 0.44$

96. Sol:- (b)

A fair coin is tossed 'n' times then the number of heads and tails can be Shown below

H	T
n	0
n-1	1
n-2	2
⋮	⋮
2	n-2
1	n-1
0	n

∴The difference between the number of heads and tails can be 'n' or (n -2) or (n–4) …. But it cannot be (n – 1) or (n – 3) or (n – 5)…..

∴The required probability = 0

97. Ans :- 0.14

Let X be the random variable, which denotes the number of dots on a face of the die. Then probability distribution table is shown below.

X	1	2	3	4	5	6
P(X)	K	2K	3K	4K	5K	6K

∑ P(X) = 1 (Where 'K' is the proportionality constant)

i.e., 21 K = 1

$\Rightarrow K = \frac{1}{21}$

∴The required probability = 3 K $= \frac{3}{21} = \frac{1}{7} = 0.14$

98. Sol:- 0.4

Given $F(x) = \begin{cases} 0.2\ for -1 \le X \le 1 \\ 0.1\ for\ x \quad \in [-4,-1) \cup (1,4\,] \\ 0\ ,\ otherwise \end{cases}$

$\therefore P(0.5 < X < 5) = \int_{0.5}^{1} (0.2)dx + \int_{1}^{4} (0.1)dx + \int_{4}^{5} 0\,dx$

99. Sol:- 0.4

$$\int_1^2 f(x)\,dx = 1$$

$$\int_1^2 kx^2\,dx = 1$$

$$k\left(\frac{X^3}{3}\right)_1^2 = 1$$

$$k\left(\frac{8}{3} - \frac{1}{3}\right) = 1$$

$$\frac{7k}{3} = 1 \Rightarrow k = \frac{3}{7} = 0.428$$

100. Sol:- 2

Total probability = $\int_{-\infty}^{\infty} f(x)\,dx = 1$

i.e., $\int_0^{\infty} \frac{e^{-\frac{X}{2}}}{K}\,dx = 1$

$$\frac{1}{K}\left(\frac{e^{-\frac{X}{2}}}{\left(-\frac{1}{2}\right)}\right)_0^{\infty} = 1$$

$$-\frac{2}{K} = \left(e^{-\frac{X}{2}}\right)_0^{\infty} = 1$$

$$-\frac{2}{K}(0-1) = 1$$

$$\therefore k = 2$$

101. Sol:- 0.893

The Probability that the input job fails at both

A and B = 0.15 × 0.05 = 0.0075

The Probability that the input job is fed to C = 1 – 0.0075 = 0.9925

∴ Required probability = 0.9925 × 0.9

= 0.89325

102. Sol:- (d)

Var = $\sigma^2 = \sum x^2 f(x) - [\sum x f(x)]^2$

= [(1×0.3) + (4×0.6) + (9×0.1)] – [(1×0.3) + (2×0.6) + (3×0.1)]^2

= (0.3 + 2.4 + 0.9) – (0.3 + 1.2 + 0.3)^2

= 3.6 – (1.8)^2

= 3.6 – 3.24 = 0.36

$\therefore$ S.D = $\sigma = \sqrt{0.36} = 0.6$

103. Sol:- (a)

Required probability = $\frac{15_{C_2}}{25_{C_2}} = \frac{7}{20}$

104. Sol:- 0.26

- Let P = $\frac{1}{3}$ = probability of getting red colour q = $\left(1-\frac{1}{3}\right)=\frac{2}{3}$

= probability of not getting red colour n = 3

X = number of times red colour appeared

$\therefore$ Required probability = P(X ≥ 2)

= P(X = 2) + P(X = 3)

$= 3_{C_2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + 3_{C_2}\left(\frac{1}{3}\right)^2$

$= \frac{6}{27}+\frac{1}{27}=\frac{7}{27}=0.2593$

105. Sol:- 0.65

Let P(M) = probability of selecting a man = $\frac{1}{2}$

Similarly

P(W) = probability of selecting a woman = $\frac{1}{2}$

P(E/M) = probability of selecting an employed man = 80% = $\frac{80}{100}=\frac{4}{5}$

P(E/W) = probability of selecting an unemployed woman = 50% = $\frac{50}{100}=\frac{1}{2}$

$\therefore$ Required probability = P(M) P(E/M) + P(W) P(E/W)

$= \left(\frac{1}{2}\times\frac{4}{5}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right) \qquad = \frac{1}{2}\left(\frac{4}{5}+\frac{1}{2}\right) = \frac{13}{20} = 0.65$

106. Sol:- (a)

X	0	1	2
P(X)	$\frac{1}{6}\frac{2}{3}=\frac{4}{6}\frac{1}{6}$		

$\therefore$ Mean $\mu = \sum X\,P(X)$

$= \left(0\times\frac{1}{6}\right)+\left(1\times\frac{4}{6}\right)+2\left(\frac{1}{6}\right) = \frac{6}{6} = 1$

Variance $\sigma^2 = \sum X^2P(X)-\mu^2$

$= \sum\left[\left(0\times\frac{1}{6}\right)+\left(1\times\frac{4}{6}\right)+\left(4\times\frac{1}{6}\right)\right]-1$

$= \frac{8}{6}-1 = \frac{2}{6} = \frac{1}{3}$

107. Sol:- 50

$\mu = 500$Rs ,$\sigma = 50$ Rs

$P(X > 500) = P\left(Z > \frac{500-\mu}{\sigma}\right)$

$= P(Z > 0) = 50\%$

108. Sol:- (b)

Given that $\lambda = 5.2$

$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$

$P(X < 2) = P(X = 0) + P(X = 1)$

$= e^{-\lambda} + \lambda e^{-\lambda}$

$= e^{-\lambda}(\lambda + 1)$

$= e^{-5.2}(5.2 + 1) = 0.034$

109. Sol:- 0.4

$P(2 \le E \le 4) = \int_2^4 f(E)dx$

$= \int_2^4 \frac{1}{5}dx = \frac{1}{5}(x)_2^4 = \frac{1}{5}(4-2)$

$= \frac{2}{5} = 0.4$

110. Sol:- 0.265

Given that $\lambda = 5$

Let X = number of penalties per day

$\therefore P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2}{2!}e^{-\lambda} + \frac{\lambda^3}{3}e^{-\lambda}$

$= e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6}\right)$

$= e^{-5}\left(1 + 5 + \frac{25}{2} + \frac{125}{6}\right)$

$= \frac{e^{-5}}{6}(36 + 75 + 125) = 0.265$

111. Sol:- (b)

For every tossing getting a tail is independent of the previous tosses.

$\therefore$The required probability $= \frac{1}{2}$

112. Sol:- (b)

The given integral represents the area under the normal curve to the left Side of the mean x = 'a' = 0.5 (The normal curve is symmetric about x = a).

113. Sol:- 0.27

Given that$\lambda = 240$ veh/h

$= \frac{240}{60}$ veh / min = 4 veh / min = 2 veh / 30 sec

∴The required probability = P(X = 1)

$= \lambda\, e^{-\lambda} = 2\, e^{-2} = 0.27$

114. Sol :- 1.2

- Standard error of mean = $\frac{\sigma}{\sqrt{n}}$

(σ = S.D of sample n = size of the sample) $= \frac{12}{\sqrt{100}} = 1.2$

115. Sol:- (c)

Mean $= \frac{(20\times25)+(20\times30)+(40\times35)+(20\times40)}{100}$

$= \frac{500+600+1400+800}{100} \quad = \frac{3300}{100} = 33$

Median = The average marks of 50^{th} , 51^{st} observations

$= \frac{35+35}{2} = 35$

Mode = The value of marks with highest frequency = 35.

116. Sol:- (d)

The probability of no loss on a rainy day $= \frac{7}{10} \times \frac{2}{10}$

The probability of no loss on a non- rainy day $= \frac{3}{10} \times \frac{9}{10}$

∴ From Baye's theorem, the required probability = $\frac{\left(\frac{3}{10}\times\frac{9}{10}\right)}{\left(\frac{3}{10}\times\frac{9}{10}\right)+\left(\frac{7}{10}\times\frac{2}{10}\right)} = \frac{27}{41}$

117. Sol:- 0.25

Let 'x' be the length of the shorter stick.

Now 'x' is uniformly distributed between 0 to $\frac{1}{2}$

∴probability density function

$F(x) = \frac{1}{\left(\frac{1}{2}-0\right)} = 2$

∴ Required expected length $= \int_0^{1/2} x\, f(x)\, dx$

$= \int_0^{1/2} 2x\, dx = \frac{1}{4} = 0.25$

118. Sol:- 10

For the sum to be 22 we have two possible cases

Case (i) : Two 6' s and one 4

No. of ways we can obtain this is $\frac{4!}{3!} = 4$

Case (ii) : Two 6's and two 5's

This can be obtained in $\frac{4!}{2!2!}$ = 6ways

∴Required value of X = 10

119. Sol:- 11.9

p = probability of picking '3' working computers or '4' working computers.

$= \frac{{}^4C_3\,{}^6C_1}{{}^{10}C_4} + \frac{{}^4C_4}{{}^{10}C_4} = \frac{25}{210}$

∴ Required value = 100 p

$= 100\times\frac{25}{210} = 11.9$

120. Sol:- 3.88

- Let X = length of the word drawn.

X has the following probability distributions.

X	3	4	5
P(X)	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

∴Expected length = E(X)

$= \left(3\times\frac{4}{9}\right) + \left(4\times\frac{2}{9}\right) + \left(5\times\frac{3}{9}\right) = 3.88.$

121. Ans :- 0.26

Number of integers in the set which are divisible by 2 or 3 or 5

= n(2) + n(3) + n(5) – n(2^3) – n(3^5) – n(5^2) + n(2^3^5)

= 50 + 33 + 20 – 16 – 10 – 6 + 3 = 76

∴The number of integers between 1 and 100,

which are not divisible by 2 or 3 or 5

= 100 – 74 = 26

∴Required probability $= \frac{26}{100} = 0.26$

122. Sol:- 0.25

Given that$(A \cup B) = S$

$\Rightarrow P(A \cup B) = P(S) = 1$

$\Rightarrow P(A) + P(B) = 1$(∵ A, B are mutually exclusive)

Let p(A) = x

$\Rightarrow P(B) = (1-x)$

Let f(x) = x(1-x)

F'(x) = 1 – 2x

F'(x) = 0 $\Rightarrow$ x = $\frac{1}{2}$

F''(x) = -2 < 0 at x = $\frac{1}{2}$

$\therefore$ f(x) is maximum at x = $\frac{1}{2}$

$\therefore$ maximum value of P(A).P(B) = x(1-x) = $\frac{1}{2}\left(1-\frac{1}{2}\right)$ = 0.25

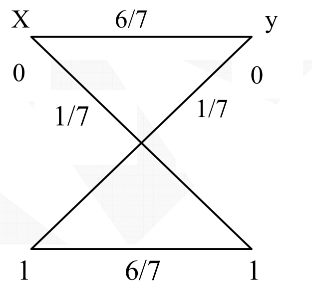
123 **Sol.** Option **(C)** is wrong since $P(A \cup B) = P(A) + P(B) - P(A).P(B)$ for independent values

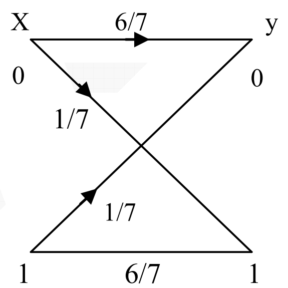
124. The input X to the binary symmetric channel (BSC) shown in figure is '1' with the probability 0.8. The cross over probability is $1/7$. If the received bit Y = 0, the conditional probability that '1' was transmitted is

Sol. **(0.4)**

P[X=0] = 0.2

P[X=1] = 0.8





P[X=0] = 0.2

P[X=1] = 0.8

$$P\left(\frac{X=1}{Y=0}\right) = \frac{P\left(\frac{Y=0}{X=1}\right).P(X=1)}{P(Y=0)}$$

$$P\left(\frac{Y=0}{X=1}\right)=1/7; P(X=1)=0.8$$

$$P(Y=0)=0.2\times\frac{6}{7}+0.8\times\frac{1}{7}=\frac{2}{7}$$

$$\Rightarrow P(X=1/Y=0)=\frac{\frac{1}{7}(0.8)}{2/7}=0.4$$

125). Ans : - (d)

Sol :- The point of Intersection both PDF's (Threshold value,v) should occur between -1 and +1. Hence probability of error is [7+v]/36 and it should be minimum.

by taking v as -1 we get Minimum probability of error which is 1/6

126) **Sol.** **(1/4)**

P(Ram) $=\frac{1}{6}$; P(Ramesh) $=\frac{1}{8}$

Here the events are independent

$$P(only\,one)=P(Ram)\times P(\overline{Ramesh})+P(Ramesh)\times P(\overline{Ram})$$

$$=\frac{1}{6}\times\frac{7}{8}+\frac{1}{8}\times\frac{5}{6}=\frac{1}{4}$$

127. **Ans : (d)**

Given $P(X=0)=p\Rightarrow P(X=1)=1-p$

$P(Y=0)=q\Rightarrow p(Y=1)=1-q$

Now $P(X+Y)\geq 1=1-[p(X+Y)=0]$

$=1-[p(X=0,Y=0)]$

$=1-pq$

128. Sol. (1.5)

Let X is a random variable denotes number of tossed to get two heads

$P(X=2) = HH = \frac{1}{2}\times\frac{1}{2}$

$P(X=2) = HHH = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$

$P(X=4) = HHHH = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$

$\text{Now}, E(X) = 2\left(\frac{1}{2}\times\frac{1}{2}\right)+3\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+4\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\ldots\ldots\ldots$

129.**Sol.** **(1.7854)**

P(getting 3) = $\frac{1}{6}$

P(not getting 3) = $\frac{5}{6}$

Expected number of times to get 3 first is

E(X) = $\frac{1}{6}+2\left(\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right)+3\left(\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right)+---$

$= \frac{1}{6}.\left[1-\left(\frac{5}{6}\right)^2\right]^{-2} = \frac{1}{6}\left(\frac{11}{36}\right)^{-2} = 1.7851$

130. **Sol.** **(6)**

Given $f(x) = \frac{1}{2}|x|e^{-|x|}$

V(X) = $E[X^2] - \{E[X]\}^2$

E(X) = $\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} \frac{1}{2}|x|e^{|x|}dx$ =0 ($\because$ f(x) is odd)

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{\infty} x^2\frac{1}{2}|x|e^{-|x|}\,dx$

$= \frac{2}{3}\int_{-\infty}^{\infty} x^3 e^{-x}dx$ ($\because$ f is even)

$= 3! = 6$

131. **Sol.** **(A)**

Denominator ${}^4C_1.{}^5C_1 = 20$

Favourable cases = 4 = required probability = $4/20 = 1/5 = 0.2$

132. **Sol.** **(A)**

P(at least two) – P(exact 2) = 0.5 – 0.4 = 0.1

0.75 = p + m + c + 0.1 – (0.5+0.11 $\times$2) $\Rightarrow$ p + m + c = 0.65+0.7 = 1.35 = $27/20$

133. A random variable X has a p.d.f $f(x) = \begin{cases} a+bx: & 0<x<1 \\ 0 & otherwise \end{cases}$ If the expected value E(X) = $2/3$ then $P(X < 0.5)$ is

Sol. **(0.25)**

We have $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 (a+bx)\,dx = 1 \Rightarrow a + \frac{b}{2} = 1 \Rightarrow 2a+b=2$ -----(1)

Given $E(X) = 2/3 \Rightarrow \int_0^1 x.[a+bx]\,dx = 2/3 \Rightarrow a/2 + b/3 = 2/3 \Rightarrow 3a+2b=4$ --(2)

By solving 1 and 2 a = 0, b = 2

Now $P(X<0.5) = \int_0^{0.5} f(x)dx = 2\int_0^{0.5} xdx = 0.25$

134. **Sol.** **(D)**

P(A wins) = $1/6$ P(B wins) = $1/6$

P (A fails) = $5/6$ P(B fails) = $5/6$

Chance of A	Chance of B
$1/6$	$5/6 . 1/6$
$(5/6)^2 \times 1/6$	$(5/6)^3 \times 1/6$
$(5/6)^4 \times 1/6$	$(5/6)^5 \times 1/6$

P(B) = $5/6 \times 1/6\left[1+(5/6)^2+(5/6)^4+---\right] = 5/6 \times 1/6 \cdot \left(\frac{1}{1-(5/6)^2}\right) = 6/11$

135.

Sol. **(D)**

p.d.f of normal distribution is $f(x)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\left(\frac{x-\mu}{4}\right)^2}$

σ^2 is lowest $\Rightarrow \sigma$ is least

$\Rightarrow$ if σ decreses $\Rightarrow$ f(x) increase

$\Rightarrow$ f(x) is in highest peak

136. **Sol.** **(B)**

n =4: p = $1/6 : \varepsilon = 5/6$

$P(x\geq 2)=1-P(x<2)=1-\left[P(x=0)+p(x=1)\right]$

$=1-\left[{}^4C_0\left(1/6\right)^0\left(5/6\right)^4+{}^4C_1\left(1/6\right)^1\left(5/6\right)^3\right]=\frac{19}{144}$

137. **Sol.** **(0.97)**

P(at least one will meet specification) = 1 – P(none)=1-(1-0.8)(1-0.7)(1-0.5) = 1 – 0.2$\times$ 0.3$\times$0.5 = 0.97

138. **Sol.** **(B)**

A: student pass the exam

B: student gets above 90%

P(A) = 20% : $P(A\cap B)$=5%

$P\left(B/A\right)=\frac{P(A\cap B)}{P(A)}=\frac{5\%}{20\%}=\frac{1}{4}$

139. **Sol.** **(D)**

X = {HHT, HHH} $\Rightarrow$ Y depends on X

Z = {TTH, TTT}

Clearly X and Z are independent

140. **Sol.** **(C)**

$P\left(\frac{Y}{X}\right)=\frac{P(X\cap Y)}{P(X)}=\frac{1/12}{1/4}=\frac{1}{3}$

141. Consider the following probability mass function of a random variable X

$$p(x,\varepsilon)=\begin{cases}\varepsilon & if\ X=0\\ 1-\varepsilon & if\ X=1\\ 0 & otherwise\end{cases}$$

If $\varepsilon=0.4$, the variance of X is

Sol. **(0.24)**

$$P(x,\varepsilon)=\varepsilon \quad if\ x=0$$
$$=1-\varepsilon \quad if\ x=1$$
$$=0 \quad otherwise$$

Given $\varepsilon=0.4 \Rightarrow$

$$P(x,\varepsilon)=0.4 \quad if\ x=0$$
$$=0.6 \quad if\ x=1$$
$$=0 \quad otherwise$$

$$Variance\ V(X)=E\left(X^2\right)-[E(x)]^2$$

$$E(X)=\Sigma x_i P_i=0\times 0.4+1\times 0.6=0.6$$

$$E(X^2)=\Sigma {x_i}^2 P_i=0^2\times 0.4+1^2\times 0.6=0.6$$

$$V(X)=0.6-(0.6)^2=0.6-0.36=0.24$$

142. **Sol.** **(B)**

Since first two cards are kings, third card must be from remaining two kings out of 50 cards. So, required probability is $\frac{2}{50}$

143. **Sol.** **(1.0667)**

Mean $\mu_z=E(X)=\int_0^2 \frac{x^2}{4}(4-x^2)dx=\frac{1}{4}\int_0^2(4x^2-x^4)dx=\frac{1}{4}\left(\frac{4x^3}{3}-\frac{x^5}{5}\right)_0^2=1.0667$

144. **Sol.** **(0.95)**

$|X|=2$: $|Y|=20$

Number of function from $X\to Y=20^2=400$

Number of one to one functions from X to Y is $^{20}p_2$

So, required probability is $\frac{^{20}p_2}{20^2} = \frac{380}{400} = 0.95$

145. **Ans: (A)**

Sol: For Poisson distribution mean = variance

given mean = μ

∴ variance = μ

∴ standard deviation= $\sqrt{\mu}$

146.**Ans: 0.41**

Sol: We require P(x ≥1)= 1- P(x=0)

$= 1 - 5C_0(0.1)^0 (0.9)^5 = 0.4095 \approx 0.41$

147.**Ans: (c)**

Sol: Group 'P'

Mean (μ) = 105

Standard deviation (σ_1) = 25

Pr (μ – σ ≤ x ≤ μ + σ) ≈ 0.6827

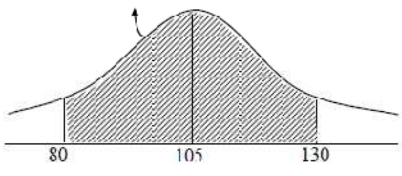
∴ 68% within one standard deviation

$\mu_1 - \sigma_1 = 105 - 25 = 80$

$\mu_1 + \sigma_1 = 105 + 25 = 130$

∴ range = 80 to 130

Distribution of P:



Group Q

Mean (μ_2) = 85

Standard deviation (σ_2) = 5

Pr ($\mu - \sigma \le x \le \mu + \sigma$) $\approx$ 0.6827

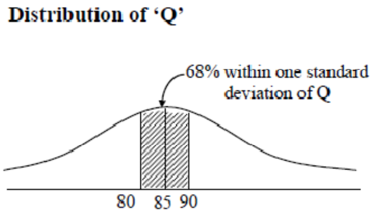
$\therefore$ 68% within one standard deviation

$\mu_2 - \sigma_2 = 85 - 5 = 80$

$\mu_2 + \sigma_2 = 85 + 5 = 90$

$\therefore$ range of Q in one standard deviation is 80 to 90

68% within one standard deviation of Q is narrower



$\therefore$ 68% within one standard deviation of Q means most students of group Q.

$\therefore$ Most students of group 'Q' scored marks in a narrower range than students in group 'P'

148. Ans: (A)

Sol: Required probability = $\frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_3} = \frac{16}{5525}$

149.**Ans: (99.73)**

Sol: In the standard normal curve the area between –3 &3 is 0.9973

$\therefore$ Percentage of area is 99.73

150. Ans: $\lambda = 1$

Sol: $E(x^2) = 2$

$V(X) = E(X^2) - (E(X))^2$

Let mean of the poission random variable be x

$x = 2 - x^2$

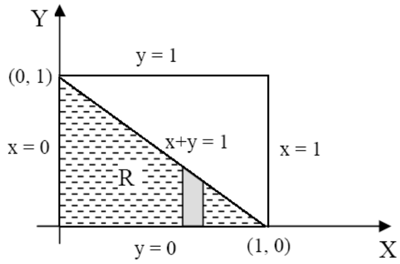
$x^2 + x - 2 = 0$

$x = 1, -2$

$\therefore$ Mean is $\lambda = 1$

151. **Ans:** 0.33

Sol: $(P(X + Y \le 1) = \int_R f(x, y)dxdy$



$$= \int_{x=0}^{1}\int_{y=0}^{1-x}(x, y)dxdy$$

$$= \int_0^1 \left(xy + \frac{y^2}{2} \right)_0^{1-x} dx$$

$$= \int_0^1 \left[x(1-x) + \frac{(1-x)^2}{2} \right] dx$$

$= 0.33$

152.**Ans: 0.07203**

Sol: $P = (0.7)^4(0.3)$

$= 0.07203$

153.Ans: (a)

Sol: $f(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$

For a : $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\int_{-\infty}^{0} ae^{4x}dx + \int_{0}^{\infty} \frac{3}{2}e^{-3x}dx = 1$$

$$\frac{a}{4} + \frac{1}{2} = 1$$

$$a = 2$$

$$p(x \le 0) = \int_{-\infty}^{0} ae^{4x}dx$$

$$= 2\int_{-\infty}^{0} e^{4x}dx$$

$$= 2\left(\frac{e^{4x}}{4}\right)_{-\infty}^{0}$$

$$= \frac{2}{4} = \frac{1}{2}$$

$$\therefore \left(2, \frac{1}{2}\right)$$

154.Ans. (A)

Soln:

$$f(x) = \begin{cases} 0.25 & if\ 1 \le x \le 5 \\ 0 & otherwise \end{cases}$$

$$P\ (X\ \le 4) = \int_{-\infty}^{4} F(x)d(X)$$

$$= \int_{-\infty}^{1} f(x)d(x) + \int_{1}^{4} f(x)d(x)$$

$$= \int_{1}^{4} \frac{1}{4}dx = \frac{1}{4}(x)_1^4 = \frac{3}{4}$$

155.Ans: (A)

Soln: $P(X) = 0.40\, P(XUY^C) = 0.7$

$P(Y) = ?$

$P(XUY^C) = P(X) + P(Y^C) - P(X)P(Y^C)$

$= P(X) + P(Y^C)(1 - P(X))$

$0.7 = 0.4 + P(Y^C)(0.6)$

$0.7 - 0.4 = P(Y^C)$

$\frac{0.3}{0.6} = P(Y^C) \Rightarrow P(Y^C) = 0.5$

$P(Y) = 0.5$

$P(XUY) = P(X) + (Y) - P(X)P(Y)$

$= 0.4 + 0.5 - 0.4 \times 0.5 = 0.9 - 0.2 = 0.7$

156.Ans:. (B)

Soln:

$$E(f(x)) = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_{-a}^{0} \left(\frac{x^2}{a} + x\right)dx + \int_0^a \left(\frac{-x^2}{a} + x\right)dx$$

$$= \left(\frac{x^3}{3a} + \frac{x^2}{2}\right)_{-a}^{0} + \left(\frac{-x^3}{3a} + \frac{x^2}{2}\right)_0^a$$

$$= -\left(\frac{-a^2}{3} + \frac{a^2}{2}\right) + \left(\frac{-a^2}{3} + \frac{a^2}{2}\right)$$

$$= 0$$

$$E\left(g(x)\right) = \int_{-a}^{0} \frac{-x^2}{a}\, dx + \int_0^a \frac{x^2}{a}\, dx$$

$$= \left(-\frac{x^3}{3a}\right)_{-a}^{0} + \left(\frac{x^3}{3a}\right)_0^a$$

$$= -\left(\frac{a^2}{3}\right) + \frac{a^2}{3} = 0$$

V (f (x))= E (X^2) – $(E(X))^2$

$$E(X^2) = \int_{-a}^{0} \left(\frac{x^3}{a} + x^2\right)dx + \int_0^a \left(\frac{-x^3}{a} + x^2\right)dx$$

$$= \left(\frac{x^4}{4a} + \frac{x^3}{3}\right)_{-a}^{0} + \left(-\frac{x^4}{4a} + \frac{x^3}{3}\right)_{0}^{a}$$

$$= -\left(\frac{a^3}{4} - \frac{a^3}{3}\right) + \left(-\frac{a^3}{4} + \frac{a^3}{3}\right)$$

$$E(X^2) = \frac{a^3}{6}$$

$$V\left(f(x)\right) = \frac{a^3}{6}$$

V (g(x))= E(x^2) – (E(X))^2

$$E(X^2) = \int_{-a}^{0} -\frac{x^3}{a}\,dx + \int_{0}^{a} \frac{x^3}{a}\,dx$$

$$\left(-\frac{x^4}{4a}\right)_{-a}^{0} + \left(\frac{x^4}{4a}\right)_{0}^{a}$$

$$= (0) - \left(-\frac{a^3}{4}\right) + \frac{a^3}{4}$$

$$= \frac{2a^3}{4} = \frac{a^3}{2}$$

$$V\left(g(X)\right) = \frac{a^3}{2}$$

157.Ans (B)

Soln:

Load	failure for cycles
80	100
40	10000

There is one failure 5000 cycles load must be between 80 and 40

∴ 46.02

158.Ans: (A)

Soln: Type II error means acceptance of the null hypothesis when it is false and should be rejected.

159.Ans: 0.55

Sol: A→ event of selection of type –I bulb

B → event of selection of type -2 bulb

E → event of selection of bulb glow for more than 100 hours

$$We\ require\ P(A)P\left(\frac{E}{A}\right)+P(B)P\left(\frac{E}{B}\right)=\frac{1}{2}\times 0.7+\frac{1}{2}\times 0.4=0.55$$

(OR)

The probability that the bulb is type I and lasting more than 100 hours = $\frac{1}{2}(0.7)$

The probability that the bulb is type II and lasting more than 100 hours = $\frac{1}{2}(0.4)$

Required probability = $\frac{1}{2}(0.7)+\frac{1}{2}(0.4)=0.55$

160.Ans : 0.5

soln: Given f(x) = $\begin{cases}\frac{1}{x^2}\ for\ a\ \le x\le 1\\ 0\quad otherwise\end{cases}$

$$\int_{-\infty}^{\infty} f(x)dx=1 \Rightarrow \int_{a}^{1}\frac{1}{x^2}dx=1$$

$$\Rightarrow \left[\frac{-1}{x}\right]_{-a}^{1}=1 \Rightarrow \frac{1}{a}-1=1$$

161.**Ans:** 233

$$\begin{bmatrix}x(n)\\x(n-1)\end{bmatrix}=\begin{bmatrix}1&1\\1&0\end{bmatrix}^n\begin{bmatrix}1\\0\end{bmatrix},\ n\ge 2$$

n = 2

$$\begin{bmatrix}x(2)\\x(1)\end{bmatrix}=\begin{bmatrix}1&1\\1&0\end{bmatrix}^2\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$$

x(2) = 2, x(1) = 1

n = 3

$$\begin{bmatrix}x(3)\\x(2)\end{bmatrix}=\begin{bmatrix}1&1\\1&0\end{bmatrix}^3\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3&2\\2&1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$$

x(3) = 3, x(2) = 2

From the above values we can write the recursive relation as

x(n) = x(n–1) + x(n –2)

x(2) = x(1) + x(0) = 1 +1 = 2

x(3) = x(2) + x(1) = 2 +1 = 3

x(4) = x(3) + x(2) = 3 + 2 = 5

x(5) = x(4) + x(3) = 5 + 3 = 8

x(6) = x(5) + x(4) = 8 + 5 = 13

x(7) = x(6) + x(5) = 13+ 8 = 21

$x(8) = x(7) + x(6) = 21 + 13 = 34$

$x(9) = x(8) + x(7) = 34 + 21 = 55$

$x(10) = x(9) + x(8) = 55 + 34 = 89$

$x(11) = 89 + 55 = 144$

$x(12) = 144 + 89 = 233$

CHAPTER - 4
CALCULUS

01. The function $f(x,y) = x^2y - 3xy + 2y + x$ has **(GATE-93[ME])**

(a) No local extreme

(b) One local maximum but no local minimum

(c) One local minimum but no local maximum

(d) One local minimum and one local maximum

02. $\lim_{x\to 0}\frac{x(e^x - 1) + 2(\cos x - 1)}{x(1-\cos x)} =$ _ **(GATE-93[ME])**

03. The value of the double integral $\int_0^1\int_x^{1/x}\frac{x}{1+y^2}dxdy =$ _____ **(GATE-93)**

04. The value of $\int_0^\infty e^{-y^3}.y^{1/2}dy$ is_____ **(GATE-94[ME])**

05. The integration of $\int \log x dx$ has the value **(GATE-94)**

(a) (x log x-1) (b) log x-x (c) x (logx-1) (d) none of the above

06. The volume generated by revolving the area bounded by the parabola y = 8x and the line x=2 about y-axis is **(GATE-94)**

(a) $\frac{128\pi}{5}$ (b) $\frac{5}{128\pi}$ (c) $\frac{127}{5\pi}$ (d) None of the above

07. The function $y = x^2 + \frac{250}{x}$ at x = 5 attains **(GATE-94)**

(a) Maximum (b) Minimum (c) Neither (d) 1

08. The value of in the mean value theorem of f(b)-f(a) = (b-a) f for f(x) =Ax +Bx+C in (a,b) is **(GATE-94)**

(a) $b+a$ (b) $b-a$ (c) $\frac{b+a}{2}$ (d) $\frac{b-a}{2}$

09. Given $y = \int_1^{x^2}\cos t dt$, then $\frac{dy}{dx} =$ _____ **(GATE-95[PI])**

10. If at every point of a certain curve , the slope of the tangent equals $\frac{-2x}{y}$, the curve is ________ **(GATE-95[CS])**

(a) A straight line (b) A parabola (c) A circle (d) An ellipse

11. $\underset{x\to\infty}{Lim}\dfrac{x^3-\cos x}{x^2+(\sin x)^2}=$ ______ **(GATE-95[CS])**

12. The area bounded by the parabola 2y=x and the lines x=y-4 is equal to **(GATE-95[ME])**

(a) 6 (b) 18 (c) ∞ (d) none

13. By reversing the order of integration $\int_0^2\int_{x^2}^{2x} f(x,y)dydx$ may be represented as **(GATE-95)**

(a) $\int_0^2\int_{x^2}^{2x} f(x,y)dydx$ (b) $\int_0^2\int_y^{\sqrt{y}} f(x,y)dxdy$ (c) $\int_0^4\int_{y/2}^{\sqrt{y}} f(x,y)dxdy$ (d) $\int_{x^2}^{2x}\int_0^2 f(x,y)dydx$

14. The third term in the Taylors series expansion of e^x about 'a' be **(GATE-95)**

(a) $e^a(x-a)$ (b) $\dfrac{e^a}{2}(x-a)^2$ (c) $\dfrac{e^a}{2}$ (d) $\dfrac{e^a}{6}(x-a)^3$

15. $\underset{x\to 0}{Lim}\, x\sin 1/x=$ ___________ **(GATE-95)**

(a) ∞ (b) 0 (c) 1 (d) does not exist

16. The function $f(x)=|x+1|$ on the interval $[-2,0]$ is_______ **(GATE-95)**

(a) Continuous and differentiable

(b) Continuous on the interval but not differentiable at all points

(c) Neither continuous nor differentiable

(d) Differentiable but not continuous

17. The function $f(x)=x^3-6x^2+9x+25$ has **(GATE-95)**

(a) a maxima at x=1 and a minimum at x=3

(b) a maxima at x=3 and a minima at x=1

(c) no maxima, but a minima at x=3

(d) a maxima at x=1 ,but no minima

18. If f (0) =2 and $f'(x)=\dfrac{1}{5-x^2}$, then the lower and upper bounds of f(1) estimated by the mean value theorem are **(GATE-95)**

(a) 1.9, 2.2 (b) 2.2, 2.25 (c) 2.25, 2.5 (d) none of the above

19. If a function is continuous at a point its first deriv2ative **(GATE-96)**

(a) May or may not exist (b) Exists always

(c) Will not exist (d) Has a unique value

20. What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval [0, 2]? **(GATE-97[CS])**

(a) 6 (b) 10 (c) 12 (d) 5.5

21. Area bounded by the curve $y = x^2$ and the lines x = 4 and y = 0 is given by **(GATE-97)**

(a) 64 (b) $\frac{64}{3}$ (c) $\frac{128}{3}$ (d) $\frac{128}{4}$

22. The curve given by the equation $x^2 + y^2 = 3axy$ is **(GATE-97)**

(a) Symmetrical about x-axis (b) Symmetrical about y-axis

(c) Symmetrical about the line y = x (d) Tangential to x = y = a/3

23. $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta}$ where m is an integer, is one of the following: **(GATE-97)**

(a) m (b) $m\pi$ (c) $m\theta$ (d) 1

24. If $y = |x|$ for x < 0 and y = x for $x \geq 0$ then **(GATE-97)**

(a) $\frac{dy}{dx}$ is discontinuous at x = 0 (b) y is not discontinuous at x = 0

(c) y is not defined at x = 0 (d) Both y and $\frac{dy}{dx}$ are discontinuous at x=0

25. If $\phi(x) = \int_0^{x^2} \sqrt{t}\,dt$ then $\frac{d\phi}{dx} =$ ___ **(GATE-97)**

(a) $2x^2$ (b) $\sqrt{x}$ (c) 0 (d) 1

26. Find the points of local maxima and minima if any of the following function defined in $0 \leq x \leq 6, x^3 - 6x^2 + 9x + 15$ **(GATE-98[CS])**

27. The infinite series $1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots\ldots\ldots\infty$ **(GATE-98[CE])**

(a) converges (b) diverges (c) oscillates (d) unstable

28. The continuous function f (x,y) is said to have a saddle point at (a,b) if **(GATE-98)**

(a) $f_x(a,b) = f_y(a,b) = 0$, $f_{xy}^2 - f_{xx}f_{yy} < 0$ at (a,b)

(b) $f_x(a,b) = 0, f_y(a,b) = 0$, $f_{xy}^2 - f_{xx}f_{yy} > 0$ at (a,b)

(c) $f_x(a,b)=0, f_y(a,b)=0$ $f_{xy}^2 - f_{xx}f_{yy} < 0$ at (a,b) (d) $f_x(a,b)=0, f_y(a,b)=0$ $f_{xy}^2 - f_{xx}f_{yy} = 0$ at (a,b)

29. The Taylor's series expansion of sin x is_____ **(GATE-98)**

(a $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots\cdots$ (b) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\cdots$

(c) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots\cdots$ (d) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\cdots$

30. A discontinuous real function can be expressed as **(GATE-98)**

(a) Taylor's series and Fourier's series

(b) Taylor's series but not Fourier's series

(c) Neither Taylor's series nor Fourier's series

(d) Not by Taylor's series but by Fourier's series

31. Number of the inflection points for the curve $y = x + 2x^4$ is_____ **(GATE-99[CE])**

(a) 3 (b) 1 (c) n (d) $(n+1)^2$

32. The infinite series $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$ **(GATE-99[CE])**

(a) converges (b) diverges (c) is unstable (d) oscillate

33. $\lim_{x\to 0}\frac{1}{10}\frac{1-e^{-j5x}}{1-e^{-jx}} =$__________ **(GATE-99[IN])**

(a) 0 (b) 1.1 (c) 0.5 (d) 1

34. Limit of the function, $\lim_{n\to\infty}\frac{n}{\sqrt{n^2+n}}$ **(GATE-99)**

(a) $\frac{1}{2}$ (b) 0 (c) ∞ (d) 1

35. The function $f(\mathrm{x}) = \mathrm{e}^x$ is____________ **(GATE-99)**

(a) Even (b) odd (c) neither even nor odd (d) None

36 . Value of the function $\lim_{x\to a}(x-a)^{x-a}$ is_____ **(GATE-99)**

(a)1 (b) 0 (c)∞ (d) a

37. The Taylor series expansion of sin x about $x = \frac{\pi}{6}$ is given by **(GATE-2000[CE])**

(a) $\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)-\frac{1}{4}\left(x-\frac{\pi}{6}\right)^2-\frac{\sqrt{3}}{12}\left(x-\frac{\pi}{6}\right)^3+$......... (b) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$

(c) $\frac{\left(x-\frac{\pi}{6}\right)}{1!}-\frac{\left(x-\frac{\pi}{6}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{6}\right)^5}{5!}-$.............. (d) $\frac{1}{2}$

38. $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin(x+y)dxdy$ **(GATE-2000)**

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2

39. Limit of the function $f(x)=\frac{1-a^4}{x^4}$ as x→∞ is given by **(GATE-2000)**

(a) 1 (b) e^{-a^4} (c) ∞ (d) 0

40. If $f(x,y,z)=\left(x^2+y^2+z^2\right)^{-1/2}$, $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$ is equal to____**(GATE-2000)**

(a) 0 (b) 1 (c) 2 (d) $-3(x^2+y^2+z^2)^{-5/2}$

41. Consider the following integral $\lim_{a\to\infty}\int_1^{\infty} x^{-4}dx$ ________ **(GATE-2000)**

(a) Diverges (b) converges to 1/3

(c) Converges to $-1/a^3$ (d) converges to 0

42. Limit of the following series as x approaches $\frac{\pi}{2}$ is $f(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-$..............

(GATE-01[CE])

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 1

43. $\lim_{x\to\frac{\pi}{4}}\frac{\sin 2\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}=$______________ **(GATE-01[IN])**

(a) 0 (b) 1/2 (c) 1 (d) 2

44. The value of the integral is $I = \int_0^{\frac{\pi}{4}} \cos^2 x dx$ **(GATE-01)**

(a) $\frac{\pi}{8}+\frac{1}{4}$ (b) $\frac{\pi}{8}-\frac{1}{4}$ (c) $-\frac{\pi}{8}-\frac{1}{4}$ (d) $-\frac{\pi}{8}+\frac{1}{4}$

45. The following function has local minima at which value of x, $f(x) = x\sqrt{5-x^2}$ **(GATE-02[CE])**

(a) $\frac{-\sqrt{5}}{2}$ (b) $\sqrt{5}$ (c) $\sqrt{\frac{5}{2}}$ (d) $-\sqrt{\frac{5}{2}}$

46. Limit of the following sequence as n→∞ is ____ $x_n = n^{\frac{1}{n}}$ **(GATE-02[CE])**

(a) 0 (b) 1 (c) ∞ (d) -∞

47. Which of the following functions is not differentiable in the domain [-1, 1]? **(GATE-02)**

(a) $f(x) = x^2$ (b) $f(x) = x-1$ (c) $f(x) = 2$ (d) $f(x) = \max imum(x, -x)$

48. The value of the following definite integral in = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin 2x}{1+\cos x} dx =$ ____ **(GATE-02)**

(a) $-2\log 2$ (b) 2 (c) 0 (d) none

49. The value of the following improper integral is $\int_0^1 x \log x dx =$ ______ **(GATE-02)**

(a) 1/4 (b) 0 (c) -1/4 (d) 1

50. The function $f(x) = 2x^2 + 2xy - y^3$ has **(GATE-02)**

(a) Only one stationary point at (0,0) (b) Two stationary points at (0,0) and (1/6, -1/3)

(c) Two stationary points at (0,0) and (1,-1) (d) no stationary point

51. $\lim_{x\to 0} \frac{\sin^2 x}{x} =$ ________ **(GATE-03)**

(a) 0 (b) ∞ (c) 1 (d) -1

52. The area enclosed between the parabola $y = x^2$ and the straight line y=x is **(GATE-04)**

(a) $1/8$ (b) $1/6$ (c) $1/3$ (d) $1/2$

53. If $x = a(\theta + \sin\theta)$ and $y = a(1-\cos\theta)$ then $\frac{dy}{dx}$ =____ **(GATE-04)**

(a) $\sin \theta/2$ (b) $\cos \theta/2$ (c) $Tan\,\theta/2$ (d) $Cot\,\theta/2$

54. The volume of an object expressed in spherical co-ordinates is given by $V = \int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^1 r^2 \sin\varphi\, dr\, d\varphi\, d\theta.$ the value of the integral is **(GATE-04)**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$

55. The value of the function, $f(x) = \lim_{x\to 0}\frac{x^3+x^2}{2x^3-7x^2}$ is _______ **(GATE-04)**

(a) 0 (b) -1/7 (c) 1/7 (d) ∞

56. The function $f(x) = 2x^3 - 3x^2 - 36x + 2$ has its maxima at **(GATE-04)**

(a) x = -2 only (b) x = 0 only (c) x = 3 only (d) both x = -2 and x = 3

57. $\int_{-a}^{a}\left[\sin^6 x + \sin^7 x\right]dx$ is equal to **(GATE-05[ME])**

(a) $2\int_0^a \sin^6 x dx$ (b) $2\int_0^a \sin^7 x dx$ (c) $2\int_0^a (\sin^6 x + \sin^7 x)dx$ (d) zero

58. For the function $f(x) = x^2 e^{-x}$, the maximum occurs when x is equal to **(GATE-05[EE])**

(a) 2 (b) 1 (c) 0 (d) -1

59. The value of the integral $\int_{-1}^{1}\frac{1}{x^2}dx$ is **(GATE-05[IN])**

(a) 2 (b) does not exists (c) -2 (d) ∞

60. The value of the integral $I = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{-x^2/8}dx$ is __________ **(GATE-05[EC])**

(a) 1 (b) π (c) 2 (d) 2π

61. Changing the order of integration in the double integral $I = \int_0^8\int_{x/4}^{2} f(x,y)\,dy\,dx$ leads to $I = \int_r^s\int_p^q f(x,y)\,dy\,dx.$ what is q? **(GATE-05)**

(a) 4y (b) $16y^2$ (c) x (d) 8

62. By a change of variables x (u,v) = uv, y(u,v) = v/u in a double integral, the integral f(x, y) Changes to f(uv, u/v). then ϕ(u,v) is _________ **(GATE-05)**

(a) 2v/u (b) 2uv (c) v^2 (d) 1

63. If $S = \int_1^\infty x^{-3} dx$ then S has the value **(GATE-05[EE])**

(a) -1/3 (b) 1/4 (c) 1/2 (d) 1

64. For real x, the maximum value of $\frac{e^{\sin x}}{e^{\cos x}}$ is **(GATE-07[IN])**

(a) 1 (b) e (c) $e^{\sqrt{2}}$ (d) ∞

65. Consider the function $f(x) = |x|^3$, where x is real. Then the function f(x) at x = 0 is **(GATE-07[IN])**

(a) continuous but not differentiable. (b) once differentiable but not twice.
(c) twice differentiable but not thrice. (d) thrice differentiable.

66. The minimum value of function $y = x^2$ in the interval [1,5] is **(GATE-07[ME])**

(a) 0 (b) 1 (c) 25 (d) undefined

67. $\lim_{x\to 0} \frac{e^x - (1 + x + \frac{x^2}{2})}{x^3} =$ **(GATE-07[ME])**

(a) 0 (b) 1/6 (c) 1/3 (d) 1

68. If $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots..\infty}}}$ then y(2) = _________ **(GATE-07[PI])**

(a) 4 or 1 (b) 4 only (c) 1 only (d) undefined

69. What is the value of $\lim_{x\to\frac{\pi}{4}} \frac{\cos x - \sin x}{x - \frac{\pi}{4}}$ **(GATE-07[PI])**

(a) $\sqrt{2}$ (b) 0 (c) - $\sqrt{2}$ (d) limit does not exist

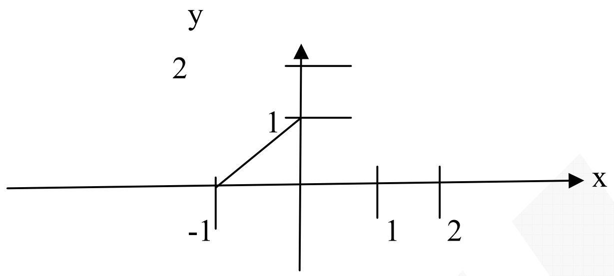
70. For the function $f(x, y) = x^2 - y^2$ defined on R^2, the point (0,0) is **(GATE-07[PI])**

(a) a local minimum (b) Neither a local minimum nor a local maximum
(c) a local maximum (d) Both a local minimum and a local maximum

71. $\lim_{\theta\to 0} \frac{\sin(\theta/2)}{\theta}$ is **(GATE-07[EC])**

(a) 0.5 (b) 1 (c) 2 (d) not defined

72. The following plot shows a function y which varies linearly with x. the value of the Integral $I = \int_1^2 y dx$ **(GATE-07[EC])**



(a)1 (b) 2.5 (c) 2 (d) 5

73. For the function e^{-x}, the linear approximation around x = 2 is **(GATE-07[EC])**

(a) $(3-x)e^{-2}$ (b) (1-x) (c) $\left[3+2\sqrt{2}-(1+\sqrt{2})x\right]e^{-2}$ (d) e^{-2}

74. For $|x| << 1, \coth(x)$ can be approximated as **(GATE-07[EC])**

(a) x (b) x^2 (c) $1/x$ (d) $\frac{1}{x^2}$

75. Consider the function $f(x) = x^2 - x - 2$. the maximum value of f(x) in the closed interval [-4,4] is **(GATE-07[EC])**

(a) 18 (b) 10 (c) -2.25 (d) indeterminate

76. The value of $\int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} dxdy$ is **(GATE-07[IN])**

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\pi}$ (c) π (d) $\frac{\pi}{4}$

77. Consider the function $f(x) = (x^2 - 4)^2$ where x is a real number. Then the function has **(GATE-07[EE])**

(a) Only one minimum (b) Only two minima

(c) Three minima (d) Three maxima

78. The integral $\frac{1}{2\pi}\int_0^{2\pi} \sin(t-\tau)\cos\tau d\tau$ equals **(GATE-07[EC])**

(a) Sintcost (b) 0 (c) $\frac{1}{2}\cos t$ (d) $\frac{1}{2}\sin t$

79. $\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x}=$ _____ **(GATE-08[CS])**

(a) 1 (b) -1 (c) ∞ (d) $-\infty$

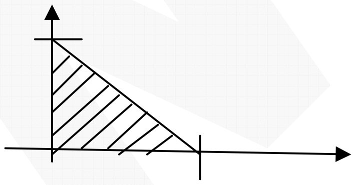
80. A point on the curve is said to be an extremum if it is a local minimum (or) a local Maximum the number of distinct extrema for the curve $3x^4-16x^3+24x^2+37$ is _______ **(GATE-08[CS])**

(a) 0 (b) 1 (c) 2 (d) 3

81. The value of $\int_0^3\int_0^x (6-x-y)dxdy$ is _________ **(GATE-08[CS])**

(a) 13.5 (b) 27.0 (c) 40.5 (d) 54.0

82. Consider the shaded triangular region P shown in the figure. What is $\iint_P xydxdy$? **(GATE-08[ME])**



(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (c) $\frac{7}{16}$ (d) 1

83. Given $y=x^2+2x+10$ the value of $\left.\frac{dy}{dx}\right|_{x=1}$ is equal to **(GATE-08[IN])**

(a) 0 (b) 4 (c) 12 (d) 13

84. $\lim_{x\to 0}\frac{\sin x}{x}$ is **(GATE-08[IN])**

(a) indeterminate (b) 0 (c) 1 (d) ∞

85. The expression $e^{-\ln x}$ for x > 0 is equal to **(GATE-08[ME])**

(a) –x (b) x (c) $-x^{-1}$ (d) $-x^{-1}$

86. Consider the function $y=x^2-6x+9$. The maximum value of y obtained when x varies over the interval 2 to 5 is **(GATE-08[IN])**

(a) 1 (b) 3 (c) 4 (d) 9

87. For real values of x, the minimum value of function $f(x)=e^{x}+e^{-x}$ is **(GATE-08[EC])**

(a) 2 (b) 1 (c) 0.5 (d) 0

88. Which of the following function would have only odd powers of x in its Taylor series Expansion about the point x = 0 ? **(GATE-08[EC])**

(a) $\sin\left(x^{3}\right)$ (b) $\sin\left(x^{2}\right)$ (c) $\cos\left(x^{3}\right)$ (d) $\cos\left(x^{2}\right)$

89. In the Taylor series expansion of $e^{x}+\sin x$ about the point x = π, the coefficient of $(x-\pi)^{2}$ is **(GATE-08[EC])**

(a) e^{π} (b) $0.5e^{\pi}$ (c) $e^{\pi}+1$ (d) $e^{\pi}-1$

90. The value of the integral of the function $g(x,y)=4x^{3}+10y^{4}$ along the straight line segment from the point (0,0) to the point (1,2) in the xy-plane is **(GATE-08[EC])**

(a) 33 (b) 35 (c) 40 (d) 56

91. In the Taylor series expansion of e^{x} about x = 2, the co-efficient of $(x-2)^{4}$ is **(GATE-08[ME])**

(a) $\frac{1}{4!}$ (b) $\frac{2^{4}}{4!}$ (c) $\frac{e^{2}}{4!}$ (d) $\frac{e^{4}}{4!}$

92. The value of $\lim_{x\to 8}\frac{x^{1/3}-2}{x-8}$ is **(GATE-08[ME])**

(a) $\frac{1}{16}$ (b) $\frac{1}{12}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

93. Which of the following integral is unbounded? **(GATE-08[ME])**

(a) $\int_{0}^{\frac{\pi}{4}}\tan x\,dx$ (b) $\int_{0}^{a}\frac{1}{1+x^{2}}dx$ (c) $\int_{0}^{a}xe^{-x}dx$ (d) $\int_{0}^{1}\frac{1}{1-x}dx$

94. The length of the curve $y=\frac{2}{3}x^{3/2}$ between x = 0 & x = 1 is **(GATE-08[ME])**

(a) 0.27 (b) 0.67 (c) 1 (d) 1.22

95. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x\cos x)dx$ is **(GATE-08[PI])**

(a) 0 (b) $\pi-2$ (c) π (d) $\pi+2$

96. The value of the expression $\lim_{x\to 0}\left[\frac{\sin(x)}{e^{x}x}\right]$ is **(GATE-08[PI])**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{1+e}$

97. If (x,y) is continuous function defined over $(x, y) \in [0,1] \times [0,1]$ given two constraints, $x > y^2$ and $y > x^2$, the volume under f(x,y) is **(GATE-09[EE])**

(a) $\int_{y=0}^{y=1} \int_{x=y^2}^{x=\sqrt{y}} f(x, y) dxdy$ (b) $\int_{y=x^2}^{y=1} \int_{x=y^2}^{x=1} f(x, y) dxdy$

(c) $\int_{y=0}^{y=1} \int_{x=0}^{x=\sqrt{y}} f(x, y) dxdy$ (d) $\int_{y=0}^{y=\sqrt{x}} \int_{x=0}^{x=\sqrt{y}} f(x, y) dxdy$

98. A parabolic cable is held between two supports at the same level. The horizontal span between the supports is L. The sag at the mid-span is h. the equation of the parabola is $y = 4h\frac{x^2}{L^2}$, where x is the horizontal coordinate with the origin at the centre of the cable. The expression for the length of the cable is **(GATE-10[CE])**

(a) $\int_0^L \sqrt{1 + 64\frac{h^2x^2}{L^4}} dx$ (b) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^3x^2}{L^4}} dx$

(c) $\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}} dx$ (d) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}} dx$

99. The parabolic arc $y = \sqrt{x}, 1 \le x \le 2$ is revolved around the x-axis. The volume of the solid of revolution is **(GATE-10[ME])**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

100. The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is **(GATE-09[ME])**

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 2

101. The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is **(GATE-09[ME])**

(a) $\frac{16}{3}$ (b) 8 (c) $\frac{32}{3}$ (d) 16

102. The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at x =π is given by **(GATE-09[EC])**

(a) $1+\frac{(x-\pi)^2}{3!}+$.................. (b) $-1-\frac{(x-\pi)^2}{3!}+$..................

(c) $1-\frac{(x-\pi)^2}{3!}+$.................. (d) $-1+\frac{(x-\pi)^2}{3!}+$..................

103. The total derivative of the function 'xy' is **(GATE-09[PI])**

(a) $xdy + ydx$ (b) $xdx + ydy$ (c) $dx + dy$ (d) $dxdy$

104. At t = 0, the function $f(t)=\frac{\sin t}{t}$ has **(GATE-10[EE])**

(a) a minimum (b) a discontinuity

(c) a point of inflection (d) a maximum

105. The value of the quantity, where $P=\int_0^1 xe^x dx$ is **(GATE-10[EE])**

(a) 0 (b) 1 (c) e (d) 1/e

106. If $e^y = x^{1/x}$ then y has a **(GATE-10[EC])**

(a) maximum at x = e (b) minimum at x = e

(c) maximum at x= 1/e (d) minimum at x = 1/e

107. If $f(x)=\sin|x|$ then the value of $\frac{df}{dx}$ at x = -π/4 is **(GATE-10[PI])**

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) -$\frac{1}{\sqrt{2}}$ (d) 1

108. The integral $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx$ is equal to **(GATE-10[PI])**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) ∞

109. What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$? **(GATE-10[CS])**

(a) 0 (b) e^{-2} (c) $e^{-1/2}$ (d) 1

110. The $\lim_{x\to 0}\frac{\sin(\frac{2}{3}x)}{x}$ is **(GATE-10[CE])**

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{3}{2}$ (d) ∞

111. Given a function $f(x,y)=4x^2+6y^2-8x-4y+8$, the optimal values of f(x,y) is

(GATE-10[CE])

(a) A minimum equal to 10/3 (b) A maximum equal to 10/3

(c) A minimum equal to 8/3 (d) A maximum equal to 8/3

112. The infinite series $f(\text{x})=\text{x}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+$................ converges to **(GATE-10[ME])**

(a) $\cos(x)$ (b) $\sin(x)$ (c) $\sinh(x)$ (d) e^x

113. The value of the integral $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$ **(GATE-10[ME])**

(a) $-\pi$ (b) $-\pi/2$ (c) $\pi/2$ (d) π

114. The function $y=|2-3x|$ **(GATE-10[ME])**

(a) is continuous $\forall x\in R$ and differential $\forall x\in R$

(b) is continuous $\forall x\in R$ and differential $\forall x\in R$ except at x = 3/2

(c) is continuous $\forall x\in R$ and differential $\forall x\in R$ except at x = 2/3

(d) is continuous $\forall x\in R$ except at x = 3 and differential $\forall x\in R$

115. The integral is continuous $\forall x\in R$ and differential $\int_{-\infty}^{\infty}\frac{\text{dx}}{1+\text{x}^2}\left(\text{t}-\frac{\pi}{6}\right)6\sin(\text{t})\text{dt}$ evaluates to

(GATE-10[IN])

(a) 6 (b) 3 (c) 1.5 (d) 0

116. What should be the value of λ such that the function defined below is continuous at $x=\frac{\pi}{2}$? **(GATE-11[CE])**

$$f(x)=\begin{cases}\dfrac{\lambda\cos x}{\frac{\pi}{2}-x}; x\neq\dfrac{\pi}{2}\\ 1; x=\dfrac{\pi}{2}\end{cases}$$

(a) 0 (b) 2π (c) 1 (d) π/2

117. What is the value of the definite integral $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$? **(GATE-11[CE])**

(a) 0 (b) $\frac{a}{2}$ (c) a (d) 2a

118. A series expansion for the function $\sin\theta$ is ________ **(GATE-11[ME])**

(a) $1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-$................. (b) $\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-$.................

(c) $1+\frac{\theta^2}{2!}+\frac{\theta^4}{4!}+$................. (d) $\theta+\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+$................

119. If f(x) is even function and a is a positive real number, then $\int_{-a}^{a} f(x)dx$ equals_______ **(GATE-11[ME])**

(a) 0 (b) a (c) 2a (d) $2\int_0^a f(x)dx$

120. What is $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$ equal to? **(GATE-11[ME])**

(a) θ (b) $\sin\theta$ (c) 0 (d) 1

121. The series $\sum_{m=0}^{\alpha}\frac{1}{4^m}(x-1)^{2m}$ converges for **(GATE-11[IN])**

(a) $-2<X<2$ (b) $-1<X<3$ (c) $-3<X<1$ (d) $X<3$

122. Roots of the algebraic equation $x^3+x^2+x+1=0$ are **(GATE-11[EE])**

(a) (1,j,-j) (b) (1,-1,1) (c) (0,0,0) (d) (-1,j,-j)

123. The function $f(x)=2x-x^2+3$ has **(GATE-11[EE])**

(a) A maxima at x = 1and a minima at x = 5 (b) A maxima at x = 1 and a minima at x = -5

(c) Only a maxima at x = 1 (d) Only a minima at x = 5

124. Given $i=\sqrt{-1}$, what will be the equation of the definite integral $\int_0^{\pi/2}\frac{\cos x+i\sin x}{\cos x-i\sin x}dx$?

(a) 0 (b) 2 (c) –i (d) i

125. Consider the function $f(x) = |x|$ $in\ the\ interval -1 \leq x \leq 1.$
At the point x = 0, f(x) is **(GATE-12 [ME, PI])**
(a) Continuous and differentiable. (b) non-continuous and differentiable.
(c) continuous and non-differentiable. (d) neither continuous nor differentiable.

126. $\lim_{x \to 0} \left(\frac{1-cosx}{x^2}\right)$ is **(GATE-12 [ME, PI])**
(a) 1/4 (b) 1/2 (c) 1 (d) 2

127. At x = 0, the function f(x) = $x^3 + 1\ has$ **(GATE-12 [ME, PI])**
(a) a maximum value (b) a minimum value (c) a singularity (d) a point of inflation

128. A Political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch in follows the equation Y= 2X-0.1 X^2 where Y is is the arch of the meters. The maximum possible height of the arch is
(GATE-12 [ME, PI])
(a) 8 meters (b) 10 meters (c) 12 meters (d) 14 meters

129. The infinite series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots..$ corresponds to **(GATE -12 [CE])**
(a) sec x (b) e^x (c) cos x (d) 1+ $\sin^2 x$

130. A function Y = $5x^2 + 10x$ is defined over an open interval X = (1, 2). At least at one point in this interval, $\frac{dY}{dx}$ is exactly **(GATE -2013 [EE])**
(a)20 (b)25 (c) 30 (d) 35

131. The value of the definite integral $\int_1^e \sqrt{x}\ 1n\ (x) dx\ is$ **(GATE -13 [ME])**
(a) $\frac{4}{9}\sqrt{e^3} + \frac{2}{9}$ (b) $\frac{2}{9}\sqrt{e^3} - \frac{4}{9}$ (c) $\frac{2}{9}\sqrt{e^3} + \frac{4}{9}$ (d) $\frac{4}{9}\sqrt{e^3} - \frac{2}{9}$

132. The solution for $\int_0^{\pi/6} cos^4\ 3\theta\ sin^3\ 6\theta\ d\theta\ is$: **(GATE -13 [CE])**
(a) 0 (b) $\frac{1}{15}$ (c) 1 (d) $\frac{8}{3}$

133. Which one of the following functions is continuous at x = 3? **(GATE -13 [CE])**
(a) f(x) = $\begin{cases} 2, & if \quad x = 3 \\ x-1 & if \quad x > 3 \\ \frac{x+3}{3}, & if \quad x < 3 \end{cases}$ (b) f(x) = $\begin{cases} 4, & if \quad x = 3 \\ 8-x & if \quad x \neq 3 \end{cases}$
(c) f(x) = $\begin{cases} x+3, & if \quad x \leq 3 \\ x-4 & if \quad x > 3 \end{cases}$ (d) f(x) = $\frac{1}{x^3-27'}$ $if\ x \neq 3$

134. The Taylor series expansion of 3 sin x + 2cos x is **(GATE -14 – EC –Set 1)**

(a) $2 + 3X - X^2 - \frac{X^3}{2} +$ ------- (b) $2 - 3X + X^2 - \frac{X^3}{2} +$ -------

(c) $2 + 3X + X^2 + \frac{X^3}{2} +$ ------- (d) $2 - 3X + X^2 + \frac{X^3}{2} +$ -------

135. The volume under the surface Z(X, Y) = X + Y and above the triangle in XY plane defined by { $0 \leq Y \leq X$ and $0 \leq X \leq 12$ } is ___________ **(GATE -14 – EC –Set 1)**

136. For $0 \leq t < \infty$, the maximum value of the function $f(t) = e^{-1} - 2e^{-2t}$ occurs at **(GATE-14–EC–Set 2)**

(a) T= $\log_e 4$ (b) t = $\log_e 2$ (c) t = 0 (d) t = $\log_e 8$

137. The values of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ is **(GATE-14–EC–Set 2)**

(a) *ln2* (b) 1.0 (c) e (d) ∞

138. The maximum value of the function f(x) = 1n(1+x) – x (where x > -1) occurs at x = ____ . **(GATE-14–EC–Set 3)**

139. If z = xy 1n(xy), then **(GATE-14–EC–Set 3)**

(a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ (b) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(c) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$ (d) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

140. The maximum value of $f(x) = 2x^2 - 9x^2 + 12x - 3$ in the interval $0 \leq x \leq 3$ is _________. **(GATE-14–EC–Set 3)**

141. The series $\sum_{n=0}^{\infty}\frac{1}{n!}$ converges to **(GATE-14–EC–Set3)**

(a) 2 In 2 (b) $\sqrt{2}$ (c) 2 (d) e

142. For a right angled triangle, if the sum of the lengths of the hypotenuse And a side is kept constant, in order to have maximum area of the triangle, the angle between the hypotenuse and the side is **(GATE-14–EC–Set 4)**

(a) 12^o (b) 36^o (c) 60^o (d) 45^o

143. Let $f(x) = xe^{-x}$. The maximum value of the function in the interval (0, ∞) is **(GATE-14–EE–Set 4)**

(a) e^{-1} (b) e (c) $1 - e^{-1}$ (d) $1 + e^{-1}$

144. Minimum of the real valued function f(x) = $(x - 1)^{2/3}$ occurs at x equal to

(GATE-14–EE–Set2)

(a) $-\infty$ (b) 0 (c) 1 (d) $1 + e^{-1}$

145. To evaluate the double integral $\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x-y}{2}\right) dx \right) dy$, we make the substitution U = $\left(\frac{2x-y}{2}\right)$ and v = $\frac{y}{2}$. The integral will reduce to **(GATE -14–EE–Set2)**

(a) $\int_0^4 \left(\int_0^2 2udu \right) dv$

(b) $\int_0^4 \left(\int_0^1 2udu \right) dv$

(c) $\int_0^4 \left(\int_0^1 udu \right) dv$

(d) $\int_0^4 \left(\int_0^{21} 2udu \right) dv$

146. The minimum value of the function $f(x) = x^2 - 3x^2 - 24x + 100$ in the interval [-3, 3] is

(GATE-14–EE–Set 2)

(a) 20 (b) 28 (c) 16 (d) 32

147. A particle, starting from origin at t = 0 s, is traveling along x – axis with velocity $V = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) m/s$ At t = 3 s, the difference between the distance covered by the particle and the magnitude of displacement from the origin is___.**(GATE-14–EE–Set 2)**

148. Given x(t) = 3 sin $(1000\pi t)$ and Y(t) = 5cos $(1000\pi t\frac{\pi}{t})$ The x- y plot will be (GATE -14–IN–Set 1)

(a) A circle

(b) a mulit – loop closed curve

(c) a hyperbola

(d) an ellipse

149. $\underset{x \to 0}{Lt} \frac{x - \sin x}{1 - \cos x}$ is **(GATE-14–ME–Set1)**

(a) 0 (b) 1 (c) 3 (d) not defined

150. $\underset{x \to 0}{Lt} \left(\frac{e^{2x} - 1}{\sin(4x)}\right)$ is equal to **(GATE-14–ME–Set2)**

(a) 0 (b) 0.5 (c) 1 (d) 2

151. If a function is continuous at a point,

(a) the limit of the function may not exist at the point

(b) the function must be derivable at the point

(c) the limit of the function at the point tends to infinity

(d) the limit must exist the point and the value of limit should be same as

The value of the function at the point. **(GATE-14–ME–Set3)**

152. The value of the integral $\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2+\cos(x-1)} dx$ is **(GATE-14–ME–Set4)**

(a) 3 (b) 0 (c) -1 (d) -2

153. $\lim_{x \to \infty} \left(\frac{x+\sin x}{x}\right)$ equal to **(GATE-14–CE–Set 1)**

(a) $-\infty$ (b) 0 (c) 1 (d) ∞

154. With reference to the conventional Cartesian (x,y) coordinate system, the vertices of a triangle have the following coordinates:

$(x_1, y_1) = (1, 0)$; $(x_2, y_2) = (2, 2)$; and $(x_3, y_3) = (4, 3)$.

The area of the triangle is equal to **(GATE -14–CE–Set1)**

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ d) $\frac{5}{2}$

155. The expression $\lim_{a-0} \frac{x^a-1}{a}$ is equal to **(GATE-14–CE–Set 2)**

(a) log x (b) 0 (c) x log x (d) ∞

156. Let the function f(θ) $\begin{vmatrix} sin\theta & cos\theta & tan\theta \\ sin\left(\frac{\pi}{6}\right) & cos\left(\frac{\pi}{6}\right) & tan\left(\frac{\pi}{6}\right) \\ sin\left(\frac{\pi}{3}\right) & cos\left(\frac{\pi}{3}\right) & tan\left(\frac{\pi}{3}\right) \end{vmatrix}$ where

$\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and f '(θ) denote the derivative of f with respect to θ.

Which of the following statements is/are TRUE? **(GATE-14–CS–Set 1)**

(I) There exists $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that f '(θ) ≠ 0.

(II) There exists $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that f '(θ) ≠ 0.

(a) I only (b) II only (c) Both I and II (d) neither I nor II

157. The function f(x) = x sin x satisfies the following equation: F " (x) + f (x) + t cos x = 0. The value of t is ____________. **(GATE-14–CS–Set 1)**

158. A function f(x) is continuous in the interval [0, 2]. It is known that f(0) = f(2) = -1 and f(1) = 1. Which one of the following statements must be true? **(GATE-14–CS–Set 1)**

(a) There exists a 'y' in the interval (0, 1) such that f(y) = f(y+1)

(b) For every 'y' in the interval (0, 1), f(y) = f(2 – y)

(c) The maximum value of the function in the interval (0, 2) is 1

(d) Threre exists a y in the interval (0, 1) such that f(y) = -f(2 – y)

159. If $\int_0^{2\pi} |x \sin x| \, dx = k\pi$, Then the values of k is equal to_____. **(GATE-14–CS–Set3)**

160. The value the integral given below is $\int_0^{\pi} x^2 \cos x \, dx$ **(GATE-14–CS–Set3)**

(a) -2π (b) π (c) $-\pi$ (d) 2π

161. The value of the integral $\int_0^2 \int_0^x e^{x+y} dy\, dx$ is **(GATE-14–ME–Set4)**

(a) $\frac{1}{2}(e-1)$ (b) $\frac{1}{2}(e^2-1)^2$ (c) $\frac{1}{2}(e^2-e)$ (d) $\frac{1}{2}\left(e-\frac{1}{e}\right)^2$

162. The expression v = $\int_0^H \pi R^2 \left(1-\frac{h}{H}\right)^2 dh$ for the volume of a cone is equal to _________.

(a) $\int_0^R \pi R^2 \left(1-\frac{h}{H}\right)^2 dr$ (b) $\int_0^R \pi R^2 \left(1-\frac{h}{H}\right)^2 dh$ **(GATE -06 – EE)**

(b) $\int_0^H 2\pi\, rH \left(1-\frac{r}{R}\right)^2 dh$ (d) $\int_0^R 2\pi\, rH \left(1-\frac{r}{R}\right)^2 dr$

163. $f = a_0 x^n + a_1 x^{n-1} y + - - + a_{n-1} x y^{n-1} + a_n y^n$ Where a_i $(i = 0 \text{ to } n)$ are constants then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is **(GATE -06 – EE)**

(a) $\frac{f}{n}$ (b) $\frac{n}{f}$ (c) n f (d) $\sqrt[n]{f}$

164. A function f(x) = $1 - x^2 + x^3$ is defined in [-1, 1]. The value of 'x' in open interval (-1, 1) for which the mean value theorem satisfied is **(GATE-EC-15)**

A) $-1/2$ B) $-1/3$ C) $1/3$ D) $1/2$

165. The maximum area (in square units) of a rectangle whose vertices lie in the ellipse $x^2 + 4y^2 = 1$ is **(GATE-EC-15)**

166. The value of $\int_{-\infty}^{\infty} 12\cos(2\pi t).\frac{\sin(4\pi t)}{4\pi t} dt$ is **(GATE-EC-15)**

167. The contour of XY plane, where the partial derivative of $x^2 + y^2$ w.r.t y is equal to partial derivative of 6y+4x w.r.t 'x' is **(GATE-EE-15)**

A) y = 2 B) x = 2 C) x + y = 4 D) x – y =0

168. If a continuous function f(x) does not have a root in the interval [a, b] then which of the following statements is true? **(GATE-EE-15)**

A) f(a).f(b)=0 B) f(a).f(b)<0 C) f(a).f(b)>0 D) f(a).f(b) ≤ 0

169. Consider the function f(x)=1- $|x|$ on $-1 \le x \le 1$, the value of 'x' at which te function attains maxima and the maximum value is **(GATE-EE-15)**

A) 0, -1 B) -1, 0 C) 0, 1 D) -1, 2

170. The value of $\underset{x \to 0}{Lt} \dfrac{1-\cos(x^2)}{2x^4}$ is **(GATE-ME-15)**

A) 0 B) $\dfrac{1}{2}$ C) $\dfrac{1}{4}$ D) Undefined

171. The value of $\int_c (3x-8y^2)dx + (4y-6xy)dy$ where 'c' is a boundary of the region bounded by x =0, y = 0 and x + y = 1 is **(GATE-ME-15)**

172. If $i = \sqrt{-1}$, the value of the definite integral $I = \int_0^{\pi/2} \dfrac{\cos x + i\sin x}{\cos x - i\sin x} dx$ is **(GATE-CE-15)**

A) 1 B) -1 C) i D) –i

173. $\underset{x \to \infty}{Lt} \left(1+\dfrac{1}{x}\right)^{2x}$ is equal to **(GATE-CE-15)**

A) e^{-2} B) e C) 1 D) e^2

174. While minimizing the function f(x), necessary and sufficient conditions for a point x_0 to be a minima are **(GATE-CE-15)**

A) $f'(x_0) > 0$ *and* $f''(x_0) = 0$ B) $f'(x_0) < 0$ *and* $f''(x_0) = 0$

C) $f'(x_0) = 0$ and $f''(x_0) < 0$ D) $f'(x_0) = 0$ and $f''(x_0) > 0$

175. $\underset{x \to \infty}{Lt} x^{1/x}$ is **(GATE-CS-15)**

A) ∞ B) 0 C) 1 D) Not defined

176. $\int_{1/\pi}^{2/\pi} \dfrac{\cos\left(1/\pi\right)}{x^2}$ **(GATE-CS-15)**

177. Let f(x) = $x^{-1/3}$ and A denote the area of a region bounded by f(x) and X-axis, when x varies from -1 to 1, which of the following is true **(GATE-CS-15)**

I) f is continuous in [-1, 1]

II) f is not bounded in [-1, 1]

III) A is non zero finite

A) II only B) III only C) II and III only D) I, II and III

178. Consider the function $(x) = 2x^3 - 3x^2$ in the domain $[-1, 2]$. The global minimum of $f(x)$)is __________ **(GATE-ME-16)**

179. The values of x for which the function $f(x) = \frac{x^2-3x-4}{x^2+3x-4}$ is **NOT** continuous at

(GATE-ME-16)

(A) 4 and −1 (B) 4 and 1 (C) −4 and 1 (D) −4 and −1

180. $\underset{x\to 0}{\text{Lt}} \frac{\log_e(1+4x)}{e^{3x}-1}$ *is equal to* **(GATE-ME-16)**

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{4}{3}$ (D) 1

181. $\underset{x\to\infty}{\text{Lt}} \sqrt{x^2+x-1} - x$ *is* **(GATE-ME-16)**

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{4}{3}$ (D) 1

182.The integral $\frac{1}{2\pi}\iint_D (x+y+10)\,dxdy$, where D denotes the disc: $x^2 + y^2 \le 4$, evaluates to __________ **(GATE-EC-16)**

183. Which one of the following is a property of the solutions to the Laplace equation: $\nabla^2 f = 0$?

(A) The solutions have neither maxima nor minima anywhere except at the boundaries

(B) The solutions are not separable in the coordinates

(C) The solutions are not continuous

(D) The solutions are not dependent on the boundary conditions **(GATE-EC-16)**

184.As x varies from – 1 to 3, which of the following describes the behaviour of the function $f(x) = x^3 - 3x + 1$

(A) f(x) increases monotonically

(B) f(x) increases, then decreases and increases again

(C) f(x) decreases, then increases and decreases again **(GATE-EC-16)**

(D) f(x) increases and then decreases

185. How many distinct values of x satisfy the equation sin(x) = x/2, where x is in radians?

(A) 1 (B) 2 (C) 3 (D) 4 or more

(GATE-EC-16)

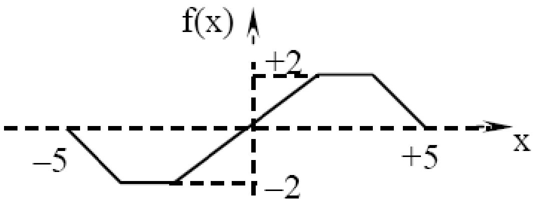
186. Consider the time-varying vector $I = \hat{x}15\cos(\omega t) + \hat{y}\sin(\omega t)$ in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude |I| is at its minimum value, the angle Θ that I makes with the x axis (in degree, such that $0 \le \Theta \le 180$) is ______

(GATE-EC-16)

187. The integral $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to ______ **(GATE-EC-16)**

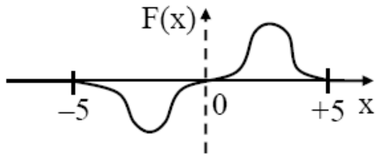
188. A triangle in the xy-plane is bounded by the straight lines 2x = 3y, y = 0 and x = 3. The volume above the triangle and under the plane x + y + z = 6 is _______. **(GATE-EC-16)**

189. Consider the plot of f(x) versus x as shown below. **(GATE-EC-16)**

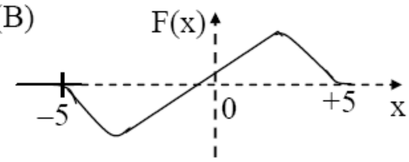


Suppose $F(x) = \int_{-5}^{x} f(y)dy$. Which one of the following is a graph of F(x) ?

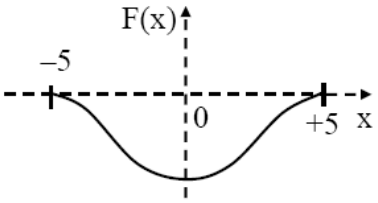
(A)



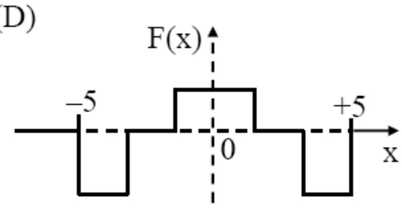
(B)



(C)



(D)



190. The region specified by $\{(\rho, \varphi, Z) : 3 \le \rho \le 5, \frac{\pi}{8} \le \phi \le \frac{\pi}{4}, 3 \le z \le 4.5\}$ in cylindrical coordinates has volume of ———. **(GATE-EC-16)**

191.Given the following statements about a function f: R→R, select the right option: P : If f(x) is continuous at $x = x_0$, then it is also differentiable at $x = x_0$

Q : If f(x) is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R : If f(x) is differentiable at $x = x_0$, then it is also continuous at $x = x_0$

(A) P is true, Q is false, R is false (B) P is false, Q is true, R is true

(C) P is false, Q is true, R is false (D) P is true, Q is false, R is true **(GATE-EC-16)**

192.The value of line integral $\int_c \left(2xy^2dx + 2x^2ydy + dz\right)$ along a path joining the origin (0, 0, 0) and the point (1, 1, 1) is **(GATE-EC-16)**

a) 0 b) 2 c) 4 d) 6

193. The value of the integral $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$ is equal to **(GATE-EE-16)**

a) 0 b) 0.5 c) 1 d) 2

194. The line integral of the vector field $F = 5xz\hat{i}+\left(3x^2+2y\right)\hat{j}+x^2z\hat{k}$ along a path from (0, 0, 0) to (1,1,1) parameterized by (t, t^2, t) *is*_________ **(GATE-EE-16)**

195.The maximum value attained by the function f(x) = x(x –1) (x –2) in the interval [1, 2] is ___ **(GATE-EE-16)**

196. If $f(x) = 2x^7+3x-5$, which of the following is a factor of f(*x*)? **(GATE-CE-16)**

(A) (x^3+8) (B) $(x-1)$ (C) $(2x-5)$ (D) $(x+1)$

197. The optimum value of the function $f(x) = x^2 - 4x + 2$ is **(GATE-CE-16)**

(A) 2 (maximum) (B) 2 (minimum) (C) −2 (maximum) (D) −2 (minimum)

198.What is the value of $\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2+y^2}$? **(GATE-CE-16)**

(A) 1 (B) −1 (C) 0 (D) Limit does not exist

ANS: . (D)

199. The angle of intersection of the curves $x^2 = 4y$ and $y^2 = 4x$ at point (0, 0) is **(GATE-CE-16)**

(A) 0^o (B) 30^o (C) 45^o (D) 90^o

200. The area between the parabola $x^2 = 8y$ and the straight line $y = 8$ is _________. **(GATE-CE-16)**

201.The quadratic approximation of $f(x) = x^3 - 3x^2 - 5$ at the point x = 0 is **(GATE-CE-16)**

(A) $3x^2 - 6x - 5$ (B) $-3x^2 - 5$

(C) $-3x^2 + 6x - 5$ (D) $3x^2 - 5$

202. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is

(A) $\frac{59}{6}$ (B) $\frac{9}{2}$ (C) $\frac{10}{3}$ (D) $\frac{7}{6}$ **(GATE-CE-16)**

203. If $f(x) = 2x^7+3x-5$, which of the following is a factor of $f(x)$? **(GATE-CE-16)**

(A) (x^3+8) (B) $(x-1)$ (C) $(2x-5)$ (D) $(x+1)$

204. $\text{Lim}_{x\to 4} \frac{\sin(x-4)}{x-4} =$ ------ **(GATE-CE-16)**

CALCULUS SOLUTIONS

01. Given $f(x,y)=x^2+3xy+2y+x$

$\Rightarrow \frac{\partial f}{\partial x}=2xy-3y+1=0, \frac{\partial f}{\partial y}=x^2-3x+2=0$. By solving $(1,1),(2,-1)$ are stationary points.

Here $\left[\frac{\partial^2 f}{dx^2}\times\frac{\partial^2 f}{\partial y^2}\right]-\left[\frac{\partial^2 f}{\partial x\partial y}\right]^2<0 \Rightarrow f(x)$ has no extrema. So optional is correct.

02. $$\lim_{x\to 0}\frac{x(e^x-1)+2(\cos x-1)}{x(1-\cos x)}=\lim_{x\to 0}\frac{(e^x-1)+xe^x-2\sin x}{(1-\cos x)+x(\sin x)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{e^x+e^x+xe^x-2\cos x}{\sin x+\sin x+x\cos x}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{e^x+e^x+e^x+xe^x+2\sin x}{\cos x+\cos x+\cos x-x\sin x}$$

$$=\frac{1+1+1+0+0}{1+1+1-0}=1$$

03. $$\int_0^1 x\left[Tan^{-1}y\right]_x^{1/x}dx=\int_0^1 x\left[Tan^{-1}\left(\frac{1}{x}\right)-Tan^{-1}x\right]dx$$

$$=\int_0^1 x\left[\frac{\pi}{2}-2Tan^{-1}x\right]dx=1-\frac{\pi}{4}$$

04. Convert the problem into Gamma function

Put $y^3=t\Rightarrow 3y^2dy=dt\Rightarrow y^{1/2}dy=\frac{1}{3}y^{-3/2}dt=\frac{1}{3}t^{-1/2}dt$

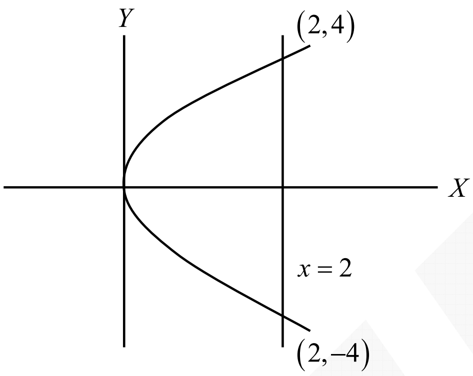
$$\int_0^\infty e^{-t}\frac{1}{3}t^{1-1/2}dt=\frac{1}{3}\sqrt{\frac{1}{2}}=\frac{1}{3}\sqrt{\pi}$$

05. Sol: (a)

$\int \log x dx = x\log x-\int\frac{1}{x}x.dx$ [Integration by parts]

$= x\log x - x$

06. Sol: (d)



Volume $= \int_{y_1}^{y_2} \pi x^2 dy = \int_{-4}^{4} \pi \left[\frac{y^2}{8}\right]^2 dy$

$= \frac{32\pi}{5}$

07. Sol: (b)

Given $y = x^2 + \frac{250}{x}$

$\Rightarrow y' = 2x - \frac{250}{x^2} = 0 \Rightarrow x = 5$ is the stationary point

Now $y''' = 2 + \frac{500}{x^3} \Rightarrow y''(5) = 2 + 4 = 6 > 0 \Rightarrow$ y is minimum at x = 5

08. Sol: (c)

Using Lagrange's mean value theorem,

$f'(c) = \frac{f(b) - f(a)}{b - a}$ where $f(x) = Ax^2 + Bx + C$ in (a,b)

$\Rightarrow 2Ac + B = \frac{(Ab^2 + Bb + C) - (Aa^2 + Ba + C)}{b - a}$

$\Rightarrow c = \frac{b + a}{2}$

09. Sol:

Given $y = \int_{1}^{x^2} \cos t dt \Rightarrow \frac{dy}{dx} = \left(\cos t\right)_1^{x^2}$

$$\Rightarrow \frac{dy}{dx} = \cos(x)^2 \frac{d}{dx}(x^2)\cos('1)\frac{d}{dx}'1$$

$$= 2x\cos x^2$$

10. Sol:(a)

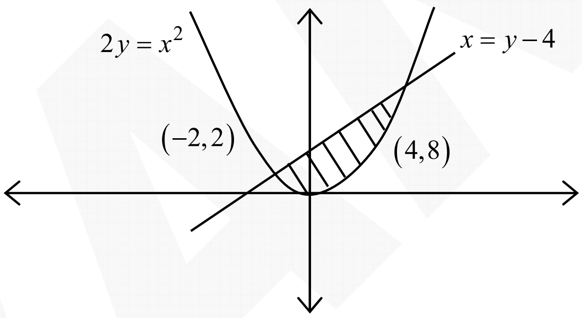
From the given data $\frac{dy}{dx} = \frac{-2x}{4}$

$\Rightarrow \frac{x^2}{C} + \frac{y^2}{2C} = 1$ is an ellipse

11. Sol: (a)

$$\lim_{n\to\infty} \frac{3x^2 + \sin x}{2x + \sin 2x} = \lim_{x\to\infty} \frac{3x + \frac{\sin x}{x}}{2 + \frac{\sin 2x}{x}} = \infty$$

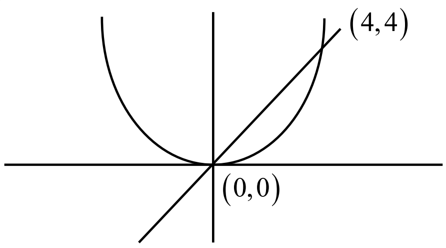
12. Sol: (b)



Points of intersection (-2, 2), (4, 8)

Area $= \int_{-2}^{4} \left[(4+x) - \frac{x^2}{2}\right] dx = 18$

13. Sol: (c)



Given limits are

$y = x^2$ to y = 2x and x = 0 to x = 2 : Variable limits for y

To change the order of integration write the variable limits for X

x = y/2 to $x = \sqrt{y}$ and y = 0 to y = 4

$$\therefore \int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y)\, dx\, dy$$

14. Sol: (b)

Third term is $\frac{(x-a)^2}{2!} f''(a) = \frac{e^a}{2}(x-a)^2$

15. Sol: (b)

$= \lim_{t \to \alpha} \frac{\sin t}{t} = 0$ (put1/x = t)

16. Sol: (b)

We know that |x+a| is continuous every where

|x+a| is differentiable every where except at x = -a

|x+1| is not differentiable at x = -1 ∈ [-2, 0]

17. Sol: (a)

Given $f(x) = x^3 - 6x^2 + 9x + 25$

$f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow x = 13$

$f''(x) = 6x - 12$

$f''(1) = -6 < 0 \Rightarrow$ max. at x = 1

$f''(3) = 6 > 0 \Rightarrow$ min. at x = 3

18. Sol: (b)

Let f(x) be defined in [0,1] by Lagrange's mean value theorem,

$\exists$ c $\in (0,1)$ such that $f''(c) = \frac{f(1) - f(0)}{1-0}$

$\Rightarrow \frac{1}{5-c^2} = \frac{f(1)-2}{1}$

We have $0 < c < 1$

$\Rightarrow 0 < c^2 < 1$

$\Rightarrow 0 > -c^2 > -1$

$\Rightarrow 5 > 5 - c^2 > 4$

$\Rightarrow \frac{1}{5} < \frac{1}{5-c^2} < \frac{1}{4} \Rightarrow \frac{1}{5} < f(1) - 2 < \frac{1}{4}$

$\Rightarrow 2.2 < f(1) < 2.25$

19. Sol: (a)

Every differentiable function is continuous but a continuous function may or may not be Differentiable.

20. Sol: (c)

Given $f(x) = 2x^2 - 2x + 2$

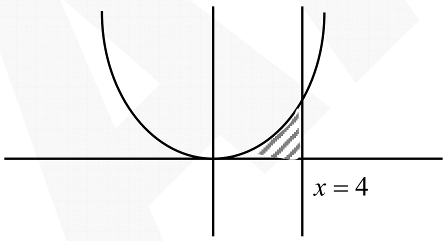
$\Rightarrow \quad f'(x) = 4x - 2 = 0 \Rightarrow x = \frac{1}{2}$

$f''(x) = 4 > 0 \Rightarrow f(x)$ is minimum at x =1/2

$\therefore f(0) = 6, f(2) = 12$

$\therefore$ Max value = 12

21. Sol: (b)



Area = $\int_0^4 x^2 dx = \frac{64}{3}$

22. Sol: (c)

f(x, y) = f(y, x) ⇒ curve is symmetric about the line y = x

23. Sol: (a)

Standard limit formula, $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = m$

24. Sol: (c)

$\frac{dy}{dx} = \begin{cases} 1 for x > 0 \\ -1 x < 0 \end{cases}$ And $\frac{dy}{dx}$ is not defined when x = 0

$$\therefore \lim_{x\to 0^-}\frac{dy}{dx}=-1, \lim_{x\to 0^+}\frac{dy}{dx}=1$$

$\therefore \frac{dy}{dx}$ is discontinuous at x = 0

25. Sol: (a)

$$\phi(x)=\int_0^{x^2}\sqrt{t}\,dt=\left(\frac{\in^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^{x^2}=\frac{2}{3}x^2$$

Now $\frac{d\phi}{dx}=2x^2$

26. Sol:

Given $f(x)=x^3-6x^2+9x+15\ in\ 0\le x\le 6$

$f'(x)=3x^2-12x+9=0 \Rightarrow x=1,2$

$f''(1)=-6<0\Rightarrow$ maxima at x = 1

$f''(2)=6>0\Rightarrow$ minima at x = 2

27. Sol: (b)

$\sum\frac{1}{n^p}$ is convergent if p>1 and divergent if p≤1 $(P-test)$

28. Sol: (b)

From the definition of manima and minima

29. Sol: (d)

By Taylor series

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\cdots\cdots$$

Let f(x) = sin x

$$\therefore \sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\cdots$$

30. Sol: (d)

Taylor's series exist only for continuous and differentiable functions and

Fourier series exist even though the function have finite no. of discontinuous

Points.

31. Sol: (b)

To obtain points of inflections equation 2nd derivative is zero

$$\Rightarrow \frac{d^2y}{dx^2} = 0 \Rightarrow x = 0, y = 0$$

$\therefore$ (0,0) is the only point of inflection

32. Sol: (a)

Apply Ratio test:

$$\upsilon_n = \frac{(n!)^2}{2n!} \Rightarrow \upsilon_{n+1} = \frac{[(n+1)!]^2}{(2n+2)!}$$

$$\frac{\upsilon_{n+1}}{\upsilon_n} = \frac{(n+1)^2}{(2n+2)(2n+1)}$$

$$\lim_{n\to\infty} \frac{\upsilon_{n+1}}{\upsilon_n} = \frac{1}{4} < 1 \Rightarrow \sum \upsilon_n \text{ is converges..}$$

33. Sol: (c)

Using L'Hospital rule

$$\lim_{x\to 0} \frac{1}{10} \times \frac{j5e^{-j5x}}{je^{-jx}} = \frac{5}{10} = 0.5$$

34. Sol: (d)

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+1/n}} = \frac{1}{\sqrt{1+0}} = 1$$

35. Sol: (c)

$f(-x) = e^{-x} \neq f(x)$ (or) –f(x)

36. Sol: (a)

Let $y = (x-a)^{x-a} \Rightarrow \log y = (x-a)\log(x-a)$

$$\Rightarrow \lim_{x\to a} (\log y) = \lim_{x\to a} (x-a)\log(x-a) \backslash$$

$$\Rightarrow \log(\lim_{x\to a} y) = \lim_{x\to a} \frac{\log(x-a)}{1/x-a} \left(\frac{\infty}{\infty} form \right) = 0$$ (<- Hospital rule)

$$\Rightarrow \lim_{x\to a} (x-a)^{x-a} = e^0 = 1$$

37. Sol: (a)

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + ...$$

$$f(x) = f\left(\frac{\pi}{6}\right) + \left(x - \frac{\pi}{6}\right) f\left('\frac{\pi}{6}\right) + \frac{\left(x - \frac{\pi}{6}\right)^2}{2!} f''\left(\frac{\pi}{6}\right) + \cdots\cdots$$

38. Sol: (d)

$$\int_0^{\pi/2} [-\cos(x+y)]_0^{\pi/2} dy = -\int_0^{\pi/2} \left[\cos\left(\frac{\pi}{2} + y\right) - \cos y\right] dy$$

$$= -\int_0^{\pi/2} [-\sin y - \cos y] dy = [-\cos y + \sin y]_0^{\pi/2}$$

$$= 2$$

39. Sol: (d)

$$\lim_{x \to \infty} \frac{1 - a^4}{x^4} = 0$$

40. Sol: (a)

Let $r^2 = x^2 + y^2 + z^2, \frac{\partial r}{\partial x} = x/r, \frac{\partial r}{\partial y} = y/r, \frac{\partial r}{\partial z} = z/r$

$$f = \frac{1}{r} \Rightarrow f_x = \frac{-1}{r^2}.\frac{x}{r} = -x/r^3$$

$$\Rightarrow f_{xx} = -\left[\frac{1}{r^3} + x.\frac{-3}{r^4}\frac{x}{r}\right] = -\frac{1}{r^3} + \frac{3x^2}{r^5}$$

$$\therefore f_{xx} + f_{yy} + f_{zz} = -\frac{3}{r^3} + \frac{3(x^2 + y^2 + z^2)}{r^5} = 0$$

41. Sol: (b)

$$\lim_{a \to \infty} \int_1^a x^{-4} dx = \lim_{a \to \infty} \left[\frac{x^{-3}}{-3}\right]_1^a = -1/3 \lim_{a \to \infty} \left[\frac{1}{a^3} - 1\right] = 1/3$$

42. Sol: (d)

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\cdots = \sin x$$

$$\lim_{x \to \frac{\pi}{2}} \sin x = 1$$

43. Sol: (d)

Let $x-\frac{\pi}{4}=t, \lim_{t\to 0}\frac{\sin 2t}{t}=2$

44. Sol: (a)

$$I=\int_0^{\pi/4}\left[\frac{1+\cos 2x}{2}\right]dx=\left[\frac{x}{2}+\frac{\sin 2x}{4}\right]_0^{\pi/4}$$

$$=\frac{\pi}{8}+\frac{1}{4}$$

45. Sol: (d)

Given $f(x)=x\sqrt{5-x^2}$

$$f'(x)=0\Rightarrow x=\pm\sqrt{\frac{5}{2}}$$

$f''\left(-\frac{\sqrt{5}}{2}\right)>0\Rightarrow f(x)$ has a minimum at $x=-\sqrt{\frac{5}{2}}$

46. Sol: (b)

$$\log x_n=\frac{\log_n}{n}\Rightarrow \lim_{n\to\infty}\log x_n=0\Rightarrow \lim_{n\to\infty}x_n=1$$

47. Sol: (d)

$$f(x)=\max(x_1-x)=\begin{Bmatrix} x & if & x>0 \\ -x & if & x<0 \\ 0 & if & x=0 \end{Bmatrix}=|x|$$

48. Sol: (c)

$f(x)=\frac{\sin 2x}{1+\cos x}$ This is an odd function ($\because$ f(-x)=-f(x))

$\Rightarrow f(x)$ *is odd function*

$$\therefore \int_{-\pi/2}^{\pi/2}f(x)dx=0$$

49. Sol: (c)

$\int_0^1 x\log x dx=\log x.\frac{x^2}{2}-\int\frac{x^2}{2}\frac{1}{x}dx$ (Integration byparts)

$$=\left[\frac{x^2}{2}\log x-\frac{x^2}{4}\right]_0^1$$

$$=(0-1/4)-\lim_{x\to 0}\left[\frac{x^2\log x}{2}\right]-0$$

$$=-1/4$$

50. Sol: (b)

Given $f(x,y)=2x^2+2xy-y^3$

$f_x=4x+2y,\quad f_y=2x-3y^2$
$=0\qquad\qquad =0$

$\Rightarrow 2x=-y\ and\ 2x-3y^2=0$

$\Rightarrow -y-3y^2=0$

$\Rightarrow y=0,-1/3$

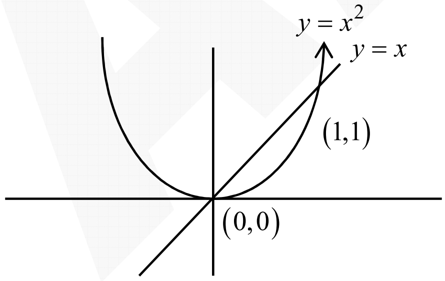
$\Rightarrow x=0,1/6$

$(0,0),(1/6,-1/3)$ *are the stationary points*

51. Sol: (a)

$$\lim_{x\to 0}\frac{\sin x}{x}\times\sin x=1\times 0=0$$

52. Sol: (b)



$$\text{Area}=\int_0^1\left(x-x^2\right)dx=\frac{1}{6}$$

53. Sol: (c)

$$\frac{dx}{d\theta}=a(1+\cos\theta),\quad \frac{dy}{d\theta}=a\sin\theta$$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a\sin\theta}{a(1+\cos\theta)}=\tan\theta/2$$

54. Sol: (a)

55. Sol: (b)

$$\lim_{x\to 0}\frac{x^3+x^2}{2x^3-7x^2}=\lim_{x\to 0}\frac{x+1}{2x-7}=\frac{-1}{7}$$

56. Sol: (a)

$$f(x)=2x^3-3x^2-36x+2$$

$$f'(x)=6x^2-6x-36=0 \Rightarrow x^2-x-6=0$$

$$\Rightarrow (x-3)(x+2)=0 \Rightarrow x=3,-2$$

$$f''(x)=12x-6$$

$$f''(3)=6>0 \Rightarrow \min imum$$

$$f''(-2)=-30<0 \Rightarrow \max imum\ at\ x=-2$$

57. Sol: (a)

$$\int_{-a}^{a}\left(\sin^6 x+\sin^7 x\right)dx=2\int_0^a \sin^6 x dx\ \ (\because\ \sin^7 x \text{ is odd function})$$

58. Sol: (a)

Given, $f(x)=x^2e^{-x}$

$$f'(x)=e^{-x}\left[x^2+2x\right]=0 \Rightarrow x=0, x=-2$$

$$f''(x)=e^{-x}\left[x^2+2x+2x+2\right]$$

$$f''(0)=2>0 \Rightarrow \min imum\ at\ x=0$$

$$f''(-2)=e^{-2}\left[-2\right]<0 \Rightarrow \max imum\ at\ x=-2$$

59. Sol: (d)

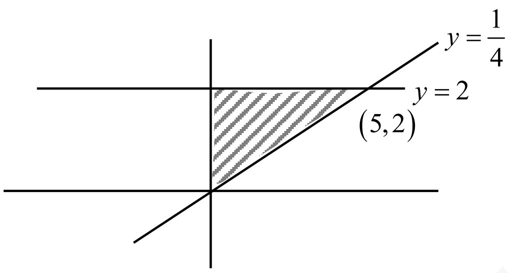
$$\int_{-1}^{1}\frac{1}{x^2}dx=\int_{-1}^{0}\frac{1}{x^2}dx+\int_0^1\frac{1}{x^2}dx=\infty$$

60. Sol: (a)

$$Let\ \frac{x}{\sqrt{8}}=y \Rightarrow dx=\sqrt{8}\,dy$$

$$I=\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-y^2}\sqrt{8}dy=\frac{1}{\sqrt{2\pi}}\times\sqrt{8}\times\frac{\sqrt{\pi}}{2}=1$$

61. Sol: (a)



Given limits are $\left.\begin{matrix} x=0 \text{ to } x=8 \\ y=x/4 \text{ to } y=2\end{matrix}\right\}$ variable limit for y

From the figure by changing the order of integration: $y:0\sim 2$

$x:0\sim 4y$

$$\therefore \quad I=\int_0^2\int_0^{4y} f\left(x,y\right)dxdy$$

62. Sol: (a)

$$\iint f(x,y)dx\ dy=\iint f\left(uv,\frac{u}{v}\right)\phi(u,v)\,du\,dv$$

Where $\phi(u,v)=J\left[\frac{x,y}{u,v}\right]=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}$

$$=\begin{vmatrix} v & u \\ -v/u^2 & 1/u\end{vmatrix}=\frac{v}{u}+\frac{v}{u}=\frac{2v}{u}$$

63. Sol: (c)

$$\int_1^\alpha x^{-3}dx=\left[\frac{x^{-2}}{-2}\right]_1^\alpha=-1/2[0-1]=1/2$$

64. Sol: (c)

$$f(x) = \frac{e^{\sin x}}{e^{\cos x}} = e^{\sin x - \cos x}$$

$$f'(x) = e^{\sin x - \cos x}[\cos x + \sin x] = 0 \Rightarrow x = \frac{3\pi}{4}, -\pi/4$$

$$f''(3\pi/4) < 0 \Rightarrow \max imum\ at\ x = 3\pi/4$$

$$Max\ value = f(3\pi/4) = e^{\sqrt{2}}$$

65. Sol: (c)

$$f(x) = |x|^3 = \begin{cases} x^3 & if\ x > 0 \\ -x^3 & if\ x < 0 \\ 0, if & x = 0 \end{cases}$$

$$f''(x) = \begin{cases} 6x\ if & x > 0 \\ -6x & if\ x < 0 \\ 0,\ if & x = 0 \end{cases}$$

Similarly,

$$and \quad f'''(x) = \begin{cases} 6\ if\ x > 0 \\ -6\ if\ x < 0 \\ does\ not\ exist\ if\ x = 0 \end{cases}$$

66. Sol: (b)

$y = x^2\ in\ [1,5]$

y is minimum when x is minimum in [1,5]

y is minimum at x = 1 (function is increasing)

minimum value of y = 1

67. Sol: (b)

68. Sol: (b)

Given $y = x + \sqrt{x + \sqrt{x + \sqrt{x + ...\alpha}}}$

$$\Rightarrow (y - x)^2 = y \Rightarrow y^2 - 2xy + x^2 = y$$
$$at\ x = 2,\ y^2 - 4y + 4 = y \Rightarrow y^2 - 5y + 4 = 0$$

y =1 (or) 4

but at x = 2, y>1 ⇒ y=4 only

69. Sol: (c)

$$\lim_{x \to \pi/4} \frac{\cos x - \sin x}{x - \pi/4}\left(\frac{0}{0} form\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$$

70. Sol: (b)

$$f(x,y) = x^2 - y^2$$
$$f_x = 2x, f_y = 2y \Rightarrow f_x = 0, f_y = 0 \Rightarrow (0,0) \text{ is stationary point}$$
$$f_{xx} = 2,\ f_{yy} = 0,\ f_{xy} = -2$$
$$at\ (0,0)\ f_{xx}f_{yy} - (f_{xy})^2 = -4 < 0 \Rightarrow \text{neither maxima nor minima exist.}$$

71. Sol: (a)

$$\lim_{x\to 0}\frac{\sin mx}{x} = m\ [\text{Standard limit}]$$
$$\therefore \lim_{\theta\to 0}\frac{\sin(\theta/2)}{\theta} = 1/2$$

72. Sol: (b)

73. Sol: (a)

$$f(x) = e^{-x}$$

Linear approximation = f(2)+(x-2)f'(2)

$$= e^{-2} + (x-2)(-e^{-2})$$
$$= (3-x)e^{-2}$$

74. Sol: (c)

75. Sol: (a)

$$f(x) = x^2 - x - 2\ in\ [-4,4]$$
$$f'(x) = 2x - 1 = 0 \Rightarrow x = 1/2 \text{ is stationary point}$$
$$f''(x) = 2 > 0 \Rightarrow \text{minimum at } x = 1/2$$

The greatest value lies at extreme points

F(-4) = 18, f(4) = 10.

Maximum at x = -4

Max.value = 18

76. Sol: (d)

$$\int_0^{\alpha}\int_0^{\alpha} e^{-x^2} e^{-y^2} dx\, dy = \int_0^{\alpha}\int_0^{\alpha} e^{-(x^2+y^2)} dx\, dy$$

$put\ x = r\cos\theta\ ,\ y = r\sin\theta\ ,\ |J| = r$

$$= \int_0^{\pi/2}\int_0^{\alpha} e^{-r^2} r\ dr\ d\theta = \frac{\pi}{4}$$

77. Sol: (b)

$f(x) = (x^2-4)^2\ where\ x \in R$

$f'(x) = 2(x^2-4)2x = 0 \Rightarrow x = 0,2,-2$

$f''(x) = 4(3x-4)$

$f''(0) = -16 < 0 \Rightarrow \max imum$

$$\left.\begin{array}{l} f''(2) = 32 > 0 \\ f''(-2) = 32 > 0 \end{array}\right\} \Rightarrow \min imum$$

78. Sol: (b)

79. Sol: (a)

$$\lim_{x\to\infty} \frac{1-\frac{\sin x}{x}}{1+\frac{\cos x}{x}} = 1$$

80. Sol: (b)

$f'(x) = 12x^3 - 48x^2 + 48x = 0 \Rightarrow x = 0,2,2$

$f''(0) = 48 > 0 \Rightarrow \min ima\ \ at\ \ x = 0$

$f''(2)=0 \Rightarrow\ no\ extremum\ at\ x = 2$

81. Sol: (a)

$$\int_0^3 \left[6y - xy - \frac{y^2}{2}\right]_0^x dx = \int_0^3 \left[6x - \frac{3x^2}{2}\right] dx = 13.5$$

82. Sol: (a)

$$\int_0^2 \int_{y=0}^{2-\frac{\lambda}{2}} xydxdy = \frac{\lambda}{2}\left(y^2\right)_0^{\frac{2-x}{2}} dx = \int_0^2 \left(4 + x^2 - 4x\right) dx = \frac{1}{6}$$

83. Sol: (b)

$y = x^2 + 2x + 10$

$$\frac{dy}{dx} = 2x+2 \Rightarrow \left.\frac{dy}{dx}\right]_{x=1} = 4$$

84. Sol: (c)

85. Sol: (c)

$e^{-\log_e^x} = 1/x = x^{-1}$

86. Sol: (c)

$v = x^2 - 6x + 9$

$y(2) = 1,\ y(3) = 0, y(4) = 1,\ y(5) = 4$

Maximum value of y is 4

87. Sol: (a)

$f(x) = e^x + e^{-x}$ *where* $x \in R$

$f'(x) = e^x - e^{-x} = 0 \Rightarrow e^x = e^{-x} \Rightarrow x = 0$

$f''(x) = e^x + e^{-x} \Rightarrow f''(0) = 1+1 = 2 > 0 \Rightarrow \min imum$

$\min imum\ value\ of\ f(0) = 2$

88. Sol: (a)

$$\sin x^3 = x^3 - \frac{(x^3)^3}{3!} + \frac{(x^3)^5}{5!} - ....$$

89. Sol: (b)

$f(x) = e^x + \sin x$ *about* $x = \pi$

$$Co-efficient\ of\ (x-\pi)^2 = \frac{f''(\pi)}{2!} = \frac{e^\pi}{2}$$

90. Sol: (a)

Equation of straight line segment from (0,0) to (1,2) is y = 2(x-1)

$$\int g(x,y)dx = \int_0^1 (4x^3 + 10y^4)dx = \int_0^1 \left[4x^3 + 10\times 16(x-1)^4\right]dx$$

$= 33$

91. Sol: (c)

$f(x) = e^x$ *about* $x = 2$

$$co-efficient\ of\ (x-2)^4 = \frac{f^{iv}(2)}{4!} = \frac{e^2}{4!}$$

92. Sol: (b)

$$\lim_{x\to 8}\frac{x^{1/3}-8^{1/3}}{x-8}=1/12\ \left(Formula\ \underset{x\to a}{Lt}\frac{x^m-a^m}{x-a}=ma^{m-1}\right)$$

93. Sol: (d)

$$\left(\because\ \int_0^1 \frac{1}{1-x}dx=\left[\log(1-x)\right]_0^1=\cos 0-\cos 1=\alpha\right)$$

94. Sol: (d)

$$y=\frac{2}{3}x^{3/2}$$

$$length=\int_0^1\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=1.22$$ (Formula from radius of curvature)

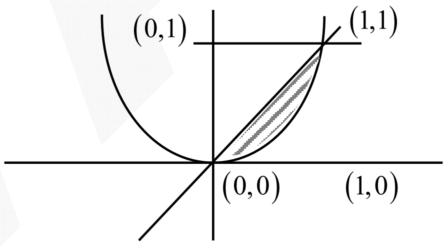
95. Sol: (a)

$$\int_{-\pi/2}^{\pi/2} x\cos x\,dx=0$$ (Integration byparts)

96. Sol: (c)

$$\lim_{x\to 0}\frac{\sin x}{e^x x}=\underset{x\to 0}{Lt}\frac{\sin x}{x}=1.1=1$$

97. Sol: (a)



98. Sol: (d)

$$Total\ \ Length=2\int_0^{L/2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$=2\int_0^{L/2}\sqrt{1+64\frac{h^2x^2}{L^4}}\,dx$$

99. Sol: (d)

$$Volume = \int_{x_1}^{x_2} \pi y^2 dx = \int_1^2 \pi\left(\sqrt{x}\right)^2 dx = \frac{3\pi}{2}$$

100. Sol: (a)

Let $f = x^2 + y^2 + z^2 \Rightarrow f = x^2 + y^2 + 1 + xy$: From maxima and minima, minimum value is 1

101. Sol: (a)

$$Area = \int_0^4 \left[2\sqrt{x} - \frac{x^2}{4}\right] dx$$

$$= \frac{16}{3}$$

102. Sol: (d)

103. Sol: (a)

$$Let\ f(x, y) = xy$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$= vdx + xdy$$

104. Sol: (d)

$$f(t) = \frac{\sin t}{t}$$

$$f'(t) = \frac{t\cos t - \sin t}{t^2} = 0 \Rightarrow t = 0$$

$$f''(0) < 0 \Rightarrow f(t)\ is\ \max imum\ at\ t = 0$$

105. Sol: (b)

(Integration by parts)

106. Sol: (a)

(Verify from the options)

107. Sol: (c)

$f(x) = \sin|x|$

$\frac{df}{dx} = \cos|x| . \frac{|x|}{x}$

$At\ x = -\frac{\pi}{4},\ \frac{df}{dx} = \cos\left|-\pi/4\right| \times (-1) = -1/\sqrt{2}$

108. Sol: (c)

(This is total area under the curve from normal distribution)

109. Sol: (b)

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = \lim_{n\to\infty}\left[\left(1+\frac{-1}{n}\right)^{n}\right]^{2} = \left(e^{-1}\right)^{2} = e^{-2}$$

110. Sol: (a)

$$\lim_{x\to 0}\frac{\sin\left(\frac{2}{3}x\right)}{x} = \frac{2}{3}$$

111. Sol: (a)

(From the option (a) is suitable)

112. Sol: (b)

Expansion of sin x is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...\infty$

113. Sol: (d)

$$\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx = 2\int_{0}^{\infty}\frac{1}{1+x^2}dx = 2\ Tan^{-1}x\Big]_0^{\infty} = 2\left[\pi/2 - 0\right] = \pi$$

114. Sol: (b)

$y = |2-3x|$ is continuous $\forall$ x$\in$ R and is not differentiable at x = 2/3

115. Sol: (b)

$$\int_{-\infty}^{\infty} f(t)\delta(t-a) = f(a)\ where\ a > 0$$

$$\therefore \int_{-\infty}^{\infty} 6\ \delta\left(t-\frac{\pi}{6}\right)\sin(t)dt = 6\sin\frac{\pi}{6} = 3$$

116. Sol: (c)

$\because f$ is continuous at $x = \frac{\pi}{2} \Rightarrow f\left(\frac{\pi}{2}\right) = \underset{x \to \frac{\pi}{2}}{Lt} + f(x)$

$$1 \pm \underset{x \to \frac{\pi}{2}}{Lt} \frac{-\lambda \sin x}{-1} \Rightarrow \lambda = 1$$

117. Sol: (b)

118. Sol: (b)

$Let \quad f(x) = \sin x$

Using $f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + ...$ (Taylor series)

119. Sol: (d)

$$\int_{-\infty}^{\infty} f(x)dx = \begin{cases} 2\int_0^a f(x)\ dx\ if\ \ f(x)\ is\ even\ \ function \\ 0\ if\ \ f(x)\ is\ odd\ \ function \end{cases}$$

120. Sol: (d)

Standard limit formulae. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

121. Sol: (b)

$1 + \frac{1}{4}(x-1)^2 + \frac{1}{4^2}(x-1)^4 + ...$ This series converses of $x \in (-1,3)$

122. Sol: (a)

$root\ of\ \ x^3 + x^2 + x + 1 = 0\ is\ \ x = -1$

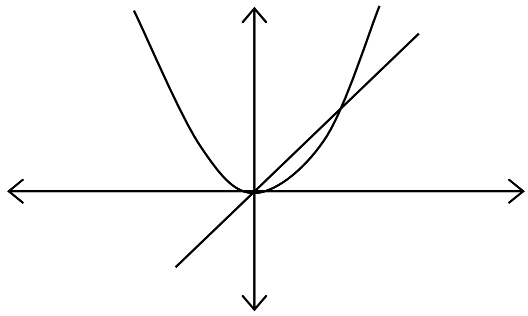
$\Rightarrow (x+1)(x^2+1) = 0$

$\Rightarrow x = -1, j, -j$

123. Sol: (c)

F"(x) = -2 < 0 maximum at x = 1

124. Sol:- (a)



Area = $\int_0^1 (x - x^2)\ dx = \left.\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \quad = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

125. Sol:- (c)

The function is continuous in [-1, 1]

It is also differentiable in [-1, 1] except at x = 0.

Since Left derivative = -1 and Right derivative = 1 at x = 0

126. Sol:- (b)

$\lim_{x-0} \frac{1-\cos x}{x^2} = 1/2$

(Apply L – Hospital rule)

127. Sol:- (d)

f(x) = $x^3 + 1$

$f^1(x) = 3x^2 = 0 \Rightarrow x = 0$

$f^{11}(x) = 6x$

$f^{11}(0) = 0$

So f(x) has a point of inflection at x = 0.

128. Sol:- (b)

y = 2x – (0. 1)x^2

$y^1 = 2 - 2(0.1)x = 0$ => x = 10

$y^{11} = -2(0.1) < 0$

Y is maximum at x = 0

Maximum height = y = 2(10) – (0.1)(100) = 10 m.

129. Sol:- (b)

$e^x = 1 + x + \frac{x^2}{2!} + \ldots\ldots$

130. Sol:- (b)

Given y = $5x^2$ = 10x

$\frac{dy}{dx}$ = 10x +10

131. Ans : (c)

$I = \int_1^e \sqrt{x}\, In\,(x) dx = \int_1^e \log x \sqrt{x}\; dx$

$= \left[\left(\log x \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) - \int \frac{1}{x}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} dx\right]_1^e \qquad = \left[\frac{2}{3}x^{\frac{3}{2}}\log(x) - \frac{2}{3}\int x^{\frac{1}{2}}\,dx\right]_1^e$

$= \left[\frac{2}{3}x^{\frac{3}{2}}\log(x) - \frac{2}{3}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^e \qquad = \left[\frac{2}{3}x^{\frac{3}{2}}\log x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^e$

$= \left(\frac{2}{3}e^{\frac{3}{2}} - \frac{4}{9}e^{\frac{3}{2}}\right) - \left(0 - \frac{4}{9}\right) = \frac{2}{9}\sqrt{e^3} + \frac{4}{9}$

132. Sol:- (b)

$I = \int_0^{\frac{\pi}{6}} cos^4\,3\theta\;\; sin^3\,6\theta\; d\theta$

Put $3\theta = t \Rightarrow d\theta = \frac{dt}{3}$

At $\theta = 0,\; t = 0$

$\theta = \frac{\pi}{6},\; k = \frac{\pi}{2}$

$I = \int_0^{\pi/2} cos^4(t) sin^3(2t)\frac{dt}{3} \qquad = \int_0^{\pi/2} cos^4(t)\,(2\sin t\;\cos t)^3\frac{dt}{3}$

$= \frac{8}{3}\int_0^{\pi/2} cos^4\,t\; sin^3 t\; dt \qquad = \frac{8}{3}\left(\frac{(6\times4\times2)(2)}{10\times8\times6\times4\times2}\right) = \frac{1}{15}$

133. Sol: (a)

If $\underset{x \to a^-}{Lt} f(x) = \underset{x \to a^+}{Lt} f(x) = f(a)$ then f(x) is continuous at x = a

Here $\underset{x \to 3^-}{Lt} f(x) = \underset{x \to 3}{Lt} \frac{x+3}{3} = 2,$

$\underset{x \to 3^+}{Lt} f(x) = \underset{x \to 3}{Lt} x - 1 = 2$

And f(3) = 2

∴ Option (a) is correct.

134. Sol:- (A)

3 sin x + 2 cos x $= 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots \dots\right) + 2\left(x - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \dots\right)$

$= 2 + 3x - x^2 - \frac{x^3}{2} + \dots\dots\dots$

135. Sol: (867)

The required volume = $\int_0^{12}\int_0^{x} z\, dy\, dx$

$= \int_0^{12}\int_0^{x} (x+y) dy\, dx$

$= \int_0^{12} \left(xy + \frac{y^2}{2}\right)_0^x dx$

$= \int_0^{12} \left(x^2 + \frac{y^2}{2}\right)_0^x dx$

$= \frac{3}{2}\left(\frac{x^3}{3}\right)_0^{12}$

$= \frac{3}{2}\left(\frac{12\times12\times12}{3}\right) = 864$

136. Ans (A)

f(t) = $e^{-t} - 2e^{-2t}$

F'(t) = $(-e^{-t} + 4e^{-2t})$

F'(t) = 0 => $e^{-t}(-1 + 4e^{-t}) = 0$

=> $e^{-t} = \frac{1}{4}$

=> -t = log $\left(\frac{1}{4}\right)$

=> t = log 4

F'' (t) = $(e^{-1} - 8\, e^{-2t})$

At t = log 4 => f'' (t) = $-\frac{1}{4} < 0$

∴ At t = log 4, f(t) has maximum value.

137. Ans (C)

$\underset{x\to\infty}{Lt}\left(1 + \frac{1}{x}\right)^x$ = e (standard limit).

138. Ans : 0

f(x) = log (1 + x) – x

F '(x) = $\frac{1}{1+x}$ - 1

F'(x) = 0 => x = 0

F'' (x) = $\frac{-1}{(1+x)^2}$ $is - ve\ at\ x = 0$

∴ f(x) is maximum at x = 0

139. Sol:- (C)

$$\frac{\partial z}{\partial x} = y\left[\frac{x}{xy}y + 1n(xy).1\right] = y(1 + \ln xy)$$

$$\frac{\partial z}{\partial y} = x\left[y\frac{1}{xy}(x) + 1n(xy).1\right] = x[1 + 1n\ xy]$$

$$x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$$

140. Sol:- 6

f(x) = $(2x^3 - 9x^2 + 12x - 3)$

F '(x) = $(6x^2 - 18x + 12)$

F'(x) = 0 => $6(x^2 - 3x + 2) = 0$

F'' (x) = 1, 2 ∈ $(0, 3)$

F'' (x) = (12x – 18)

At x = 1 => F'' (x) = -6 < 0

=> maximum exists

At x = 2 => F''(x) = 6 > 0

=> minimum exists

∴ f(1) = 2

But f(3) = 6

∴ Global maximum of f(x) in [0, 3] = max {f(1), f(3)} = 6.

141. We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ ....$

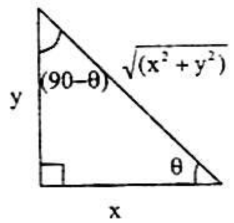
$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Put x = 1 => $e^1 = \sum_{n=0}^{\infty} \frac{1}{n!}$

142. Ans : (C)

Given that x + $\sqrt{(x^2 + y^2)} = C$ (constant)

=> $y^2 = (C^2 - 2Cx)$ ……….(1)



Area of the triangle A = $\frac{1}{2}xy$

Let $A^2 = \frac{x^2y^2}{4} = \frac{x^2}{4}(C^2 - 2Cx)$ = f(x) (say)

f' (x) = $\frac{1}{4}(2C^2x - 6Cx^2)$

f' (x) = 0 => x = $\frac{C}{3}$

At x = $\frac{C}{3} \Rightarrow F'(x) < 0$

∴ Area is maximum at x = $\frac{C}{3}$

Put x = $\frac{C}{3}$ in (1)

$y^2 = \left(c^2 - \frac{2c^2}{3}\right) = \frac{c^2}{3}$

∴ y = $\frac{C}{\sqrt{3}}$

Tan $\theta = \left(\frac{y}{x}\right) = \sqrt{3}$

$\theta = 60^o$

143. Ans :- (a)

F' (x) = $(-x\,e^{-x} + e^{-x})$

F' (x) = 0 => x = 1

At x = 1, F'' (x) < 0

∴ maximum exists at x = 1 and is equal to f(1) = e^{-1}.

144. Ans :- (c)

f(x) = $(x-1)^{\frac{2}{3}}$

= $\left[(x-1)^{\frac{1}{3}}\right]^2$ has no stationary points

$\therefore f(x) \geq 0$

∴ minimum value is '0' and occurs at x = 1

145. Sol: (B)

Since v = $\frac{y}{2} \Rightarrow dv = dy$

U = $\left(\frac{2x-y}{2}\right) \Rightarrow du = 2dx$ (∴ y is constant)

As x : $\frac{y}{2} \to \frac{y}{2} + u : 0 \to 1$

And y : 0 → 8 $\Rightarrow v : 0 \to 4$

$$\therefore \int_0^8 \left[\int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x-y}{2}\right) dx\right] dy \text{ becomes } \int_0^4 \left(\int_1^2 2\,u\,du\right) dv$$

146. Sol: (B)

f(x) = $(x^3 - 3x^2 - 24x + 100)$

F'(x) = $(3x^2 - 6x - 24)$

F'(x) = 0 => x = -2, 4

F'(x) = (6x – 6)

At x = -2 ; F'(x) < 0 we get maximum

At x = 4 ; F'(x) > 0 we get minimum

But x = 4 ∉ [-3, 3]

∴ Global minimum of f(x) = min {f(-3), f(3)}

But f(-3) = 118 and f(3) = 28

∴ Required minimum = 28

147. Displacement x = $\int v\, dt$

$= \sin\left(\frac{\pi t}{2}\right) + C$

At = 0 => x = 0 ∴ C = 0

=> x = $\sin\left(\frac{\pi t}{2}\right)$

After = 3s => x = -1

i.e., the particle is moved '1' unit left of the origin

But x = $\sin\left(\frac{\pi t}{2}\right)$ is oscillating

After t = 3s, the total distance moved by the particle is '3' units

Since At t = 0 => x = 0

At t = 1 => x = 1

At t = 2 => x = 0

At t = 3 => x = -1

∴ The required difference = 3 - $|-1|$

= 3 – 1 = 2

148. Ans :- (D)

Form the given data

$\frac{x}{3} = sin\ \phi$ … … … … … … … … … … … … …..(1)

$\frac{y}{5} = \frac{1}{\sqrt{2}}(\cos\phi - sin\ \phi)$ … … … … ….…..(2)

Where (ϕ = 1000 πt)

Using (1) in (2)

$\frac{y}{5} = \frac{1}{\sqrt{2}}\left(\cos\phi - \frac{x}{3}\right)$ => $\left(\frac{\sqrt{2}}{5}y + \frac{x}{3}\right) = cos\ \phi$……………(3)

Squaring & adding eqn (1) and (3)

$$\frac{x^2}{9} + \left[\frac{2}{25}y^2 + \frac{2\sqrt{2}}{15}xy + \frac{x^2}{9}\right] = 1$$

i.e., $\frac{2x^2}{9} + \frac{2\sqrt{2}}{15}xy + \frac{2}{25}y^2 - 1 = 0$ … … … … … … … …...(4)

It is in the form

$ax^2 + 2hxy + dy^2 + 2gx + 2fy + c = 0$ in eqn (4)

$$ab - h^2 = \left(\frac{4}{225} - \frac{2}{225}\right) = \frac{2}{225} > 0$$

$$\text{And } \Delta = \begin{vmatrix} \frac{2}{9} & \frac{\sqrt{2}}{15} & 0 \\ \frac{\sqrt{2}}{15} & \frac{2}{25} & 0 \\ 0 & 0 & -1 \end{vmatrix} = \frac{2}{225} \neq 0$$

This represents an ellipse if ab - $h^2 > 0$

$$\text{And } \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$$

149. Sol:- (A)

$$\lim_{x \to 0} \frac{x - \sin x}{1 - \cos x} \left(\frac{0}{0} form\right)$$

$$= \lim_{x \to 0} \frac{1-\cos x}{\sin x} \text{ (Apply L – Hospital rule)}$$

$$= \lim_{x \to 0} \frac{\sin x}{\cos x}$$

$$= \frac{0}{1} = 0$$

150. Ans :- (B)

$$\lim_{x \to 0} \left(\frac{e^{2x}-1}{\sin 4x}\right) \left(\because \frac{0}{0} form\right)$$

$$= \lim_{x \to 0} \frac{2e^{2x}}{4\cos 4x} \text{ (Apply L – Hospital rule)}$$

$$= \frac{1}{2} = 0.5$$

151. Sol:- (D)

If a function f(x) is continuous at x = a then $\lim_{x \to 0}$ f(x) = f(a)

i.e., limit exists and is equal to function value.

152. Sol:- (B)

Let (x – 1)

x = (t + 1)

As x : $0 \to 2 \Rightarrow t : 1 \to 1$

$$\therefore \int_0^2 \frac{(x-1)^2 sin(x-1)}{(x-1)^2+cos(x-1)}\, dx = \int_{-1}^{1} \left(\frac{t^2 \sin t}{t^2+\cos t}\right) dt$$

$$\circ = 0 \ (\because \ Integrand\ is\ an\ odd\ function)$$

(OR)

Use the property

$$\circ \int_0^2 f(x)dx = 0 \ \{if\ f\ (2a - x) = -f(x)\}$$

153. Sol:- (C)

$$\lim_{x \to \infty} \left(\frac{x+\sin x}{x}\right) = \lim_{x \to \infty} \left(1 + \frac{\sin x}{x}\right)$$

= 1 + 0 = 1

154. Sol:- (A)

$$= \frac{1}{2}\begin{vmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,(2-3) + 1(6-8) = -2 = -\frac{3}{2}$$

Since area is always +ve, answer is $\frac{3}{2}$

155. Ans :- (A)

$$\underset{x \to 0}{Lt}\ \frac{x^a - 1}{a}\left(\frac{0}{0}\, form\right) \qquad = \underset{x \to 0}{Lt}\ \frac{x^a \log x}{1}$$

(Apply L – Hospital rule by treating 'x' as constant and differentiate w.r.t 'a' both numerator and denominator)

= 1. Log x

= log x

156. Ans :- (C)

$$f'(\theta) = \begin{vmatrix} \sin\theta & -\sin\theta & sec^2\theta \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} & \tan\frac{\pi}{6} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} & \tan\frac{\pi}{3} \end{vmatrix}$$

F (θ) is continuous in $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$ and is differentiable in $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

$f\left(\frac{\pi}{6}\right) = f\left(\frac{\pi}{3}\right)$

$\therefore$ By Rolle's theorem, there exists at least one value $\theta \in \left[\frac{\pi}{6},\frac{\pi}{3}\right]$ such that f ' $(\theta) = 0$ i.e., (1) is true.

f ' (θ) is always zero if $f(\theta)$ is a constant function.

Since $f(\theta)$ is not a constant function.

$\therefore$ (II) is also true.

157. Ans :- (2)

f(x) = x sin x

F'(x) = x cos x + sin x

F''(x) = -x sin x + 2 cos x

Given f'' (x) + f(x) + t cos x = 0

-x sin x + 2 cos x + x sin x + t cos x = 0

$\therefore$ t = -2

158. Ans (A)

(A) As Y ϵ (0, 1) ; f(y) Varies from -1 to 1

Similarly f(Y+1) Varies from 1 to -1

$\therefore$ Let g(x) = f(y) – f(y+1) ; Y ϵ (0, 1)

We get g(x) = 0 for some value of 'X'

i.e., f(y) = f(y+1) for some Y ϵ (0, 1)

option A is true

(B) f(Y) = f(2-Y) only at Y = 0 & y = 1

$\therefore$ In (0, 1) we can not conclude that the maximum value of f(x) is '1' in (0, 2)

(D) As Y ϵ (0, 1) ; f(Y) varies from -1 to 1 and –f(2-y) varies from 1 to -1

$\therefore$ Let g(x) = f(Y) + f(2-Y) ; Y ϵ (0, 1)

$\therefore$ g(x) = 0 for some value of 'X'

i.e., f(y) = -f(2, -Y) for some Y ϵ (0, 1)

But the difference between y (2 – y) should be less than the length of

The interval '2' is not possible.

Hence (D) is false.

159. Ans (4)

$$\int_0^{2\pi} |x\sin x|\,dx = k\pi \Rightarrow \int_0^{2\pi} x|\sin x|\,dx = k\pi$$

$$\pi \int_0^{2\pi} |\sin x|\,dx = k\pi$$

$$\left(\therefore \int_0^{a} xf(x)dx = \frac{a}{2}\int_0^{a} f(x)dx \; if\; f\,(a-x) = f(x)\right)$$

$$\pi\, 2 \int_0^{\pi} |\sin x| dx = k\pi$$

$$\left(\therefore \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \; if\; f(2a-x) = f(x)\right)$$

$$2\;\pi \int_0^{\pi} \sin x\; dx = k\pi$$

$4\pi = k\pi \Rightarrow k = 4$

160. Ans : -2π

$$\int_0^\pi x^2 \cos x \, dx$$

$$= \left(x^2(\sin x) - 2x(-\cos x) + 2(-\sin x)\right)_0^\pi$$

$$= 2\pi \cos \pi = -2\pi$$

161. Ans : (B)

$$\int_0^2 \int_0^x e^{x+y}\, dy\, dx = \int_0^2 e^x (e^y)_0^x \, dx$$

$$= \int_0^2 e^x(e^x - 1)dx = \int_0^2 (e^{2x} - e^x)dx$$

$$= \left[\left(\frac{e^{2x}}{2}\right) - e^x\right]_0^2$$

$$= \left[\left(\frac{e^4}{2} - e^2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{e^4}{2} - e^2 + \frac{1}{2} = \frac{(e^2-1)^2}{2}$$

162. Sol:- (D)

Integrating option (d) we obtain the volume of cone as $v = \frac{1}{3}\pi R^2 H$

163. Sol:- (C)

Given $f = a_0x^n + a_1x^{n-1}y + - - + a_{n-1}xy^{n-1} + a_ny^n$

F is a homogenous polynomial in x and y of degree 'n'

∴ By Euler's theorem for homogenous function, we have

$$x\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = n\,f$$

164. Sol. By Lagranges Mean Value Theorem,

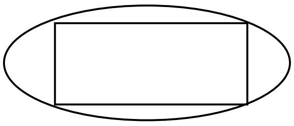
$$f'(x) = \frac{f(1) - f(-1)}{1-(-1)} = \frac{2}{2} = 1$$

$$\Rightarrow -2x + 3x^2 = 1 \Rightarrow 3x^2 - 2x - 1 = 0$$

$\Rightarrow x = 1, -1, -\frac{1}{3}$ *but* $x = -\frac{1}{3} \in (-1, 1)$

165. Sol. **(1)**

Figure:



Let 2x, 2y be length, breadth respectively of a rectangle of a rectangle inscribed in ellipse $x^2 + 4y^2 = 1$ then

Area of a rectangle (2x)(2y) i.e 4xy

Consider $f(x, y) = (Area)^2 = 16x^2y^2 = 4x^2(1 - x^2)$

Now, $f'(x) = 0 \Rightarrow x(1 - 2x^2) = 0 \Rightarrow x = \frac{1}{\sqrt{2}}$

$$y^2 = \frac{1}{8} \Rightarrow y = \frac{1}{\sqrt{8}}$$

$$f''(x) = 8 - 48x^2 < 0 \quad \text{when } x = \frac{1}{\sqrt{2}}$$

$\therefore$ f is maximum at $x = \frac{1}{\sqrt{2}}$ and the maximum area is $4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{8}}\right)$ *i.e* 1

166. Sol. **()**

$$\int_{-\infty}^{\infty} 12\cos(2\pi t).\frac{\sin(4\pi t)}{4\pi t}\,dt$$

$$= \frac{12}{4\pi}\int_{-\infty}^{\infty}\frac{\cos(2\pi t)\sin(4\pi t)}{t}dt = \frac{3}{\pi}\left[\int_0^{\infty}\frac{\sin(6\pi t)}{t}dt + \int_0^{\infty}\frac{\sin(2\pi t)}{t}dt\right]$$

$$= \frac{3}{\pi}\left[\int_0^{\infty} e^{0t}\,\frac{6\sin(6\pi t)}{t}dt + \int_0^{\infty} e^{0t}\,\frac{\sin(2\pi t)}{t}dt\right]$$

$$= \frac{3}{\pi}\left[L\left\{\frac{6\sin(6\pi t)}{t}\right\} + L\left\{\frac{\sin(2\pi t)}{t}\right\}\right]$$

$$= \frac{3}{\pi}\left[6\pi.\frac{1}{6\pi}\left[Tan^{-1}\left(\frac{s}{6\pi}\right)\right]_s^{\infty} + 2\pi.\frac{1}{2\pi}\left[Tan^{-1}\left(\frac{s}{2\pi}\right)\right]_s^{\infty}\right]_{s=0}$$

$$=\frac{3}{\pi}\left[Tan^{-1}\infty - Tan^{-1}\left(s/_{6\pi}\right) + Tan^{-1}\infty - Tan^{-1}\left(s/_{2\pi}\right)\right]$$

$$=\frac{3}{\pi}\left[\frac{\pi}{2} - Tan^{-1}0 + \frac{\pi}{2} - Tan^{-1}0\right] = \frac{3}{\pi}[\pi] = 3$$

167. Sol. **()**

Partial derivative of $x^2 + y^2$ w.r.t y is 0+2y=2y

"6y + 4x w.r.t x is 0 + 4 = 4

Both are equal $\Rightarrow$ 2y = 4 $\Rightarrow$ y=2

168. Sol.

(C) by intermediate value theorem if f(x) has a root in $[a,b)$

$\Rightarrow$ f(a).f(b)<0 since f(x) doesnot have a root means f(a).f(b)>0

169. Sol.

f(x) = $1-|x| \;\; on -1 \le x \le 1$

$$= 1 - x \quad for \quad x \ge 0$$
$$1 + x \quad for \quad x < 0$$

Input '0' output 1 so, option C

170. Sol.

$$\underset{x\to 0}{Lt}\frac{1-\cos(x^2)}{2x^4} = \frac{0}{0}$$

Using L-hospital rule

$$\underset{x\to 0}{Lt}\frac{\sin(x^2).2x}{-8x^3} = \frac{0}{0}$$

$$= \underset{x\to 0}{Lt}\frac{\cos(x^2).2x.2x + \left(\sin x^2\right).2}{24x^2}$$

$$= \underset{x\to 0}{Lt}\frac{\cos(x^2).4x^2 + 2\left(\sin x^2\right)}{24x^2}$$

$$= \underset{x\to 0}{Lt}\frac{\cos(x^2).4x^2 + 2\left(\sin x^2\right)}{24x^2}$$

$$= \underset{x\to 0}{Lt}\frac{\left(-\sin(x^2)\right).4x^2 + \cos x^2(8x) + 2\left(\cos x^2\right).2x}{48x} = \frac{0}{48} = 0$$

171. Sol. (3.66)

By Green's theorem

$$\oint_c Mdx + Ndx = \iint \frac{\partial N}{\partial x}\frac{\partial M}{\partial y}dxdy = \int_0^1\int_0^{y=1-x}(-6y+16y)dxdy = \frac{11}{3} = 3.66$$

172. Sol.

Let $I = \int_0^{\pi/2}\frac{\cos x + i\sin x}{\cos x - i\sin x}dx = \int_0^{\pi/2}\frac{e^{ix}}{e^{-ix}}dx = \int_0^{\pi/2}e^{2ix}dx = \left(\frac{e^{2ix}}{2}\right)_0^{\pi/2} = \frac{1}{2}(-2) = -1$

173. Sol.

$$\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^{2x} = \underset{x\to\infty}{Lt}\left\{\left(1+\frac{1}{x}\right)^x\right\}^2 = \left\{\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^x\right\}^2 = e^2$$

174. Sol.

Option D is a property of the point of minima

175. Sol. (C)

Let y= $\underset{x\to\infty}{Lt}\ x^{1/x}$

$\Rightarrow Lny = \underset{x\to\infty}{Lt}\frac{1}{x}\ln x = \underset{x\to\infty}{Lt}\frac{-1/x}{1} = 0 \Rightarrow y = 1$

176.

$$\int_{1/\pi}^{2/\pi}\frac{\cos\left(1/\pi\right)}{x^2}$$

Put $1/x = t$

$\frac{-1}{x^2}dx = dt$

$x = 2\pi \Rightarrow t = \pi/2$

$x = \frac{1}{\pi} \Rightarrow t = \pi$

$$\int_{1/\pi}^{2/\pi}\frac{\cos\left(1/\pi\right)}{x^2} = \int_{\pi/2}^{\pi}\cos t dt = (\sin t)_{\pi/2}^{\pi} = \sin\pi - \sin\pi/2 = -1$$

177. Sol.

Since $f(0) \to \infty$ f is not bounded in [-1, 1] and hence it is not continuous in [-1, 1]

$$A = \int_{-1}^{1} f(x)dx = \int_{-1}^{0} x^{-1/3}dx + \int_{0}^{1} x^{1/3}dx = \frac{3}{2} + \frac{3}{2} = 3$$, which is non zero finite 0.

178.**Ans: (–5)**

Sol: $f(x) = 2x^3 - 3x^2$ in [-1, 2]

$f'(x) = 0 \Rightarrow 6x^2 - 6x = 0$

$6x(x-1) = 0$

$X = 0 \;\&\; 1$

$F(-1) = -5,\ f(1) = -1,\ f(2) = 4$

Global minimum = -5

179.**Ans: (C)**

Sol: The function, $f(x) = \dfrac{x^2 - 3x - 4}{x^2 + 3x - 4}$ is not defined at $x = 1$ and $x = -4$

$\therefore$ The function f(x) is not continuous at $x = -4, 1$.

180.**Ans: (C)**

Sol: $\displaystyle \underset{x\to 0}{\text{Lt}} \frac{\log(1+4x)}{e^{3x}-1} = \lim_{x\to 0} \frac{\frac{1}{1+4x}(4)}{3e^{3x}} = \frac{4}{3}$

181. **Ans: (C)**

Sol:

$$\underset{x\to\infty}{\text{Lim}} \sqrt{x^2+x-1} - x = \lim_{x\to\infty} \left(\sqrt{x^2+x-1} - x\right) \times \frac{\left(\sqrt{x^2+x-1} - x\right)}{\left(\sqrt{x^2+x-1} + x\right)}$$

$$= \lim_{x\to\infty} \frac{(x^2+x-1-x^2)}{\sqrt{x^2+x-1}+x}$$

$$= \lim_{x\to\infty} \frac{x\left(1-\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{1}{x}-\frac{1}{x^2}}+1\right)} = \frac{1}{2}$$

182. **Ans: 20**

Sol: Converting to polar coordinates, we get

$$\frac{1}{2\pi}\iint_D (x+y+10)dxdy = \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r\cos\theta + r\sin\theta + 10)\, r dr d\theta$$

$$= \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r^2\cos\theta + r^2\sin\theta + 10r)\, dr\, d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi} \left\{\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\sin\theta + 5r^2\right\}_0^2 d\theta$$

$$= \frac{1}{2\pi}\int_{0}^{2\pi} \left\{\frac{8}{3}\cos\theta + \frac{8}{3}\sin\theta + 20\right\} d\theta$$

$$= \frac{1}{2\pi}\left\{\frac{8}{3}\sin\theta - \frac{8}{3}\cos\theta + 20\theta\right\}_0^{2\pi}$$

$$= \frac{1}{2\pi}\left\{\left(-\frac{8}{3} + 40\pi\right) - \left(-\frac{8}{3}\right)\right\}$$

$$= 20$$

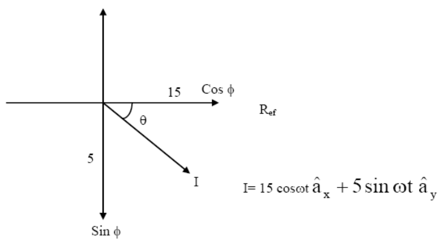
183.**Ans: (A)**

184. **Ans: (B) ;;; Sol:** Since, f(-1) = -3, f(0) = 1, f(1) = -1, f(2) = -3, f(3) = 1

185.**Ans: (C) ;;;Sol:** Sin x = $\frac{x}{2}$ touches at 3 points

186.**Ans: 90°**

Sol:



If $\theta = 0$

$$|I| = 15 \left| \begin{array}{l} 0 < \theta < \frac{\pi}{2} \\ 15 \le \theta \le 5 \end{array} \right.$$

If $\theta = \frac{\pi}{2}$

|I| = 5

187.**Ans: 2**

Sol: $\int_0^1 \frac{dx}{\sqrt{(1-x)}} = \left\{-2\sqrt{1-x}\right\}_0^1$

$= -2[(0)-1] = 2$

188.**Ans: 10**

Sol: Volume = $\iint z\,dxdy \quad \int_{x=0}^{3}\int_{y=0}^{\frac{2}{3}x}(6-x-y)dxdy = 10$

189.**Ans: (C)**

Sol: Integration of ramp is parabolic, integration of step is ramp.

190.**Ans: 4.714**

Sol: Given region of cylinder

$3 \le \rho \le 5$,

$\frac{\pi}{8} \le \phi \le \frac{\pi}{4}$,

$3 \le z \le 4.5$

The differential volume of cylinder in given by

$dv = \rho d\rho d\phi dz$

Volume, $\nu = \int_{\rho=3}^{5} \int_{\phi=\frac{\pi}{8}}^{\frac{\pi}{4}} \int_{z=3}^{4.5} \rho d\rho d\phi dz$

$$= \left[\frac{\rho^2}{2}\right]_3 \times [\phi]_{\frac{\pi}{8}}^{\frac{\pi}{8}} \times [Z]_3^{4.5} \quad = \frac{1}{2}(25-9)\times\left(\frac{\pi}{4}-\frac{\pi}{8}\right)\times(4.5-3)$$

$\therefore \ \nu = 4.71 \text{ m}^3$

191. **Ans: (B)**

Sol: Since continuous function may not be differentiable. But differentiable function is always continuous.

192. Ans: (b)

Sol: (0,0,0) to (1,1,1)

Equation of straight line $\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{3-0}{1-0} = t$

x = t, y = t, z = t

dx = dt, dy = dt, dz = dt

t = 0, t = 1

$$\int_0^1 \left(4t^3+1\right) d\left[\frac{4t^3}{4}+t\right]^1 = 1+1 = 2$$

193.Ans: (d)

Sol: $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt = \frac{2\times 2}{\pi}\times\int_0^{\infty}\left[\frac{\sin 2\pi t}{t}\right]dt$

$$= \frac{4}{\pi}\times\frac{\pi}{2} = 2$$

194. Ans: 4.4167

Sol: $F = 5xza_x + \left(3x^2+2y\right)a_y + x^2 z a_z$

i.e $F = 5xz\bar{i} + \left(3x^2+2y\right)\bar{j} + x^2 z\bar{k}$

$(0,0,0)$ to $(1,1,1)$

$$\int_L F.dl = \int 5xzdx + \int\left(3x^2+2y\right)dy + \int x^2 zdz$$

x = t, $\quad y = t^2 \quad$ z = t

dx = dt, $\quad$ dy = 2tdt, $\quad$ dz = dt

$$= \int_{x=0}^{1} 5xzdx + \int_{y=0}^{1}\left(3x^2+2y\right)dy + \int_{z=0}^{1} x^2 zdz$$

$$= \int_{t=0}^{1} 5t^2dt + \left(3t^2+2t^2\right)2tdt + t^3dt$$

$$= \int_{t=0}^{1} \left(5t^2 + 10t^3 + t^3\right) dt$$

$$= \left(5\frac{t^3}{3} + 11\frac{t^4}{4}\right)_0^1$$

$$= \frac{5}{3} + \frac{11}{4} = \frac{53}{12} = 4.4167$$

195.Ans: (0)

Sol: $f(x) = x(x - 1)(x - 2)$ in [1,2]

$f'(x) = 3x^2 - 6x + 2 = 0 \Rightarrow x = \frac{3 \pm \sqrt{3}}{3}$

$f''(x) = 6x - 6$

$f''\left(\frac{3+\sqrt{3}}{3}\right) = 3.4 > 0 \Rightarrow$ minimum

$f''\left(\frac{3-\sqrt{3}}{3}\right) = -3.4 < 0 \Rightarrow$ maximum

$f(1) = 0$, $f(2) = 0$

Max value = 0

196.Ans: (B)

Soln: option (A)

$X^3 + 8 = 0.x^3 = -8.x = -2$

Sub x =-2, in F (x) $=2x^7 + 3x-5$

$F(-2) = 2(-2)^7 + 3(-2) - 5$

$=-256 - 6 - 5 = -267$

$\therefore$ This is not a factor of f (x)

Option (B)

X-1 =0, X=1

Sub @ x =1, in f (x) = $2x^7 + 3x-5$

$F(1) = 2(1)^7 + 3(1) - 5 = 5-5=0$

$\therefore$ this is a factor of f(x)

Option (C)

2x –f=0, x =5/2

Sub @ x = 5/2 in f (X)= $2x^7 + 3x-5$

$f = \left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right)^7 + 3\left(\frac{5}{2}\right) - 5$

=-2-3-5=-10

$\therefore$ This is not a factor of f(x)

$\therefore$ Option ‘B’ is true

197. Ans: (D)

Soln: $f(x) = x^2 - 4x + 2$

$F^{|}(x) = 2x - 4 = 0$

$\Rightarrow x=2$

f (x) = 2 >0 minimum at x = 2

$\therefore$ min value is f (2)= 4-8-+2=-2

198. Soln:

$$\lim_{x\to 0}\frac{xy}{x^2+y^2}$$

$$\textit{it is in the form}\ \frac{0}{0}$$

Put y = mx

As y →0, mx →0

$$\lim_{x\to 0}\frac{x^2 m}{(1+m^2)x^2}=\frac{1}{1+m^2}$$

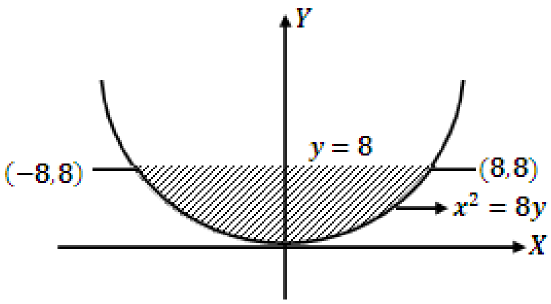
For different value of m we get different limits it is not unique therefore limit does not exist.

199.Ans :- (D)

Sol: Angle between the curves is angle between the tangents at the point of intersection

200.Ans. 85.33

Soln:



Area

$$2\int_0^8 \sqrt{8y}\,dy$$

$$= 2\sqrt{8}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\infty} = 85.33$$

201.Ans: (B)

Soln: Quadratic Approximation

$$= f(0) + (x-0)f'(0) + \frac{(x-0)^2}{2!}f''(0)$$

$f^{|}(x)= x^3 -3x^2 -5 \Rightarrow f(0)=-5$

$f'(x)= 3x^2- 6x \Rightarrow f'(0)=0$

$f'' = 6x - 6$

$\Rightarrow 8''(0) = -6$

$\therefore$ Equation is = $-5 + x(0) + \frac{x^2}{2}(-6)$

$= -3x^2-5$

202.Ans : (B)

Soln:

$y = x^2 + 1$; $x + y = 3$ area bounded by parabola

$x + y = 3$

$\Rightarrow x + x^2 + 1 = 3$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x = -2, 1$

$$A = \iint dy\, dx$$

$$= \int_{-2}^{1} \int_{x^2-1}^{-x} dy dx$$

$$= \int_{-2}^{1} [y]_{x^2-1}^{3-x}\, dx$$

$$= \int_{-2}^{1} (3 - x - x^2 - 1)\, dx$$

$$= \int_{-2}^{1} (2 - x - x^2) dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{2}\right]_{-2}^{1}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$$

$$= \left(\frac{12 - 3 - 2}{6}\right) - \left(\frac{10}{3}\right)$$

$$= \frac{7}{6} + \frac{10}{3} = \frac{27}{6} = \frac{9}{2}$$

203.Ans. (B)

Soln: $F(X) = 2x^7 + 3x - 5$ for X =1 the equations is satisfied. The factor is (X-1)

204.Ans: 1

Soln:

$$\lim_{x \to 4} \frac{\sin(x-4)}{x-4} \quad let\ x - 4 = t$$

$$= \lim_{t \to 0} \frac{\sin t}{t} = 1$$

(OR)

The given limit is in 0/0 form Applying L. Hospital's rule, we get

The limiting value = 1

CHAPTER- 5

NUMERICAL METHODS QUESTIONS

1. Given the differential equation y' = x-y with initial conditions y(0) = 0. The value of y(0.1) calculated numerically up to the third place of decimal by the second order Runge-Kutta method with step size h = 0.1 is **(GATE-1993-All)**

2. Back ward Euler method for solving the differential equation $\frac{dy}{dx} = f(x, y)$ is specified by **(GATE-1994-CS)**

   (a) $y_{n+1} = y_n + hf(x_n, y_n)$ (b) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$

   (c) $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$ (d) $y_{n+1} = (1 = h) f(x_{n+1}, y_{n+1})$

3. In the interval $[0, \Pi]$ the equation $x = \cos x$ has **(GATE-1995-CS)**

   (a) No solution (b) Exactly one solution

   (c) Exactly two solutions (d) an infinite number of solutions

4. The iteration formula to find the square root of a positive real number by using the Newton- Raphson method is **(GATE-1995-CS)**

   (a) $x_{k+1} = \frac{3(x_k + b)}{2x_k}$ (b) $x_{k+1} = \frac{x_k^{\ 2} + b}{2x_k}$ (c) $x_{k+1} = \frac{x_k - 2x_k - 1}{x_k^{\ 2} + b}$ (d) none

5. Let $f(x) = x - \cos x$. using Newton Raphson method at the $(n+1)^{th}$ iteration. The point x_{n+1} is Computed from x_n as **(GATE-1995-CS)**

6. The formula used to compute an approximation for the second derivative of a function f at a point x_0 is **(GATE-1996-CS)**

   (a) $\frac{f(x_0 + h) + f(x_0 - h)}{2}$ (b) $\frac{f(x_0 + h) + f(x_0 - h)}{2h}$

   (c) $\frac{f(x_0 + h) + 2f(x_0) + f(x_0 - h)}{h^2}$ (d) $\frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$

7. The Newton-Raphson iteration formula for finding $3\sqrt{c}$, where c > 0 is,

   (a) $x_{n+1} = \frac{2x_n^{\ 3} + 3\sqrt{c}}{3x_n^{\ 2}}$ (b) $x_{n+1} = \frac{2x_n^{\ 3} - 3\sqrt{c}}{3x_n^{\ 2}}$ **(GATE-1996-CS)**

(c) $x_{n+1} = \dfrac{2x_n^3 + c}{3x_n^2}$ (d) $x_{n+1} = \dfrac{2x_n^3 - c}{3x_n^2}$

8. The Newton-Raphson method is used to find the root of the equation x^2-2. If the iterations are started from -1, then the iteration will **(GATE-1997-CS)**

(a) converges to -1 (b) converges to √2

(c) converges to - √2 (d) not converge 2

9. The Newton- Raphson method is to be used to find the root of the equation and f'(x) is **(GATE-1999)**

(a) always (b) only if f is a polynomial

(c) only if f(x0)<0 (d) none of the above

10. Given a>0, we wish to calculate its reciprocal value 1/a by using Newton-Raphson method for f(x) = 0. For a=7 and starting wkith x0 = 0.2. the first 2 iterations will be **(GATE-2005)**

(a) 0.11, 0.1299 (b) 0.12, 0.1392 (c) 0.12, 0.1416 (d) 0.13, 0.1428

11. Given a>0, we wish to calculate its reciprocal value 1/a by using Newton-Raphson method for f(x) = 0. The Newton-Raphson algorithm for the function will be **(GATE-2005)**

(a) $x_{k+1} = \dfrac{1}{2}\left(x_k + \dfrac{a}{x_k}\right)$ (b) $x_{k+1} = x_k + \dfrac{a}{2}x_k^2$

(c) $x_{k+1} = 2x_k - ax_k^2$ (d) $x_{k+1} = x_k - \dfrac{a}{2}x_k^2$

12. Starting from x_0=1, one step of Newton-Raphson method in solving the equation $x^3 + 3x - 7 = 0$ Gives the next value x_1 as **(GATE-2005)**

(a) x_1 = 0.5 (b) x_1 = 1.406 (c) x_1= 1.5 (d) x_1 = 2

13. For solving algebraic and transcendental equation which one of the following is used? **(GATE-2005)**

(a) Coulomb's theorem (b) Newton-Rapson method

(c) Euler's method (d) Stroke's method

14. Newton-Raphson formula to find the roots of an equation f(x) = 0 is given by **(GATE-2005)**

(a) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ (b) $x_{n+1} = x_n + \dfrac{f(x_n)}{f'(x_n)}$

(c) $x_{n+1} = \frac{f(x_n)}{x_n f'(x_n)}$ (d) $x_{n+1} = \frac{x_n f(x_n)}{f'(x_n)}$

15. The real root of the equation $xe^x = 2$ is evaluated using Newton-Raphson's method. If the first approximation of the value of x is 0.8679 the second approximation of the value of x correct to three decimal places is **(GATE-2005)**

(a) 0.865 (b) 0.853 (c) 0.849 (d) 0.838

16. Match the following and choose the correct combination **(GATE-2005)**

Group-I	**Group-II**
E. Newton-Raphson method non-linear equations	1. Solving
F. Rungue-kutta method linear simultaneous equations	2. Solving
G. Simpson's Rule ordinary differential equations	3. Solving
H. Gauss elimination integration method	4. Numerical
	5. Interpolation
	6. Calculation of Eigen values

(a) E-6, F-1, G-5, H-3 (b) E-1, F-6, G-4, H-3

(c) E-1, F-3, G-4, H-2 (d) E-5, F-3, G-4, H-1

17. The polynomial P(x) = x5+x+2 has **(GATE-2007)**

(a) All real roots (b) 3 real and 2 complex roots

(c) 1 real and 4 complex roots (d) all complex roots

18. Identify the Newton-Raphson iteration scheme for finding the square root of 2 **(GATE-2007)**

(a) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ (b) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{2}{x_n}\right)$

(c) $x_{n+1} = \left(\frac{2}{x_n}\right)$ (d) $x_{n+1} = \sqrt{2 + x_n}$

19. Given that one root of the equation $x^3 - 10x^2 + 31x - 30 = 0$ is 5 then other roots are **(GATE-2007)**

(a) 2 and 3 (b) 2 and 4 (c) 3 and 4 (d) -2 and -3

20. The following equation needs to be numerically solved using the Newton-Raphson method $x^3 + 4x - 9 = 0$. The iterative equation for this purpose is (k indicates the iteration level) **(GATE-2007)**

(a) $x_{k+1} = \dfrac{2x_k^3 + 9}{3x_k^2 + 4}$ (b) $x_{k+1} = \dfrac{3x_k^3 + 9}{2x_k^2 + 4}$

(c) $x_{k+1} = x_k^3 - 3x_k^2 + 4$ (d) $x_{k+1} = \dfrac{4x_k^2 + 3}{9x_k^2 + 2}$

21. Match the exercise and choose the correct one out of the alternatives A,B,C,D **(GATE-2007)**

Group-I	**Group-II**
P. Second order differential equations	1. Runge-kutta method
Q. Non-linear algebraic equations	2. Newton-Raphson method
R. Linear algebraic equations	3. Gauss elimination
S. Numerical integration	4. Simpson's rule

(a) P-3, Q-2, R-4, S-1 (b) P-2, Q-4, R-3, S-1

(c) P-1, Q-2, R-3, S-4 (d) P-1, Q-3, R-2, S-4

22. The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton- Raphson method. If x = 2 taken as the initial approximation of the solution then the next approximation using this method will be **(GATE-2007)**

(a) 2/3 (b) 4/3 (c) 1 (d) 3/2

23. Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then after one step of Newton's method estimate x_1 of the solution will be given by **(GATE-2008)**

(a) 0.71828 (b) 0.36784 (c) 0.20587 (d) 0.0000

24. It is known that two roots of the non-linear equation $x^3 - 6x^2 + 11x - 6 = 0$ are 1 and 3. The third Root will be **(GATE-2008)**

(a) j (b) – j (c) 2 (d) 4

25. The recursion relation to solve using Newton-Raphson method is **(GATE-2008)**

(a) $x_{n+1} = e^{-x_n}$ (b) $x_{n+1} = x_n - e^{-x_n}$

(c) $x_{n+1} = \dfrac{(1 + x_n)e^{-x_n}}{(1 + e^{-x_n})}$ (d) $x_{n+1} = \dfrac{x_n^2 - (1 + x_n)e^{-x_n} - 1}{x_n - e^{-x_n}}$

26. The Newton-Raphson iteration $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{R}{x_n}\right)$ can be used to compute the

(GATE-2008)

(a) Square of R (b) Reciprocal of R (c) Square root of R (d) logarithm of R

27. Let x2-117 = 0. The iterative steps for the solution using Newton-Raphson's method is given by **(GATE-2009)**

(a) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$ (b) $x_{k+1} = \left(x_k - \frac{117}{x_k}\right)$

(c) $x_{k+1} = \left(x_k - \frac{x_k}{117}\right)$ (d) $x_{k+1} = x_k - \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

28. During the numerical solution of a first order differential equation using the Euler (also known as Euler Cauchy) method with step size h, the local truncation error is of the order of **(GATE-2009)**

(a) h^2 (b) h^3 (c) h^4 (d) h^5

29. Consider a differential equation $\frac{dy(x)}{dx} - y(x) = x$ with initial condition $y(0) = 0$. Using euler's first order method with a step size of 0.1 then the value of $y(0.3)$ is

(GATE-2010)

(a) 0.01 (b) 0.031 (c) 0.0631 (d) 0.1

30. Newton-Rap son method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value the approximation after one iteration is **(GATE-2010)**

(a) 3.575 (b) 3.677 (c) 3.667 (d) 3.607

31. A numerical solution of the equation $f(x) = x + \sqrt{x} - 3$ can be obtained using Newton-Raphson method. If the starting value is x = 2 for the iteration then the value of x that is to be used in the next step is **(GATE-2011)**

(a) 0.306 (b) 0.739 (c) 1.694 (d) 2.306

32. The square root of a number N is to be obtained by applying Newton-Raphson iteration to the equation $x^2 - N = 0$ If i denote the iteration index, the correct iterative scheme will be **(GATE-2011)**

(a) $x_{i+1} = \frac{1}{2}\left[x_i + \frac{N}{x_i}\right]$ (b) $x_{i+1} = \frac{1}{2}\left[x_i^2 + \frac{N}{x_i^2}\right]$

(c) $x_{i+1} = \frac{1}{2}\left[x_i + \frac{N^2}{x_i}\right]$ (d) $x_{i+1} = \frac{1}{2}\left[x_i - \frac{N}{x_i}\right]$

33. Solution, the variable x_1 and x_2 for the following equations is to be obtained by employing the Newton-Raphson iteration method **(GATE-2011)**

Equation $\begin{matrix} 10x_2 - \sin x_1 - 0.8 = 0 \\ 10x_2^2 - 10x_2 - \cos x_1 - 0.6 = 0 \end{matrix}$

Assuming the initial values $x_1 = 0.0$ and $x_2 = 1.0$ the Jacobean matrix is

(a) $\begin{bmatrix} 10 & -0.8 \\ 0 & -0.6 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -0.8 \\ 10 & -0.6 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 0 \\ 10 & -10 \end{bmatrix}$

34. The estimate of $\int_{0.5}^{1.5} \frac{dx}{x}$ obtained using simpson's rule with three-point function evaluation exceeds the exact value by

(a) 0.235 (b) 0.068 (c) 0.024 (d) 0.012

35. When the Newton-Raphson method is applied to solve the euqaiton $f(x) = x^3 + 2x - 1 = 0$, the solution at the end of the first iteration with the initial value as $x_0 = 1.2$ is

(a) -0.82 (b) 0.49 (c) 0.705 (d) 1.69

36. While numerically solving the differential equation $\frac{dy}{dx} + 2xy^2 = 0,\ y(0) = 1$ using Euler's predictor corrector (improved Euler-Cauchy) method with a step size of 0.2, the value of y after the first step is

(a) 1.00 (b) 1.03 (c) 0.97 (d) 0.96

37. Match the application to appropriate numerical method

Applications:

P1: Numerical integration

P2: Solution to a transcendental equation

P3: Solution to a system of linear equations

P4: Solution to a differential equation

Numerical Method:

M1: Newton-Raphson Method

M2: Runge-Kutta Method

M3: Simpson's 1/3-rule

M4: Gauss Elimination Method

38. The function $f(x) = e^x - 1$ is to be solved using Newton-Rapshon method. If the initial value of x_0 is taken 1.0, then the absolute error observed at 2nd iteration is

39. The iteration step in order to solve for the cube roots of a given number 'N' using the Newton-Raphson's method is

(a) $x_{k+1} = x_k + \frac{1}{3}(N - x_k^3)$ (b) $x_{k+1} = \frac{1}{3}\left(2x_k + \frac{N}{x_k^2}\right)$

(c) $x_{k+1} = x_k - \frac{1}{3}(N - x_k^3)$ (d) $x_{k+1} = \frac{1}{3}\left(2x_k - \frac{N}{x_k^2}\right)$

40. The real root of the equation $5x - 2\cos x = 0$ (up to two decimal accuracy) is

41. Consider an ordinary differential equation $\frac{dy}{dx} = 4t + 4$. If $x = x_0$ at $t = 0$, the increment in x calculated using Runge-Kutta fourth order multi-step method with a step size of $\Delta t = 0.2$ is

(a) 0.22 (b) 0.44 (c) 0.66 (d) 0.88

42. If the equation $\sin(x) = x^2$ is solved by Newton Raphson's method with the initial guess of $x = 1$, then the value of x after 2 iterations would be

43. A non-zero polynomial $f(x)$ of degree 3 has roots at $x = 1, x = 2$ *and* $x = 3$. which one of the following must be TRUE?

(a) $f(0)f(4) < 0$ (b) $f(0)f(4) > 0$ (c) $f(0) + f(4) > 0$ (4) $f(0) + f(4) < 0$

44. In the Newton-Raphson method, an initial guess of $x_0 = 2$ is made the sequence $x_0, x_1, x_2, ...$ is obtained for the function

Consider of the statement

(I) $x_3 = 0$

(II) The method converges to a solution in a finite number of iterations

Which of the following is TRUE?

(a) Only I (b) Only II (c) Both I and II (d) Neither I nor II

45. Simpson's $\frac{1}{3}$ rule is used to integrate the function f(x) = $\frac{3}{5}x^2 + \frac{9}{5}$ b/w x =0 and x =1 using least numbers of subintervals the value of integral is **(GATE – ME-15)**

46. The values of the function f(x) at 5 discrete points are given below **(GATE – ME-15)**

x	0	0.1	0.2	0.3	0.4
f(x)	0	10	40	90	160

Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)dx$

47. Using a unit stpe size the value of $\int_1^2 x \ln x dx$ by trapezoidal rule **(GATE – ME-15)**

48. N-R method is used to find the roots the equation $x_3 + 2x^2 + 3x - 1 = 0$. If the initial guesses x_0=1, then value of x after 2nd iterations is **(GATE – ME-15)**

49. The quadratic equation $x^2 - 4x + 4 = 0$ is to be solved numerically, starting with the initial guess x_0=3. The N-R method is applied once to get new estimate and secant method is applied once using initial guess and this new estimate. The estimated value of root after the application of second method is **(GATE – CE-15)**

50. In N-R iterative method, the initial guess value x_{ini} is considered as '0' while finding the roots of the equation $f(x) = -2 + 6x - 4x^2 + 0.5x^3$. The correction Δx to be added to x_{ini} in the first iteration is **(GATE – CE-15)**

51. For stepsize Δx =0.4 the value of the following integral using simpson's $\frac{1}{3}$ rule

$\int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)dx$ **(GATE – CE-15)**

52. The solution of a nonlinear equation $x^3 - x = 0$ is to be obtained by N-R method. If the initial guess is x = 0.5, the method converges to which one of the following values **(GATE – CE-15)**

A) -1 B) 0 C) 1 D) 2

53.The root of the function $f(x) = x^3+x-1$ obtained after first iteration on application of Newton-Raphson scheme using an initial guess of x_0=1 is **(GATE – ME-16)**

(A) 0.682 (B) 0.686 (C) 0.750 (D) 1.000

54.Solve the equation $x = 10\cos(x)$ using the Newton-Raphson method. The initial guess is $x = \pi/4$. The value of the predicted root after the first iteration, up to second decimal, is ______

(GATE – ME-16)

55. Gauss-Seidel method is used to solve the following equations (as per the given order):

$x_1 + 2x_2 + 3x_3 = 5$

$2x_1 + 3x_2 + x_3 = 1$

$3x_1 + 2x_2 + x_3 = 3$ **(GATE – ME-16)**

Assuming initial guess as $x_1 = x_2 = x_3 = 0$, the value of x_3 after the first iteration is ____

56. Numerical integration using trapezoidal rule gives the best result for a single variable function, which is **(GATE – ME-16)**

(A) linear (B) parabolic (C) logarithmic (D) hyperbolic

57. The error in numerically computing the integral $\int_0^{\pi}(\sin x + \cos x)\,dx$ using the trapezoidal rule with three intervals of equal length between 0 and π is __________ **(GATE – ME-16)**

58. Consider the first order initial value problem **(GATE – EC-16)**

$y' = y + 2x - x^2, \quad y(0) = 1, \ (0 \le x < \infty)$

with exact solution $y(x) = x^2 + e^x$. For x = 0.1 the percentage difference between the exact solution and the solution obtained using a single iteration of the second-order Runge-Kutta method with step-size h = 0.1 is _______

59. Newton-Raphson method is to be used to find root of equation $3x - e^x + \sin x = 0$. If the initial trial value for the root is taken as 0.333, the next approximation for the root would be ________ **(GATE – CE-16)**

(*note: answer up to three decimal*)

NUMERICAL METHODS SOLUTIONS

1Sol.

Given $y^1 = x - y ....(1)$

$\frac{dy}{dx} = f(x,y)$ where $f(x,y) = x - y$

Also given $y(0) = 0.......(2)$

and $h = 0.1$

$y(0.1) = ?$

Let 2[nd] order rungue-Kulta method is given by $y_1 = y(x_1) = y_0 + \frac{1}{2}(k_1 + k_2)$

where $k_1 = hf(x_0, y_0)$ and $k_2 = hf(x_0 + h,\ y_0 + k_1)$

$k_1 = (0.1)[x_0 - y_0] = (0.1)(0-0) = 0$

$k_2 (0.1)[(x_0 + h) - (y_0 + k_1)]$

$= (0.1)[0 + 0.1 - (0+0)] = 0.01$

$y_1 = y(0.1) = 0 + \frac{1}{2}(0 + 0.01)$

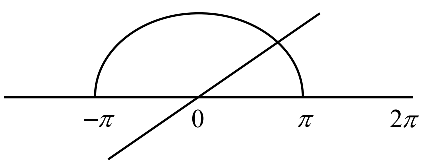
$= \frac{0.01}{2} = \frac{1}{2\times 100} = \frac{1}{200}$

2Sol. (a)

Back ward Euler method is given by $y_{n+1} = y_n + hf(x_{n-1,} y_{n-1})$

3Sol. (b)

Let $f(x) = x - \cos x = 0$



The curves $y = x$ and $y = \cos x$ intersect at only one point in the interval $(0, \pi)$

$\therefore$ The equation has exactly one solution in $[0,\pi]$

4sol. (b)

Let $x=\sqrt{b}\ (or)\ x^2-b=0$

Let $f(x)=x^2-b\ and\ f^1(x)=2x$

$$x_{n+1}=x_n-\frac{f(x_n)}{f^1(x_n)}=x_n-\frac{(x_n^2-b)}{2x_n}$$

$$=\frac{2x_n^2-x_n^2+b}{2x_n}=\frac{x_n^2+b}{2x_n}$$

5Sol.

Given $f(x)=x-\cos x$

$\Rightarrow f^1(x)=1+\sin x$

Newton-Raphson method is given by

$$X_{n+1}=x_n-\frac{f(x_n)}{f^1(x_n)}$$

$$x_{n+1}=x_n-\frac{[x_n-\cos(x_n)]}{[1+\sin(x_n)]}$$

6Sol. (d)

The finite difference approximation for the 2nd derivative of a function f at a apint x_0 is

$$f^{11}(x)=\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}$$

7Sol. (c)

Let $x=\sqrt[3]{c}\ then\, x^3-c=0$

$\Rightarrow f(x)=x^3-c=0\ and\ f^1(x)=3x^2$

Newton-Raphson method is givne by

$$x_{n+1}=x_n-\frac{f(x_n)}{f^1(x_n)}$$

$$= x_n - \frac{\left(x_n^3 - c\right)}{3x_n^2} = \frac{3x_n^3 - x_n^3 + c}{3x_n^2}$$

$$x_{n+1} = \frac{2x_n^3 + c}{3x_n^2}$$

8sol. (c)

Let $f\left(x\right) = x^2 - 2$ and $x_0 = -1$

$$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f^1\left(x_n\right)}$$

$$= x_n - \frac{\left(x_n^2 - 2\right)}{2x_n} = \frac{\left(x_n^2 + 2\right)}{2x_n}$$

$$x_{n+1} = \frac{\left(x_n^2 + 2\right)}{2x_n}$$

$$x_1 = \frac{x_0^2 + 2}{2x_0} = \frac{1+2}{-2} = \frac{-3}{2} = -.15$$

$$x_2 = \frac{x_1^2 + 2}{2x_1} = \frac{\left(-1.5\right)^2 + 2}{2\left(-1.5\right)}$$

$$= \frac{\frac{9}{4} + 2}{-3} = \frac{\frac{17}{4}}{-3} = -1.4166$$

$$x_3 = \frac{x_2^2 + 2}{2x_2} = \frac{\left(-1.4166\right)^2 + 2}{2\left(-1.4166\right)}$$

$$= \frac{2.0067 + 2}{-2.8332} = -1.4141$$

Continuous like this, the value convency ~ -1.4141

9Sol. (d)

Newton Raphson formula converges provided the initial approximation x_0 is chosen sufficiently close to the root

10Sol. (b)

Let $x = \frac{1}{a}(or)\frac{1}{x} - a = 0$

Taking $f(x) = \frac{1}{x} - a$

$\Rightarrow f^1(x) = -\frac{1}{x^2}$

Newton Raphson formula gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$= x_n - \frac{\left(\frac{1}{x_n} - a\right)}{-x_n^{-2}} = 2x_n - ax_n^2$$

$x_{n+1} = 2x_n - ax_n^2 \; for \; n = 0,1,2,....$

Given $a = 7$ and $x_0 = 0.2$

$x_1 = 2x_0 - ax_0^2 = 2(0.2) - 7(0.2)^2$

$= 0.4 - 7(0.04) = 0.12$

$x_1 = 0.12$

$x_2 = 2x_1 - ax_1^2 = 2(0.12) - 7(0.12)^2$

$= 0.24 - 7(0.0144)$

$= 0.24 - 0.1008 = 0.1392$

$\therefore \; x_2 = 0.1392$

11Sol. (c)

Let $x = \frac{1}{a}(or)\frac{1}{x} - a = 0$

Then take $f(x) = \frac{1}{x} - a$ and $f^1(x) = -\frac{1}{x^2}$

Now the Newton Raphson formula gives $X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$

$$= X_n - \frac{\left(\frac{1}{X_n} - a\right)}{-\frac{1}{X_n^2}} = X_n + \left(\frac{1}{X_n} - a\right)X_n^2$$

$$X_{n+1} = 2X_n - aX_n^2 \ \ for\ n = 0,1,2,3,.....$$

12Sol. (c)

Given $f(x) = x^3 + 3x - 7$ *and* $x_0 = 1 \Rightarrow f^1(x) = 3x^2 + 3,$

Newton Raphson's formula is

$$X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$$

$$X_1 = X_0 - \frac{f(X_0)}{f^1(X_0)} = 1 - \frac{f(1)}{f^1(1)}$$

$$= 1 - \frac{(-3)}{6} = \frac{3}{2} = 1.5$$

$\therefore \ X_1 = 1.5$

13Sol. (b)

Newton Raphson method is one of the method to solve algebraic and transcendental equations.

14Sol. (a)

Newton-Raphson formula is

$$X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)} n = 0,1,2,3,.......$$

15Sol. (b)

Given $f(x) = xe^x - 2 = 0$

and $x_1 = 0.8679 \quad \left(\because f^1(x) = xe^x + e^x\right)$

Newton-Raphson's method is given by

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)}$$

$$= 0.8679 - \frac{\left(0.8679e^{0.8679} - 2\right)}{\left(0.8679e^{0.8679} + e^{0.8679}\right)}$$

$= 0.853$

16Sol. (c)

$E-1, F-3, G-4, H-2$

17Sol. (c)

Given $p(x) = x^5 + x + 2$

The signs of all the terms $p(x)$ are same. Then $p(x) = 0$ does one change of sign from $-$ to +

$\therefore$ $p(x) = 0$ has at most one negative root

The degree of $p(x) = 0$ is odd then the equation has at least one real root.

But the total roots of $p(x) = 0$ are five. Out of five roots one is negative real root and remaining 4 are complex roots.

18Sol. (a)

Let $x = \sqrt{2}$ (or) $x^2 - 2 = 0$

Taking $f(x) = x^2 - 2$ *and* $f^1(x) = 2x$

Newton-Raphson's formula is

$$X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$$

$$= X_n - \frac{\left(X_n^2 - 2\right)}{2X_n} = \frac{2X_n^2 - X_n^2 + 2}{2X_n}$$

$$\therefore \quad X_{n-1} = \frac{X_n^2 + 2}{2X_n} = \frac{1}{2}\left[X_n + \frac{2}{X_n}\right]$$

19Sol. (a)

By verification, 2 and 3 are the roots of a given equation

$x^3 - 10x^2 + 31x - 30 = 0$

$i.e. f(2) = 2^3 - 10(2)^3 + 31(2) - 30 = 0$

and $f(3) = 3^3 - 10(3)^2 + 31(3) - 30 = 0$

By synthetic division we have

$x = 5$	1	-10	31	-30
	0	5	-25	30
	1	-5	6	0

$(x-5)(x^2 - 5x + 6) = 0$

$(x-5)[(x-3)(x-2)] = 0$

$\therefore \quad x = 5, 3, 2$

20Sol. (a)

Given $f(x) = x^3 + 4x - 9 = 0$

$\Rightarrow f^1(x) = x^2 + 4$

Newton-Raphson formula is

$$X_{k+1} = X_k - \frac{f(X_k)}{f^1(X_k)}$$

$$= X_k - \frac{(X_k^3 + 4X_k - 9)}{(3X_k^2 + 4)}$$

$$= \frac{3X_k^3 + 4X_k - X_k^3 - 4X_k + 9}{(3X_k^2 + 4)}$$

$$\therefore \quad X_{k+1} = \frac{2X_k^3 + 9}{3X_k^2 + 4}$$

21Sol. (c) P-1, Q-2, R-3, S-4

(1) Simpson's Rule is one of the numerical integration technique (method).

(2) Gauss-elimination method is used to solve only system of linear algebraic equations

(3) Runge-Kutta method is used to solve the ordinary differential equations.

(4) Newton-Raphson method is used to solve the linear and non-linear algebraic equation

22Sol. (b)

Newton-Raphson formula

is $X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$

Given $f(x) = x^3 - x^2 + 4x - 4 = 0$ and $x_0 = 2$

$f'(x) = 3x^2 - 2x + 4$

$$X_1 = X_0 - \frac{f(X_0)}{f^1(X_0)}$$

$$X_1 = 2 - \frac{(2^3 - 2^2 + 4(2) - 4)}{3(2)^2 - 2(2) + 4}$$

$$= 2 - \frac{8}{12}$$

$$\therefore \ x_1 = \frac{4}{3}$$

23Sol. (a)

Given $f(x) = e^x - 1 = 0$ *and* $x_0 = -1$

$\Rightarrow f^1(x) = e^x$

Newton-Raphson's formula is

$$X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$$

$$X_1 = X_0 - \frac{f(X_0)}{f^1(X_0)} = -1 - \frac{(e^{-1} - 1)}{e^{-1}}$$

$$= -1 \frac{\left(\frac{1}{e} - 1\right)}{\frac{1}{e}} = -1 - (1 - e)$$

$$x_1 = -2 + e$$

$$x_1 = 0.71828$$

24Sol. (c)

For the equation $ax^3 + bx^2 + cx + d = 0$, product of roots $= \frac{d}{a}$

$\therefore$ For the given equation let X be the third root

$\therefore\ 1 \times 3 \times X = 6$

$\therefore\ X = 2$

25Sol. (c)

Given $f(x) = x - e^{-x} = 0$

$\Rightarrow f^1(x) = 1 + e^{-x}$

Newton – Raphson formula is

$$X_{n+1} = x_n - \frac{f(X_n)}{f^1(X_n)}$$

$$= X_n - \frac{(X_n - e^{-X_n})}{(1 + e^{-X_n})}$$

$$= \frac{X_n + X_n e^{-X_n} - X_n + e^{-X_n}}{(1 + e^{-X_n})}$$

$$= \frac{e^{-X_n}(1 + X_n)}{(1 + e^{-X_n})}$$

26Sol. (c)

The given Newton's iterative formulae $X_{n+1} = \frac{1}{2}\left(X_n + \frac{R}{X_n}\right)$

Let us suppose the formula converges to the root after n iterations

Then $X_n = X_{n+1} = x(root)$

The formula becomes $X = \frac{1}{2}\left(X + \frac{R}{X}\right)$

$\therefore\ X = \sqrt{R}$

27Sol. (a)

Given $f(x) = x^2 - 117 = 0$

$\Rightarrow f^1(x) = 2x$

Newton-Raphson's formula is

$$x_{k+1} = x_k - \frac{f(x_k)}{f^1(x_k)}$$

$$x_{k+1} = x_k - \frac{(x_k^2 - 117)}{2x_k}$$

$$= \frac{2x_k^2 - x_k^2 + 117}{2x_k} = \frac{x_k^2 + 117}{2x_k}$$

$$\therefore \quad x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$$

28Sol. (a)

In the Euler method, the truncation error is proportional to h^2 denoted as $0(b^2)$

29Sol. (b)

Given $\frac{dy}{dx} - y = x \rightarrow (1)$

and $y(0) = 0 \rightarrow (2)$

Also given $h = 0.1$

$y(0.3) = ?$

from (2), we have

$x_0 = 0, y_0 = 0$ *and* $f(x, y) = \frac{dy}{dx} = y + x$

$x_1 = x_0 + 1h = 0 + 0.1 = 0.1$

$x_2 = x_0 + 2h = 0 + 2(0.1) = 0.2$

$x_3 = x_0 + 3h = 0 + 3(0.1) = 0.3$

Euler's first order method is given by

$$y_1 = y_0 + hf(x_0, y_0) = y_0 + h\left(\frac{dy}{dx}\right)_p$$

$$y_1 = 0 + (0.1)(x_0 + y_0) = 0.1[0 + 0] = 0.0$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 0.0 + (0.1)[1 + 0.0] = 0.0 + 0.01 = 0.01$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$= 0.01 + (0.1)[0.2 + 0.01] = 0.01 + 0.021$$

$$= 0.031$$

30Sol. (d)

Given $x^2 - 13 = f(x) = 0$ and $x_0 = 3.5$

Newton-Raphson formula is

$$X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$$

$$X_1 = X_0 - \frac{f(X_0)}{f^1(X_0)}$$

$$= 3.5 - \frac{\left[(3.5)^2 - 13\right]}{2(3.5)} \left(\because f^1(x) = 2x\right)$$

$$= 3.5 - \left[(12.25) - 13 = 3.5 - \frac{(-0.75)}{7}\right.$$

$$x_1 = \frac{24.5 + 0.75}{7} = \frac{25.25}{7} = 3.607$$

31So. ()

Given $f(x) = x + \sqrt{x} - 3 = 0$ & $x_0 = 2$

$$f^1(x) = 1 + \frac{1}{2\sqrt{x}}$$

Newton-Raphson formula is

$$X_{n+1} = X_n - \frac{f(X_n)}{f^1(X_n)}$$

$$\Rightarrow X_1 = X_0 - \frac{f(X_0)}{f^1(X_0)}$$

$$= 2 - \frac{\left(2+\sqrt{2}-3\right)}{\left(1+\frac{1}{2\sqrt{2}}\right)} = 1.6939$$

$$X_1 = 1.6939$$

32Sol. (a)

Given $f(x) = x^2 - N = 0$

$\Rightarrow f^1(x) = 2x$

Newton-Raphson formula is

$$X_{i+1} = X_i - \frac{f(X_i)}{f^1(X_i)}$$

$$= X_i - \frac{\left(X_i^2 - N\right)}{2X_i} = \frac{2X_i^2 - X_i^2 + N}{2X_i}$$

$$X_{i+1} = \frac{1}{2}\left[X_i + \frac{N}{X_i}\right]$$

33Sol. (b)

Given $10x_2 \sin x_1 - 0.8 = 0$

$10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0$

and $x_{1_0} = 0.0,\ x_{2_0} = 1.0$

let $u(x_1, x_2) = 10x_2 \sin x_1 - 0.8$

and $v(x_1, x_2) = 10x_2^2 - 10x_2 \cos x_1 - 0.6$

Then the Jacobian matrix 'J' is given by

$$J_k = \begin{bmatrix} u_{x_1} & u_{x_2} \\ v_{x_1} & v_{x_2} \end{bmatrix}$$

$$J = \begin{bmatrix} 10x_2 \cos x_1 & 10\sin x_1 \\ 10x_2 \sin x_1 & 20x_2 - 10\cos x_1 \end{bmatrix}_{(x_{1_0}, x_{2_0})}$$

$$J = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

34Sol. (d)

X	0.5	1	1.5
$f(X) = \frac{1}{X}$	2	1	$\frac{2}{3}$

By simpson's rule $\int_{0.5}^{1.5} \frac{1}{X} dx = \frac{h}{3}(y_0 + 4y_1 + y_2)$

$$= \frac{1}{6}(2 + 4 + 0.666) = 1.111$$

By direct integration

$$\int_{0.5}^{1.5} \frac{1}{X} dX = \log_e 3$$

Estimated value – Exact value

$$= 1.111 - 1.0986 = 0.012$$

35Sol. (c)

Given $f(x) = x^2 + 2x - 1 = 0$

$\Rightarrow f^1(x) = 3x^2 + 2x$

$x_0 = 1.2$

The Newton-Raphson iterative formula is

$$x_{n+1} = x_n = -\frac{f(x_n)}{f^1(x_n)} f^1(x_n) \neq 0 \; \& \; n0,1,2,.....$$

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)} = 1.2 - \frac{\left[(1.2)^3 + 2(1.2) - 1\right]}{3(1.2)^2 + 2}$$

$= 0.705$

36Sol. (d)

$$\frac{dy}{dx} = -2xy^2 = f(x,y)$$

$$f(0,1) = 0$$

$$y_1^p = y_0 + hf(x_0, y_0) = 1 + 0.2(0) = 1$$

$$y_1^c = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^p)\right]$$

$$= 1 + \frac{0.2}{2}(0 - 0.4) = 0.96$$

37Sol. (b)

The Numerical methods described the corresponding applications

38Sol. (0.06)

$$f'(x) = e^x$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \left(\frac{e-1}{e}\right) = \frac{1}{e}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{1}{e} - \left(\frac{e^{1/e} - 1}{e^{1/e}}\right) = 0.06$$

39Sol. (b)

Let $N = \sqrt[3]{x} \Rightarrow x^3 = N$

Let $f(x) = x^3 - N = 0$

$$f'(x) = 3x^2$$

$$\therefore\ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\left(x_n^3 - N\right)}{3x_n^2}$$

$$= \frac{1}{3}\left(\frac{2x_n^3 + N}{x_n^2}\right)$$

40Sol. (0.54)

Let $f(x) = (5x - 2\cos x - 1)$

$f'(x) = 5 + 2\sin x$

Apply Newton-Raphson method iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let $x_0 = 1 then\ x_1 = 0.5631$

$x_2 = 0.5426, x_3 = 0.5425$

41Sol. (d)

Given that $\frac{dx}{dt} = (4t + 4)$

At $t = 0, x = x_0 = 0$

$h = 0.2, f(t, x) = (4t + 4)$

By R-K fourth order method

$$x_1 = x_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

where

$K_1 = hf(t_0, x_0) = 0.2 \times 4 = 0.8$

$$K_2 = hf\left(t_0 + \frac{h}{2}, x_0 + \frac{K_1}{2}\right)$$

$= (0.2)\left[4(0 + 0.1) + 4\right] = 0.88$

$$K_3 = hf\left(t_0 + \frac{h}{2}, x_0 + \frac{K_2}{2}\right)$$

$$=(0.2)\left[4(0+0.1)+4\right]=0.88$$

$$K_4=hf\left(t_0+h,x_0+K_3\right)$$

$$=(0.2)\left[4(0+0.2)+4\right]=0.96$$

$$\therefore\ x_1=0+\frac{1}{6}\left[0.8+2(0.88)+2(0.88)+0.96\right]=0.88$$

42Sol. (0.73)

$$f(x)=x^2-\sin x$$

$$f'(x)=2x-\cos x$$

$$x_0=1$$

$$\therefore\ x_1=x_0-\frac{f(x_0)}{f'(x_0)}=1-\left(\frac{1-\sin 1}{2-\cos 1}\right)=0.8915$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

$$=0.8915-\left[\frac{1-\sin(0.89)}{2-\cos(0.89)}\right]=0.73\ \text{(approximately)}$$

43Sol. (a)

Let $f(x)=k(x-1)(x-2)(x-3)(k\neq 0)$

Now $f(0)$ *and* $f(4)$ have opposite signs

$\therefore\ \ f(0).f(4)<0$

44Sol. (a)

Let $f(x)=\left(0.75x^3-2x^2-2x+4\right)=0$

$$f'(x)=\left(2.25x^3-4x-2\right)$$

given $x_0=2$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=0$$

Similarly $x_2 = x_3 = 0, x_4 = 2, x_5 = 0.....$

$\therefore$ The iterations will not converge to root.

45. Sol:

X	0	$\frac{1}{2}$	1
Y	$\frac{9}{5}$	$\frac{39}{20}$	$\frac{12}{5}$

$$\int_0^1 ydx = \frac{\frac{1}{2}}{2}.\left[\left(\frac{9}{5}+\frac{12}{5}\right)+4\left(\frac{39}{20}\right)\right] = 0.0208$$

46 Sol: By Trapezoidal rule

$$\int_0^{0.4} f(x)dx = \frac{h}{2}\left[\left(y_0+y_4\right)+2\left(y_1+y_2+y_3\right)\right] \quad = \frac{0.1}{2}\left[\left(0+160\right)+2(10+40+90)\right] = 22$$

47. Sol:

X	1	2
xlnx	0	2ln2

By trapezoidal rule $\int_1^2 x\ln x dx = \frac{1}{2}\left[0+2\ln 2\right] = \ln 2 = 0.69$

48. Sol: By N-R method $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{5}{10} = \frac{1}{2}$

2nd iteration $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.3043$

49. Sol: $f(x) = x^2 - 4x + 4$, x_0=3 $f'(x) = 2x - 4$

By N-R method $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{1}{2} = 2.5$

For secant method $x_0 = 2.5$ and x_1=3

By secant method

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 3 - \frac{(3-2.5)}{f(3).f(2.5)} \times f(3) = 3 - \frac{0.5}{1-0.25} \times 1 = 2.333$$

50. Sol: $f(x) = -2 + 6x - 4x^2 + 0.5x^3$, f(0)=-2

$f'(x) = 6 - 8x + 1.5x^2$, $f'(0)$=6

By N-R method $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{(-2)}{6} = 0.333$

51. Sol: Given h = Δx =0.4

X	0	0.4	0.8
f(x)	0.2	24.456	-126.744

By Simpson's $\frac{1}{3}$ rule $\int_0^{0.8} f(x)dx = \frac{0.4}{3}[(0.2-126.744)+4(24.456)] = -3.8293$

52. Sol: $f(x) = x^3 - x$ initial guess $x_0 = 0.5$

$f'(x) = 3x^2 - 1$, First iteration $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{(0.5)^3 - 0.5}{3(0.5)^2 - 1} = -1 \Rightarrow x_1 = -1$

So, it converges to -1

53. **Ans: (C)**

Sol: Let f(x) = x^3 + x -1 & x_0 = 1

Then f' (x) = $3x^2$ + 1

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(1+1-1)}{(3+1)} = 0.75$$

54. **Ans: (1.564)**

Sol: $Let\ f(x) = x - 10cos(x) \& x_0 = \left(\frac{\pi}{4}\right)$

$Then\ f'(x) = 1 + 10sin(x)$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{\pi}{4} - \frac{\left(\frac{\pi}{4} - \frac{10}{\sqrt{2}}\right)}{\left(1 + \frac{10}{\sqrt{2}}\right)}$$

$\Rightarrow \frac{\pi}{4} + \frac{(6.2857)}{(8.0711)}$ =1.564

55. **Ans: (– 6)**

Sol: Let x + 2y +3z =5

2x + 3y + z = 1

3x+2y + z = 3 and $x_0 = 0$, $y_0 = 0$, $z_0 = 0$

Then first iteration will be

$x_1 = 5 - 2y_0 - 3z_0 = 5 - 0 - 0 = 5$

$$x_2 = y_1 = \frac{1}{3}(1 - 2x_1 - z_0) = \frac{1}{3}(1 - 10 - 0) = -3$$

$X_3 = z_1 = 3- 3x_1 – 2y_1 = 3 – 15 + 6 = -6$

$\therefore\ x_3 = -6$

56. **Ans: (A)**

Sol: f(x) is a linear function

57.**Ans: 0.1862**

Sol: $I = \int_0^{\pi} (\sin x + cosx)dx$

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π
f(x)	1	$\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$	-1

$$h = \frac{b-a}{n} = \frac{\pi - 0}{3} = \frac{\pi}{3}$$

By trapezoida, rule, we have

$$\int_0^{\pi} f(x)dx = \frac{h}{2}[(y_0 + y_3) + 2(y_1 + y_2)]$$

$$= \frac{\pi}{6}\left[(1-1) + 2\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\right]$$

= 1.813799364 (approximate value)

By Exact method

$$I = \int_0^{\pi} (\sin(x) + \cos(x))dx$$

$$= [-\cos(x) + sinx]_0^{\pi}$$

= cos(0) – cos(π)+sin(π) – sin 0

= 1-(-1)= 2 (Exact value)

∴ Error = Exact value – approximate value

= 2- 1.813799364 = 0.1862

58.**Ans: 0.06**

Sol: $\frac{dy}{dx} = y + 2x - x^2 \qquad y(0) = 1, \ (0 \le x < \infty)$

Given $f(x, y) = y + 2x - x^2, \quad x_0 = 0, \quad y_0 = 1, \quad h = 0.1$

$k_1 = hf(x_0, y_0) = 0.1(1 + 2(0) - 0^2) = 0.1$

$k_2 = hg(x_0 + h, y_0 + k_1) = 0.1(y_0 + k_1) + 2(x_0 + h) - (x_0 + h)^2)$

$= 0.1(1 + 0.1) + 2(0.1) - (0.1)^2$

$= 0.1(1.1 + 0.2 - 0.01)$

$= 0.129$

$\therefore \quad y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$

$= 1 + \frac{1}{2}(0.1 + 0.129)$

= 1+0.1145 = 1.1145

Exact solution, $y(x) = x^2 + e^x$

$y(0.1) = (0.1)^2 + e^{0.1}$

= 0.01 + 1.1052 = 1.1152

ERROR = 1.1152 – 1.1145

= 0.00062

Percentage Error = 0.00062×100 = 0.06%

59.**Ans: 0.3601**

Soln: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 0.333 - \frac{f(0.333)}{f'(0.333)}$

f(0.333) = 3(0.333)– $e^{0.333}$+sin(0.333)

= 0.999 – 1.3951+0.3268

= – 0.0693

f ′(0.333)=3– $e^{0.333}$+ cos(0.333)

=3– 1.3951+0.9450

=2.5499

$$\therefore x_1 = 0.333 + \frac{0.0693}{2.5499}$$
$$= 0.333 + 0.02717$$
$$= 0.3601$$

CHAPTER- 6
DIFFERENTIAL EQUATIONS

01. The differential equation $y^{11}+y = 0$ is subjected to the conditions $y(0) = 0$, $y(\lambda) = 0$. In order that the equation has non-trivial solutions. The general value of λ is **(GATE-93[ME])**

(a) $y = C_2\cos\lambda$ (b) $y = C_2\sin\lambda$ (c) $y = C_1+C_2\lambda$ (d) None

02. The differential equation $d^2y/dx^2+dy/dx+\sin y = 0$ is **(GATE-93[ME])**

(a) linear (b) non-linear (c) homogeneous (d) of degree 2

03. The necessary and sufficient condition for the differential equation of the form $M(x,y)dx+ N(x,y)\,dy = 0$ to be exact is **(GATE-94)**

(a) $M = N$ (b) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$ (c) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ (d) $\frac{\partial^2 M}{\partial x^2}=\frac{\partial^2 N}{\partial y^2}$

04. The differential equation $\frac{d^4y}{dx^4}+P\frac{d^2y}{dx^2}+ky=0$ is **(GATE-94)**

(a) Linear of fourth order (b) Non-linear of fourth order

(c) Non-homogeneous (d) Linear and fourth degree

05. For the differential equation $\frac{dy}{dt}+5y=0$ with $y(0) = 1$, the general solution is **(GATE-94[ME])**

(a) e^{5t} (b) e^{-5t} (c) $5e^{-5t}$ (d) $e^{\sqrt{-5t}}$

06. Solve for y if $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with $y(0)=1$ and $y'(0) = -2$ **(GATE-94[PI])**

(a) $(1-t)e^{-t}$ (b) $(1+t)e^{t}$ (c) $(1+t)e^{t}$ (d) $(1-t)e^{t}$

07. $y=e^{-2x}$ is solution of the differential equation $y''+y'-2y=0$ **(GATE-94[EC])**

(a) TRUE (b) FALSE

(c) Cannot be determined (d) None

08. Match each of the items A,B,C with an appropriate item from 1,2,3,4 and 5 **(GATE-94[EC])**

(A) $a_1\frac{d^2y}{dx^2}+a_2x\frac{dy}{dx}+a_3x^2y=a_4$ (B) $a_1\frac{d^3y}{dx^3}+a_2y=a_3$

(C) $a_1\frac{d^2y}{dx^2}+a_2x\frac{dy}{dx}+a_3x^2y=0$ (D) None of these

(1) Non-linear differential equation

(2) Linear-differential equation with constant co-efficient

(3) Linear-homogeneous differential equation

(4) Non-linear homogeneous differential equation

(5) Non-linear first order differential equation

(a) A-1, B-2, C-3 (b) A-3, B-4, C-2 (c) A-2, B-5, C-3 (d) A-3, B-1, C-2

09. Solve for y if $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with y(0)=1 and y'(0) = +2 **(GATE-94[ME])**

(a) $(1-3t)e^{t}$ (b) $(1-3t)e^{-t}$ (c) $(1+3t)e^{t}$ (d) $(1+3t)e^{-t}$

10. If H(x,y) is homogeneous function of degree n then $x\frac{\partial H}{\partial x}+y\frac{\partial H}{\partial y}=nH$ **(GATE-94[ME])**

(a) TRUE (b) FALSE

(c) Cannot be determined (d) None

11. The differential equation $y''+(s^3\sin x)^5y'+y=\cos x^3$ is **(GATE-95)**

(a) Homogeneous (b) Non-Linear

(c) Second order linear (d) Non-homogeneous with constant co-efficient

12. The solution to the differential equation $f''(x)+4f'(x)+4f(x)=0$ **(GATE-95[ME])**

(a) $f_1(x)=e^{-2x}$ (b) $f_1(x)=e^{2x}, f_2(x)=e^{-2x}$

(c) $f_1(x)=e^{-2x}, f_2(x)=xe^{-2x}$ (d) $f_1(x)=e^{-2x}, f_2(x)=e^{-x}$

13. A differential equation of the form $\frac{dy}{dx}=f(x,y)$ is homogeneous if the function f(x,y)

Depends only on the ratio of $\frac{y}{x}(or)\frac{x}{y}$ **(GATE-95[ME])**

(a) TRUE (b) FALSE

(c) Cannot be determined (d) None

14. The solution of a differential equation $y''+3y'+2y=0$ is of the form **(GATE-95)**

(a) $c_1e^{x}+c_2e^{2x}$ (b) $c_1e^{-x}+c_2e^{3x}$ (c) $c_1e^{-x}+c_2e^{-2x}$ (d) $c_1e^{-2x}+c_2e^{-x}$

15. Solve $\frac{d^4v}{dx^4}+4\lambda^4 v=1+x+x^2$ **(GATE-96)**

(a) $v=e^{-\lambda x}\left(C_1\cos\lambda x+C_2\sin\lambda x\right)+\left(C_3\cos\lambda x+C_4\sin\lambda x\right)+\frac{\left(1+x+x^2\right)}{4\lambda 4}$ (c) $\left(1+x+x^2\right)4\lambda^4$

(b) $v=e^{-\lambda x}\left(C_1\cos\lambda x+C_2\cos\lambda x\right)$ (d) None

16. The particular solution for the differential equation $\frac{d^2y}{dt^2}+3\frac{dy}{dx}+2y=sx$

(GATE-96[ME])

(a) 0.5cos x +1.5 sin x (b) 1.5 cosx+0.5 sin x

(c) 1.5 sin x (d) 0.5 cos x

17. For the differential equation $f(x,y)\frac{dy}{dx}+g(x,y)=0$ to be exact is **(GATE-97[CE])**

(a) $\frac{\partial f}{\partial y}=\frac{\partial g}{\partial x}$ (b) $\frac{\partial f}{\partial x}=\frac{\partial g}{\partial y}$ (c) f=g (d) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial^2 g}{\partial y^2}$

18. The differential equation $\frac{dy}{dx}+Py=Q$, is a linear equation of first order only if,

(GATE-97[CE])

(a)P is a constant but Q is a function of y (b) P and Q are functions of y or constants

(c) P is a function of y but Q is a constant (d) P and Q are functions of y or constants

19. Solve $\frac{d^4y}{dx^4}-y=15\cos 2x$ **(GATE-98[CE])**

(a) $C_1e^x+C_2e^{-x}+C_3\text{Cos}x+C_4\sin x+\cos 2x$ (b) $\cos 2x$ (c) $C_1e^x+C_2e^{-x}$ (d) None

20. The general solution of the differential equation $x^2\frac{d^2y}{dx^2}-\frac{dy}{dx}+y=0$ is **(GATE-98)**

(a) $Ax+Bx^2$ (A and B are constants) (b) $Ax+B\log x$ (A and B are constants)

(c) $Ax+Bx^2\log x$ (A and B are constants) (d) $Ax+Bx\log x$ (A and B are constants)

21. The radial displacement in a rotating disc is governed by the differential equation $\frac{d^2u}{dx^2}+\frac{1}{x}\frac{du}{dx}-\frac{u}{x^2}=8x$ where u is the displacement and x is the radius. If u=0 at x=0 And u=2 at x=1, calculate the displacement at x=1/2 **(GATE-98)**

22. The equation $\frac{d^2y}{dx^2}+(x^2+4x)\frac{dy}{dx}+y=x^8-8$ is a **(GATE-99)**

(a) Partial differential equation (b) Non-linear differential equation

(c) Non-homogeneous differential equation (d) Ordinary differential equation

23. If c is a constant, then the solution of $\frac{dy}{dx}=1+y^2$ is **(GATE-99[CE])**

(a) $y=\sin(x+c)$ (b) $y=\cos(x+c)$ (c) $y=\tan(x+c)$ (d) $y=e^x+c$

24. Find the solution of the differential equation $\frac{d^2y}{dx^2}+\lambda^2 y=\cos(wt+k)$ with initial conditions $y(0)=0, \frac{dy(0)}{dt}=0$, Here λ,w and k are constants. Use either the method of undetermined Co-efficient or the operator (D=d/dt) based method. **(GATE-2000)**

25. The solution for the differential equation with boundary conditions y(0)=2 and $y'(1)=-3$ is where $\frac{d^2y}{dx^2}=3x-2$ **(GATE-01[CE])**

(a) $y=\frac{x^3}{3}-\frac{x^2}{2}\neq 3x-2$ (b) $y=3x^3-\frac{x^2}{2}-5x+2$

(c) $y=\frac{x^3}{3}-x^2-\frac{5x}{2}+2$ (d) $y=x^3-\frac{x^2}{2}+5x+\frac{3}{2}$

26. Solve the differential equation $\frac{d^2y}{dx^2}+y=x$ with the following conditions

(1) at x=0, y=1 (2) at x=0, y' =1 is **(GATE-2001)**

(a) x (b) cosx (c) x + cosx (d) None

27. The solution of the differential equation $\frac{dy}{dx}+y^2=0$ is **(GATE-03[ME])**

(a) $y=\frac{1}{x+c}$ (b) $y=\frac{-x^3}{3}+c$

(c) ce^x (d) Unsolvable as equation is non-linear

28. Bio transformation of an organic compound having concentration (x) can be modeled using an ordinary equation $\frac{dx}{dt}+kx^2=0$, where k is the reaction rate constant. If x=a at t=0 then solution of the equation is **(GATE-04[CE])**

(a) $x = ae^{-kt}$ (b) $\frac{1}{x} = \frac{1}{a} + kt$ (c) $x = a(1-e^{-kt})$ (d) $x = a + kt$

29. The differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = c^2\left[\frac{d^2y}{dx^2}\right]^2$ is of **(GATE-05[PI])**

(a) second order and third degree (b) third order and second degree

(c) second order and second degree (d) third order and third degree

30. The general solution of the differential equation $(D^2 - 4D + 4)y = 0$ is of the form (given $D = \frac{d}{dx}$ & C_1, C_2 are constants) **(GATE-05)**

(a) C_1e^{2x} (b) $C_1e^{2x} + C_2e^{-2x}$ (c) $C_1e^{2x} + C_2e^{2x}$ (d) $C_1e^{2x} + C_2xe^{2x}$

31. The solution of the first order differential equation $y(t) = -3x(t), x(0) = x_0$ is **(GATE-05[EE])**

(a) $x(t) = x_0e^{-3t}$ (b) $x(t) = x_0e^{-3}$ (c) $x(t) = x_0e^{-\frac{t}{3}}$ (d) $x(t) = x_0e^{-t}$

32. For the equation $\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 5$, the solution x(t) approaches the following values As $t \to \infty$ **(GATE-05[EE])**

(a) 0 (b) 5/2 (c) 5 (d) 10

33. Transformation to linear form by substituting $v = y^{1-n}$ of the equation $\frac{dy}{dt} + p(t)y = q(t)y^n, n > 0$ Will be **(GATE-05[CE])**

(a) $\frac{dv}{dt} + (1-n)pv = (1-n)q$ (b) $\frac{dv}{dt} + (1+n)pv = (1+n)q$

(c) $\frac{dv}{dt} + (1+n)pv = (1-n)q$ (d) $\frac{dv}{dt} + (1+n)pv = (1+n)q$

34. The solution $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 0; y(0) = 1, \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 0$ in the range $0 < x < \frac{\pi}{4}$ is given by **(GATE-05[EC])**

(a) $e^{-x}\left[\cos 4x + \frac{1}{4}\sin 4x\right]$ (b) $e^{x}\left[\cos 4x - \frac{1}{4}\sin 4x\right]$

(c) $e^{-4x}\left[\cos 4x - \frac{1}{4}\sin 4x\right]$ (d) $e^{-4x}\left[\cos 4x - \frac{1}{4}\sin 4x\right]$

35. If $x^2\left(\frac{dy}{dx}\right) + 2xy = \frac{2\ln x}{x}$ and y(1) = 0 then what is y(e) ? **(GATE-05[ME])**

(a) e (b) 1 (c) $\frac{1}{e}$ (d) $\frac{1}{e^2}$

36. The complete solution of the ordinary differential equation $\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy=0$ is $y=C_1e^{-x}+C_2e^{-3x}$ then p and q are **(GATE-05[ME])**

(a) p = 3, q = 3 (b) p = 3, q = 4 (c) p = 4, q = 3 (d) p = 4, q = 4

37. Which of the following is a solution of the differential equation $\frac{d^2y}{dx^2}+p\frac{dy}{dx}+(q+1)=0$? Where p = 4, q = 3 **(GATE-05[ME])**

(a) e^{-3x} (b) xe^{-x} (c) xe^{-2x} (d) x^2e^{-2x}

38. The following differential equation has $3\frac{d^2y}{dt^2}+4\left(\frac{dy}{dt}\right)^3+y^2+2=x$ **(GATE-05[EC])**

(a) degree = 2, order = 1 (b) degree = 1, order = 2
(c) degree = 4, order = 3 (d) degree = 2, order = 3

39. A solution of the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$ is given by **(GATE-05[EC])**

(a) $y=e^{2x}+e^{-3x}$ (b) $y=e^{2x}+e^{3x}$ (c) $y=e^{-2x}+e^{3x}$ (d) $y=e^{-2x}+e^{-3x}$

40. The solution of the differential equation $x^2\frac{dy}{dx}+2xy-x+1=0$ given that at x = 1, y = 0 is **(GATE-06[CE])**

(a) $\frac{1}{2}-\frac{1}{x}+\frac{1}{2x^2}$ (b) $\frac{1}{2}-\frac{1}{x}-\frac{1}{2x^2}$ (c) $\frac{1}{2}+\frac{1}{x}+\frac{1}{2x^2}$ (d) $-\frac{1}{2}+\frac{1}{x}+\frac{1}{2x^2}$

41. For initial value problem $\ddot{y}+2\dot{y}+(101)y=(10.4)e^x, y(0)=1.1$ and y(0) = -0.9. Various solutions are written in the following groups. Match the type of solution with the correct expression.

GROUP-I	**GROUP-II**
P. General solution of homogeneous equations	(1) $0.1e^x$
Q. Particular integral	(2) $e^{-x}[A\cos 10x+B\sin 10x]$
R. Total solution satisfying boundary conditions	(3) $e^{-x}\cos 10x+0.1e^x$

(GATE-06[IN])

(a) P-2, Q-1, R-3 (b) P-1, Q-3, R-2 (c) P-1, Q-2, R-3 (d) P-3, Q-2, R-1

42. For the differential equation $\frac{d^2y}{dx^2} + k^2 y = 0,$ the boundary conditions are y = 0 for x = 0 and y = 0 for x = a. the form of non-zero solution of y (where m varies over all integers) are **(GATE-06[EC])**

(a) $y = \sum_m A_m \sin\left(\frac{m\pi x}{a}\right)$ (b) $y = \sum_m A_m \cos\left(\frac{m\pi x}{a}\right)$

(c) $y = \sum_m A_m x^{\left(\frac{m\pi}{a}\right)}$ (d) $y = \sum_m A_m e^{-\left(\frac{m\pi x}{a}\right)}$

43. The solution of the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with y(0) = 1 is

(GATE-06[ME])

(a) $(1+x)e^{x^2}$ (b) $(1+x)e^{-x^2}$ (c) $(1-x)e^{x^2}$ (d) $(1-x)e^{-x^2}$

44. For $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x},$ the particular integral is **(GATE-06[ME])**

(a) $\frac{1}{15}e^{2x}$ (b) $\frac{1}{5}e^{2x}$ (c) $3e^{2x}$ (d) $C_1e^{-x} + C_2e^{-3x}$

45. The degree of the differential equation $\frac{d^2x}{dt^2} + 2x^3 = 0$ is **(GATE-07[CE])**

(a) 0 (b) 1 (c) 2 (d) 3

46. The solution for the differential equation $\frac{dy}{dx} = x^2 y$ with the condition that y = 1 at X = 0 is **(GATE-07[EC])**

(a) $y = e^{\frac{1}{2x}}$ (b) $\ln(y) = \frac{x^3}{3} + 4$ (c) $\ln(y) = \frac{x^2}{2}$ (d) $y = e^{\frac{x^3}{3}}$

47. The solution of $\frac{dy}{dx} = y^2$ with initial value y(0) = 1 bounded in the interval is

(GATE-07[ME])

(a) $-\infty \le x \le \infty$ (b) $-\infty \le x \le 1$ (c) $x < 1,\ x > 1$ (d) $-2 \le x \le 2$

48. The solution of the differential equation $k^2\frac{d^2y}{dx^2} = y - y_2$ under the boundary conditions (i) $y = y_1$ at $x = 0$ (ii) $y = y_2$ at $x = \infty$ where k, y1, y2 are constant is **(GATE-07[EC])**

(a) $y = (y_1 - y_2)e^{-x/k^2} + y_2$ (b) $y = (y_1 - y_2)e^{-x/k} + y_1$

(c) $y = (y_1 - y_2)\sinh\left(\frac{x}{k}\right) + y_1$ (d) $y = (y_1 - y_2)e^{-x/k} + y_2$

49. A body originally at 60^0 cools down to 40 in 15 minutes when kept in air at a temperature of 25^0 C. what will be the temperature of the body at the end of 30 minutes? **(GATE-07[CE])**

(a) $35.2^0 C$ (b) $31.5^0 C$ (c) $28.7^0 C$ (d) $15^0 C$

50. Consider the differential equation $\frac{dy}{dx} = 1 + y^2$. which one of the following can be particular solution of this differential equation? **(GATE-08[IN])**

(a) $y = \tan(x+3)$ (b) $y = \tan^{-1}(x+3)$ (c) $x = \tan(y+3)$ (d) $x = \tan^{-1}(y+3)$

51. Which of the following is a solution to the differential equation $\frac{d}{dt}x(t) + 3x(t) = 0, x(0) = 2$? **(GATE-08[EC])**

(a) $x(t) = 3e^{-t}$ (b) $x(t) = 2e^{-3t}$ (c) $x(t) = \frac{-3}{2}t^2$ (d) $x(t) = 3t^2$

52. Given that $\ddot{x} + 3x(t) = 0$ and x(0)=1, what is x(1)=_____________ **(GATE-08[ME])**

(a) -0.99 (b) -0.16 (c) 0.16 (d) 0.99

53. It is given that $y''+2y'+y = 0, y(0) = 0, y(1) = 0$ what is y (0.5)? **(GATE-08[ME])**

(a) 0 (b) 0.37 (c) 0.62 (d) 1.13

54. The solutions of the differential equation $\frac{d^{2y}}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ are **(GATE-08[PI])**

(a) $e^{-(1+i)x}, e^{-(1-i)x}$ (b) $e^{(1+i)x}, e^{(1-i)x}$ (c) $e^{-(1+i)x}, e^{(1+i)x}$ (d) $e^{(1+i)x}, e^{-(1+i)x}$

55. Match each differential equation in group I to its family of solution curves from Group II **(GATE-09[EC])**

GROUP-I	**GROUP-II**
P: $\frac{dy}{dx} = \frac{y}{x}$	1. Circles
Q: $\frac{dy}{dx} = \frac{-y}{x}$	2. Straight lines

R. $\frac{dy}{dx}=\frac{x}{y}$ 3. Hyperbolas

S. $\frac{dy}{dx}=\frac{-x}{y}$

(a) P-2,Q-3,R-3,S-1 (b) P-1,Q-3,R-2,S-1 (c) P-2,Q-1,R-3,S-3 (d) P-3,Q-2,R-1,S-2

56. Solution of the differential equation $3y\frac{dy}{dx}+2x=0$ represents a family of

(GATE-09[CE])

(a) ellipses (b) circles (c) parabolas (d) hyperbolas

57. The order of differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+y^4=e^{-1}$ is **(GATE-09[EC])**

(a) 1 (b) 2 (c) 3 (d) 4

58. The solution of $x\frac{dy}{dx}+y=x^4$ with condition $y(1)=6/5$ **(GATE-09[ME])**

(a) $y=\frac{x^4}{5}+\frac{1}{x}$ (b) $y=\frac{4x^4}{5}+\frac{4}{5x}$ (c) $y=\frac{x^4}{5}+1$ (d) $y=\frac{x^5}{5}+1$

59. The homogeneous part of the differential equation (p,q,r are constants) has real distinct roots if **(GATE-09[PI])**

(a) $p^2-4q>0$ (b) $p^2-4q<0$ (c) $p^2-4q=0$ (d) $p^2-4q=r$

60. The solution of the differential equation $\frac{d^2y}{dx^2}=0$ with boundary conditions

$\frac{dy}{dx}=1\ at\ x=0\ and\ x=1$ is_______ **(GATE-09[PI])**

(a) y = 1 (b) y = x

(c) y = x + c where c is an arbitrary constant

(d) $y=C_1x+C_2$ where C_1,C_2 are arbitrary constants

61. For the differential equation $\frac{d^2x}{dt^2}+6\frac{dx}{dt}+8x=0$ with initial conditions x(0)=1 and

$\left(\frac{dx}{dt}\right)_{t=0}$ The solution is _____ **(GATE-10[EE])**

(a) $2e^{-2}t+e^{-4t}$ (b) $2e^{-2t}+e^{-4t}$ (c) $e^{-4t}-2e^{-2t}$ (d) None

62. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x$ with the condition that y=1 at x=1 is

(GATE-11[CE])

(a) $x^2+\frac{2}{x}$ (b) $3\left(x^2+\frac{2}{x}\right)$ (c) $\frac{1}{3}\left(x^2+\frac{2}{x}\right)$ (d) None

63. The G.S of the D.E $\frac{dy}{dx}$-$y^2=1, y(0)=1$ **(GATE-10-PI)**

(a) $y=\tan\left(x+\frac{\pi}{3}\right)$ (b) $y=\tan\left(x-\frac{\pi}{3}\right)$ (c) $y=\tan\left(x-\frac{\pi}{4}\right)$ (d) $y=\tan\left(x+\frac{\pi}{4}\right)$

64. The Differential Equation of $y=\left(5\cos\frac{\pi}{3}\right)\sin 3x+\left(5\sin\frac{\pi}{3}\right)\cos 3x$ is **(GATE-10-PI)**

(a) $\frac{d^2y}{dx^2}-4y$ (b) $\frac{d^2y}{dx^2}+4y=0$ (c) $\frac{d^2y}{dx^2}+9y=0$ (d) $\frac{d^2y}{dx^2}-9y=0$

65. The order and Degree of the Differential Equation $\frac{d^3y}{dx^3}+4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}=0$ are

(GATE-10-CE)

(a) Order = 3, Degree =2 (b) Order = 2, Degree = 3

(c) Order = 2, Degree = 2 (d) None

66. The General Solution of the Differential Equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$ is **(GATE-10-CE)**

(a) $y=C_1 e^{-3x}+C_2e^{-2x}$ (b) $y=C_1e^{3x}+C_2 e^{2x}$

(c) $y = C_1e^{-3x}+C_2e^{2x}$ (d) $y=c_,e^{3x}+c^2 e^{-2x}$

67. The General Solution of the Differential Equation $\frac{dy}{dx}+y=e^x, y(0)=1$ is **(GATE-10-IN)**

(a) $y(1)=\frac{e}{2}-\frac{e-1}{2}$ (b) $y(1)=\frac{e}{2}+\frac{e^{-1}}{3}$

(c) $y(1)=\frac{e}{2}+\frac{e^{-1}}{2}$ (d) None of these

68. The solution of the D. E $\frac{dy}{dx}=e^{-3x}$ is **(GATE-11-EE)**

(a) $y=-\frac{e^{-3x}}{3}+K$ (b) $y=\frac{e^{-3x}}{3}+k$

(c) $y=\frac{e^{-3x}}{3}+K$ (d) $y=-\frac{e^{-3x}}{3}+K$

69. The Solution of the D. E $\frac{dy}{dx}=ky, y(0)=C$ is **(GATE-11-EC)**

(a) $y=Ce^{-kx}$ (b) $y=\frac{e^{-3x}}{3}+k$ (c) $y=ce^{kx}$ (d) None of these

70. The Solution of $y^{11}+2y^{1}+y=0,\ y(0)=1,\ y(1)=0$ is **(GATE-11-IN)**

(a) e^{-1} (b) $-e^{-1}$ (c) $-e^{-2}$ (d) e^{-2}

71. The Solution of $(D^2+6D+9)y=9x+6$ is **(GATE-11-PI)**

(a) $y(C_1x+C_2)e^{-3x}$ (b) $y(C_1x+C_2)e^{3x}+x$ (c) $y=(C_1x+C_2)e^{-3x}$ (d) None

72. The Solution of $\frac{dy}{dx}=(1+y^2)x$ is **(GATE-11-ME)**

(a) $y=\tan\left(\frac{x^2}{2}\right)+C$ (b) $y=\tan\left(\frac{x}{2}\right)+C$ (c) $y=\tan^2 C\frac{x^2}{2}+C$ (d) $y=\tan\left(\frac{x^2}{2}+c\right)$

73. The Solution of $\frac{dy}{dx}+\frac{y}{x}=x$ and $y=1$ at $x=1$ is **(GATE-11-CE)**

(a) $y=\frac{x}{3}+\frac{2}{3x}$ (b) $\frac{x^2}{3}+\frac{2}{x}$ (c) $y=\frac{x^2+2}{3}$ (d) $y=\frac{x^2}{3}+\frac{2}{3x}$

74. The Solution of $\frac{dx}{dt}+x=t, x(1)=0.5$ is **(GATE-12-EC/EE/IN)**

(a) $x(t)=\frac{t}{2}$ (b) $x(t)=t^2$ (c) $x(t)=\frac{t^2}{2}$ (d) None of these

75. The Solution of the D.E $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$ is **(GATE-13-CE)**

(a) $x=(a+bt)e^{t}$ (b) $(a+bt)e^{-t}$ (c) $(a+bt)e^{2t}$ (d) $(a+bt)e^{-2t}$

76. The Solution of $(D+2)^2y=0,\ y(0)=1$, and $y^1(0)=1$ is **(GATE-14-IN)**

(a) 0.341 (b) 0.441 (c) 0.541 (d) 0.641

77. The Solution of $\left(D^2+9\right)x=0, x\left(0\right)=1, \frac{dx}{dt}=1 \text{ at } 1=0$ **(GATE-14-CE)**

(a) cos3t (b) cos3t-sin3t (a)cos3t+$\frac{1}{3}\sin 3t$ (d) $\frac{1}{3}\cos 3t$+sin3t

78. The solution of D.E $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with $y(0)=y'(0)=1$ is **(GATE - EC -15)**

A) $(2-t)e^t$ B) $(1+2t)e^t$ C) $(2+t)e^{-t}$ D) $(1-2t)e^t$

79. General solution of D.E $\frac{dy}{dx}=\frac{1-\cos 2y}{1+\cos 2x}$ is **(GATE - EC -15)**

A) $\tan y-\cot x=c$ B) $\tan y+\cot x=c$ C) $\tan x-\cot y=c$ D) $\tan x+\cot y=c$

80. Consider the D.E $\frac{dx}{dt}=10-2x$ with initial condition x(0)=1. The response x(t) for t>0 is

(GATE - EC -15)

A) $2-e^{-0.2t}$ B) $2-e^{0.2t}$ C) $50-49e^{-0.2t}$ D) $50-49e^{0.2t}$

81. Consider the D.E $\frac{d^2x(t)}{dt^2}+3\frac{dx(t)}{dt}+2x(t)=0$ given $x(0)=20$, $x(1)=10/e$ where $e=2.71$, the value of x(2) is **(GATE - EE -15)**

82. The solution of D.E $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=0$ is such that y(0)=2 and $y(1)=\frac{1-3e}{e^3}$. The value of $\frac{d}{dt}$ (0) is **(GATE - EE -15)**

83. If $y=(x)$ satisfies the boundary value problem $y''+9y=0$, $y(0)=0$, $y(\pi/2)=\sqrt{2}$, then $y(\pi/4)$ is ________ **(GATE - ME -16)**

84. The ordinary differential equation $\frac{dx}{dt}=-3x+2$, with x(0)=1 is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is _______ ` **(GATE - EC -16)**

85. The particular solution of the initial value problem given below is **(GATE - EC -16)**

$\frac{d^2y}{dx^2}+12\frac{dy}{dx}+36y=0$ with y(0) = 3 and $\frac{dy}{dx}|_{x=0}=-36$

(A) $(3-18x)e^{-6x}$ (B) $(3+25x)e^{-6x}$ (C) $(3+20x)e^{-6x}$ (D) $(3-12x)e^{-6x}$

86. A function y(t), such that y(0) = 1 and y(1) = $3e^{-1}$, is a solution of the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$. Then y(2) is **(GATE - EE -16)**

a) $5e^{-1}$ b) $5e^{-2}$ c) $7e^{-1}$ d) $7e^{-2}$

87.The respective expressions for complimentary function and particular integral part of the solution of the differential equation **(GATE - EE -16)**

$$\frac{d^4y}{dx^4}+3\frac{d^2y}{dx^2}=108x^2\ are$$

(A) $\lfloor c_1+c_2x+c_3 sin\sqrt{3}x+c_4 cos\sqrt{3}x\rfloor and\ \lfloor 3x^4-12x^2+c\rfloor$

(B) $\lfloor c_2x+c_3 sin\sqrt{3}x+c_4 cos\sqrt{3}x\rfloor and\ \lfloor 5x^4-12x^2+c\rfloor$

(C) $\lfloor c_1+c_3 sin\sqrt{3}x+c_4 cos\sqrt{3}x\rfloor and\ \lfloor 3x^4-12x^2+c\rfloor$

(D) $\lfloor c_1+c_2x+c_3 sin\sqrt{3}x+c_4 cos\sqrt{3}x\rfloor and\ \lfloor 5x^4-12x^2+c\rfloor$

88. The type of partial differential equation **(GATE -CE -16)**

$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+3\frac{\partial^2 p}{\partial x\,\partial y}+2\frac{\partial p}{\partial x}-\frac{\partial p}{\partial y}=0$ is

(A) elliptic (B) parabolic (C) hyperbolic (D) none of these

89.The solution of the partial differential equation $\frac{\partial u}{\partial t}=\alpha\frac{\partial^2 u}{\partial x^2}$ *is of the form*

(GATE -CE -16)

(A) $C\cos(kt)\left[C_1e^{(\sqrt{k/\alpha})x}+C_2e^{(\sqrt{k/\alpha})x}\right]$

(B) $Ce^{kt}\left[C_1e^{(\sqrt{k/\alpha})x}+C_2e^{-(\sqrt{k/\alpha})x}\right]$

(C) $Ce^{kt}[C_1\cos(\sqrt{k/\alpha})x+C_2\sin(-\sqrt{k/\alpha})x]$

(D) $C\sin(kt)[C_1\cos(\sqrt{k/\alpha})x+C_2\sin-(\sqrt{k/\alpha})x]$

DIFFERENTIAL EQUATIONS SOLUTIONS

1. Sol: Given $y'' + y = 0$ ---- (1) and y (0) ------ (2), $y(\lambda) = 0$ ------ (3)

$\Rightarrow (D^2+1)y = 0$

$\Rightarrow D^2 + 1 = 0$

$\Rightarrow D = \pm i$

$\therefore$ Solution of (1) is given by

y $= C_1 \cos x + C_2 \sin x$ ---- (4)

using (2),(4) becomes

$0 = C_1 + 0$

$\therefore$ $C_1 = 0$

Using (3),(4) becomes

$0 = C_1 \cos\lambda + C_2 \sin\lambda$

$\Rightarrow C_2 \sin\lambda = 0$

$\Rightarrow \sin\lambda = 0$

$\Rightarrow \lambda = n\pi, n \in z$

$\therefore$ has non-trivia solution for $\lambda \neq n\pi, n \in z$ and it is given by $y = C_2 \sin\lambda$

2. Sol: Answer is (b)

Given equation is a non-linear differential equation.

3. Sol: Answer is C

Mdx+Ndy=0 is exact $\Leftrightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

4. Ans (a)

The given differential equation is a linear differential equation of fourth order

5. Answer is (b)

$\Rightarrow \frac{dy}{dt} + 5y = 0$ ------ (1) and $Y(0) = 1$ ----- (2)

$\Rightarrow \log y = -5t + C$

$\Rightarrow y = e^{-5t} + C$

$\Rightarrow y = e^{-5t}K$ ------- (3) where $K = e^c$

By (2), (3) becomes

$1 = e^0 K$

K=1 $\therefore$ the general solution is $y = e^{-5t}$

6. Sol: Answer is (a)

$(D^2 + 2D + 1)y = 0$ ------- (1) and y(0)=1 ------- (2)

$y'(0) = -2$ ----- (3)

$\Rightarrow D^2 + 2D + 1 = 0$

$\Rightarrow (D+1)^2 = 0$

$\Rightarrow$ D = -1, -1

Equal roots and real

Solution is $y = (C_1 + C_2 t)e^{-t}$ -------- (4)

& $\frac{dy}{dt} = (C_1 + C_2 t)(-e^{-t}) + C_2 e^{-t}$ -------- (5)

Using (2), (4) becomes

$1 = (C_1 + 0)e^0$

$\therefore C_1 = 1$

Using (3), (5)

$-2 = C_1 + C_2$

$\therefore C_2 = -1$

Hence the general solution is $y = (1-t)e^{-t}$

7. Sol: Answer is a

Given $y'' + y' - 2y = 0$

$\Rightarrow (D^2 + D - 2)y = 0$

$\Rightarrow D^2 + D - 2 = 0$

$\therefore y = e^{-2x}$ is a solution

8. Sol: Answer is (a)

9. Sol: Given $(D^2+2D+1)y=0$ ----- (1), y(0) = 1 ------- (2), $y'(0)=2$ ------ (3)

$\Rightarrow y=(C_1+C_2t)e^{-t}$ ------- (4)

$\Rightarrow y'=-(C_1+C_2t)e^{-t}+C_2e^{-t}$ ----- (5)

By (2), we have C_1=1

By (3), we have C_2=3

$\therefore y=(1+3t)e^{-t}$ is a solution of (1)

10. Sol: Answer is a

By Euler's theorem, option a is correct

11. Sol: Answer is c

The given differential equation is 2nd order linear, non-homogeneous differential equation

12. Sol: Answer is c

Given $f''(x)+4f'(x)+4f(x)=0$

$\Rightarrow (D^2+4D+4)f(x)=0$

$\Rightarrow D^2+4D+4=0$

$\Rightarrow (D+2)^2=0$

$\Rightarrow D=-2,-2$

$\therefore f(x)=e^{-2x}(C_1+C_2x)$

And $f_1(x)=e^{-2x}, f_2(x)=xe^{-2x}$

13. Sol: Answer is a

By the definition of homogeneous differential equation. Theoption a is correcct .

14. Sol: Answer is c

Given $y''+3y'+2y=0$

$\Rightarrow (D^2+3D+2)y=0$

$\Rightarrow D^2+3D+2=0$

$D=-1,-2$

$\therefore y=c_1e^{-x}+c_2e^{-2x}$

15. Sol: $D^4 v + 4\lambda^4 v = 1 + x + x^2$

$\Rightarrow \left(D^4 + 4\lambda^4\right) v = 1 + x + x^2$ ----- (1) (f(D)v=Q(x), where $f(D) = D^4 + 4\lambda^4, Q = 1 + x + x^2$)

Solution of (1) is

$\Rightarrow D^4 + 4\lambda^4 = 0$

$\Rightarrow (D^2 + 2\lambda^2)^2 - 4D^2\lambda^2 = 0$

$\Rightarrow (D^2 + 2\lambda^2 - 2D\lambda)(D^2 + 2\lambda^2 + 2D\lambda) = 0$

$\Rightarrow D = -\lambda \pm \lambda i, \lambda \pm \lambda i$

$\therefore v_C = e^{-\lambda x}[C_1 \cos \lambda x + C_2 \sin \lambda x] + e^{\lambda x}[C_3 \cos \lambda x + C_4 \sin \lambda x]$

$$v_p = P.I = \left(\frac{1}{f(D)}\right) Q(x)$$

$$= \left(\frac{1}{D^4 + 4\lambda^2}\right)(1 + x + x^2)$$

$$= \frac{1}{4\lambda^4}\left[1 + \frac{D^4}{4\lambda^4}\right]^{-1}(1 + x + x^2)$$

$$= \frac{1}{4\lambda^4}\left[1 - \frac{D^4}{4\lambda^4} + \frac{D^8}{16\lambda^8} - - - -\right](1 + x + x^2)$$

$$v_p = \frac{1}{4\lambda^4}(1 + x + x^2)(\Theta D^4(x^2) = 0, D^4(x^1) = 0, D^4(1) = 0)$$

Hence the complete solution of a given equation is

$v = v_c + v_p = C.F + P.I$

$$= e^{-\lambda x}[C_1 \cos \lambda x + C_2 \sin \lambda x] + e^{\lambda x}[C_3 \cos \lambda x + C_4 \sin \lambda x] + \frac{(1 + x + x^2)}{4\lambda^4}$$

Ans:- A

16. Sol: Answer is a

Given $(D^2 + 3D + 2)Y = 5\cos x$

$\Rightarrow f(D)Y = Q(x)\, where\, f(D) = D^2 + 3D + 2, Q(x) = 5\cos x$

$$P.I. = \left(\frac{1}{f(D)}\right)Q(x) = \frac{1}{D^2+3D+2}(5\cos x)$$

$$= \frac{1}{(3D+1)}\frac{(3D-1)}{(3D-1)}\left(5\cos x\right) = \frac{(3D-1)}{9D^2-1}5\cos x$$

$$P.I. = \frac{15(-\sin x)-5\cos x}{-10} = 1.5\sin x + 0.5\cos x$$

17. Sol: Answer is b

Given $f(x,y) = \frac{dy}{dx} + g(x,y) = 0$

$\Rightarrow f(x,y)dy + g(x,y)dx = 0$

$\Rightarrow g(x,y)dx + f(x,y)dy = 0$

This D.E. is exact iff $\frac{\partial g}{\partial y} = \frac{\partial f}{\partial x}$

18. Sol: Answer is d

According to general form of linear differential equation in y, the function P and Q must be either functions of x (or) constants.

19. Sol: The solution of given equation $(D^4-1)y = 15\cos 2x \, is \; y = y_c + y_p$

f(D)y=Q(x) where f(D) $= D^4 - 1$ and Q(x) = 15 cos2x

f(D)y=0

$\Rightarrow D^4 - 1$

$\Rightarrow (D^2-1)(D^2+1) = 0$

$\Rightarrow D = \pm 1, \pm i$

$y_c = C_1e^x + C_2e^{-x} + e^{0x}[C_3\cos x + C_4\sin x]$

$$y_p = \left[\frac{1}{f(D)}\right]Q(x) = \frac{1}{D^4-1}(15\cos 2x)$$

$$\Rightarrow y_p = \frac{1}{(D^2)^2-1}(15\cos 2x) = \frac{1}{(-4)^2-1}(15\cos 2x)$$

$\therefore y_p = \cos 2x$

Hence the solution is $y = y_c + y_p$

$y = C_1e^x + C_2e^{-x} + e^{0x}[C_3 \cos x + C_4 \sin x] + \cos 2x$

Ans: (a)

20. **Ans: (d)**

Given $(x^2D^2 - xD + 1)y = 0$

Put $x = e^t\ (or) \log x = t, xD = D, x^2D^2 = D(D-1)$

$[D(D-1) - D + 1]y = 0$

$\Rightarrow (D^2 - D - D + 1)y = 0$

$\Rightarrow (D^2 - 2D + 1)y = 0$

$\Rightarrow (D-1)^2 y = 0$

$\Rightarrow f(D)y = 0 \ where\ f(D) = (D-1)^2$

f(D) = 0

$\Rightarrow (D-1)^2 = 0$

$\Rightarrow$ D = 1, 1

$\therefore$ Solution is $y = (A + Bt)e^t = (A + B\log x)x$

$$y = Ax + Bx\log x$$

21. Sol: Similar to the above problem

22. Sol: Answer is d

This given differential equation is linear non-homogeneous ordinary differential equation

23. Sol: Answer is c

Given $\frac{dy}{dx} = 1 + y^2$

$\Rightarrow \int \frac{1}{1+y^2} dy = \int dx + c$

$\Rightarrow \tan^{-1}(y) = x + c$

$\therefore y = \tan(x + c)$

24. Sol: Given $\frac{d^2y}{dt^2} + \lambda^2 y = \cos(wt + k)$ ------ (1) and

$y(0) = 0$ ------ (2) and

$\frac{dy(0)}{dt} = 0$ ------ (3)

$(D^2 + \lambda^2)y = \cos(wt + k)$

$\Rightarrow f(D)y = Q(x)$ *when* $f(D) = D^2 + \lambda^2$ & $Q(x) = \cos(wt + k)$

$\Rightarrow f(D) = 0$

$\Rightarrow D^2 + \lambda^2 = 0$

$\Rightarrow D = \pm\lambda i$

$y_c = C_1 \cos(\lambda t) + C_2 \sin(\lambda t)$

$$y_p = \frac{1}{f(D)}Q(x) = \frac{1}{D^2 + \lambda^2}\cos(wt + k)$$

$$\therefore y_p = \frac{1}{-w^2 + \lambda^2}\cos(wt + k)$$

Hence the solution of (1) is given by $y = y_c + y_p$

$$y = C_1 \cos(\lambda t) + C_2 \sin(\lambda t) + \frac{1}{-w^2 + \lambda^2}\cos(wt + k) \quad \text{---- (4)}$$

$$\frac{dy}{dt} = -C_1\lambda \sin(\lambda t) + C_2\lambda \cos(\lambda t) + \frac{1}{-w^2 + \lambda^2}\sin(wt + k)$$

By using (2), (4) becomes

$$0 = C_1 + 0 + \frac{1}{-w^2 + \lambda^2}\cos(k)$$

$$\therefore C_1 = \frac{\cos k}{w^2 - \lambda^2}$$

By using (3), (5) becomes

$$0 = 0 + C_2\lambda + \frac{w\sin k}{w^2 - \lambda^2}$$

$$\therefore C_2 = \frac{w\sin k}{\lambda(\lambda^2 - w^2)}$$

Now the solution is $y = \frac{\cos k}{w^2 - \lambda^2}\cos(\lambda t) + \frac{w\sin k}{\lambda(\lambda^2 - w^2)}\sin(\lambda t) + \frac{\lambda}{(\lambda^2 - w^2)}\cos(wt + k)$

25. Sol: Answer is C

Given $\frac{d^2y}{dx} = 3x - 2$ -------- (1), &

$y(0) = 2 \rightarrow (2):$ $y'(1) = -3 \rightarrow (3)$

$(1) \Rightarrow \frac{dy}{dx} = 3x^2/2 - 2x + C_1$

$\Rightarrow y = \frac{x^3}{2} - x^2 + C_1 x + C_2$ -------- (4)

And $\frac{dy}{dx} = \frac{3x^2}{2} - 2x + C_1$ ----- (5)

Using (2), (4) becomes

$2 = 0 - 0 + 0 + C_2$

$\therefore C_2 = 2$

Again using (3), (5) becomes

$-3 = \frac{3}{2} - 2 + C,$

$C_1 = \frac{-5}{2}$

$\therefore$ The solution of a given differential equation is $y = \frac{x^3}{2} - x^2 + x\left(\frac{-5}{2}\right) + 2$

26. Given $\frac{d^2y}{dx^2} + y = x$ -------- (1)

X = 0, y =1 ------ (2)

X =0 , $y' = 1$ ----- (3)

$(D^2 + 1)y = x$

$\Rightarrow f(D)y = Q(x)$ *where* $f(D) = D^2 + 1 \,\&\, Q(x) = x$

Complementary function:

f(D) =0

$D^2 + 1 = 0$

$D = \pm i$

$\therefore y_c = C_1 \cos x + C_2 \sin x$

Particular integral,

$$y_p = \frac{1}{f(D)} Q(x) = \frac{1}{(D^2+1)} x = (1+D^2)^{-1} x = [1 - D^2 + D^4 ....] x$$

$y_p = x - 0 + 0 + ...$

$y_p = x$

$\therefore$ Solution is $y = y_c + y_p = C_1 \cos x + C_2 \sin x + x$ ------ (4)

By using (2), (4) becomes

$1 = C_1 + 0 + 0$

$\therefore C_1 = 1$

From (4), $\frac{dy}{dx} = -C \sin x + C_2 \cos x + 1$ ------- (5)

Using (3) , (5) becomes

$1 = -0 + C_2 + 1$

$C_2 = 0$

Hence the solution of (1) is $y = x + \cos x$

27. Sol: Answer is a

$$\frac{dy}{dx} + y^2 = 0$$

$$\Rightarrow \int \frac{dy}{y^2} = -\int dx + c$$

$$\Rightarrow -\frac{1}{y} = -x - c$$

$$\Rightarrow \frac{1}{y} = x + c$$

$$\Rightarrow y = \frac{1}{x+c}$$

Ans: (C)

28. Sol: Answer is b

Given $\frac{dx}{dt} = -kx^2$ ----- (1) and x =a, at k = 0 ---- (2)

$$\Rightarrow \int \frac{dx}{x^2} = -k\int dt - c$$

$$\Rightarrow -\frac{1}{x} = -kt - c$$

$$\Rightarrow \frac{1}{x} = kt + c$$

By (2), (3) becomes

$$\frac{1}{a} = 0 + c$$

$\therefore$ The solution of (1) is $\frac{1}{x} = kt + \frac{1}{a}$

29. Sol: Answer is c

Given $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = C^2\left[\frac{d^2y}{dx^2}\right]^2$

From this order is 2 and degree is 3

30. **Ans: (d)**

Sol: Answer is a

Given $(D^2 - 4D + 4)y = 0$

$$\Rightarrow f(D)y = 0 \ \ where \ \ f(D) = D^2 - 4D + 4$$

$F(D) = 0$

$$\Rightarrow D^2 - 4D + 4 = 0 \quad \Rightarrow (D-2)^2 = 0 \Rightarrow D = 2,2$$

$\therefore y_c = (C_1 + C_2x)e^{2x}$

31. Sol: Answer is a

Given $\frac{dx}{dt} = -3x$ ---- (1) and $x(0) = x_0$ ----- (2)

$$\Rightarrow \int \frac{dx}{dt} = -3\int dt + C$$

$$\Rightarrow \log x = -3t + C \Rightarrow x = e^{-3t} + C \Rightarrow x = e^{-3t} + C \Rightarrow x = e^{-3t}.e^c$$

$$\Rightarrow x = e^{-3t}k \text{ ---- (3)}$$

Using (2), (3) becomes $x_0 = e^{-3(0)}k$

$\therefore k = x_0$

Hence, the solution of (1) is $x = e^{-3t}.x_0$

32. Sol: Answer is B

Given $\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 5$

$$\Rightarrow (D^2 + 3D + 2)x = 5e^0 t$$

$$\Rightarrow f(D)y = Q(t) \quad where\, f(D) = D^2 + 3D + 2 \;\&\, Q(t) = 5$$

$C.F : f(D) = 0$

$$\Rightarrow D^2 + 3D + 2 = 0$$

$$\Rightarrow (D+1)(D+2) = 0$$

$$\Rightarrow D = -1, -2$$

$\therefore x_c = C_1 e^{-t} + C_2 e^{-2t}$

$$P.I : x_p = \frac{1}{f(D)}Q(t) = \frac{1}{D^2 + 3D + 2}(5e^{0.t}) = \frac{5}{2} \qquad (\because f(0) \neq 0)$$

Solution is $x = C_1 e^{-t} + C_2 e^{-2t} + \frac{5}{2}$

But, As $t \to \infty, x \to 5/2$

33. Sol: Answer is A

$$\frac{dy}{dt} + p(t)y = q(t)y^n; n > 0 \text{ ---- (1)}$$

Put $y^{1-n} = V$ ---- (2)

From (2), we have

$$\frac{dv}{dt} = (1-n)y^{1-n-1}\frac{dy}{dt}$$

$$\frac{1}{(1-n)}\frac{dv}{dt} = y^{-n}\frac{dy}{dt} \text{ -----(3)}$$

Using (3) and (2), (1) becomes

$$\frac{1}{(1-n)}\frac{dv}{dt}+P(t)v=q(t)$$

$$\frac{dv}{dt}+P(t)V(1-n)=q(t)(1-n)$$

34. Sol: Answer is a

Given $(D^2+2D+17)y=0$ ---- (1) and y(0) = 1 ---- (2), $\frac{dy}{dx}=0$ ----- (3)

$$\Rightarrow f(D)y=Q(x) \qquad where\, f(D)=D^2+2D+17 \;and\; Q(x)=0$$

f(D) = 0

$$\Rightarrow D^2+2D+17=0$$

$$\Rightarrow D=-1\pm 4i$$

$\therefore$ Solution is $y=(C_1\cos 4x+C_2\sin 4x)e^{-x}$ ------- (4)

$$\frac{dy}{dx}=-e^{-x}[C_1\cos 4x+C_2\sin 4x]+e^{-x}[-4C_1\sin 4x+C_2 4\cos 4x] \text{ --- (5)}$$

By using (2), (4) becomes

$1=C_1$

By using (3), (5) becomes

$$0=-e^{-\pi/4}[-C_1+0]+e^{-\pi/4}[0-C_2 4]$$

$$0=-e^{-\pi/4}[-1]+-e^{-\pi/4}(-C_2 4)$$

$$0=-e^{-\pi/4}[1-C_2 4]$$

$$1-C_2 4=0$$

$\therefore$ The solution is $y=e^{-x}\left[\cos 4x+\frac{1}{4}\sin 4x\right]$

35. Sol: Answer is D

Given $x^2\left(\frac{dy}{dx}\right)+2xy=\frac{2\log x}{y}$ ---- (1) and y(1) = 0 ---- (2)

$$\Rightarrow \frac{dy}{dx}+\left(\frac{2}{x}\right)y=\frac{2\log x}{x^3} \qquad \left(\Theta \frac{dy}{dx}P(x)y=Q(x)\right)$$

$$I.F=e^{\int Pdx}=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$$

Solution is given by y.(I.F) = Q(x) dx + C

$$y.x^2 = \int x^2 \frac{2\log x}{x^3} dx + C$$

$$y.x^2 = 2\frac{(\log x)^2}{2} + C$$

$yx^2 = (\log x)^2$ +C----- (3)

Using (2), (3) becomes

0 = log 1 + C

$\therefore C = 0 \ and \ yx^2 = (\log x)^2$

Hence $y(e) = \frac{(\log_e e)^2}{e^2} = \frac{1}{e^2}$

36. Sol: Answer is C

Given $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + 2y = 0$ --- (1) and its solution is

$y = C_1e^{-x} + C_2e^{-3x}$ ---- (2)

From (2), the roots of f(D) = 0 are -1 and -3

$\therefore (D+1)(D+3) = 0$

$\Rightarrow D^2 + 4D + 3 = 0$ ---- (3)

Comparing (1) and (3), we have

P = 4 and q = 3

37. Sol: Answer is C

Given $\left[D^2 + 4D + (3+1)\right]y = 0$

$\Rightarrow \left(D^2 + 4D + 4\right)y = 0$

$\Rightarrow (D+2)^2 = 0$

$D = -2, -2 \ \therefore y = (C_1 + C_2x)e^{-2x} = C_1e^{-2x} + C_1xe^{-2x}$

Hence $e^{-2x} \ and \ xe^{-2x}$ are independent solutions.

38. Sol: Answer is b

The order and degree of a differential equation are 2 and 1.

39. Sol: Answer is b

Given $(D^2-5D+6)y=0$

$\Rightarrow D^2-5D+6=0$

$\Rightarrow D=2,3$

$\therefore y=c_1e^{2x}+c^2e^{3x}\ (or)\ y=e^{2x}+e^{3x}$

40. Sol: Answer is a

Given $(2xy-x+1)dx+x^2dy=0\ \&\ x=1, y=0$

$M_y=2x=N_x$

$\therefore$ Solution of a given equation is

$$\int(2xy-x+1)dx+\int x^2dy=C$$

$$x^2y-\frac{x^2}{2}+x=C \text{ ---- (3)}$$

Using (2), (3) becomes

$$C=\frac{1}{2}$$

$$\therefore y=\frac{1}{2x^2}+\frac{1}{2}-\frac{1}{x}$$

41. Sol: Answer is a

Given $(D^2+2D+101)y=10.4e^x$

C.F. is $y_c=e^{-x}[c_1\cos 10x+c_2\sin 10x]$

$$y_p=\frac{1}{(D^2+2D+101)}(10.4)e^x$$

$$y_p=\frac{1}{(1+2+101)}(10.4)e^x=\frac{(10.4)e^x}{104}=\frac{104}{10\times104}e^x=0.1e^x$$

$\therefore y=y_c+y_p$

$$y=e^{-x}[c_1\cos 10x+c_2\sin 10x]+0.1e^x$$

$$y'=-e^{-x}[c_1\cos 10x+c_2\sin 10x]+0.1e^x+e^{-x}[-c_1 10\sin 10x+c_2 10\cos 10x]$$

But y(0) = 1.1 and $y'(0)=-0.9$

$$1.1=c_1+0.1$$

$c_1 = 1.1 - 0.1 = 1$

-0.9=-[1+0]+0.1+[10c_2]

10c_2=-0.9+1-0.1=0

c_2=0

$\therefore y = e^{-x}[\cos(10)x] + (0.1)e^x$

Hence P-2, Q-1, R-3

42. Sol: Answer is a

Given $(D^2 + k^2)y = 0$

$\Rightarrow (D^2 + k^2) = 0$

$D = \pm ki$

$\therefore y = c_1 \cos(kx) + c_2 \sin(kx)$

Also given y =0 for x =0

And y =0 for x = a

$0 = c_1$

$\therefore y = c_2(\sin kx)$

$0 = c_2 \sin(ka)$

For non –trivial solution, we have

$c_2 \neq 0, \sin(ka) = 0$

$ka = n\pi, n \in z$

$k = \frac{n\pi}{a}, n \in z$

$\therefore y = c_2 \sin \frac{n\pi x}{a}$

i.e. $y = \sum A_m \sin\left(\frac{m\pi x}{a}\right)$

43. Sol: Answer is b

Given $\frac{dy}{dx} + 2xy = e^{-x^2}$ ---- (1)

And y(0) = 1 ---- (2)

$$I.F = e^{\int 2x dx} = e^{x^2}$$

$$y.e^{x^2} = \int e^{x^2} e^{-x^2} dx + C$$

$$y.e^{x^2} = x + c$$

$$1 = 0 + c$$

C=1

$$\therefore y = xe^{-x^2} + e^{-x^2} = (x+1)e^{-x^2}$$

44. Sol: Answer is b

Given $(D^2 + 4D + 3)y = 3e^{2x}$

$$D^2 + 4D + 3 = 0$$

$$D = -1, -3$$

$$y_c = c_1 e^{-x} + c_2 e^{-3x}$$

$$y_p = \frac{3e^{2x}}{D^2 + 4D + 3} = \frac{3e^{2x}}{4+8+3} = \frac{e^{2x}}{5}$$

$$\therefore y_p = \frac{e^{2x}}{5}$$

45. Sol: Answer is (b)

By the definition degree is 1

46. Sol: Answer is (d)

Given $\frac{dy}{dx} = x^2 y$ --- (1) and y =1 at x = 0 --- (2)

$$\Rightarrow \frac{dy}{y} = x^2 dx$$

$$\Rightarrow \log y = \frac{x^3}{3} + c$$

$$\Rightarrow y = e^{x^3/3} + e^c$$

$$y = e^{x^3/3} k \text{ --- (3)}$$

By (2), (3) becomes

$$1 = e^0 k$$

$\therefore k = 1$

Hence $y = e^{x^3/3}$ is a solution of (1)

47. Sol: Answer is (c)

Given $\frac{dy}{dx} = y^2$ --- (1) and y(0) = 1 ---- (2)

$$\Rightarrow \int \frac{1}{y^2} = \int dx + c$$

$$\Rightarrow -\frac{1}{y} = x + c \text{ ---- (3)}$$

Using (2), (3) becomes

$$\frac{-1}{1} = 0 + c$$

$$\Rightarrow c = -1$$

$\therefore$ Solution is $\frac{-1}{y} = x - 1 \ (or)\ y = \frac{1}{1-x}$

$y = \frac{1}{1-x}$ is not defined at x = 1

$\therefore y = \frac{1}{1-x}$ is bounded in the interval $x < 1, x > 1$

48. Sol: Answer is D

Given $k^2 \frac{d^2y}{dx^2} = y - y_2$ ---- (1)

And (i) $y = y_1 \ at\ x = 0$ ----- (2)

(ii) $y = y_2 \ at\ x = \infty$ ----- (3)

$$(1) \Rightarrow k^2 \frac{d^2y}{dx^2} - y = -y_2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{k^2} = \frac{-y_2}{k^2}$$

$$\Rightarrow \left(D^2 - \frac{1}{k^2}\right) y = \frac{-y_2}{k^2}$$

F(D)= 0, where $f(D) = D^2 - \frac{1}{k^2}$

$$\Rightarrow D^2 - \frac{1}{k^2} = 0$$

$$\Rightarrow (D + \frac{1}{k})(D - \frac{1}{k}) = 0$$

$$\Rightarrow D = \frac{-1}{k}, \frac{1}{k}$$

$$y_c = C_1 e^{\frac{-x}{k}} + C_2 e^{\frac{x}{k}}$$

$$y_p = \frac{1}{f(d)} Q(x) \quad Q(x) = \frac{-y_2}{k^2}$$

$$= \frac{1}{\left(D^2 - \frac{1}{k^2}\right)} \left(\frac{-y_2}{k^2}\right) C^{0.x} = \frac{K^2}{1} \frac{y_2}{k^2}$$

$$y_p = y_2$$

$\therefore solution\, is\; y = C_1 e^{\frac{-x}{k}} + C_2 e^{\frac{x}{k}} + y_2$ ------ (4)

Using (2), (4) becomes

$$y_1 = c_1 + c_2 + y_2$$

$\Rightarrow c_1 + c_2 = y_1 - y_2$ ---- (5)

Again using (3), (4) becomes

$$y_2 = c_1(0) + c_2(\infty) + y_2$$

$$c_2(\infty) = 0$$

$$c_2 = \frac{0}{\infty} = 0 \times 0 = 0$$

$$c_1 = y_1 - y_2$$

$$\therefore y = (y_1 - y_2) e^{\frac{x}{k}} + y_2$$

49. Sol: Answer is (b)

By Newton's law of cooling, we have

$\frac{dT}{dt} = -k(T - T_0)$ where $T_0 \rightarrow$ Temperature of air

$\Rightarrow T = T_0 + e^{-kt}c \rightarrow (1)$ T – Temperature of body

T = Time

Given, At t = 0, T = $60^0 \rightarrow$ (2)

At t = 15, T = $40^0 \rightarrow (3)$

At t =30, T =?

And also, given $T_0 = 25$

Using (2), (1) becomes

$60 = 25 + e^0 c$

$\Rightarrow c = 35$

$\therefore T = T_0 + e^{-kt} 35$ --- (4)

Using (3), (4) becomes

$40 = 25 + e^{-k/5} 35$

$-k = \frac{1}{15}\log\left(3/7\right)$

$\therefore T = T_0 + e^{\frac{t}{15}\log(3/7)} 35$

$T = 25 + e^{\frac{30}{15}\log(3/7)} 35$

$T = 25 + e^{\log(3/7)^2} 35$

$T = 25 + \frac{9}{49} \times 35$

$T = \frac{220}{7} = 31.42^0 C$ t = 30

50. Sol: Answer is a

Given $\frac{dy}{dx} = 1 + y^2$ --- (1)

$\Rightarrow \frac{dy}{1+y^2} = dx$

$\Rightarrow \tan^{-1}(y) = x + c$

$\Rightarrow y = \tan(x+c)$

For c =3, $y = \tan(x+c)$ is a particular solution of (1)

51. Sol: Answer is b

Given $\frac{d}{dt}x(t) + 3x(t) = 0 \rightarrow$ (1) and $x(0) = 2 \rightarrow$ (2)

$\Rightarrow \frac{dx}{x} = -3dt$

$\Rightarrow \log x = -3k + c$

$x = e^{-3t}k \rightarrow (3)$

Using (2), (3) becomes

$2 = e^{0}k$

$\therefore k = 2$

Hence $x(t) = 2e^{-3t}$ is a solution of (1)

52. **Ans: (d)**

Sol.

Given $\frac{d^2x}{dt^2} + 3x = 0 \rightarrow (1)$ and $x(0) = 1 \rightarrow$ (2)

$\Rightarrow (D^2 + 3)x = 0$ $\quad\quad \frac{dx}{dt}(0) = 1 \rightarrow (3)$

$\Rightarrow f(D)x = 0, f(D) = D^2 + 3$

Now f(D) = 0

$\Rightarrow D^2 + (\sqrt{3}) = 0$

$\Rightarrow D = \pm i\sqrt{3}$

$x = C_1\cos(\sqrt{3}t) + C_2\sin(\sqrt{3}t) \rightarrow (4)$

And $x = -\sqrt{3}C_1\sin(\sqrt{3}t) + \sqrt{3}C_2\cos(\sqrt{3}t) \rightarrow (5)$

By (2), (4) becomes

$1 = C_1$

By (3), (5) becomes

$1 = -\sqrt{3}C_1(0) + \sqrt{3}C_2$

$C_2 = 1/\sqrt{3}$

$\therefore x = \cos(\sqrt{3}t) + \frac{1}{\sqrt{3}}\sin(\sqrt{3}t)$

Hence $x(1) = \cos\sqrt{3} + \frac{1}{\sqrt{3}}\sin\sqrt{3} =$

Ans: (d)

53. Sol: Answer is a

Given $(D^2 + 2D + 1)y = 0 \rightarrow (1)$

And y(0)=0 $\rightarrow$ (2)

Y(1) = 0 $\rightarrow$ (3)

$f(D)y = 0 \ where \ f(D) = D^2 + 2D + 1$

$f(D) = 0$

$\Rightarrow D^2 + 2D + 1 = 0 \ \Rightarrow (D+1)^2 = 0 \ \Rightarrow D = -1, -1$

$y = (C_1 + C_2 x)e^{-x}$ --- (4)

By using (2), (4) becomes

$0 = (C_1 + 0)$

$C_1 = 0$

By using (3), (4) becomes

$0 = (C_1 + C_2)e^{-1}$

$C_1 + C_2 = 0$

$C_2 = -C_1 = 0$

$\therefore y = 0 \ and \ y(0.5) = 0$

54. Sol: Answer is (a)

Given $(D^2 + 2D + 2)y = 0$

$\Rightarrow f(D)y = 0$

Now f(D) = 0

$\Rightarrow D^2 + 2D + 2 = 0$

$$\Rightarrow D = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm i2}{2} = -1 \pm i = -(1-i), -(1+i)$$

$\therefore y = (C_1 \cos x + C_2 \sin x)e^{-x}$ (or) $y = C_1 e^{-(1-i)x} + C_2 e^{-(1+i)x}$

Here $e^{-(1-i)x}$ *and* $e^{-(1+i)x}$ are independent solution.

55. Sol: Answer is (a)

P: $\frac{dy}{dx} = \frac{y}{x} \quad \Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x} + \log c$

$\Rightarrow \log y = \log x + \log c$

$\Rightarrow y = x_c \rightarrow$ straight lines

Q: $\frac{dy}{dx} = \frac{-y}{x} \quad \Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{x} + \log c$

$\Rightarrow \log y = -\log x + \log c$

$\Rightarrow y = \frac{c}{x} \rightarrow$ Hyperbola

R: $\frac{dy}{dx} = \frac{x}{y} \quad \Rightarrow \int y dy = \int x dx$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

$\Rightarrow \frac{x^2}{2} - \frac{y^2}{2} = k \rightarrow$ Hyperbola

S: $\frac{dy}{dx} = -\frac{x}{y} \Rightarrow \int y dy = -\int x dx$

$\Rightarrow \frac{y^2}{2} = -\frac{x^2}{2} + c$

$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = c \rightarrow$ Circle

56. Sol: Answer is a (a)

Given $\frac{dy}{dx} 3y + 2x = 0$

$$\Rightarrow 3y dy + 2x dx = 0 \Rightarrow \frac{3y^2}{2} + \frac{2x^2}{2} = C \Rightarrow \frac{x^2}{1} + \frac{y^2}{\left(3/2\right)} = C$$

The above equation represents a family of ellipse

57. Sol: Answer is (b)

By the definition of order of a differential equation, the order of a given equation is two

58. Sol: Answer is a

Given $\frac{dy}{dx}+\frac{1}{x}y=x^3 \rightarrow (1\ )$ and $y(1)=6/5 \rightarrow (2)$

$$I.F = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$$xy = \frac{x^5}{5} + c \rightarrow (3)$$

Using (2), (3) becomes

$$\frac{6}{5} = \frac{1}{5} + c \quad \Rightarrow c = 1$$

$\therefore$ Solution is $y = \frac{x^4}{5} + \frac{1}{x}$

59. Sol: Answer is a (a)

Given $\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = r$

$$\Rightarrow (D^2 + pD + q)y = r$$

Now f(D) = 0

$$\Rightarrow D^2 + pD + q = 0$$

$$D = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

If $p^2 - 4q > 0$ then the roots of f(D) = 0 are real and different

60. Sol: Answer is (c)

Given $\frac{d^2y}{dx^2} = 0 \rightarrow (1)$ and $\frac{dy}{dx} = 1\, at\, x = 0 \rightarrow (2)$

$\frac{dy}{dx} = 1\, at\, x = 1 \rightarrow (3)$ and $\frac{dy}{dx} = C \rightarrow (4)$

$\Rightarrow y = Cx + k \rightarrow (5)$ Where c,k are arbitary constants

Using (3) and (1), (2), (4) becomes

$\therefore y = x + k$ Where k is arbitary constant

61. Sol: Answer is (b)

Given $(D^2 + 6D + 8)y = 0$ --- (1)

And x(0) = 1 -------- (2)

$\left(\frac{dx}{dt}\right)_{t=0} = 0$ ------- (3)

f(D)=0

$\Rightarrow D^2 + 6D + 8 = 0$

$\Rightarrow D = -2, -4$

$x = C_1 e^{-2t} + C_2 e^{-4t}$ ---- (4)

$\frac{dx}{dt} = -2C_1 e^{-2t} - 4C_2 e^{-4t}$ --- (5)

Using (2), (4) becomes

$1 = C_1 + C_2$ ----- (6)

Using (3), (5) becomes

$0 = -2C_1 - 4C_2$ ---- (7)

Solving (6) & (7), we get

$(C_1 = 2), (C_2 = -1)$

$\therefore$ Solution is $x(t) = 2e^{-2t} - e^{-4t}$

62. Sol: Answer is (a)

$\frac{dy}{dx} + \frac{y}{x} = x$

$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = Q(x)$

$\therefore P(x) = \frac{1}{x}, Q(x)$

I.F = $e^{\int pdx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

The Required General Solution is $ye^{\int pdx} = \int Qe^{\int pdx}dx + c$

$$\Rightarrow y.x = \int x \times x dx + c = \int x^2 dx + c = \frac{x^3}{3} + c$$

$$\Rightarrow y = \frac{x^2}{3} + \frac{c}{x} \text{..................................(1)}$$

$$y(1) = 1 : (1) \Rightarrow = \frac{1}{3} + C \Rightarrow C = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore y = \frac{x^2}{3} + \frac{2}{3x} = \frac{1}{3}\left(x^2 + \frac{2}{x}\right)$$

Ans: (c)

63. Sol: Answer is (D)

Given $\frac{dy}{dx} - y^2 = 1 \quad \Rightarrow \frac{dy}{dx} = 1 + y^2$ ------ (1)

And y(0) = 1 ---- (2)

$$(1) \Rightarrow \int \frac{dy}{1+y^2} = \int dx + c$$

$$\Rightarrow \tan^{-1}(y) = x + c$$

Using (2), (1) becomes

i.e, $\tan^{-1}(1) = 0 + C \Rightarrow \pi/4 = C$

$\therefore$ Solution is $y = \tan\left(x + \frac{\pi}{4}\right)$

64. Sol: Answer (C)

$$y = \left(5\cos\frac{\pi}{3}\right)\sin 3x + \left(5\sin\frac{\pi}{3}\right)\cos 3x$$

$$y = C_1 \sin 3x + C_2 \cos 3x \quad where\, C_1 = 5\cos\frac{\pi}{3}, C_2 = 5\sin\frac{\pi}{3}$$

$$\frac{dy}{dx} = 3C_1 \cos 3x - 3C_2 \sin 3x$$

$$\frac{d^2y}{dx^2} = -9C_1 \sin 3x - 9C_2 \cos 3x$$

$$\frac{d^2y}{dx^2} = -9\left[C_1 \sin 3x + C_2 \cos 3x\right]$$

$$\frac{d^2y}{dx^2} = -9y$$

$$\frac{d^2y}{dx^2} + 9y = 0$$

65. Sol: Given $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0 \Rightarrow \frac{d^3y}{dx^3} = -4\left(\left(\frac{dy}{dx}\right)^3 + y^2\right)^{\frac{1}{2}}$

$$\Rightarrow \frac{d^3y}{dx^3} = 16\left(\frac{dy}{dx}\right)^3 + 16y^2$$

$\therefore$ Order = 3 and degree = 2

66. Sol: Answer is (c)

Given $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

$\Rightarrow (D^2 + D - 6)y = 0$

$\Rightarrow f(D) = y$

The auxiliary equation is f(D) = 0

$\Rightarrow D^2 + D - 6 = 0$

$\Rightarrow D = 2, -3$

$\therefore y = C_1e^{2x} + C_2e^{-3x} \quad (or) \quad y = C_1e^{-3x} + C_2e^{2x}$

67. Sol: Answer is (c)

$\frac{dy}{dx} + y = e^x$ --- (1) and y(0) = 1 ---- (2)

$I.F = \int e^{\int dx} = e^x$

$y(I.F) = \int (I.F)e^x dx + C$

$y.e^x = \int e^x e^x dx + C$

$ye^x = \frac{e^{2x}}{2} + C$ ---- (3)

Using (2), (3) becomes

$1 = \frac{1}{2} + C$

$\therefore C = \frac{1}{2} \; and \; ye^{x} = \frac{e^{2x}}{2} + \frac{1}{2}$

$\therefore y(1) = \frac{e}{2} + \frac{e^{-1}}{2}$

68. Ans (a)

Given $\frac{dy}{dx} = e^{-3x}$ ------ (1)

$$\Rightarrow \int dy = \int e^{-3x} + K \Rightarrow y = \frac{e^{-3x}}{-3} + K$$

69. Ans: (c)

Sol: Given $\frac{dy}{dx} = Ky$ --- (1)

And y(0) = c --- (2)

$$\Rightarrow \int \frac{1}{y} dy = K \int dx + c_1$$

$$\Rightarrow \log y = Kx + c_1 \;\; \Rightarrow y = e^{Kx + c_1}$$

$$\Rightarrow y = e^{Kx} + c_2 \text{ ---- (3)}$$

Where $c_2 = e^{c_1}$

Using (2), (3) becomes

$c = c_2$

$\therefore y = e^{kx} c$

70. Ans: (c)

Sol: Given $y'' + 2y' + y = 0$ ---- (1) and

y(0) = 1 ---- (2), y(1) = 0 ---- (3)

(1) $\Rightarrow (D^2 + 2D + 1) y = 0$

Now f(D)=0 where f(D) = $D^2 + 2D + 1$=0

$$\Rightarrow D^2 + 2D + 1$$

$$\Rightarrow D = -1, -1$$

$\therefore y = (C_1 + C_2 x)e^{-x}$ ---- (4)

Using (2), (4) becomes

$1 = C_1$

Using (3), (4) gives $0 = (1 + C_2)e^{-1}$

$\therefore C_2 = -1$

Hence the solution is $y = (1-x)e^{-x}$

$\therefore y(2) = (1-2)e^{-2} = -e^{-2}$

71. Ans: (c)

Sol: Given $(D^2 + 6D + 9)y = 9x + 6$

Solution is $y = y_c + y_p$

$$y_c : D^2 + 6D + 9 = 0$$

$$\Rightarrow D = -3, -3$$

$\therefore y_c = (C_1 x + C_2)e^{-3x}$

$$y_p = \frac{1}{f(D)}(Q(x)) = \frac{1}{(D+3)^2}(9x+6) = \frac{1}{9}\left[1 + \frac{D}{3}\right]^2 (9x+6)$$

$$= \frac{1}{9}\left[1 - 2\frac{D}{3} + 3\left(\frac{D}{3}\right)^2 + - - \right](9x+6)$$

$$= \frac{1}{9}(9x+6) - \frac{2}{3}.\frac{1}{9}(9) = x + \frac{2}{3} - \frac{2}{3}$$

$y_p = x$

$\therefore y = (C_1 x + C_2)e^{-3x} + x$

72. Ans: (d)

Sol: Given $\frac{dy}{dx} = (1 + y^2)x$ ---- (1)

$$\Rightarrow \int \frac{dy}{1+y^2} = \int x dx + C$$

$$\Rightarrow \tan^{-1}(y) = \frac{x^2}{2} + C$$

$$\therefore y = \tan\left(\frac{x^2}{2} + C\right)$$

73. Ans: (d)

Sol: Given $\frac{dy}{dx} + \frac{y}{x} = x$ ---- (1) and

Y=1 at x =1 ---- (2)

$$I.F = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

The general solution of (1) is

$$xy = \frac{x^3}{3} + C \text{ ---- (3)}$$

Using (2), (3) becomes

$$1 = \frac{1}{3} + C \Rightarrow C = 2/3$$

$$\therefore y = \frac{x^2}{3} + \frac{2}{3x}$$

74. Ans: ©

Sol: $t\frac{dx}{dt} + x = t \Rightarrow \frac{dx}{dt} + \frac{1}{t}x = 1$

$$\Rightarrow I.F = e^{\int \frac{1}{t}dt} = e^{\log t} = t$$

$$\therefore Sol.\ is\ xt = \int t\,dt + c \Rightarrow xt = \frac{t^2}{2} + c$$

Given x(1) = 0.5 $\Rightarrow 0.5 = \frac{1}{2} + c \Rightarrow c = 0$

$$\therefore xt = \frac{t^2}{2}$$

75. Ans: (b)

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$$

i.e., $(D^2+2D+1)x=0$

$(D+1)^2x=0$

AE has roots -1, -1

$\therefore$ General solution is $x=(a+bt)e^{-t}$

76. Ans: (c)

Sol: Given that $(D+2)^2y=0$

A.E has roots -2, -2

$\therefore$ solution is $y=(C_1+C_2x)e^{-2x}$ --- (1)

Given that y(0) = 1

$1=C_1$ ----- (2)

$$\frac{dy}{dx}=-2C_1e^{-2x}+C_2(-2xe^{-2x}+e^{-2x})$$

Apply $y'(0)=1$

$1=-2+C_2(0+1)$

$\Rightarrow C_2=3$ --- (3)

Using (2) & (3) in (1)

$y=(1+3x)e^{-2x}$

$\therefore$ At $x=1\Rightarrow y=4e^{-2}=0.541$

77. Ans: (c)

Sol: Given that $(D^2+9)x=0$

A.E has roots $\pm 3i$

$\therefore$ Solution is $x=C_1\cos 3t+C_2\sin 3t$ ---(1)

X(0) gives $C_1=1$

$$\frac{dx}{dt}=(-3C_1\sin 3t+3C_2\cos 3t)$$

Apply $\frac{dx}{dt}=1$ at t =0, we get $1=-3C_2$

$\Rightarrow C_2=\frac{1}{3}$

$\therefore x=\left(\cos 3t+\frac{1}{3}\sin 3t\right)$

78. Sol: $A \in (D^2+2D+1)=0 \Rightarrow D=-1,-1$

$y=(c_1+c_1t)e^{-t}$

$y'(t)=c_2e^{-t}+(c_1+c_2t)e^{-t}$

$y(0)=1: y'(0)=1 \Rightarrow c_1=1$ *and* $c_2+c_1(-1)=1 \Rightarrow c_2=2$

$\therefore$ General solution is $y(t)=(1+2t)e^{-t}$

79.

Sol: Given $\frac{dy}{dx}=\frac{1-\cos 2y}{1+\cos 2x} \Rightarrow \frac{dy}{1-\cos 2y}=\frac{dx}{1+\cos 2x} \Rightarrow \frac{dy}{2\sin^2 y}=\frac{dx}{2\cos^2 x}$

$\Rightarrow \int \cos ec^2 y dy=\int \sec^2 x dx+c$

$\Rightarrow -\cot y=\tan x+c$

$\Rightarrow -\tan x-\cot y=c \Rightarrow \tan x+\cot y=c$

80. **Ans: (c)**

Sol: Given D.E is $\frac{dx}{dt}=10-2x : x(0)=1 \Rightarrow \frac{dx}{dt}+(0.2)x=10$

A.E is $m+0.2=0 \Rightarrow m=-0.2$

Complimentary function $x_c=ce^{-0.2x}$

$x_p=\frac{1}{D+0.2}10e^{0t}=50e^{0t}=50$

G.S is $x=x_c+x_p=ce^{-0.2t}+50$

$x(0)=1 \Rightarrow c+50=1 \Rightarrow c=-49 \Rightarrow x=50-49e^{-0.2t}$

81. Sol: Given $x''(t)+3x'(t)+2t=0$

A.E is $m^2+3m+2=0 \Rightarrow m=-1,-2$

G.S is $x=c_1e^{-t}+c_2e^{-2t}$

Given $x(0)=20 \Rightarrow c_1+c_2=20 \Rightarrow c_1=20-c_2$

$x(1)=10/e \Rightarrow \frac{c_1}{e}+\frac{c_2}{e^2}=\frac{10}{e}$

$\Rightarrow \frac{20-c_2}{e}+\frac{c_2}{e^2}=\frac{10}{e} \Rightarrow c_2=\frac{10e}{e-1}$ and $c_1=\frac{10e-20}{e-1}$

Now $x(t)=\frac{10e-20}{e-1}e^{-t}+\frac{10e}{e-1}e^{-2t}$

$$x(2)=\left(\frac{10e-20}{e-1}\right)e^{-2}+\left(\frac{10e}{e-1}\right)e^{-4}=0.8556$$

82. Sol: A.E is $m^2+5m+6=0 \Rightarrow m=-2,-3$

G.S is $y(t)=c_1e^{-2t}+c_2e^{-3t}$

Given $y(0)=2 \Rightarrow c_1+c_2=2$

$y(1)=\frac{1-3e}{e^3} \Rightarrow c_1e^{-3}+c_2e^{-2}=\frac{1-3e}{e^3}$

By solving $c_1=-1 : c_2=-3$

$\therefore y(t)=-e^{-3t}+3e^{-2t}$

$\frac{d}{dt}y(t)=3e^{-3t}-6e^{-2t}, \frac{d}{dt}y(0)=3-6=-3$

83.**Ans: (-1)**

Sol: y`` + 9y = 0

A.E is $m^2 + 9 = 0$

m = ± 3i

$y = y_c + y_p$

$y = c_1 \cos 3x + c_2 \sin 3x$ -------- (1)

$(\therefore Y_p = 0)$

If x = 0, y= 0

(1) $0 = C_1 (1) + C_2 (0) \Rightarrow C_1 = 0$

If x = π/2 $y = \sqrt{2}$

(2) $\sqrt{2} = C_1(0) + C_2 \sin(3\pi/2) = C_2(-1)$

$\therefore\ y = -\sqrt{2}\sin 3x$

If $x = \pi/4$

$y(\pi/4) = -\sqrt{2}\sin(3\pi/4)$

$= -\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -1$

84.**Ans:** 0.66

Sol: $\frac{dy}{dx} = -3y + 2,\ y(0) = 1$

If $|1 - 3h| < 1$ then solution of differential equation is stable

$\Rightarrow\ -1 < 1 - 3h < 1$

$\Rightarrow\ -2 < -3h < 0$

$\Rightarrow\ 0 < 3h < 2$

$\Rightarrow\ 0 < h < \frac{2}{3}$

$\therefore$ If $0 < h < \frac{3}{2}$ then we get stable.

85.**Ans: (A)**

Sol: $D^2 + 12D + 36 = 0 \ \Rightarrow\ = -6, -6$

The solution is $y = C_1 e^{-6x} + C_2 x e^{-6x} \rightarrow (1)$

$y(0) = 3 \ \Rightarrow\ 3 = C_1$

$(1)\ \Rightarrow y = e^{-6x} + C_2 x e^{-6x}$

$\frac{dy}{dx} = -18e^{-6x} + C_2\left\{-6xe^{-6x} + e^{-6x}\right\} \ \Rightarrow\ \frac{dy}{dx}|_{x=0} = -18 + C_2 \ \Rightarrow\ -36 = -18 + C_2$

$\Rightarrow C_2 = -18$

$\therefore$ The solution is $y = 3e^{-6x} + 18xe^{-6x}$

86.Ans: (b)

Sol: Given equation $m^2+2m+1 = 0$

$(m+1)^2 = 0$

$y(t) = (c_1+c_2t)e^{-t}$

Given $y(0) = 1$

$1 = c_1$

Given $y(1) = 3e^{-1}$

$3e^{-1} = (1+c_2)e^{-1}$

$3 = 1 + c_2$

$c_2 = 2$

$\therefore\ y(t) = (1+2t)e^{-t}$

$y(2) = 5e^{-2}$

87.**Ans: (A)**

Sol: $\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2$

$A = m^4 + 3m^2 = 0$

$\Rightarrow m = 0, 0 \pm \sqrt{3}i$

$C.F = c_1 + c_2 x + c_3 \sin\sqrt{3x} + c_4 \cos\sqrt{3x}$

$$PI = \left(\frac{1}{D^4 + 3D^2}\right)108x^2$$

$$= \frac{1}{3D^2\left[1 + \frac{D^4}{3D^2}\right]}108x^2$$

$$= \frac{1}{3D^2}\left[1 + \frac{D^2}{3}\right]^{-1}108x^2$$

$$= \frac{1}{3D^2}\left[1 - \frac{D^2}{3} + ...\right]108x^2$$

$$= \frac{1}{3D^2}\left[108x^2 - 72\frac{x^2}{2}\right] = 3x^4 - 12x^2$$

$$y(x) = c_1 + c_2 x + c_3 \sin\sqrt{3x} + c_4 \cos\sqrt{3}x + 3x^4 - 12x^2$$

88.Ans: (C)

Soln: $B^2 - 4ac = 9 - 4$

$= 5 > 0$

$\therefore$ PDE is Hyperbolic

89.**Ans: (B)**

Soln: $\frac{\partial y}{\partial t} = \alpha \frac{\partial^2 y}{\partial x^2}$ ------(1)

Let $u = XT \Rightarrow \frac{\partial u}{\partial t} = XT' \;\&\; \frac{\partial^2 u}{\partial x^2} = TX''$

Substituting in (1)

$XT' = \alpha TX''$

$\frac{T'}{T} = \alpha \frac{X''}{X} = K$ say

$\frac{T'}{T} = K$ and $\alpha \frac{X''}{X} = K$

$\frac{1}{T}\frac{dT}{de} = K$

$\int \frac{1}{T}\, dT = \int K\, dt$

$\log T = Kt + \log C_1$

$T = C_1 e^{Kt}$ --------------(2)

$\frac{d^2X}{dX^2} = \frac{K}{\alpha} X$

$\frac{d^2X}{dX^2} - \frac{K}{\alpha} X = 0$

$A.E = m^2 - \frac{K}{\alpha} = 0$

$m = \pm\sqrt{\frac{K}{\alpha}}$

$X = C_2 e^{\sqrt{\frac{K}{\alpha}}x} + C_3 e^{-\sqrt{\frac{K}{\alpha}}x}$ -------(3)

Substitute (2) & (3) in (1)

$u = XT$

$u = C_1 e^{Kt}\left(C_2 e^{\sqrt{\frac{K}{\alpha}}x} + C_2 e^{-\sqrt{\frac{K}{\alpha}}x}\right)$

CHAPTER- 7
COMPLEX VARIABLES

01. The real part of the complex number $z = x + iy$ is **(GATE-94[IN])**

(a) $\text{Re}(z) = z - z^*$ (b) $\text{Re}(z) = \frac{z - z^*}{2}$ (c) $\text{Re}(z) = \frac{z + z^*}{2}$ (d) $\text{Re}(z) = z + z^*$

02. $\cos\phi$ can be represented as **(GATE-94[IN])**

(a) $\frac{e^{i\phi} - e^{-i\phi}}{2}$ (b) $\frac{e^{i\phi} - e^{-i\phi}}{2i}$ (c) $\frac{e^{i\phi} + e^{-i\phi}}{i}$ (d) $\frac{e^{i\phi} + e^{-i\phi}}{2}$

03. i^i, where $i = \sqrt{-1}$ is given by **(GATE-96[ME])**

(a) 0 (b) $e^{\frac{-\pi}{2}}$ (c) $\frac{\pi}{2}$ (d) 1

4. The complex number $z = x + iy$ which satisfy the equation $|z+1| = 1$ lie on **(GATE-97[IN])**

(a) a circle with (1,0) as the centre and radius 1

(b) a circle with (-1,0) as the centre and radius 1

(c) y-axis (d) x-axis

05. e^z is a periodic with a period of **(GATE-97[CE])**

(a) 2π (b) $2\pi i$ (c) π (d) $i\pi$

06. The bilinear transformation $w = \frac{z-1}{z+1}$ **(GATE-02[IN])**

(a) Maps the inside of the unit circle in the z-plane to the left half of the w-plane

(b) Maps the outside of the unit circle in the z-plane to the left half of the w-plane

(c) Maps the inside of the unit circle in the z-plane to the right half of the w-plane

(d) Maps the outside of the unit circle in the z-plane to the right half of the w-plane

07. Consider likely applicability of Cauchy's integral theorem to evaluate the following Integral counter clock wise around the unit circle C. I = $\oint_c \sec z\,dz$, z being a complex Variable. The value of I will be **(GATE-05[CE])**

(a) I=0; Singularities set = ϕ

(b) I=0; singularities set = $\left\{\pm\frac{(2n+1)}{2}\pi / n = 0,1,2,3\cdots\cdots\right\}$

(c) $I = \frac{\pi}{2}$; singularities set = $\{\pm n\pi; n = 0,1,2,3\cdots\cdots\}$

(d) none of the above

08. Consider the circle $|Z-5-5i| = 2$ in the complex number plane (x,y) with z=x+iy. The minimum distance from the origin to the circle is **(GATE-05[IN])**

(a) $5\sqrt{2}-2$ (b) $\sqrt{54}$ (c) $\sqrt{34}$ (d) $5\sqrt{2}$

09. Let $z^3 = \bar{z}$, where z is a complex number not equal to zero. Then Z is a solution of **(GATE-05[IN])**

(a) $z^2 = 1$ (b) $z^3 = 1$ (c) $z^4 = 1$ (d) $z^9 = 1$

10. The function $w = u + iv = \frac{1}{2}\log\left(x^2 + y^2\right) + i\tan^{-1}\left(\frac{y}{x}\right)$ is not analytic at the point. **(GATE-05[PI])**

(a) (0,0) (b) (0,1) (c) (1,0) (d) (2,α)

11. The value of the counter integral $\int_{|z-j|=2} \frac{1}{z^2+4}dz$ in the positive sense is **(GATE-2006[EC])**

(a) $\frac{j\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{-j\pi}{2}$ (d) $\frac{\pi}{2}$

12. For the function of a complex variable (where $w = u + jv$ and $z = x + iy$) the constant lines get mapped in the z-plane as **(GATE-2006[EC])**

(a) set of radial straight lines (b) set of concentric circles

(c) set of confocal hyperbolas (d) set of confocal ellipses

13. Using Cauchy's integral theorem, the value of the integral (Integration being taken in counter clock wise direction) $\int_c \frac{z^3-6}{3z-1}dz$ where C is $|z| = 1$ **(GATE-2006[CE])**

(a) $\frac{2\pi}{81} - 4\pi i$ (b) $\frac{\pi}{8} - 6\pi i$ (c) $\frac{4\pi}{81} - 6\pi i$ (d) 1

14. Let $j = \sqrt{-1}$. Then one value of j^j is **(GATE-2007[IN])**

(a) $\sqrt{3}$ (b) -1 (c) $\frac{\sqrt{1}}{2}$ (d) $e^{-\frac{\pi}{2}}$

15. For the function of a complex variable z, the point z = 0 is **(GATE-2007[IN])**

(a) a pole of order 3 (b) a pole of order 2

(c) a pole of order 1 (d) not a singularity

16. Potential function is given as $\phi = x^2 + y^2$. What will be the stream function with the condition at x = 0, y=0? **(GATE-2007[CE])**

(a) 2xy (b) $x^2 + y^2$ (c) $x^2 + y^2$ (d) $2x^2y^2$

17. If and are functions with continuous second derivatives then $2\phi(x,y) + i\psi(x,y)$ can be expressed as an analytic Function of x+iy ($i = \sqrt{-1}$) when **(GATE-2007[ME])**

(a) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial x}, \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y}$ (b) $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial x}, \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y}$

(c) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1$ (d) $\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

18. If a complex number $z = \frac{\sqrt{3}}{2} + i\frac{1}{2}$ then z^4 is **(GATE-2007[PI])**

(a) $2\sqrt{2} + 2i$ (b) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2} - i\frac{1}{2}$ (d) $\frac{\sqrt{3}}{8} - i\frac{1}{8}$

19. The value of $\oint_c \frac{1}{(1+z^2)} dz$ where C is the contour $\left|z - i/2\right| = 1$ is **(GATE-2007[EC])**

(a) $2\pi i$ (b) л (c) $\tan^{-1}(z)$ (d) $\pi i \tan^{-1} z$

20. If the semi circular contour D of radius 2 as shown in the figure. Then the value of the integral $\oint_D \frac{1}{s^2 - 1} ds$ is **(GATE-2007[EC])**

(a) $i\pi$ (b) $-i\pi$ (c) $-\pi$ (d) π

21. The value of the expression $\frac{-5 + i10}{3 + 4i}$ **(GATE-2008[PI])**

(a) $1 - 2i$ (b) $1 + 2i$ (c) $2 - i$ (d) $2 + i$

22. The residue of the function $f(z) = \frac{1}{(z+2)^2(z-2)^2}$ at z = 2 is **(GATE-2008[EC])**

(a) $-1/32$ (b) $-1/16$ (c) $1/16$ (d) $1/32$

23. The integral $\oint f(z)dz$ evaluated around the unit circle on the complex plane for $f(z)=\frac{\cos z}{z}$ is **(GATE-2008[ME])**

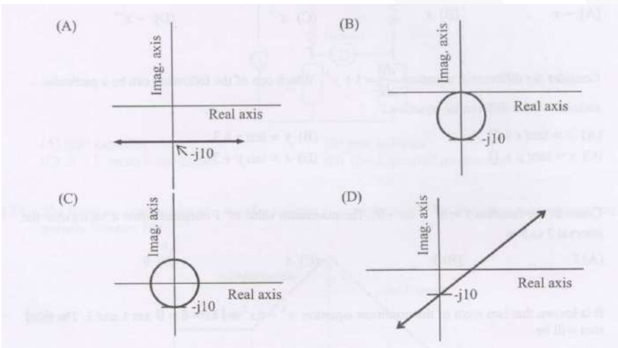
(a) $2\pi i$ (b) $4\pi i$ (c) $-2\pi i$ (d) 0

24. The equation Sin (z) =10 has **(GATE-2008[EC])**

(a) No real (or) Complex solution (b) Exactly two distinct complex solutions.

(c) A unique solution (d) An infinite number of complex solutions.

25. A complex variable $z = x + j(0.1)$ has its real part x varying in the range -∞ to ∞. Which one of the following is the locus (shown in thick lines) of 1/z in the complex plane? **(GATE-2008[IN])**



26. Given $X(z)=\frac{z}{(z-a)^2}$ with $|z|>a$, the residue of $X(z)z^{n-1}$ at z=a for n≥0 will be **(GATE-2008[EE])**

(a) a^{n-1} (b) a^n (c) na^n (d) na^{n-1}

27. The analytical function has singularities at, where $f(z)=\frac{z-1}{z^2+1}$ is **(GATE-2009[CE])**

(a) 1 and -1 (b) 1 and i (c) 1 and -i (d) i and -i

28. The value of the integral $\int_c \frac{\cos(2\pi z)}{(2z-1)(z-3)}dz$ where C is a closed curve given by Z ≥1 is **(GATE-2009[CE])**

(a) $-\pi i$ (b) $\pi i/5$ (c) $2\pi i/5$ (d) πi

29. If $f(z)=C_0+C_1 z^{-1}$ then $\oint_{unit} \frac{1+f(z)}{z}dz$ is given **(GATE-2009[EC])**

(a) $2\pi C_1$ (b) $2\pi(i+C_0)$ (c) $2\pi j C_1$ (d) $2\pi j(1+C_0)$

30. If $Z = x + jy$ where x,y are real then the value of $\left|e^{jz}\right|$ is **(GATE-2009[IN])**

(a) 1 (b) $e^{\sqrt{x^2+y^2}}$ (c) e^{y} (d) e^{-y}

31. One of the roots of equation $x^3 = j$, where j is the positive square root of -1 is **(GATE-2009[IN])**

(a) j (b) $\frac{\sqrt{3}}{2}+\frac{j}{2}$ (c) $\frac{\sqrt{3}}{2}-\frac{j}{2}$ (d) $-\frac{\sqrt{3}}{2}-\frac{j}{2}$

32. The value of $\oint_a \frac{\sin z}{z}dz$, where the contour of the integration is a simple closed curve around the origin is **(GATE-2009[IN])**

(a) 0 (b) 2лj (c) ∞ (d) 1/2лj

33. An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x, y)$ where $i = \sqrt{-1}$. If u=xy then the expression for v Should be **(GATE-2009[ME])**

(a) $\frac{(x+y)^2}{2}+k$ (b) $\frac{x^2-y^2}{2}+k$ (c) $\frac{x^2+y^2}{2}+k$ (d) $\frac{(x-y)^2}{2}+k$

34. The product of complex numbers results in **(GATE-2009[PI])**

(a) 1+6i (b) 9-8i (c) 9+8i (d) 17+6i

35. If $f(x+iy) = x^3 - 3xy^2 + i\phi(x, y)$ where $i = \sqrt{-1}$ and f(x+iy) is an analytic function then $\phi(x/y)$ is **(GATE-2010[PI])**

(a) $y^3 - 3x^2y$ (b) $3x^2y - y^3$ (c) $x^4 - 4x^3y$ (d) $xy - y^2$

36. The modulus of the complex number $\frac{3+4i}{1-2i}$ is **(GATE-2010[CE])**

(a) 5 (b) $\sqrt{5}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{5}$

37. If a complex number ω satisfies the equation $\omega^3 = 1$ then the value of $1+\omega+\frac{1}{\omega}$ is ___________ **(GATE-2012[PI])**

(a) 0 (b) 1 (c) 2 (d) 4

38. The contour C in the adjoining figure is described by $x^2 + y^2 = 16$. then the value of

$\oint_c \frac{z^2+8}{(0.5)z-(1.5)j}dz$ **(GATE-2010[IN])**

(a) -2πj (b) 2πj (c) 4πj (d) -4πj

39. The residues of a complex function $X(z) = \frac{1-2z}{z(z-1)(z-2)}$ at it poles **(GATE-2010[EC])**

(a) $\frac{1}{2}, \frac{-1}{2}, 1$ (b) $\frac{1}{2}, \frac{-1}{2}, -1$ (c) $\frac{1}{2}, 1, \frac{-3}{2}$ (d) $\frac{1}{2}, -1, \frac{3}{2}$

40. The value of the integral $\oint_c \frac{-3z+4}{z^2+4z+5}dz$, when C is the circle $|z| = 1$ is given by

(GATE-2011[EC])

(a) 0 (b) 1/10 (c) 4/5 (d)1

41. The contour integral $\int_c e^{1/z}dz$ with C as the counter clock-wise unit circle in the z-plane is equal to **(GATE-2011[IN])**

(a) 0 (b) 2л (c) $2\pi\sqrt{-1}$ (d)∞

42. The product of two complex numbers 1+i & 2-5i is **(GATE-2011[ME])**

(a) 7-3i (b) 3-4i (c) -3-4i (d) 7+3i

43. For an analytic function $f(x+iy) = u(x,y) + iv(x,y)$, is given by $u = 3x^2 - 3y^2$. The expression for v, considering k is to be constant is **(GATE-2011[CE])**

(a) $3y^2 - 3x^2 + k$ (b) $6x - 6y + k$ (c) $6y - 6x + k$ (d) $6xy + k$

44. The value of $\oint_c \frac{z^2}{z^4-1}dz$, using Cauchy's integral around the circle |z+1|=1 where Z= x+iy is **(GATE-2011[PI])**

(a) $2\pi i$ (b) $-\frac{\pi i}{2}$ (c) $-\frac{3\pi i}{2}$ (d) $\pi^2 i$

45. For a complex number z = x+iy the locus of all points of |z| < 1 for the transformation w = 1/z lies in **(GATE-2012)**

(a) First Quadrant (b) Second Quadrant (c) Third Quadrant (d) Fourth Quadrant

46. If x = $\sqrt{-1}$ then the value of x^x **(GATE-2012-EC/EE/IN)**

(a) 1 (b) $e^{\pi/2}$ (c) 0 (d) $e^{-\pi/2}$

47. If $f(z)=\frac{1}{(z+1)(z+3)}$ then $\frac{1}{2\pi j}\int_c f(z)\,dz$ is **(GATE-2012-EC/EE/IN)**

(a) 0 (b) -1 (c) 1 (d) None

48. Square root of –i, where is $\sqrt{-1}$,are **(GATE-2013-EE)**

(a) $e^{\pi/4}$ (b) $e^{i\pi/4}$ (c) $e^{-i\pi/4}$ (d) $e^{-\pi/4}$

49. Find $I=\oint_C \frac{z^2-4}{z^2+4}dz$ is, where 'C' is a circle |z-i| = 2 is **(GATE-2013-EE)**

(a) 4π (b) -4 π (c)2 π (d) π

50. Evaluate $\oint_c \frac{z^2-z+4j}{\left[\left(z-\left(-2j\right)\right)\right]}dz$, where 'C' is a circle |z| = 3 **(GATE-2014-EC)**

(a) -4 π(3+2j) (b) 2 π(3+2j) (c) π(2-3j) (d) 4 π(3+2j)

51. For an analytic function f(z) = u+iv if u =e^{-y}cosx , find its harmonic conjugate u? **(GATE-2014-EC)**

(a) u = e^{y} sin x+c (b) e^{-y}sinx+c (c) e^{y}cosx+c (d) None

54. Evaluate $\int_{|z-1|=1} \frac{z^2}{z^2-1}$dz **(GATE-2014-EE)**

(a) -πi (b) πi (c) 0 (d) π

55. Find the argument of $\frac{1+i}{1-i}$ **(GATE-2014-ME)**

(a) π/2 (b) – π/2 (c) πi/2 (d) π/4

56. For an analytic function f(z) = u+iv if the real part u = 2xy, then find it imaginary part v. **(GATE-2014-ME)**

(a) v =x^2+y^2+K (b) $-x^2+y^2$+K (c) x^2-y^2+K (d) None

57. For an analytic function f(z) = u+iv if the real part u = x^2+y^2 find its harmonic conjugate v. **(GATE-2014-EE)**

(a) v = -2xy+c (b) v= x^2-y^2+c (c) 2xy+c (d) None

58. Evaluate $\int_5^{3i} \frac{dz}{z}$ **(GATE-2014-EE)**

59. represent the complex number $z = \frac{2-3i}{-5+i}$ in the form z = a+ib. **(GATE-2014-IN)**

60. Let z = x + iy is complex variable. Consider the contour integration is perform along unit circle in anticlockwise direction. Which of the following is not true?

(GATE – EC-15)

(A) The residue of $\frac{z}{z^2-1}$ at z = 1 is $1/2$

(B) $\oint_c z^2 dz = 0$

(C) $\frac{1}{2\pi i}\oint_c \frac{1}{z} dz = 0$

(D) $\bar{z}$ is an anylitic function

61. Let $f(z) = \frac{az+b}{cz+d}$, If $f(z_1) = f(z_2) \forall z_1 \neq z_2, a = 2 : b = 4 : c = 5$, then d is equal to

(GATE – EC-15)

62. If C denotes the counter clockwise unit circle, the value of contour integral

$\frac{1}{2\pi i}\oint_c \text{Re}\, a(z) dz$ is **(GATE –15)**

63. If 'C' is a circle of radius 'r' with centre z_0, in the complex plane if n is a nonzero, then

$\oint \frac{dz}{(z-z_0)^{n+1}}$ equal to **(GATE – -15)**

(A) $2n\pi i$ (B) 0 (C) $\frac{ni}{2\pi}$ (D) $2n\pi$

64. Given f(z) = g(z) + h(z), where f,g,h are complex valued function of a complex variable z, which of the following is true **(GATE – EE-15)**

(A) If f(z) is differentiable at z_0 , then g(z), h(z) are also differentiable

(B) If g, h are differentiable at z_0 , then f is differentiable at z_0

(C) If f is continuosy at z_0 , then it is differentiable at z_0

(D) If f is differentiable at z_0 , then its real and imaginary parts are differentiable

65. Given two complex numbers $z_1 = 5 + (5\sqrt{3})i$ and $z_2 = \frac{2}{\sqrt{3}} + 2i$, the argument of $\frac{z_1}{z_2}$ in degrees is **(GATE – ME-15)**

A) 0 B) 30 C) 60 D) 90

66. If the fluid velocity for a potential flow is given by V(x,y) = u + iv with usual notations, then the slope of the potential line at (x, y) is **(GATE – ME-15)**

(A) $\frac{v}{u}$ (B) $\frac{-u}{v}$ (C) $\frac{v^2}{u^2}$ (D) $\frac{u}{v}$

67. Consider the following complex function f(z) = $\frac{9}{(z-1)(z+2)^2}$ which of the following residue of the above function **(GATE – CE-15)**

(A) -1 (B) $9/16$ (C) 2 (D) 9

68. Complex valued function f(z) given below is analytic in domain D. f(z) = u(x, y) + I v(x, y): z = x + i y , which of the following is not correct **(GATE – CH-15)**

(A) $\frac{df}{dz}=\frac{\partial v}{\partial y}+i\frac{\partial u}{\partial y}$ (B) $\frac{df}{dz}=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}$ (C) $\frac{df}{dz}=\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}$ (D) $\frac{df}{dz}=\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}$

69. For the complex variable z, the value of the contour integral $\frac{1}{2\pi i}\int_c \frac{e^{-2z}}{z(z-3)}dz$ along the clockwise contour $c:|z|=2$ up to two decimal places **(GATE – CH-15)**

70.In the following integral, the contour C encloses the points 2πj and –2πj. The value of the integral $-\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3}dz$ is _______ **(GATE – EC-16)**

71. The values of the integral $\frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz$ along a closed contour c in anti-clockwise direction for **(GATE – EC-16)**

(i) the point $z_0 = 2$ inside the contour c, and

(ii) the point $z_0 = 2$ outside the contour c, respectively, are

(A)(i) 2.72,(ii) 0 (B) (i) 7.39,(ii) 0

(C) (i) 0,(ii) 2.72 (D) (i) 0,(ii) 7.39

72. $f(z) = u(x, y) + i\, v(x, y)$ is an analytic function of complex variable $z = x + i\, y$ where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ may be expressed as **(GATE – ME-16)**

(A) $-x^2 + y^2$ + constant (B) $x^2 - y^2$ + constant

(C) $x^2 + y^2$ + constant (D) $-(x^2 + y^2)$+ constant

73.The value of the integral **(GATE – ME-16)**

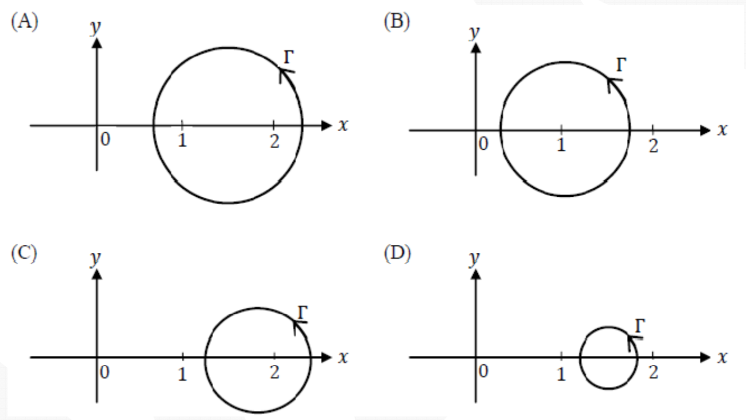
$$\int_{-\infty}^{\infty} \frac{Sinx}{x^2+2x+2}dx$$

evaluated using contour integration and the residue theorem is

(A) $-\pi\sin(1)/e$ (B) $-\pi\cos(1)/e$ (C) $\sin(1)/e$ (D) $\cos(1)/e$

74.A function f of the complex variable $z = x + i\,y$, is given as $f(x, y) = u(x, y) + (x,y)$ where $u(x, y) = 2kxy$ and $v(x, y) = x^2 - y^2$. The value of k, for which the function is analytic, is _____ **(GATE – ME-16)**

75.The value of $\oint_\Gamma \frac{3z-5}{(z-1)(z-2)}dz$ along a closed path Γ is equal to $(4\pi i)$, where $z = x + iy$ and $i = \sqrt{-1}$. The correct path Γ is **(GATE – ME-16)**



76.Consider the complex valued function $f(z) = 2z^3 + b|z|^3$ where z is a complex variable. The value of b for which the function f(z) is analytic is ___________ **(GATE – EC-16)**

77.For $f(z) = \frac{\sin(z)}{z^2}$, the residue of the pole at z = 0 is _______ **(GATE – EC-16)**

78.Consider the function $f(z) = z + z^*$ where z is a complex variable and z^* denotes its complex conjugate. Which one of the following is TRUE? **(GATE – EE-16)**

a) f(z) is both continuous and analytic

b) f(z) is continuous but not analytic

c) f(z) is not continuous but is ananlytic

d) f(z) is neither continuous nor analytic

79. The value of the integral $\oint_C \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}dz$ over the contour $|z|=1,$ taken in the anti-clockwise direction, would be **(GATE – EE-16)**

a) $\frac{24\pi i}{13}$ b) $\frac{48\pi i}{13}$ c) $\frac{24}{13}$ d) $\frac{12}{13}$

COMPLEX VARIABLES SOLUTIONS

1. Ans: c

Sol: Given z = x+iy

$$\Rightarrow \overline{z} = x - iy$$

$$x = \frac{\overline{z} + z}{2}$$

2. Ans: d

Sol: $e^{i\phi} = \cos\phi + i\sin\phi$

$$\Rightarrow e^{-i\phi} = \cos\phi - i\sin\phi$$

$$\therefore \cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}$$

3. Ans: b

Sol: $i^i = e^{\log i^i} = e^{i\log i} = e^{i\log e^{i\pi/2}} = e^{-\pi/2 \log e} = e^{-\pi/2}$

4. Ans b

Sol: The general equation of a circle is given by $|z - z_0| = r$

Where z_0 is center & radius is r. Now the equation of the given circle is

$|z+1| = 1 (or) |z-(-1)| = 1$

$\therefore$ center = (-1, 0) and radius = 1

5. Ans: b

Sol: e^z is a periodic function with period $2\pi i$

f(z)= $e^z = e^{z+2n\pi i}$

$\Rightarrow f(z) = f(z+2n\pi i)$

$\therefore e^z$ is a periodic function of period $2\pi i$.

6. Ans a

Sol: Given $w = \frac{z-1}{z+1}$

$$w = \frac{az+b}{cz+d} = f(z)$$

$\Leftrightarrow z = f^{-1}(w) = \dfrac{-dw+b}{cw-a}$

$\therefore z = \dfrac{-w-1}{w-1}$

Unit circle $|z| = 1$

Consider $|z| < 1$ which represents inside of this unit circle $|z| = 1$

$\Rightarrow \dfrac{|-w-1|}{|w-1|} < 1$

$\Rightarrow |u+iv+1| < |u+iv-1|$

$\Rightarrow |(u+1)+iv| < |(u-1)+iv|$

$\Rightarrow (u+1)^2 + v^2 < (u-1)^2 + v^2$

$\Rightarrow u^2 + 1 + 2u + v^2 < u^2 + 1 - 2u + v^2$

$\Rightarrow 4u < 0$

$\Rightarrow u < 0$

$\Rightarrow$ The function $w = \dfrac{z-1}{z+1}$ maps the inside of unit circle in the zplane to the left half of the w plane

7. Ans: a

Sol: $I = \oint_C \sec z dz = \oint_C \dfrac{1}{\cos z} dz$

Singular points of $\dfrac{1}{\cos z}$

Are $\pm(2n+1)\pi/2, n = 0,1,2,3---$

But the given region is unit circle $|z| = 1$

$\therefore \dfrac{1}{\cos z}$ has no singularities $|z| = 1$

Hence by cauchy's integral theorem, we have

$\therefore$ I = 0, singularities set = ϕ in $|z| = 1$

8. Ans: a

Sol: For a given circle center is (5, 5) and radius is 2

$\therefore$ The minimum distance from the origin to the circle is $5\sqrt{2}-2$

09. Ans: a

Sol: Given $z^3 = \bar{z}$

$$\Rightarrow z^3 z = \bar{z}z = z^2$$

$$\Rightarrow z^2 = 1$$

10. Ans: a

Sol: Putting x = z and y =0 in the given equation we get

$\therefore$ w = log z which is not defined at origin (0, 0)

$\Rightarrow$ $w = u + iv$ is not analytic at origin (0, 0).

11. Ans: d

Sol: Given $I = \int_c \frac{1}{z^2+4} dz$ where C is $|z-j| = 2$

The integrated is not analytic at $z = \pm 2i$ and z = 2i lies inside C. By cauchy's integral formula

$$= \int_c \frac{\left(\frac{1}{z+2i}\right)}{z-2i} dz = 2\pi i \left(\frac{1}{z+2i}\right)_{z=2i} = 2\pi i \left(\frac{1}{2i+2i}\right) = \frac{\pi}{2}$$

12. Ans: b

Sol: Given function is w = lnz where w = u + iv = $\cos(re^{i\theta})$

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$\Rightarrow w = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow u = \frac{1}{2}\log(x^2+y^2) \,\&\, v = \tan^{-1}\left(\frac{y}{x}\right)$$

But given that u = constant c

$$\Rightarrow \frac{1}{2}\log(x^2+y^2) = c$$

$$\Rightarrow \log(x^2+y^2)=2c$$

$$\Rightarrow x^2+y^2=e^{2c}$$

$$\Rightarrow x^2+y^2=C_1^2 \ where\, C_1=e^c$$

Which represents the set of concentric circles with center as (0, 0) and radius c_1

13. Ans: a

Sol: $I=\int_c \frac{z^3-6}{3z-i}dz=\frac{1}{3}\int_{|z|=1}\frac{z^3-6}{z-i/3}dz$

The integrated is not analytic at $z=i/3$ which lies insides C. BY cauchy's integral formula

$$=\frac{1}{3}2\pi i\left[\left(\frac{i}{3}\right)^2-6\right]=\frac{2\pi}{81}-4\pi i$$

14. Ans: d

Same as Q 3

15. Ans: b

Sol: $f(z)=\frac{\sin z}{z^3}=\frac{1}{z^3}\left[z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+-----\right]=\frac{1}{z^2}-\frac{1}{3!}+\frac{z^2}{5!}-\frac{z^4}{7!}+---$

$$=\frac{1}{(z-0)^2}-\frac{1}{3!}+\frac{(z-0)^2}{5!}-\frac{(z-0)^4}{7!}+----$$

$\therefore$ z = 0 is a pole of order 2

16. Ans: a

Sol: Given $\phi=x^2-y^2$ for f(z) = u + iv

$=\phi+i\psi \;\&\; \psi=?$

Also given $\psi(x,y)=0\, at\, x=0, y=0$

$d\psi=\psi_x dx+\psi_y dy\ (\because \psi_x=-\phi_y \;\&\, \psi_y=\phi_x)$

$=-\phi_y dx+\phi_x dy$

$=-(-2y)dx+2x(dy)$

$\psi=2xy+k$

At x = 0, y =0

$\psi(0,0) = 0 + k$

0 = 0 +k

$\therefore$ k =0 and stream function is $\psi(x, y) = 2xy$

17. Ans: b

Sol: Given $f(z) = \phi(x, y) + i\psi(x, y)$ and

$\phi(x, y), \psi(x, y)$ are continuous 2nd order derivatives

For the function $f(z) = \phi + i\psi$ to be analytic function the ϕ and ψ must satisfy cauchy riemann equations i.e. $\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$ *and* $\frac{\partial\psi}{\partial x} = -\frac{\partial\phi}{\partial y}$

18. Ans: b

Sol: $z = \frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)$

$iz = -\frac{1}{2} + \frac{\sqrt{3}}{2} i = w$

Where w is root of $x^3 = 1$

$i^4 z^4 = w^4$

$\Rightarrow z^4 = w \quad (\because w^3 = 1)$

19. Ans: b

Sol: $I = \int_c \frac{1}{1+z^2} dz \ \ where\, C\, is \ \ \left|z - \frac{i}{2}\right| = 1$

Singular points of $\frac{1}{1+z^2}$ are given by $1 + z^2 = 0$

$\Rightarrow z = \pm i$

But z = I lies in the contour $\left|z - i/2\right| = 1$

By cauchys integral formula

$$I = \int \frac{1}{(z-i)(z+i)} dz = \int \frac{1/(z+i)}{z-i} dz = 2\pi i f(i)$$

Where $f(z)=\dfrac{1}{z+i}$

$$I=2\pi i\left(\frac{1}{1+i}\right)=\pi$$

20. Ans:

Sol: $I=\oint_D \dfrac{1}{s^2-1}ds=\oint_D \dfrac{1}{(s-1)(s+1)}ds$

The singular points of $\dfrac{1}{s^2-1}$ are given by s^2-1 i.e. $S=\pm 1$

But S = 1 lies in the contour D.

By cauchys integral formula

$$\therefore I=\int \frac{1/(s+1)}{s-1}ds=2\pi i f(1) \text{ where } f(s)=\frac{1}{s+1}$$

Hence $I=2\pi i\left(\dfrac{1}{1+1}\right)=\pi i$

21. Ans: b

Sol: $\dfrac{-5+10i}{3+4i}=\dfrac{(-5+10i)(3-4i)}{(-3+4i)(3-4i)}=\dfrac{-15+20i+30i+40}{9+16}=\dfrac{25+50i}{25}=1+2i$

22. Ans: a

Sol: $f(z)=\dfrac{1}{(z+2)^2(z-2)^2}$

Z = 2 is a pole of f(z) of order 2

$\operatorname{Re}s(f(z):z=z_0)$

$$=\frac{1}{(m-1)!}\left[\underset{z\to z_0}{Lt}\frac{d^{m-1}}{dz^{m-1}}\left((z-z_0)^m f(z)\right)\right]$$

$\operatorname{Re}s(f(z):z=2)$

$$=\frac{1}{(2-1)!}\left[\underset{z\to 2}{Lt}\frac{d^{2-1}}{dz^{2-1}}\left((z-2)^2\frac{1}{(z+2)^2(z-2)^2}\right)\right]$$

$$=\underset{z\to 2}{Lt}\left[\frac{(-2)}{(z+2)^3}\right]=-1/32$$

23. Ans: a

Sol: $I = \int_c \frac{\cos z}{z} dz$

Since z = 0 lies in the unit circle

By cauchy integral formula

$$I = 2\pi i g(0) = 2\pi i \cos(0) = 2\pi i \qquad (\because g(z) = \cos z)$$

24. Ans: d

Sol: $\sin z = 10$

$$\Rightarrow \frac{e^{iz} - e^{-iz}}{2i} = 10$$

$$\Rightarrow e^{iz} - \frac{1}{e^{iz}} = 20i$$

$$\Rightarrow (e^{iz})^2 - 20i(e^{iz}) - 1 = 0$$

$$\Rightarrow e^{iz} = \frac{-(-20i) \pm \sqrt{-400+4}}{2} = \frac{20i \pm \sqrt{-396}}{2} = \frac{20i \pm 6i\sqrt{11}}{2} = 10i \pm 3\sqrt{11}i$$

$$\Rightarrow iz = \log[i(10 \pm 3\sqrt{11})] = \log i + \log(10 \pm 3\sqrt{11})$$

$$\Rightarrow iz = \log 1 + i\left(\frac{\pi}{2} \pm 2n\pi\right) + \log(10 \pm 3\sqrt{11})$$

$$\Rightarrow iz = i\left(\frac{\pi}{2} \pm 2n\pi\right) + \log(10 \pm 3\sqrt{11})$$

$$\Rightarrow z = \left(\frac{\pi}{2} \pm 2n\pi\right) - i\log(10 \pm 3\sqrt{11})$$

N = 0, 1, 2, ----

$\therefore$ sinz = 10 has infinite number of complex solutions

25. Ans: b

Sol: The straight line y = 0.1 from $-\infty$ to ∞ not passing through the origin in z-plane is transformed in to a circle passing through the origin in the w-plane under the mapping (or) transformation $w = 1/z$

Let z = x+ iy & w = u + iv

Then $w = 1/z$

$$\Rightarrow u+iv=\frac{1}{x+iy}\Rightarrow u+iv=\frac{x-iy}{x^2+y^2}$$

$$\Rightarrow u=\frac{x}{x^2+y^2}\ \&\ v=\frac{-y}{x^2+y^2}$$

$\therefore$ the points (x, y) in the 1st and 2nd quadrants are transformed to 3rd and 4th quadrants as x is positive and negative, y is only negative, u is posive and negative, v is only negative.

26. Ans: d

Sol: Given $X(z)=\frac{z}{(z-a)^2},\quad |z|>a$

Let $X(z)z^{n-1}=\frac{z}{(z-a)^2}z^{n-1}=\frac{z^n}{(z-a)^2}$

Here z = a is a pole of f(z) of order 2

$\therefore$ Res (f(z):z=a)

$$\underset{z\to a}{Lt}\left[\frac{d}{dz}\left\{(z-a)^2\frac{z^n}{(z-a)^2}\right\}\right]=\underset{z\to a}{Lt}\ nz^{n-1}=na^{n-1}$$

27. Ans: d

Sol: $f(z)=\frac{z-1}{z^2+1}$

$\therefore$ The singular points of f(z) are given by $z^2+1=0$ *i.e.* $z=\pm i$

28. Ans: c

Sol: $I=\int_c \frac{\cos(2\pi z)}{(2z-1)(z-3)}dz$

The integral has singularities $z=\tfrac{1}{2},3$

But only $z=\tfrac{1}{2}$ lies in $|z|=1$

By cauchy integral formula

$$\therefore I=\frac{1}{2}\int\frac{\{\cos(2\pi z)\}/(z-3)}{(z-1/2)}dz$$

$$=\frac{1}{2}2\pi i f\left(\frac{1}{2}\right)\ where\ \ f(z)=\frac{\cos 2\pi z}{z-3}$$

$\therefore I = \frac{2\pi i}{5}$

29. Ans: d

Sol: $I = \int_c \frac{1+f(z)}{z} dz$

Where $f(z) = C_0 + C_1 z^{-1} = C_0 + \frac{C_1}{z}$ $I = \int_c \frac{1 + C_0 + \left(\frac{C_1}{z}\right)}{z} dz = \int_c \frac{z + C_0 z + C_1}{z} dz$, C is $|z| = 1$

Here z = 0 lies insides C

By cauchy integral formula $= \frac{2\pi j}{1!} F'(0)$

Where $F(z) = z_0 + C_0 z + C_1$ & $F'(z) = 1 + C_0 + 0 = 2\pi j(+1 + C_0)$

$I = 2\pi j(C_0 + 1)$

30. Ans: d

Sol: $\left|e^{jz}\right| = \left|e^{j(x+iy)}\right| = \left|e^{jx-y)}\right| = \left|e^{-y}\right|\left|e^{jx}\right| = e^{-y}$ $\left(\therefore \left|e^{jx}\right| = \left|\cos x + j\sin x\right| = 1\right)$

31. Ans: b

Sol: $x^3 = j \;\; where \;\; j = \sqrt{-1}$

By verification option (b) will satisfy the given equation $x^3 = j$

32. Ans: a

Sol: $I = \oint_c \frac{\sin z}{z} dz = \oint_c \frac{f(z)}{z - z_0} dz = \oint_c \frac{\sin z}{z - 0} dz$

$= 2\pi j f(0) = 2\pi j(0)$ where f(z) = sinz

$\therefore I = 0$

33. Ans: c

Sol: Given $u = xy \;\; for \;\; f(z) = u(x,y) + iv(x,y)$

$dv = v_x dx + v_y dy = -u_y dx + v_y dy$

$dv = -xdx + ydy \;\; (\therefore u_x = v_y \;\&\; v_x = -u_y)$

$v = -x^2/2 + y^2/2 + k = \frac{y^2 - x^2}{2} + k$

34. Ans: d

Sol: $(3-2i)(3+4i)=9+12i-6i-8(i^2)=17+6i$

35. Ans: b

Sol: $f(x+iy)=(x^3-3x^2y)+i\phi(x,y)$

$=\psi(x,y)+i\phi(x,y)$

Where $\psi(x,y)=x^3-3xy^2$

$d\phi=\phi_x dx+\phi_y dy=-\psi_y dx+\psi_x dy$

$d\phi=-(0-6xy)dx+(3x^2-3y^2)dy$

$\phi=\frac{6x^2y}{2}+\left(-3\frac{y^3}{3}\right)+k$

$\phi=3x^2y-y^3$

36. Ans: b

Sol: $\left|\frac{3+4i}{1-2i}\right|=\frac{\sqrt{9+16}}{\sqrt{1+4}}=\frac{5}{\sqrt{5}}=\sqrt{5}$

37. Ans: a

Sol: Given $\omega^3=1$ (or) $\omega=(1)^{1/3}$

ω is the cube root of unity

But, we know that, sum of the cube roots of unity is zero

i.e., $1+\omega+\omega^2=0$

38. Ans: d

Sol:

$I=\int_c \frac{z^2+8}{1/2\,z-3/2\,j}dz$ and C is $x^2+y^2=16$ (or) $|z|=4$

$=2\int_c \frac{z^2+8}{z-3j}dz$

By cauchy integral formula

$\therefore I=2[2\pi j f(3j)]=-4\pi j$

Where $f(z)=z^2+8$

(Since z = 3 lies in the contour)

39. Ans: c

Sol: Given $X(z) = \frac{1-2z}{z(z-1)(z-2)}$

$\Rightarrow$ The poles of X(z) are 0, 1, 2 which are simple poles

$R_1 = \operatorname{Re}s(X(z) : z = 0)$

$$= \underset{z\to 0}{Lt}\left[(z-0)\frac{(1-2z)}{z(z-1)(z-2)}\right] = \frac{1}{2}$$

$R_2 = \operatorname{Re}s(X(z) : z = 1)$

$$= \underset{z\to 1}{Lt}\left[(z-1)\frac{(1-2z)}{z(z-1)(z-2)}\right] = \frac{-1}{-1} = 1$$

$R_2 = \operatorname{Re}s(X(z) : z = 2)$

$$= \underset{z\to 1}{Lt}\left[(z-2)\frac{(1-2z)}{z(z-1)(z-2)}\right] = \frac{-3}{2(1)} = \frac{-3}{2}$$

40. Ans: a

Sol: $I = \int_c \frac{-3z+4}{z^2+4z+5}dz$ where C is $|z| = 1$

The singular points of the integrated are z = - 2 + I, -2 – i

But, the given contour does not contain singular points

$\therefore$ By cauchy's integral theorem, we have $I = \int_c \frac{-3z+4}{z^2+4z+5}dz = 2\pi i(0) = 0$

41. Ans: c

Sol: $f(z) = e^{\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + -----$

$$= 1 + \frac{1}{z-0} + \frac{1}{2!(z-0)^2} + \frac{1}{3!(z-0)^3} + ----$$

$R_1 = \operatorname{Re}s(f(z) : z = 0) = 1$

(Coefficient of $1/z$ in the above expansion)

$$\therefore \int_c e^{\frac{1}{z}} dz = 2\pi i \quad R_1 = 2\pi i(1) = 2\pi\sqrt{-1}$$

42. Ans: a

Sol: (1+i)(2-5i)=2-5i+2i+5=7-3i

43. Ans: d

Sol: Given $u = 3x^2 - 3y^2$

For f(x+iy) = u (x,y)+iv((x,y)

$$dv = v_x dx + v_y dy$$

$$= -u_y dx + u_x dy \quad (\because u_x = v_y \;\&\; v_x = -u_y)$$

$dv = -(0-6y)dx + 6xdy$ which is exact differential equation. integrating v = 6xy+k

44. Ans: b

Sol: $I = \int_c \frac{z^2}{z^4-1} dz = \int \frac{z^2}{(z^2-1)(z^2+1)} dz = \frac{1}{2}\int \left(\frac{1}{(z^2-1)} + \frac{1}{(z^2+1)} \right) dz$

$I = \frac{1}{2}\oint_c \frac{1}{z^2-1} dz + \frac{1}{2}\oint_c \frac{1}{z^2+1} dz$, C is $|z+1| = 1$

$$= \frac{1}{2}\oint_c \frac{1}{(z-1)(z+1)} dz + \frac{1}{2}\oint_c \frac{1}{(z+i)(z-i)} dz$$

Only z = -1 lies inside C. By cauchy integral formula $I = \frac{1}{2} 2\pi i \left[\frac{1}{z-1} \right]_{z=-1} + \frac{1}{2} 2\pi i(0)$

$$I = \frac{-\pi i}{2}$$

45. Ans: d

Sol: Given that the point z inside of the unit circle within the first quadrant

i.e. $|z| < 1$ ----(1)

But the given transformation is $w = \frac{1}{z}$

Now $|w| > 1$, represents the region outside the circle $|w| = 1$

$\Rightarrow w = \frac{1}{z} = \left(\frac{x}{x^2+y^2} \right) - i \left(\frac{y}{x^2+y^2} \right)$=u + iv

When the point z = (x, y) lies in the first quadrant when in the corresponding point w = (u, v) lies in fourth quadrant

46. Ans: d

Sol: $i = \sqrt{-1} = x$

Let $y = x^x \Rightarrow \log y = x\log x \Rightarrow \log y = i\log i = i\log e^{\frac{i\pi}{2}}$

$\Rightarrow \log y = i \times i\frac{\pi}{2} = -\frac{\pi}{2} \Rightarrow y = e^{\frac{-\pi}{2}}$

47. Ans: c

Sol: $\frac{1}{2\pi j}\int_c f(z)dz = \frac{1}{2\pi j}\left[\int_c \frac{1}{z+1}dz - \int_c \frac{2}{z+3}dz\right] = \frac{1}{2\pi j}\int_c \frac{1}{z+1}dz - 0$

(By Cauchy integral theorem)

= f(-1) where f(z) = 1

=1

48. Ans: b

Sol: we know that $i = e^{i\pi/2}$

$\therefore -i = \frac{1}{i} = e^{-i\pi/4}$

$\Rightarrow \sqrt{-i} = (-i)^{1/2} = e^{-i\pi/4}$

And $-i = i^3 = e^{i3\pi/2}$

$\therefore$ square root of – i are $e^{-i\pi/4}$, $e^{-3\pi/4}$

$e^{-i\pi/4} = \cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)$

$e^{i3\pi/4} = \cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

49. Ans: a

Sol: Given $I = \oint_c \frac{z^2-4}{z^2+4}dz$

The singular points of $\phi(z) = \frac{z^2-4}{z^2+4}$ are given by $z^2 + 4 = 0$

$\Rightarrow z^2+4=0 \Rightarrow z=2i,-2i$

But only z = 2i lies in $|z-i|=2$

$$\therefore \int \frac{z^2-4}{z^2+4}dz = \int \frac{z^2-4/z+2i}{z^2-2i}dz$$

By cauchy integral formula

$$i = 2\pi i\left[\frac{z^2-4}{z+2i}\right]_{z=2i} = -4\pi$$

50. Ans: c

Sol: z = -2j lies inside $|z|=3$

By cauchy integral formula

$$I = \oint_c \frac{z^2-z+4j}{(z-(-2j))}dz = 2\pi j f(-2j)$$

(where f(z) = z^2-z+4j)

$=2\pi j(-4+2j+4j)$

$= 2\pi j(6j-4)$

=-4 π (3+2j)

51. Ans: c

Sol: u(x,y) = $u(x,y)=e^{-y}\cos x$

Let v(x,y) =C

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = 0$$

$$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy = 0$$

$$= e^{-y}\cos x dx - e^{-y}\sin x dy = 0$$

$$= d(e^{-y}\sin x)$$

$$\therefore v = e^{-y}\sin x + C$$

52. Ans: c

Sol: w = u +iv

$f(z) = zz^* = (z+iy)(x-iy) = x^2 + y^2$ =1=(1+0.i)

$\therefore$ All the points on 's' are mapped to the point (1, 0) in w-plane

53. Ans: b

Sol: $1^i = (\cos 2n\pi + i\sin 2n\pi)^i \quad = e^{-2n\pi}$

It is always real and non-negative

54. Ans:c

Sol: The function $f(z) = \dfrac{z^2}{(z^2-1)}$ has singular point z = 1 which lies inside $|z-1| = 1$

$$I = \int_{|z-1|=1} \frac{z^2}{z^2-1} dz = \int_{|z-1|=1} \frac{z^2}{(z+1)(z-1)} dz$$

By cauchy integral formula

$I = 2\pi i f(1)$ (where $f(z) = \dfrac{z^2}{(z+1)}$)

$$= 2\pi i \frac{1}{2} = \pi i$$

55. Ans: c

$$\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{2} = \frac{2i}{2} = i = 0 + 1i$$

Let $(x+iy) = (0+1i)$

$$\therefore Argument = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

56. Ans: c

Sol: Let $V(x,y) = C$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy = 0$$

$$= -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy = 0$$

$$= -2xdx + 2ydy = 0$$

$$V = (-x^2 + y^2 + K)$$

57. Ans: c

Sol: Let $V(x,y)=C$

$$dV=\frac{\partial V}{\partial x}dx+\frac{\partial V}{\partial y}dy=0$$

$$=-\frac{\partial u}{\partial y}dx+\frac{\partial u}{\partial x}dy=0$$

$$=2ydx+2xdx$$

$$dV=2(xdy+ydx)=2d(xy)\Rightarrow V=2xy+C$$

58. Ans: b

Sol: $\int_5^{3i}\frac{dz}{z}=(\log z)_5^{3i}=\log 3i-\log 5=\log 3+\log i-\log 5=\log\left(\frac{3}{5}\right)+\log i$

$$=-0.511+i\frac{\pi}{2}=(-0.511+1.57i)$$

59. Ans: b

Sol: $z=\frac{2-3i}{-5+i}=\frac{(2-3i)}{-5+i}\times\frac{(-5-i)}{(-5-i)}=\frac{-13+13i}{26}=\left(\frac{-1}{2}+\frac{i}{2}\right)$

60. Sol. ()

For the given data f(z)= $\bar{z}$ =x-iy

$\Rightarrow$ u = x: v = - y

u_x = 1: v_x =0

u_y =0: v_y =-1

F is not analytic. So option (D) is correct remaining all options are correct

61. Sol. (10)

$f(z)=\frac{az+b}{cz+d}$: $f(z_1)=f(z_2)\forall z_1\neq z_2$

a = 2: b = 4: c =5

$$f(z)=\frac{2z+4}{5z+d}$$

Now, $f(z_1)=f(z_2)\Rightarrow\frac{2z_1+4}{5z_1+d}=\frac{2z_2+4}{5z_2+d}$

$$\Rightarrow 10z_1z_2+20z_2+2dz_1+4d=10z_1z_2+2dz_2+4d$$

$$\Rightarrow 20(z_2 - z_1) = 2d(z_2 - z_1) \Rightarrow d = 10$$

62. Sol. $\frac{1}{2}$

$$\frac{1}{2\pi i}\oint_c \operatorname{Re} a(z)dz : where \quad c : |z| = 1$$

Put $z = e^{i\theta} \Rightarrow dz = Ce^{i\theta}$

$$\frac{1}{2\pi i}\int_0^{2\pi} \operatorname{Re}(e^{i\theta}).ie^{i\theta}d\theta$$

$$= \frac{1}{2\pi i}\int_0^{2\pi} \cos\theta.i(\cos\theta + i\sin\theta)d\theta$$

$$= \frac{1}{2\pi i}\int_0^{2\pi} \cos^2\theta d\theta - \int_0^{2\pi} \cos\theta\sin\theta d\theta = \frac{i}{2\pi i}(\pi - 0) = \frac{1}{2}$$

63. Sol. ()

By cachy integral formula $\oint_c \frac{f(z)}{(z-z_0)^{n+1}}dz = \frac{2\pi i f^n(z_0)}{n!}$

$$\Rightarrow \oint_c \frac{f(z)}{(z-z_0)^{n+1}}dz = \frac{2\pi i}{n!} \times 0 = 0$$

64. Sol. **(B)**

Option B is suitable since the sum of two differentiable function is also a differentiable function at a point.

65. Sol. **()**

$$z_1 = 5 + (5\sqrt{3})i$$

$$\arg z_1 = \tan^{-1}\left(\frac{5\sqrt{3}}{5}\right) = \tan^{-1}\left(\sqrt{3}\right) = 60^o$$

$$z_2 = \frac{2}{\sqrt{3}} + 2i \Rightarrow \arg z_2 = \tan^{-1}\left(\frac{2}{2/\sqrt{3}}\right) = \tan^{-1}\left(\sqrt{3}\right) = 60^o$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = 60^o - 60^o = 0^o$$

66. Sol. **(D)**

Given V = u + iv $\Rightarrow V = \phi + i\psi$; Where ϕ is a potential function ψ is a velocity function

But we have $u = \frac{-\partial \phi}{\partial x} \Rightarrow \frac{\partial \phi}{\partial x} = -u(x, y)$

Similarly, $\frac{\partial \phi}{\partial y} = -v(x, y)$

Now $\frac{\partial y}{\partial x} = \frac{\partial \phi}{\partial x} \times \frac{\partial y}{\partial \phi} = \frac{u}{v}$

67. Sol.

For the given function z = 1 is a simple pole

Z = -2 is a pole of order 2

(i) $|\text{Re}\, f(z)|_{z=1} = \underset{z \to 1}{Lt} \frac{9}{(z+2)^2} = 1$

(ii) $|\text{Re}\, f(z)|_{z=-2} = \frac{1}{1!} \underset{z \to -2}{Lt} \frac{9}{(z-1)} = \underset{z \to -2}{Lt} \frac{9}{(z-1)} = -1$

So, A is correct option

68. Sol. (C)

$f(z) = u + iv \Rightarrow f'(z) = u_x + iv_x$

$\Rightarrow \frac{df}{dz} = u_y - iv_y$ ($\because$ f is analytic)

69. Sol. (-0.33)

$\frac{1}{2\pi i} \int_c \frac{e^{-2z}}{z(z-3)} dz \quad , c : |z| = 2 \, , -2 < z < 2$

Poles are z=0, 3

Z = 3 lies outside the circle

$f(0) = \frac{e^{-2.0}}{0-3} = -1/3$

$\therefore$ By cauchy integral formula $\frac{1}{2\pi i} \int_c \frac{e^{-2z}}{z(z-3)} dz = f(0) = -1/3 = -0.33$

70.**Ans: – 133.8**

Sol: $-\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3}dz \quad = \frac{1}{2\pi}\times 2\pi i \frac{f''(2\pi j)}{2!}$

$f(z) = \sin z$

$f''(z) = -\sin z$

$\therefore f''(Z_0) = -\sin 2\pi j$

$\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3}dz \quad = -\frac{1}{2\pi}\times 2\pi j\left(\frac{-\sin(2\pi j)}{2}\right)$

$= j\times j\frac{\sinh 2\pi}{2} \quad = -\frac{1}{2}(\sinh 2\pi) = -133.87$

71.**Ans: (B)**

Sol: i) $\frac{1}{2\pi j}\oint_c \frac{e^z}{(z-2)}dz = \frac{1}{2\pi j}2\pi j f(2) \Rightarrow e^2 = 7.39$

ii) $\frac{1}{2\pi j}\oint_c \frac{e^z}{(z-2)}dz = 0$ ($\because$ z = 2 lies out side c)

72.**Ans: (A)**

Sol: Given u =2xy, v = ?

The Cauchy-Riemann equation

$u_x = v_y$ & $v_x = -u_y$ are satisfying with option (a) $-x^2 + y^2$ + constant

$\therefore$ $V(x,y) = -x^2 + y^2$+ constant

73. **Ans: (A)**

Sol: $I = \int_{-\infty}^{\infty} \frac{Sin(x)}{x^2+2x+2}dx$

$Let\ f(z) = \frac{I_m(e^{iz})}{Z^2+2z+2}$

Then poles of f(z) are given by $z^2 + 2z + 2 = 0$

$\therefore$ z = -1 ± i

$R_1 = Res(f(z): z = -1+i) = \lim_{z\to 1+i} \frac{e^{iz}}{[z-(-1+i)][z-(-1-i)]}$

$$= \frac{e^{i(-1+i)}}{-1+i+1+i} = \frac{e^{-i-1}}{2i}$$

$$\int_c f(z)dz = \int_c \frac{I_{m(e^{iz})}}{z^2+2z+2}dz = I_m[2\pi i(R_1)]$$

$$= I_m\left[2\pi i\left(\frac{e^{-i-1}}{2i}\right)\right]$$

$$= I_m\left[\pi e^{-1}\left(\cos(1) - i\sin(1)\right)\right]$$

$$= -\frac{\pi \sin(1)}{e}$$

74.**Ans: –1**

Sol: Given, u = 2kxy & v = $x^2 - y^2$

$u_x = v_y$

$\Rightarrow 2ky = -2y$

$\therefore$ k = –1

75.**Ans: (B)**

Sol: L The correct path is given in option (B).

$$\int_c \frac{\left(\frac{3z-5}{z-2}\right)}{z-1}dz = 2\pi i\left[\frac{3z-5}{z-2}\right]_{z=1} = 4\pi i$$

76.**Ans:** 0

Sol: f (Z) = $2z^3 + b\,|z|^3$

for b = 0, f(z) becomes polynomial

so it is analytic every where only when b = 0

77.**Ans: 1**

Sol: $\frac{\sin(z)}{z^2} = \frac{1}{z^2}\left\{z - \frac{z^3}{3!} + \frac{z^2}{5!} - \dots\dots\dots\right\}$

$$= \frac{1}{z} - \frac{z}{3!} + \frac{z^3}{5!} - \dots\dots\dots$$

Res. f(z) = 1

z = 0

78.Ans: (b)

Sol: $f(z) = z + z^* = 2x$,

z^* is a continuous but not analytic

so, f(z) is continuous but not analytic

79.Ans: (b)

Sol: $f(z) = \dfrac{2(1/2)+5}{(1/2)^2 - 4(1/2) + 5} = \dfrac{6}{13/4} = \dfrac{24}{13}$

Ans = $2\pi i\ [\text{sum of residues}]$

$= 2\pi i \times \dfrac{24}{13} = \dfrac{48\pi i}{13}$

CHAPTER- 8
VECTOR CALCULUS

01. If the linear velocity $\overline{V}$ in given by $\overline{V} = x^2 y\bar{i} - \bar{j} - yz^3\bar{k}$ then the angular velocity $\overline{W}$ at point (1,1,-1) is it _____________ **(GATE -93)**

02. The directional derivative of f(x,y) = $2x^2+3y^2+z^2$ an point P(2,1,3) in the direction of the vector $\bar{a} = \bar{i} - \overline{2x}$ is **(GATE -94)**

a) $4/\sqrt{5}$ b) $-4/\sqrt{5}$ c) $\sqrt{5}/4$ d) $-\sqrt{5}/4$

03. If $\overline{V}$ is a differentiate vector function and f is sufficiently differentiable scalar function then curl $\left(f\overline{V}\right)$=_______________ **(GATE-95(ME)]**

a) $\left(\text{gradA}\times\overline{V}+\left(\text{fcurl}\overline{V}\right)\right)$ b) 0

c) f curl $\overline{V}$ d) (gradf) x $\overline{V}$

04. The derivation of f(x,y) and point (1,2) in the direction of vector $\bar{i}+\bar{j}$ in $2\sqrt{2}$ and in the direction of the vector $-2\bar{j}$ is -3, then the derivative of f(x,y) in direction $-\bar{i}-\bar{j}$ is **(GATE-95(ME)]**

a) $2\sqrt{2}+3/2$ b) $-7/\sqrt{5}$ c) $-2\sqrt{2}-3/2$ d) $1/\sqrt{5}$

05. The expression curl (grad f) where f is a scalar function is **(GATE-96(ME)]**

a) Equal to $\nabla^2 f$

b) Equal to div (grad f)

c) A scalar of zero magnitude

d) A vector of zero magnitude

06. The directional derivative of the function f(x,y,z) = x-y at the point P(1,1,0) along the direction $\bar{i}+\bar{j}$ is **(GATE-96)**

a) $1/\sqrt{2}$ b) $\sqrt{2}$ c) $-\sqrt{2}$ d) 2

07. For the function $\phi = ax^2y - y^3$ to represent the velocity potential of an ideal fluid, $\nabla^2\phi$ should be equal to zero. In that case, the value of 'a' has to be **(GATE – 99)**

a) -1 b) 1 c) -3 d) 3

08. Given a vector field $\overline{F}$, the divergence theorem sates that **(GATE – 99)**

a) $\int_S \overline{F}.d\overline{s} = \int_v \nabla.\overline{F}dv$

b) $\int_S \overline{F}.d\overline{s} = \int_v \overline{\nabla} \times \overline{F}dv$

c) $\int_S \overline{F} \times d\overline{s} = \int_v \nabla.\overline{F}dv$

d) $\int_S \overline{F} \times d\overline{s} = \int_v \nabla.\overline{F}dv$

09. The directional derivative of the following function at (1,2) in the direction of (4i+3j) is:

$F(x,y) = x^2+y^2$ **(GATE-02)**

a) 4/5 b) 4 c) 2/5 d) 1

10. The vector fields F = $x\overline{i} - y\overline{j}$ (where $\overline{i}$ and $\overline{j}$ are unit vectors) is **(GATE – 03)**

a) Divergence free, but not irrotational

b) Irrotational, but not divergence free

c) Divergence free and irrotational

d) Neither divergence free not irrotational

11. Value of the integral $\oint_c xydy - y^2dx$, where, c is the square cut from the first quadrant by the line x = 1 and y=1 will be (Use Green's theorem to change the line integral into double integral) **(GATE -05)**

a) 1/2 b) 1 c) 3/2 d) 5/3

12. The line integral $\int$ V.dr of the vector function $V(r) = 2xyz\overline{i} + x^2z\overline{j} + x^2y\overline{k}$ from the origin to the point P(1,1,1) **(GATE-05)**

a) is 1 b) is zero c) is -1

d) cannot be determined without specifying the path

13. Stokes theorem connects **(GATE -05(ME)]**

a) a line integral and a surface integral

b) a surface integral and a volume integral

c) a line integral and a volume integral

d) gradient of a function and its surface integral

14. For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, the magnitude of the gradient at the point(1,3) is

(GATE – 05 (EE)]

a) $\sqrt{\frac{13}{9}}$ b) $\frac{9}{2}$ c) $\sqrt{5}$ d) $\frac{9}{2}$

15. If a vector $\overline{R}(t)$ has a constant magnitude then **(GATE -05 (IN)]**

a) $\overline{R} \cdot \frac{d\overline{R}}{dt} = 0$ b) $\overline{R} \cdot \frac{d\overline{R}}{dt} = 0$ c) $\overline{R} \cdot \overline{R} = \frac{d\overline{R}}{dt}$ d) $\overline{R} \cdot \overline{R} = \frac{d\overline{R}}{dt}$

16. A scalar field is given by f = $x^{2/3}+y^{2/3}$, where x and y are the Cartesian coordinates. The derivative of 'f' along the line y = x directed away from the origin at the point (8,8) is

(GATE -05 (IN)]

a) $\frac{\sqrt{2}}{3}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{3}{\sqrt{2}}$

17. Which one of the following is Not associated with vector calculus? **(GATE-05(PI)]**

a) Stoke's theorem b) Gauss Divergence theorem

c) Green's theorem d) Kennedy's theorem

18. $\nabla \times (\nabla \times P)$ where P is a vector is equal to **(GATE-05EC)]**

a) $P \times \nabla \times P - \nabla^2 P$ b) $\nabla^2 P + \nabla(\nabla \cdot P)$ c) $\nabla^2 P + (\nabla \times P)$ d) $\nabla(\nabla . P) - \nabla^2 P$

19. The directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at the point p(2,1,3) in the direction of the vector $\bar{a} = \bar{i} - 2\bar{k}$ *is* ________ **(GATE -06(CE)]**

a) -2.785 b) -2.145 c) -1.789 d) 1.000

20. The velocity vector is given as $\overline{V} = 5xy\bar{i} + 2y^2\bar{j} + 3yz^2\bar{k}$. the divergence of this velocity vector at (1,1,1) is **(GATE – 07(CE)]**

21. The area of a triangle formed by the tips of vectors, $\bar{a}, \bar{b}$ and $\bar{c}$ **(GATE – 07(ME)]**

a) $\frac{1}{2}|(\bar{a}-\bar{b})\cdot(\bar{a}-\bar{c})|$ b) $\frac{1}{2}(\bar{a}-\bar{b})\times(\bar{a}-\bar{c})$

c) $\frac{1}{2}|\bar{a}\times\bar{b}\times\bar{c}|$ d) $\frac{1}{2}(\bar{a}\times\bar{b})\cdot\bar{c}$

22. The angel (in degrees) between two planer vectors $\bar{a} = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$ and $\bar{b} = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$ is

(GATE-07(PI)]

a) 30 b) 60 c) 90 d) 120

23. Divergence of the vector field v(x,y,z) = (-x cosxy + y) $\bar{i} + (y\cos xy)\bar{j} + \left[(\sin z^2) + x^2 + y^2\right]\bar{k}$ is **(GATE -07(EE)]**

(a) $2z \cos z^2$ b) $\sin xy + 2z \cos z^2$ c) xsinxy –xcosz d) none of these

24. Consider points P and Q in xy – plane with P = (1,0) and Q = (0,1). The line integral $2\int_{\tau}^{Q} (x\,dx + y\,dy)$ along the semicircle with the line segment PQ as its diameter

(GATE -08(EC))

a) is -1 b) is 0 c) 1 d) depends on the direction (clockwise (or) anti-clockwise) of the semi circle

25. The divergence of the vector field $(x-y)\bar{i} + (y-x)\bar{j} + (x+y+z)\bar{k}$ is **(GATE -08(ME)]**

a) 0 b) 1 c) 2 d) 3

26. The directional derivative of the scalar function f(x,y,z) = x^2+2y^2+z at the point P = (1,1,2) in the direction of the vector $\bar{a} = 3\bar{i} - 4\bar{j}$ is **(GATE-08(ME)]**

a) -4 b) -2 c) -1 d) 1

27. If $\bar{r}$ is the position vector of any point on a closed surface S that encloses the volume V then $\iint_s (\bar{r} \cdot d\bar{s})$ is equal to **(GATE -08(PI)]**

a) $\frac{1}{2}$v b) V c) 2V d) 3V

28. For a scalar function f(x,y,z) = $x^2+3y^2+2z^2$, the gradient at the point P(1,2,-1) is

(GATE – 09 (CE)]

a) $2\bar{i} + 6\bar{j} + 4\bar{k}$ b) $2\bar{i} + 12\bar{j} - 4\bar{k}$ c) $2\bar{i} + 12\bar{j} + 4\bar{k}$ d) $\sqrt{2}$

29. For a scalar function f(x,y,z) = x^2-+$3y^2+2z^2$, the directional derivative at the point P(1,2,-1) in the direction of a vector $\bar{i} - \bar{j} + 2\bar{k}$ is

(GATE-09(CE)]

a) -18 b) $-3\sqrt{6}$ c) $3\sqrt{6}$ d) 18

30. If a vector field $\overline{V}$ is related to another field $\overline{A}$ through $\overline{V} = \nabla \times \overline{A}$, which of the following is true? **(GATE-09(EC)]**

Note: C and S_C refer to any closed contour and any surface whose boundary is C.

a) $\oint_C \overrightarrow{V} \cdot d\ell = \int\int_{S_C} \overrightarrow{A} \cdot \overrightarrow{ds}$

b) $\oint_C \overrightarrow{A} \cdot d\ell = \int\int_{S_C} \overrightarrow{V} . \overrightarrow{ds}$

c) $\oint_C \nabla \times \overrightarrow{V} \cdot \overrightarrow{d\ell} = \int\int_{S_C} \overrightarrow{A} \cdot \overrightarrow{ds}$

d) $\oint_C \nabla \times \overrightarrow{A} \cdot \overrightarrow{d\ell} = \int\int_{S_C} \overrightarrow{V} \cdot \overrightarrow{ds}$

31. A sphere of unit radius is centered at the origin, the unit normal at appoint (x,y,z) on the surface of the sphere is the vector. **(GATE-09(IN)]**

a) (x, y, z) b) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ c) $\left(\frac{x}{\sqrt{3}}, \frac{u}{\sqrt{3}}, \frac{z}{\sqrt{3}}\right)$ d) $\left(\frac{x}{\sqrt{2}}, \frac{y}{\sqrt{2}}, \frac{z}{\sqrt{2}}\right)$

32. The divergence of the vector field $3xz\,\hat{i} + 2xy\,\hat{j} - yz^2\,\hat{k}$ at a point (1, 1, 1) is equal to

(GATE-09(ME)]

a) 7 b) 4 c) 3 d) 0

33. $F(x.y) = \left(x^2 + xy\right)\hat{a}_x + \left(y^2 + xy\right)\hat{a}_y$. Its line integral over the straight line from (x,y) = (0,2) to (x,y) = (2,0) evaluates to **(GATE-09(EE)]**

a) -8 b) 4 c) 8 d) 0

34. The line integral of the vector function $\overline{F} = 2x\,\hat{i} + x^2\,\hat{j}$ along the x-axis from x=1 to x=2 is

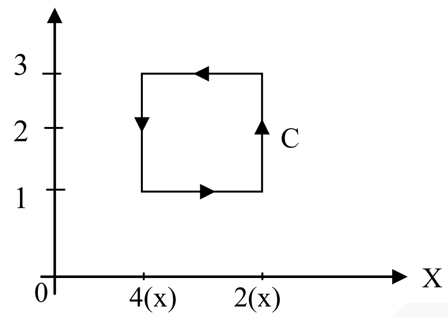
(GATE-09(PI)]

a) 0 b) 2.33 c) 3 d) 5.33

35. Divergence of the 3- dimensional radial vector field $\overline{r}$ is **(GATE-10(EE)]**

a) 3 b) $\frac{1}{r}$ c) $\hat{i} + \hat{j} + \hat{k}$ d) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

36. If $\vec{A} = xy\,\hat{a}_x + x^2\,\hat{a}_y$ then $\oint \vec{A}$ over the path shown in the figure is



a) 0 b) $\frac{2}{\sqrt{3}}$ c) 1 d) $2\sqrt{3}$

37. If $\bar{a}$ and $\bar{b}$ are two arbitrary vectors with magnitudes a and b respectively, $|\bar{a}\times\bar{b}|^2$ will be equal to **(GATE-11(CE)]**

a) $a^2b^2 - (\bar{a},\bar{b})^2$ b) ab-$\bar{a}\cdot\bar{b}$ c) $a^2b^2 + (a\cdot\bar{b})^2$ c) $ab + \bar{a}.\bar{b}$

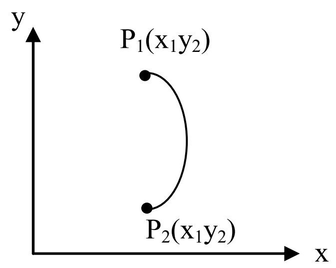
38. If A(0,4,3), B(0,0,0) and c(3,0,4) are there points declined as x, y, z coordinate system, then with one of the following vectors is perpendicular to both the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$

(GATE-11(PI))

a) $16\vec{j} + 9j - 12\overrightarrow{K}$ b) $\overrightarrow{16}i - 9\vec{j} + 12\vec{k}$

c) $16\vec{i} - 9\vec{j} - \vec{k}$ d) $16\vec{i} + 9\vec{j} + 12\vec{k}$

39. The line integral $\int_{P_1}^{P_2}(ydx + xdy)$ from ($P_1(x_1,y_2)$ to $P_2(x_2,y_2)$ along the semi-circle P_1P_2 shown in the figure is **(GATE-11(PI))**



a) $x_1y_2 - x_1y_1$

b) $\left(y_2^2 - y_1^2\right) + \left(x_2^2 - x_1^2\right)$

c) $\left(x_2 - x_1\right)\left(y_2 - y_1\right)$

d) $\left(y_2 - y_1\right)^2 + \left(x_2 - x_1\right)^2$

40. If T(X, Y, Z) = $x^2+y^2+2z^3$ defines the temperature at any location (x, y, z) then the magnitude of the temperature gradient at point (P(1,1,1) is __________. **(GATE-11(PI))**

a) $2\sqrt{6}$ b) 4 c) 24 d) $\sqrt{6}$

41. Consider a closed surface 'S' surrounding a volume V. If $\vec{r}$ is the position vector of point inside S with $\vec{n}$ the unit formal on 'S'. the value of the integral $\oiint 5\vec{r}.\hat{n}\,ds$ is

(GATE-11(EC))

a) 3V b) 5V c) 10 V d) 15 V

42. The two vectors [1,1,1] and [1,a,a^2] where $a = \frac{-i}{2} + j\frac{\sqrt{3}}{2}$ are **(GATE -11(EE)**

a) Orthonormal b) Orthogonal c) Paralist d) Collinear

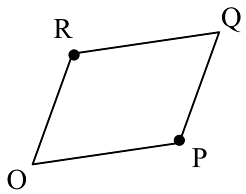
43. the direction of vector A is radially outward from the origin, with |A| = Kr^n value of n for which ∇. A=0 is **(GATE-12(EC,EE,IN))**

a) -2 b) 2 c)1 d) 0

44. For the spherical surface $x^2+y^2+z^3=1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by **(GATE-12(ME,PI))**

a) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ b) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$ c) $\hat{k}$ d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

45. For the parallelogram OPQR shown in the sketch. $\overline{OP} = a\hat{i} + b\hat{j}$ and $\overline{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is **(GATE-129CE))**



a) ad –be b0 ac + bd c) ad +bc d) ab-cd

46. Consider a vector field $\vec{A}(\bar{r})$. The closed loop line integral $\oint \vec{A}\cdot\vec{dt}$ can be expressed as

(GATE – 2013(EC))

a) $\oiint (\nabla\times\vec{A})\cdot\vec{ds}$ over the closed surface bounded by the loop

b) $\oiiint (\nabla\cdot\vec{A})\cdot\vec{dv}$ over the closed volume bounded by the loop

c) $\iiint (\nabla\cdot\vec{A})dv$ over the open volume bounded by the loop

d) $\iint (\nabla\times\vec{A})\cdot\vec{ds}$ over the open surface bounded by the loop

47. The divergence of the vector field statements is NOT TRUE? **(GATE-2013(EC))**

a) 0 b) 1/3 c) 1 d) 3

48. For a vector E, which one of the following **(GATE-2013(IN))**

a) If ∇E =0, E is called solenoidal

b) If ∇ x E = 0, E is called conservative

c) If ∇ x E = 0,E is called irrotational

d) If ∇ . E =0, E is called irrotational

49. The following surface integral is to be evaluated over a sphere for the givens steady velocity vector field F = xi + yj + zk defined with respect to a certain coordinate system having i,j and k as unit base vectors.

$$\iint_s \frac{1}{4}(F.n)dA$$

Where S is the sphere, $x^2+y^2+z^2=1$ and n is the outward unit normal vector to the sphere. The value of the surface integral is **(GATE – 2013(ME))**

a) π b) 2π c) 3 π/4 d) 4 π

50. The curl of the gradient of the scalar field defined by $V= 2x^2y+3y^2z+4z^2x$ is

a) $4xya_x+6yza_y+8zxa_z$

b) $4a_x+6a_y+8a_z$

c) $(4xy+4z^2)a_x+(2x^2+6yz)a_y+(3y^2+8zx)a_z$

d) 0

51. Given a vector field $\overline{F} = y^2 x \hat{a}_x - yz\hat{a}_y - x^2\hat{a}_z$, the line integral $\int F.dt$ evaluated along a segment on the x-axis from x=1 ω x = 2 is **(GATE-2013(EE))**

a) 2.33 b) 0 c) 2.33 d) 7

52. If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and $|\vec{r}| = r$, then $dx\left(\tau^2 \nabla(\log r)\right) =$ _______. **(GATE-14-EC-)**

53. The magnitude of the gradient for the function f(x,y,z)= $x^2+3y^2+z^2$ at the point (1,1,1) is _______. **(GATE – 14-EC)**

54. The directional derivative of $f(x,y) = \frac{xy}{\sqrt{2}}(x+y)$ at (1,1) in the direction of the unit vector at an angle of $\frac{\pi}{4}$ with y-axis, at given by ___________. **(GATE-14-EC)**

55. Given $\vec{F} = z\hat{a}_x + x\hat{a}_y + y\hat{a}_z$, if S represents the portion of the sphere $x^2+y^2+z^2=1$ for z ≥0, then $\int_s (\nabla \times \vec{F}.)\overline{ds}$ is ____________. **(GATE-14-EC)**

56. The line integral of function F = yzi, in the counterclockwise direction, along the circle $x^2+y^2=1$ at z=1 is **(GATE-14-EE)**

a) -2π b) – π c) π d) 2 π

57. Let $\nabla.(f\,V) = x^2y + y^2z + z^2x$, where f and V if $V = yi + zj + xk$, then $V.(\nabla f)$ is **(GATE-14-EE)**

a) $x^2y+y^2z+z^2x$ b) 2xy+2yz+2zx

c) x+y+z D) 0

58. A vector is defined as $f = y\hat{i} + x\hat{j} + z\hat{k}$ where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors in Cartesian (x,y,z) coordinate system. The surface integral $\oint\oint f.ds$ over the closed surface S of a cube with Vertices having the following coordinates: (0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,0,1), (1,1,1), (0,1,1), (1,1,0) is___________. **(GATE -14 –ME)**

59. The integral $\oint_C (ydx\text{-}xdy)$ is evaluated along the circle $x^2 + y^2 = \frac{1}{4}$ traversed in counter clockwise direction. The integral is equal to **(GATE -14-ME)**

a) 0 b) $-\frac{\pi}{4}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{4}$

60. Curl of vector $\overline{F} = x^2z^2\hat{i} - 2xy^2z\hat{j} + 2y^2z^3\hat{k}$ **(GATE-14-ME)**

a) $\left(4yz^3 + 2xy^3\right)\hat{i} + 2x^3z\hat{j} - 2y^2z\hat{k}$

b) $\left(4yz^3 + 2xy^3\right)\hat{i} - 2x^3z - 2y^3z\hat{k}$

c) $2xz^3\hat{i} - 4xyz\hat{j} + 6y^3z^3\hat{k}$

d) $2xz^3\hat{j} + 4xyz\hat{j} + 6y^2z^3\hat{k}$

61. Divergence of the a vector field $x^2z\hat{i} + xy\hat{j} - yz^2\hat{k}$ at $(1,-1,1)$ is **(GATE-14-ME)**

a) 0 b) 3 c) 5 d) 6

62. Directional derivative of $\phi = 2xz - y^2$ at the point (1,3,2) becomes maximum in the direction of **(GATE-14-PI)**

a) 4i+2j-3k b) 4j-6j+2k c) 2i-6j+2k d) 4i-6j-2k

63. If $\phi = 2x^3y^2z^4$ then $\nabla^2\phi$ is

a) $12x^2z^4$+4x2z^4+20x3y2z3

b) $2x^2y^2z+4x^3z^4+24x^3y^2z^2$

c) $12xy^2z^4+4x^3z^4+24x^3y^2z^2$

d) $4xy^2z+4x^2z^4+24x^3y^2z^2$

64. Vector $\overline{P}$ is given by $\overline{P} = x^3y\hat{a}_x - x^2y\hat{a}_y - x^2yz\hat{a}_z$. Which of the following is true? **(GATE - EC -15)**

A) $\overline{P}$ is solenoidal but not irrational B) $\overline{P}$ is irrotational, but not solenoidal

C) $\overline{P}$ is neither solenoidal (nor) irrational D) $\overline{P}$ is both solenoidal and irrotational

65. Consider the function $\overline{f} = \frac{1}{r^2}\overline{r}$, where r is a distance from the origin and $\overline{r}$ is a unit vector in the radial direction. The divergence of the function over a sphere of radius R. Which includes origin is **(GATE - EC -15)**

A) 0 B) 2π C) 4π D) $R\pi$

66. Match the following **(GATE - EE -15)**

P. Stoke's theorem 1. $\oiint D.ds = \phi$

Q. Gauss theorem | 2. $\oint f(z)dx = 0$

R. Divergence theorem | 3. $\iiint (\nabla . A)dy = \oiint A.ds$

S. Cauchy integral theorem | 4. $\iint (\nabla \times A)ds = \oint A.dI$

A) P-2, Q-1, R-4, S-3 B) P-4, Q-1, R-3, S-2

C) P-4, Q-3, R-1, S-2 D) P-3, Q-4, R-2, S-1

67. The velocity field of an incompressible flow is given by **(GATE - ME -15)**
$V = (a_1x + a_2y + a_3z)i + (b_1x + b_2y + b_3z)j + (c_1x + c_2y + c_3z)k$ where $a_1 = 2; c_3 = -4$ the value of b is

68. Curl of a vector $V(x, y, z) = 2x^2 i + z^2 j + y^3 k$ at x = y =z=1 is **(GATE - ME -15)**

A) -3i B) 3i C) 3i-4j D) 3i-6k

69. The surface integral $\iint_s \frac{1}{\pi}(9xi - 3yj).nds$ over the sphere $x^2 + y^2 + z^2 = 9$ is

(GATE - ME -15)

70. Let ϕ is an arbitrary constant smooth real valued scalar function and V is an arbitrary smooth vector valued function in three dimension space which of the following is identify **(GATE - ME -15)**

A) $curl(\phi \bar{V}) = \nabla(Div \bar{V})$ B) $div \bar{V} = 0$

C) $Div curl \bar{V} = 0$ D) $Div(\phi \bar{V}) = \phi Div \bar{V}$

71. The D.D of the field u(x, y, z) = $x^2 - 3yz$ in the direction of a vector I + j – 2k at a point (2, -1, 4) is **(GATE - CE -15)**

72. The velocity components of a two dimensional motion plane of a fluid are
$u = \frac{y^3}{3} + 2x - x^2 y$, which of the following is correct **(GATE - CE -15)**

A) Fluid is incompressible and flow is irrotational

B) Fluid is incompressible and flow is rotational

C) Fluid is compressible and flow is irrotational

D) Fluid is compressible and flow is rotational

73. A Scalar function in XY plane is given by $\phi(x,y) = x^2 + y^2$, if $\hat{i}$ *and* $\hat{j}$ are unit vectors in x and y directions, the direction of maximum increase in the value of ϕ at (1, 1) is a long

(GATE - CH -15)

A) $-2\hat{i}+2\hat{j}$ B) $2\hat{i}+2\hat{j}$ C) $-2\hat{i}-2\hat{j}$ D) $2\hat{i}-2\hat{j}$

74. A scalar potential φ has the following gradient: $\nabla\varphi = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$. Consider the integral $\int_C \nabla\varphi.\overrightarrow{dr}$ on the curve $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. The curve *C* is parameterized as follows:

(GATE - ME -16)

$$\begin{cases} x = t \\ y = t^2 \\ z = 3t^2 \end{cases} \quad and \ 1 \le t \le 3.$$

The value of the integral is ________

75. The value of the line integral $\oint_c \bar{F}.\bar{r}'\,ds$, where C is a circle of radius $\frac{4}{\sqrt{\pi}}$ units is ________

Here, $\bar{F}(x, y) = y\,\hat{\imath} + 2x\,\hat{\jmath}$ and $\bar{r}\,'$ is the **UNIT** tangent vector on the curve *C* at an arc length *s* from a reference point on the curve. $\hat{\imath}$ and $\hat{\jmath}$ are the basis vectors in the *x-y* Cartesian reference. In evaluating the line integral, the curve has to be traversed in the counter-clockwise direction. **(GATE - ME -16)**

76. Suppose C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anti-clockwise. The value of $\int_C \left(xy^2dx + x^2ydy\right)$ over the curve C equals _________

(GATE - EC -16)

VECTOR CALCULUS
ANSWERS WITH SOLUTIONS

01. We know angular velocity $\overline{W} = \frac{1}{2}\begin{vmatrix} i & j & \overline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xyz & -yz^2 \end{vmatrix}$

$$= \frac{1}{2}\left[i\left(-z^2 - xy\right) - j\left(0-0\right) + \overline{K}\left(yz - x^2\right)\right]$$

$$\text{At } \left(1,1,-1\right) = \frac{1}{2}\left[-2i - 2\overline{K}\right] = -\left(i + \overline{k}\right)$$

02. Sol: (b) Given scalar point function is $f(x,y) = 2x^2+3y^2+a^2$

$$\left(\nabla_f\right)_{(2,1,3)} = 4xi + 6yj + 2z\overline{k}$$

$$= 8i + 6j + 6\overline{k}$$

$$\text{Directional derivative } = \nabla t. = \frac{\overline{a}}{|\overline{a}|} = \frac{-4}{\sqrt{5}}$$

03. Sol: Option (a) is vector identity

04. Sol: (b)

Directional derivative of f in the direction of i+j = $2\sqrt{2}$

Similarly,

$$\nabla f.\frac{\overline{b}}{|\overline{b}|} = -3 \Rightarrow \left(\frac{\partial f}{\partial x}i + j\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\overline{k}\right)\cdot\frac{(i+j)}{\sqrt{2}} = 2\sqrt{2} \Rightarrow \frac{\partial f}{\partial x} + \frac{\partial f}{dy} = 4......(1)$$

Similarly,

$$\nabla f.\frac{\overline{b}}{|\overline{b}|} = -3 \Rightarrow \left(\frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}\overline{k}\right)\cdot\frac{(-2j)}{2} = -3 \Rightarrow -2\frac{\partial f}{dy} = -6 \Rightarrow \frac{\partial f}{dy} = 3......(2)$$

Sub. (2) in (1), we obtain $\frac{\partial f}{dx} = 1$

$$\therefore \nabla f\cdot\frac{\overline{c}}{|\overline{c}|} = \left(\frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{dz}\overline{k}\right).\frac{(-i-2j)}{\sqrt{5}} = \frac{-7}{\sqrt{5}}$$

05. Sol: (d)

Curl (gradϕ) = $\bar{0}$

06. Sol: (b)

f(x,y,z) = x+y P(1,1,0), $\bar{a} = i + j$

directional derivative = $\nabla f.\frac{a}{|a|} = \sqrt{2}$

07. Sol : (d)

$$\phi = ax^2y - y^2 = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

08. Sol: (a) Gauss divergence theorem

9. Sol : (b)

$F(x,y) = x^2 + y^2, \bar{a} = 4i+3j(1,2)$

Directional derivative = $\nabla f.\frac{\bar{a}}{|\bar{a}|} = (2i+4j).\frac{(4i+3j)}{5} = 4$

10. Sol: (c)

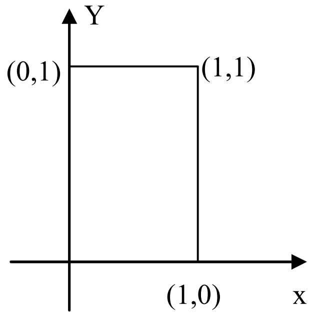
$\bar{F} = xi - yj$

div $\bar{F} = 1 - 1 = 0 \Rightarrow \bar{F}$ is solinaidal

$$\text{curl } \bar{F} = \begin{vmatrix} i & j & \bar{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix} = \bar{0} \Rightarrow \bar{F} \text{ is irroational}$$

11. Sol: (c)

$$\oint_c xydy - y^2dx$$



By Green's theorem $\oint_e Mdx - Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\, dy$

$$= \int_{x=0}^{1} \int_{y=0}^{1} (y + 2y) \text{dx dy} = \frac{3}{2}$$

12. Sol : (a)

$V(r) = 2xyz\,\bar{i} + x^2 z\,\bar{j} + x^2 y\bar{k}$

For this function clearly curl $\bar{V} = 0$

$\Rightarrow \bar{V} = \nabla \phi$

$\Rightarrow \bar{V} = i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{dy} + k\frac{\partial \phi}{\partial z}$

$\Rightarrow \phi = x^2 yz$

$\therefore \int_{(0,0,0)}^{(1,1,1)} \bar{V}.d\bar{r} = (\phi)_{(0,0,0)}^{(1,1,1)} = 1$

13. Sol : (a)

$\int_c \bar{F}.d\bar{r} = \iint_s \text{curl } \bar{F}, \text{ d}\bar{s}$

14. Ans: (c)

$\nabla u = xi + \frac{2}{3} yj$

$\nabla u_{(1,3)} = i + 2j$

Magnitude of D. D is $|\nabla u| = \sqrt{1+4} = 5$

15. Sol : (a)

$\overline{R}(t)$ is a vector with constant magnitude $\Rightarrow |R| = R$ = constant

$\overline{R}\,\overline{R} = R^2$

$\Rightarrow \frac{d}{dt}\left(\overline{R}.\overline{R}\right) = \frac{d}{dt}\left(R^2\right) = 0$

$\Rightarrow \frac{d\overline{R}}{dt}.\overline{R} + \overline{R}.\frac{d\overline{R}}{dt} = 0 \Rightarrow 2\overline{R}.\frac{d\overline{R}}{dt} = 0 \Rightarrow \overline{R}\,\frac{d\overline{R}}{dt} = 0$

16. Sol: (a)

A unit vector along the line y = x is $\hat{a} = \cos\theta i + \sin\theta\, j$, where $\theta = \pi/4$

$\hat{a} = \cos\frac{\pi}{4} i + \frac{\pi}{4} j = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} \overline{j}$

$\nabla f = \frac{2}{3} \times^{\frac{-1}{3}} \overline{i} \frac{2}{3} y^{\frac{-1}{3}} \overline{j}$

$\therefore$ Directional derivative = $\nabla f \cdot \frac{\overline{a}}{|\overline{a}|}$

$= \frac{1}{3\sqrt{2}} + \frac{1}{3\sqrt{2}} = \frac{\sqrt{2}}{3} j$

17. Sol: (d)

Kennedy's Theorem is not related to vector calculus

18. Sol: (d)

Curl (Curl P) = $\nabla \times (\nabla \times P) = grad\,(div\,P) - \nabla^2 P$

$= \nabla(\nabla.P) - \nabla^2 P$ (Vector identity)

19. Sol:(c)

$\nabla f\,]_{(2,1,3)} = 8i + 6j + 6\overline{k}$

Directional derivative = $\nabla f \frac{\overline{a}}{|\overline{a}|} = \frac{-4}{\sqrt{5}}$

20. Sol: (d)

$\overline{V} = 5xy\bar{i} + 2y^2\bar{j} + 3yz^2\overline{k}$

div $\overline{V} = 5y + 4y + 6yz$

At $(1,1,1)$ div $\overline{V} = 5 + 4 + 6 = 15$

21. Sol: (b)

Area of triangle ABC = $\frac{1}{2}|\overline{AB} \times \overline{AC}|$

$= \frac{1}{2}|(\bar{a} - \bar{b}) \times (\bar{c} - \bar{a})$

$= \frac{1}{2}|(\bar{a} - \bar{b}) \times (\bar{a} - \bar{c})|$

22. Sol : (d)

$\bar{a} = \frac{\sqrt{3}}{2}i + \frac{1}{2}j, \quad \bar{b} = -\frac{\sqrt{3}}{2}i + \frac{1}{2}j$

$\cos\theta = \frac{\bar{a}}{|\bar{a}||\bar{b}|} = \frac{-\frac{3}{4} + \frac{1}{4}}{\sqrt{\frac{3}{4} + \frac{1}{4}}\sqrt{\frac{3}{4} + \frac{1}{4}}} = -\frac{1}{2}$

$\Rightarrow \theta = 120$

23. Sol : (a)

$V(x, y, z) = -(x \cos xy + y)\bar{i} + (y \cos xy)\bar{j} + \left[\sin z^3 + x^2 + y^2\right]\bar{k}$

$\operatorname{div}\overline{V} = \frac{\partial}{\partial x}\left[-(x\cos xy + y)\right] + \frac{\partial}{\partial y}\left[\sin z^2 + x^2 + y^2\right]$

$= xy \sin xy - xy \sin xy + 2z \cos z^2$

$= 2z\cos z^2$

24. Sol : (b)\

P(1,0), Q(0,1)

$2\int_P^Q x\,dx + y\,dy = 2\int_P^Q \overline{F}.d\bar{r}.\text{where} = \overline{F}\,xi + yj$

curl $\overline{F} = \bar{0} \Rightarrow$ there exists scalar function $\phi(x, y)$

Such that $\overline{F} = \nabla\phi$

$$\therefore \phi(x,y) = \frac{x^2 + y^2}{2}$$

$$\therefore 2\int_P^0 x\,dx + y\,dy = \int_P^0 \overline{F}.dr. Where\,\overline{F} = xi + yj$$

25. Sol : (d)

$$\overline{V} = (x-y)i + (y-x)j + (x+y+z)\overline{k}$$

div $\overline{V} = 1+1+1 = 3$

26. Sol: (b)

$$f(x,y,z) = x^2 + 2y^2 + z, p(1,1,2), \overline{a} = 3i - 4j$$

$$\nabla f = 2xi + 4yj + \overline{k}$$

$$\nabla f_p = 2i + 4j + \overline{k}$$

Directional derivative = $\nabla f.\frac{\overline{a}}{|\overline{a}|} = \frac{6-16}{5} = -\frac{10}{5} = -2$

27. Sol : (d)

As ‘s’ a closed surface, using Gauss – Divergence theorem,

$$\int\int_s \overline{r}d\overline{s} = \int\int\int_v (\nabla.\overline{r})dv = 3v$$

28. Sol: (b)

$$f(x,y,z) = x^2 + 3y^2 + 2z^2 \qquad p(1,2,-1)$$

Grad $f = \nabla f = 2xi + 6yj + 4z\overline{k}$

$$\nabla f_p = 2i + 12j - 4\overline{k}$$

29. Sol: (b)

$$F(x,y,z) = x^2 + 3y^2 + 2z^2,\ P(1,2,-1),\ \overline{a} = i - j + 2\overline{k}$$

$$\nabla f = 2xi + 6yj + 4z\overline{k}$$

$$\nabla f_p = 2i + 12j - 4\overline{k}$$

Directional derivative = $= \nabla f.\frac{\bar{a}}{|\bar{a}|} = \frac{2-12-8}{\sqrt{1+1+4}} = \frac{18}{\sqrt{6}} = -3\sqrt{6}$

30. Sol: (b)

Using stokes theorem for $\bar{A}$ over the 'C' bounding an open surface S_C,

$$\oint_C \bar{A}.d\bar{l} = \int\int_{S_C} \text{curl } \bar{A}.d\bar{s}$$

$$= \int\int_{S_C} \bar{V}.d\bar{s}$$

31. Sol: (a)

Equation of sphere is $x^2+y^2+z^2=1$

Lit $\phi(x,y,z) = x^2+y^2+z^2$

Normal to the surface is $\nabla\phi = 2xi + 2yj + 2z\bar{k}$

Unit normal is $\frac{\nabla\phi}{|\nabla\phi|} = \frac{2(xi+yj+z\bar{k})}{\sqrt{4x^2+4y^2+4z^2}}$

$$= xi + yj + z\bar{k}\left[\sin ce\, x^2+y^2+z^2=1\right]$$

32. Sol : (C)

$\bar{f} = 3xz\bar{i} + 2zy\bar{j} - yz^2\bar{k}$

$div\ \bar{f} = 3z + 2y - 2yz$

$div\ \bar{f}_{(1,1,3)} = 3+2-2=3$

33. Sol: (d)

$f(x,y) = (x^2+xy)\hat{a}_x + (y^2+xy)\hat{a}_y$

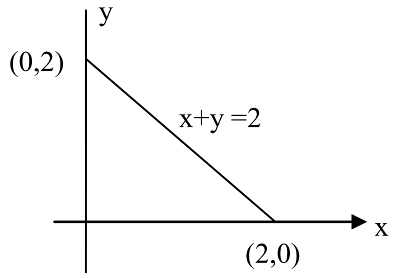
$\int \bar{F}.d\bar{r} = \int_c (x^2+xy)dx + (y^2+xy)dy$

$x+y=2 \Rightarrow dx = -dy$

$= \int_C \left[x^2 + x(2-x)\right]dx + \left[(2-x)^2 + x(2-x)\right](-dx)$

$= 0$

Ans;(c)



34. Sol:

$\bar{F} = 2x\bar{i} + x^2\bar{j}$

Along x-axis, y=0 → dy=0

$$\int_C \bar{F}.dr\bar{F} = \int_C 2xdx + x^2dy = \int_1^2 2xdx = 3$$

35. Sol: (a)

$\bar{r} = x\bar{i} + yj + z\bar{k}$

$$\text{div } \bar{r} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_2}{\partial z} = 1 + 1 + 1 = 3$$

36. Sol: (c)

$\bar{A} = xy\,\hat{a}_x + x^2\,\hat{a}_y$

$$\oint_C \bar{A}.d\bar{\mathrm{i}} = \oint_C xydx + x^2dy$$

Using green's theorem,

$$\oint_C M\,dx + Ndy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dx\,dy$$

$$= \int_{x=\frac{1}{\sqrt{3}}}^{\frac{2}{\sqrt{3}}} \int_{y=1}^{3} (2x - x)\,dy\,dx$$

$$= \int_{\frac{1}{\sqrt{3}}}^{\frac{2}{\sqrt{3}}} 2xdx = 1$$

37. Sol: (a)

$$\left|\overline{a}\times\overline{b}\right|^2 = \left|\overline{a}\right|^2\left|\overline{a}\right|^2 \sin^2\left(\overline{a},\overline{b}\right) = a^2b^2\left(1-\cos^2\left(\overline{a},\overline{b}\right)\right)$$

$$a^2b^2\left(1-\left(\overline{a}.\overline{b}\right)^2 / a^2b^2\right) = a^2.b^2 - \left(\overline{a},\overline{b}\right)$$

38. Sol: (a)

$$\int_{P_1}^{P_2} ydx + xdy = \int_{P_1}^{P_2}\left(yi+xj\right).d\overline{r}$$

$$B\overline{C} = O\overline{C} - O\overline{B} = \left(3,0,4\right) = 3j + 4\overline{K}$$

$$\left(16i+9j-12\overline{k}\right).B\overline{A} = 0$$

$$\left(16i+9j-12\overline{k}\right).B\overline{C} = 0$$

39. Sol: (a)

$$\int_{P_1}^{P_2} ydx + xdy = \int_{P_1}^{P_2}\left(yi+xj\right).d\overline{r}$$

$$\overline{\mathrm{F}} = yi + xj, curl\overline{F} = \overline{O}$$

$$\Rightarrow \overline{F} = \nabla\phi Where\phi = xy$$

$$\therefore \int_{P_1}^{P_2}\left(yi+xj\right).d\overline{r} = \phi_{P_2} - \phi_{P_1} = \left(x_2,y_2\right) - \left(x_1,y_1\right) = x_2y_2 - x_1y_1$$

40. Sol: (a)

$$\nabla T = 2x\overline{i} + 2y\overline{j} + 4z\overline{k}$$

$$\nabla T]_{P(1,1,1)} = 2\overline{i} + 2\overline{j} + 4\overline{k}$$

$$|\nabla T| = \sqrt{24} = 2\sqrt{6}$$

41. Option : D similar to Q . No -27

42. Given vectors are $\overline{a} = i + j + \overline{k} : \overline{b}, where\, w = \dfrac{-1+j\sqrt{3}}{2}$

$\overline{\mathrm{a}}\cdot\overline{\mathrm{b}} = 1 + w + w^2 = \mathrm{U} :\Rightarrow \overline{\mathrm{a}},\overline{\mathrm{b}}$ are orthogonal

Option (a) is correct

43. Option (b), if n=1 then |A| = Kr, where $r = |\overline{r}|$. Then $A = K\overline{r} = K\left(xi + yj + 3\overline{k}\right)$

Now ∇ A=$\left(\frac{\partial^2}{\partial^2} + \frac{\partial^2}{\partial y} + \frac{\partial^2}{\partial z^2}K\right).\left(Kxi + Kyj + Kz\overline{K}\right)$

$$\Rightarrow \frac{\partial^2}{\partial x^2}(Kx) + \frac{\partial}{\partial y^2}(ky) + \frac{\partial^2}{\partial z^2}(kz) = 0$$

44. Given $\phi = x^2 + y^2 ez^2 - 1$, the unit out word normal is grad of optional grad $\phi = 2x + 2yj + 2z\overline{k}at\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}},0\right), is \frac{2i+2i}{\sqrt{2}} - \sqrt{2}(1+1)$

45. Option (b) .Area of a parallel logram $\overline{OP}\overline{QR}$ = ac+bd

46. option (d) , by stokes theorem

47. $A = xi + yj + z\overline{k}$ div A = 1+1+1=3 : (option (d))

48. Option (d) , since E is irrotational $\Rightarrow \nabla \times E = 0$

49. $f = xi + ji + z\overline{k}$. By gauss divergence theorem $\int\int(\text{F-n})dA = \int\int\int \text{div} f dv$

$$= \int\int 3dv$$

$$= 3\int\int\int V. = 3\frac{4}{3}\pi r^2$$

$$= = 4\pi(1)^2$$

Now $\int\int\int -\frac{1}{4}(F.n)ds = \frac{1}{4}4\pi$: so option (a) is correct

50. Vector identity curl grad V=0: So option (d) is correct

51. Given $\overline{F} = yxi - yzi - x^2K$

$$\int_{x=1}^{x=2} F \cdot dr = \int_{x=1}^{2} = (yxi - yzi)(dx + dyj + dzk)$$

$= \int_1^2 -x^2 dz$ since integration is along X axis from 1 to 2 and y=0, z=0, →dy = 0, dz = 0

$$= \int_1^2 -x^2(0) = 0, option(b)$$

52 . Ans : 3

If $\bar{r} = xi + yz + z\bar{k} : r = x^2 + y^2 + z^2.\nabla(or\, r) = \frac{1}{r}.\frac{\bar{r}}{r} = \frac{\bar{r}}{r^2}$

Now $r^2\nabla(or\ r) = r^2\frac{\bar{r}}{r^2} = \bar{r} : div\bar{r} = 3$

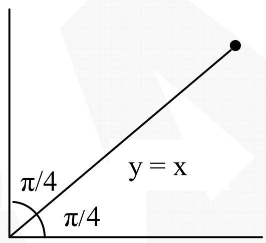
53. Ans : 7

Magnitude of gradient is $|\nabla\phi| = \sqrt{(2x^2)+(6y^2)+(3z^2)^2}$

$$|\nabla\phi|_{(1,1,1)} = \sqrt{4+36+9} = 7$$

54. Given $f = \frac{1}{\sqrt{2}}(x^2y + y^2x)$

$$\nabla f = \frac{(2xy+y^2)}{\sqrt{2}}i + \frac{(2xy+x^2)}{\sqrt{2}}j$$



$(\nabla f)(1,1) = \frac{3}{\sqrt{2}}(i+j)$

Let $\bar{r} = xi + yj = r\cos\theta i + r\sin\theta i = \sqrt{2}\left(\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j\right)$

= i + j

Now D.D $= \frac{3}{\sqrt{2}}(1+1) = 3\sqrt{2}$

55. Given $F = zi + xj + y\bar{k}$ by stoke's theorem, $\int_S (\nabla\times F)ds = \oint F\cdot dr$

$$= \oint (xi + xi + yk)$$

$$= zdx + xdy + ydz$$

$= 0$

56. Given F = yzi

Now $\oint F.dr = \int (yz)dx = \int_{\theta} (y-1)\,dx$ since z=1

Given circle of $x^2<y^2=1$

X= cosθ, y=sinθ

Dx=-sinθdθ

$= \theta \int y dx$

$= \int_0^{2\pi} \sin\theta d\theta$

$= -\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) = \frac{-1}{2}\left[(0)_0^{2\pi} - \left(\frac{\sin 2\theta}{2}\right)_0^{2\pi}\right] = \frac{1}{2}x^2$

x =-π

57. we know that $div(fv) = fdivv + gradf \cdot v$

i.e $\nabla(fV) = f(\nabla \text{v}) + \nabla f \cdot \text{V}$

$\Rightarrow x^2y + y^2z + z^2z = f\left(\frac{\partial}{\partial x}y + \frac{\partial y}{\partial y^2} + \frac{\partial}{\partial z}x\right) + \nabla.\nabla f$

$\Rightarrow x^2y + y^2z + z^2x = f(0) + v.Df$

58. Given $f = yi + xj + z\bar{k} \int\int f.dsw = \int\int\int div\, dv$ (Gauss divergence function)

$= \int_0^1\int_0^1\int_0^1 \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial x}(K)(dxi + jdy + Kz)$

$= \int\int\int 0\,\text{dv} = 0$

59. Ans : 1

$\oint (ydx - xdy) = \int\int (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})dxdy$, Green's theorem

$= \int\int(-1-1)ds - 2\int\int ds = -2(\pi r^2)$

$$= -2\left(\pi\left(\frac{1}{2}\right)^2\right) = \frac{-\pi}{2}$$

Option (c)

60. $curl\, f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{dz} \\ x^3y^3 & -2x^2y^2z^2 & 2y^2z^3 \end{vmatrix} = \left(4yz^3 + 2xy^2 i\right) + \left(2x^2z\right)j - 2zy^2z\overline{k}$

Option (c)

61. dif $F = \frac{\partial}{\partial x}\left(x^2z\right) + \frac{\partial}{\partial y}\left(xy\right) + \frac{\partial}{\partial z}\left(-yz^2\right) = 2xz + x - 2yz$

$\left(\nabla.F\right)\left(1,-1,1\right) = 2+1+2 = 5$ option – (c)

62. D.D of $\phi = xz - y^2\ at\left(1,3,2\right)$ is maximum in the direction of $\nabla\phi$ at $\left(1,3,2\right)$

i.e., $\left(\nabla\phi\right)\left(1,3,2\right) = \left(2z\right)i - 2ji + 2z\overline{k}$

$= 4i - 6j + 2\overline{k}$

Option (b)

63. $\phi = 2x^3y^2z^4$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial\phi}{\partial z^2} = 2xy^2z^4 + 4x^3z^4 + 24x^3y^2z^4$$

$$= xi + yj + z\overline{k} = \left(t^3 + 2t\right)i - 3e^{-2t}j + 2\overline{k}$$

The acceleration of the particle is $\frac{d\overline{x}}{dt} = \left(3t^2 + 2\right)i + 6e^{-2t}j$

Magnitude of a acceleration of $\left|\frac{d\overline{r}}{dt}\right| = \sqrt{\left(9t^4 + 4 + 12t^2\right) + 36e^{-4t}}$

$$\left|\frac{d\overline{r}}{dt}\right|_{t=0} = \sqrt{4+35} = \sqrt{40}$$

$= 1\sqrt{10}$ cm/sec

64. Sol. **(A)**

$$P = x^3y\hat{a}_x - x^2y\hat{a}_y - x^2yz\hat{a}_z$$

$\nabla.P = 3x^2y - 2x^2y - x^2yz = 0$

It is solenoidal

$$\nabla \times P = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -x^2y & -x^2yz \end{vmatrix} \neq 0 \Rightarrow \overline{P} \text{ is not irrational}$$

65.Sol.

$$\overline{f} = \frac{1}{r^2}\overline{r}$$

$$div\overline{f} = \nabla.\overline{f} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right).\left(\frac{1}{r^2}\overline{r}\right)$$

66. Sol. (b)

67. Ans : (c)

Sol.

V is incompressible

$$\Rightarrow div = 0 \Rightarrow \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} = 0$$
$$\Rightarrow a_1 + b_2 + c_3 \Rightarrow 2 + b_2 - 4 = 0$$
$$b_2 = 2$$

68. Ans: (a)

Sol: $$curlV = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix} = i(3y^2 - 6z) + j(0-0) + k(0-0) = (3y^2 - 6z)i_{(x,y,z)=(1,1,1)} = -3i$$

69.Sol.

By Gauss divergence theorem $\int_s F.nds = \int_v divF.dv$

Here F = 9xi-3yj

divF=9-3=6

$$\therefore \iint_s \frac{1}{\pi}(9xi - 3yj) = \frac{1}{\pi}\iiint 6dv = \frac{1}{\pi}6v = \frac{1}{\pi}6\left(\frac{4}{3}\pi r^3\right) = 216$$

70 Sol.

C is correct since it is vector identity

71.Sol.

$$\nabla u = i\frac{\partial u}{\partial x} + j\frac{\partial u}{\partial y} + k\frac{\partial u}{\partial z} = 2xi - 3zj - 3yk$$

$$\nabla u_{(2,-1,4)} = 4i - 12j + 3k$$

$$\bar{a} = i + j - 2k \quad : \quad |\bar{a}| = \sqrt{1+1+4} = \sqrt{6}$$

The required directional derivative is $\nabla u \frac{\bar{a}}{|\bar{a}|} = (4i - 12j + 3k)\frac{i+j-2k}{\sqrt{6}} = \frac{-14}{\sqrt{6}}$

72.Sol.

A since $\nabla . u = 0 \, and \, \nabla \times u = 0$

73.Sol. (B)

Direction of maximum increase $= \nabla\phi$ at (1, 1)

$= (2x\hat{i} + 2y\hat{j})$ at (1, 1)

$= 2\hat{i} + 2\hat{j}$

74.**Ans: 726**

Sol: line integral

$$L.I = \int_c \bar{f}.d\bar{r} = \int_{t=1}^{3} [yzdx + xzdy + xydz]$$

$$\int_{t=1}^{3} d(xyz) = (xyz)_{t=1}^{3} = (3t^5)_{t=1}^{3} = 726$$

75. **Ans: (16)**

76. **Ans:** 0 (Zero)

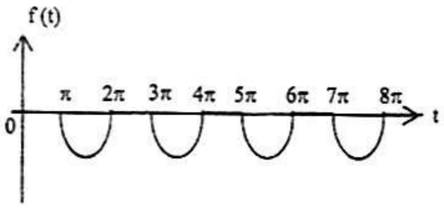
Sol: Using Green's Theorem

$$\oint_C (xy^2dx + x^2ydy) = \iint_R (2xy - 2xy)dxdy = 0$$

CHAPTER- 9
LAPLACE TRANSFORMS

01. The Laplace transform of the periodic function given below i.e.

$f(t)=\begin{cases}\sin t,\ if\ (2n-1)\pi < t < 2n\pi\ (n=1,2,3.......)\\ 0\ otherwise\end{cases}$ is________ **(GATE-1993-ME)**



02. If f(t) is a finite and continuous function for t>0 the Laplace transformation is given by

$F=\int_0^{\infty} e^{-st} f(t)dt,\ then\ for\ f(t)=\cosh mt,$ The Laplace transformation is (GATE-1994-ME)

03. The inverse Laplace transform of $(s+9)/(s^2+6s+13)$ is **(GATE-1995)**

(a) $\cos 2t+9\sin 2t$ (b) $e^{-3t}\cos 2t-3e^{-3t}\sin 2t$

(c) $e^{-3t}\sin 2t+3e^{-3t}\cos 2t$ (d) $e^{-3t}\cos 2t+3e^{-3t}\sin 2t$

04. The Laplace transform of f (t) is F(s). Given $F(s)=\frac{\omega}{s^2+\omega^2}$, the final value of f(t) is **(GATE-1995-EE)**

(a) Infinite (b) zero (c) one (d) none

05. The Laplace transform of a function f(t) is defined by $F(s)=L\{f(t)\}=\int_0^{\infty} e^{-st} f(t)dt.$

Find the inverse Laplace transform of F(s-a) **(GATE-1995-IN)**

06. Find $L\{e^{at}\cos \omega t\}\ when\ L\{\cos \omega t\}=\frac{s}{s^2+\omega^2}$ **(GATE-1995-IN)**

07. If $L\{f(t)\}=\frac{2(s+1)}{s^2+2s+2}\ then\ f(0^+)\ and\ f(\infty)$ are given by___ **(GATE-1995-EC)**

(a) 0,2 (b) 2,0 (c) 0,1 (d) $\frac{2}{5},0$

08. Using Laplace transform, solve the initial value problem $9y''-6y'+y=0$. $y(0)=3$ *and* $y'(0)=1$, where prime denotes derivative with respect to t. **(GATE-1996)**

09. The inverse Laplace transform of the function $\frac{s+5}{(s+1)(s+3)}$ is ____ **(GATE-1996-EC)**

(a) $2e^{-t}-e^{-3t}$ (b) $2e^{-t}+e^{-3t}$ (c) $e^{-t}-2e^{-3t}$ (d) $e^{-t}+2e^{-3t}$

10. The Laplace transform of $e^{\alpha t}\cos\alpha t$ is equal to _____ **(GATE-1997-EC)**

(a) $\frac{s-\alpha}{(s-\alpha)^2+\alpha^2}$ (b) $\frac{s+\alpha}{(s+\alpha)^2+\alpha^2}$ (c) $\frac{1}{(s-\alpha)^3}$ (d) None

11. Solve the initial value problem $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=0$ with y=3 and $\frac{dy}{dx}=7$ *at* $x=0$ using the Laplace transform technique? **(GATE-1997-ME)**

(a) e^t-6e^{3t} (b) $-(e^t+6e^{3t}$ (c) e^t+6e^{3t} (d) None

12. The Laplace transform of a unit step function $u_a(t)$ defined as $u_a(t)=\begin{cases}0, & for\ t<a\\ 1 & for\ t>a\end{cases}$ is

(a) e^{-as}/s (b) se^{-as} (c) Se^{as} (d) $se^{-as}-1$ **(GATE-1998)**

13. $(s+1)^{-2}$ is the Laplace transform of _____ **(GATE-1998)**

(a) t^2 (b) t^3 (c) e^{-2t} (d) te^{-t}

14. If $L\{f(t)\}=\frac{w}{s^2+w^2}$ then the value of $\lim_{t\to\infty} f(t)=$ ___ **(GATE-1998-EC)**

(a) cannot be determined (b) zero (c) unity (d) ∞

15. The Laplace transform of $(t^2-2t)u(t-1)$ is _______ **(GATE-1998-EE)**

(a) $\frac{2}{S^3}e^{-s}-\frac{2}{S^2}e^{-s}$ (b) $\frac{2}{S^3}e^{-2S}-\frac{2}{S^2}e^{-S}$ (c) $\frac{2}{S^3}e^{-S}-\frac{2}{S}e^{-S}$ (d)None

16. If $L\{f(t)\}=F(s)$ *then* $L\{f(t-T)\}$ is equal to _______ **(GATE-99-EC)**

(a) $e^{sT}F(s)$ (b) $e^{-sT}F(s)$ (c) $\frac{F(s)}{1-e^{sT}}$ (d) $\frac{F(s)}{1-e^{-sT}}$

17. The Laplace Transform of the function $\begin{aligned}f(t)&=k, 0<t<c\\ &=0, c<t<\infty,\end{aligned}$ is **(GATE-1999)**

(a) $(k/s)e^{-sc}$ (b) $(k/s)e^{sc}$ (c) ke^{-sc} (d) $(k/s)(1-e^{-sc})$

18. Laplace transform of $(a+bt)^2$ where a and b are constants is given by **(GATE-1999)**

(a) $(a+bs)^2$ (b) $1/(a+bs)^2$

(c) $(a^2/s)+(2ab/s^2)+(2b^2/s^3)$ (4) $(a^2/s)+(2ab/s^2)+(b^2/s^3)$

19. If $L\{f(t)\}=\frac{s+2}{s^2+1}$, $L\{g(t)\}=\frac{s^2+1}{(s+3)(s+2)}$, $h(t)=\int_0^t f(T)g(t-T)dT$ then L{h(T)} is__________ **(GATE-2000-EC)**

(a) $\frac{s^2+1}{s+3}$ (b) $\frac{1}{s+3}$ (c) $\frac{s^2+1}{(s+3)(s+2)}+\frac{s+2}{s^2+1}$ (d) None

20. Let $F(s)=L[f(t)]$ denote the Laplace transform of the function $f(t)$. Which of the following statements is correct?

(a) $L[df/dt]=1/s\ F(s);\ L\left\{\int_0^t f(\tau)d\tau\right\}=sF(s)-f(0)$

(b) $L[df/dt]=s\ F(s)-F(0);\ L\left\{\int_0^t f(\tau)d\tau\right\}=-dF/ds$

(c) $L[df/dt]=s\ F(s)-F(0);\ L\left\{\int_0^t f(\tau)d\tau\right\}=F(s-a)$

(d) $L[df/dt]=s\ F(s)-F(0);\ L\left\{\int_0^t f(\tau)d\tau\right\}=F(s)$

21. The inverse Laplace transform of $1/(s^2+2s)$ is **(GATE-2001)**

(a) $(1-e^{-2t})$ (b) $(1+e^{2t})/2$ (c) $(1-e^{2t})/2$ (d) $(1-e^{-2t})/2$

22. The Laplace transform of the function is $f(t)=\begin{cases}\sin t & for\ 0\le t\le\pi\\ 0 & for\ t>\pi\end{cases}$ **(GATE-2002)**

(a) $1/(1+s^2)\ for\ all\ s>0$ (b) $1/(1+s^2)\ for\ all\ s<\pi$

(c) $1/(1+e^{-\pi s})/(1+s^2)\ for\ all\ s>0$ (d) $e^{-\pi s}/(1+s^2)\ for\ all\ s>0$

23. Using Laplace Transform, solve $(d^2y/dt^2)+4y=12t$ given that y=0 and dy/dt=9 at t=0 _____ **(GATE-2002)**

(a) 3(t-sin2t) (b) 3(t+sin2t) (c) 3(t-cos2t) (d) 3(t+cos2t)

24. Let $Y(s)$ be the Laplace transform of function $Y(t)$, then the final value of the function is **(GATE-2002-EE)**

(a) $\lim_{s\to 0} Y(s)$ (b) $\lim_{s\to\infty} Y(s)$ (c) $\lim_{s\to 0} sY(s)$ (d) $\lim_{s\to\infty} sY(s)$

25. If L denotes the Laplace transform of a function, $L\{\sin at\}$ will be equal to **(GATE-2003-CE)**

(a) $\frac{a}{s^2-a^2}$ (b) $\frac{a}{s^2+a^2}$ (c) $\frac{s}{s^2+a^2}$ (d) $\frac{s}{s^2-a^2}$

26. The Laplace transform of $i(t)$ is given by $I(s)=\frac{2}{s(1+s)}$. As $t\to\infty$, the value of $i(t)$ tends to _____ **(GATE-2003-EC)**

(a) 0 (b) 1 (c) 2 (d) ∞

27. A delayed unit step function is defined as $u(t-a)=\begin{cases}0, & t<a\\ 1, & t\ge a\end{cases}$. Its Laplace transform is ____ **(GATE-2004)**

(a) $a\,e^{-as}$ (b) e^{-as}/s (c) e^{as}/s (d) e^{as}/a

28. The Laplace transform of a function f(t) is F(s) $=\frac{5s^2+23s+6}{s(s^2+2s+2)}$. $As\ t\to\infty$, f(t) approaches **(GATE-2005)**

(a) 3 (b) 5 (c) 17/2 (d) ∞

29. Laplace transform of $f(t)=\cos(pt+q)$ is **(GATE-2005)**

(a) $\frac{s\cos q-p\sin q}{s^2+p^2}$ (b) $\frac{s\cos q+p\sin q}{s^2+p^2}$ (c) $\frac{s\sin q-p\cos q}{s^2+p^2}$ (d) $\frac{s\sin q+p\cos q}{s^2+p^2}$

30. In what range should Re(s) remain so that the Laplace transform of the function $e^{(a+2)t+5}$ exists? **(GATE-2005-EC)**

(a) $\text{Re}(s)>a+2$ (b) $\text{Re}(s)>a+7$ (c) $\text{Re}(s)<2$ (d) $\text{Re}(s)>a+5$

31. The dirac delta Functions $\delta(t)$ is defined as **(GATE-2005-EC)**

(a) $\delta(t)=\begin{cases}1 & t=0\\ 0, & other\ wise\end{cases}$ (b) $\delta(t)=\begin{cases}\infty & t=0\\ 0, & other\ wise\end{cases}$

(c) $\delta(t)=\begin{cases}1 & t=0\\ 0, & other\ wise\end{cases}\ and \int_{-\infty}^{\infty}\delta(t)dt=1$

(d) $\delta(t)=\begin{cases}\infty & t=0\\ 0, & other\ wise\end{cases}\ and \int_{-\infty}^{\infty}\delta(t)dt=1$

32. Consider the function $f(t)$ having Laplace transform $F(s)=\frac{\omega_0}{s^2+\omega_0^2}, \operatorname{Re}(s)>0$. The final value of $f(t)$ would be ______ **(GATE-2006-EC)**

(a) 0 (b) 1 (c) $-1\le f(\infty)\le 1$ (d) ∞

33. If $F(s)$ is the Laplace transform of the function $f(t)$ then Laplace transform of $\int_0^t f(x)dx$ is **(GATE-2007-ME)**

(a) $\frac{1}{s}F(s)$ (b) $\frac{1}{s}F(s)-f(0)$ (c) $sF(s)-f(0)$ (d) $\int F(s)ds$

34. Laplace transform of $8t^3$ is _ **(GATE-2008)**

(a) $\frac{8}{s^4}$ (b) $\frac{16}{s^4}$ (c) $\frac{24}{s^4}$ (d) $\frac{48}{s^4}$

35. Laplace transform of sin ht is **(GATE-2008-P1)**

(a) $\frac{1}{s^2-1}$ (b) $\frac{1}{1-s^2}$ (c) $\frac{s}{s^2-1}$ (d) $\frac{s}{1-s^2}$

36. Laplace transform of $f(x)=\cosh(ax)$ is **(GATE-2009)**

(a) $\frac{a}{s^2-a^2}$ (b) $\frac{s}{s^2-a^2}$ (c) $\frac{a}{s^2+a^2}$ (d) $\frac{s}{s^2+a^2}$

37. Given that $F(s)$ is the one-sided Laplace transform of $f(t)$, the Laplace transform of $\int_0^s f(\tau)d\tau$ **(GATE-2009-EC)**

(a) $sF(s)-f(0)$ (b) $\frac{1}{s}F(s)$ (c) $\int_0^s f(\tau)d\tau$ (d) $\frac{1}{s}\left[F(s)-f(0)\right]$

38. The inverse Laplace transform of $\frac{1}{(s^2+s)}$ is **(GATE-2009-ME)**

(a) $1+e^{t}$ (b) $1-e^{t}$ (c) $1-e^{-t}$ (d) $1+e^{-t}$

39. Given $f(t)=L^{-1}\left[\frac{3s+1}{s^3+4s^2+(k-3)s}\right]$. If $\underset{t\to\infty}{Lt} f(t)=1$ then value of k is

(GATE-2010-EE)

(a) 1 (b) 2 (c) 3 (d) 4

40. The Laplace transform of $f(t)$ is $\frac{1}{s^2(s+1)}$. The function **(GATE-2010-ME)**

(a) $t-1+e^{-t}$ (b) $t+1+e^{-t}$ (c) $-1+e^{-t}$ (d) $2t+e^{t}$

41. u(t) represents the unit step function. The Laplace transform of u(t-ι) is **(GATE-2010-IN)**

(a) $\frac{1}{s\tau}$ (b) $\frac{1}{s-\tau}$ (c) $\frac{e^{-s\tau}}{s}$ (d) $e^{-s\tau}$

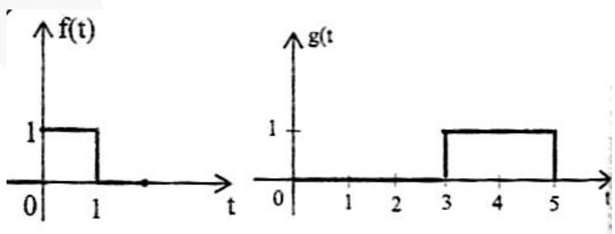
42. Given two continuous time signals $x(t)=e^{-t}$ *and* $y(t)=e^{-2t}$ which exists for t > 0. Then the convolution z(t) = f(t) * y(t) is____ **(GATE-2011)**

(a) $e^{-t}-e^{-2t}$ (b) e^{-2t} (c) e^{-t} (d) $e^{-t}+e^{-3t}$

43. If $F(s)=L\{f(t)\}=\frac{2(s+1)}{s^2+4s+7}$ then the initial and final values of $f(t)$ are respectively **(GATE-2011)**

(a) $0,2$ (b) $2,0$ (c) $0,\frac{2}{7}$ (d) $\frac{2}{7},0$

44. Given $f(t)$ *and* $g(t)$ as shown below **(GATE-2011-EE)**



(i) $g(t)$ can be expressed as

(a) $g(t)=f(2t-3)$ (b) $g(t)=f\left(\frac{t}{2}-3\right)$

(c) $g(t)=f\left(2t-\frac{3}{2}\right)$ (d) $g(t)=f\left(\frac{t}{2}-\frac{3}{2}\right)$

45. If $x[n]=(1/3)^{|n|}-(1/2)^{n}u[n]$, then the region of convergence (ROC) of its Z-transform in the Z-plane will be **(GATE-2012-EC, EE, IN)**

(a) $\frac{1}{3}<|z|<3$ (b) $\frac{1}{3}<|z|<\frac{1}{2}$ (c) $\frac{1}{3}<|z|<3$ (d) $\frac{1}{3}<|z|$

46. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $t\,f(t)$ is **(GATE-2012-EC, EE, IN)**

(a) $-\frac{s}{\left(s^2+s+1\right)^2}$ (b) $-\frac{2s+1}{\left(s^2+s+1\right)^2}$ (c) $\frac{s}{\left(s^2+s+1\right)^2}$ (d) $\frac{2s+1}{\left(s^2+s+1\right)^2}$

47. Consider the differential equation $\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\delta(t)$

with $y(t)\Big|_{t=0}=-2 \text{ and } \frac{dy}{dt}\Big|_{t=0}=0$

The numerical value of $\frac{dy}{dt}\Big|_{t=0}$, is **(GATE-2012-EC, EE, IN)**

(a) -2 (b) -1 (c) 0 (d) 1

48. The inverse Laplace transform of the function $F(s)=\frac{1}{s(s+1)}$ is given by

(GATE-2012-ME, P1)

(a) $f(t)=\sin t$ (b) $f(t)=e^{-t}\sin t$ (c) $f(t)=e^{-t}$ (d) $f(t)=1-e^{-t}$

49. The function $f(t)$ satisfies the differential equation $\frac{d^2f}{dt^2}+f=0$ and the auxiliary conditions, $f(0)=0, \frac{df}{dt}(0)=4$. The Laplace transform of $f(t)$ is given by

(GATE-2013-ME)

(a) $\frac{2}{s+1}$ (b) $\frac{4}{s+1}$ (c) $\frac{4}{s^2+1}$ (4) $\frac{2}{s^4+1}$

50. A system is described by the following differential equation, where $u(t)$ is the output of the system and y(t) is the output to the system $y(t)+5y(t)=u(t)$. When $y(0)=1$ and $u(t)$ is a unit step function $y(t)$ is **(GATE-2014-EC-Set1)**

(a) $0.2+0.8e^{-5t}$ (b) $0.2-0.2e^{-5t}$ (c) $0.8+0.2e^{-5t}$ (d) $0.8-0.8e^{-5t}$

51. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. which one of the following is the unilateral Laplace transform of $g(t)=t.f(t)$? **(GATE-2014-EC-Set4)**

(a) $\frac{-s}{(s^2+s+1)^2}$ (b) $\frac{-(2s+1)}{(s^2+s+1)^2}$ (c) $\frac{s}{(s^2+s+1)^2}$ (d) $\frac{(2s+1)}{(s^2+s+1)^2}$

52. Let $X(s)=\frac{3s+5}{s^2+10s+20}$ be the Laplace Transform of a signal $x(t)$. Then $x(0^+)$ is

(GATE-2014-EE-Set1)

(a) 0 (b) 3 (c) 5 (d) 21

53. Laplace transform of $e^{-2t}\cos(4t) is$ **(GATE-2014-ME-Set4)**

(a) $\frac{s-2}{(s-2)^2+16}$ (b) $\frac{s+2}{(s-2)^2+16}$ (c) $\frac{s-2}{(s+2)^2+16}$ (d) $\frac{s+2}{(s+2)^2+16}$

54. The bilateral Laplace transform of a function $f(t)=\begin{cases}1 & if\ a\le t\le b\\ 0 & otherwise\end{cases}$ **(GATE-EC-15)**

(A) $\frac{a-b}{s}$ (B) $\frac{e^2(a-b)}{s}$ (C) $\frac{e^{-as}-e^{-bs}}{s}$ (D) $\frac{e^{s(a-b)}}{s}$

55. The Laplace transform of $f(t)=2\sqrt{\frac{t}{\pi}}$ is $s^{-3/2}$. Laplace transform of $g(t)=\sqrt{\frac{1}{\pi t}}$ is

(GATE-EE-15)

(A) $\frac{3}{2}s^{-5/2}$ (B) $s^{-1/2}$ (C) $s^{1/2}$ (D) $s^{3/2}$

56. Laplace transform of e^{ijt} $where\, i=\sqrt{-1}$ is **(GATE-ME-15)**

(A) $\frac{s-5i}{s^2-25}$ (B) $\frac{s+5i}{s^2+25}$ (C) $\frac{s+5i}{s^2-25}$ (D) $\frac{s-5i}{s^2+25}$

57. Laplace Transform of f(t) is given by $F(S)=L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt$ Laplace transform of the function shown below is $f(t)=\begin{cases}2, & 0<tC_1\\ 0, & Other\ wise\end{cases}$ **(GATE-ME-15)**

A) $\frac{1-e^{-2s}}{s}$ B) $\frac{1-e^{-s}}{s}$ C) $\frac{2-2e^{-s}}{s}$ D) $\frac{1-2e^{-s}}{s}$

58.If $f(t)$ is a function defined for all $t \geq 0$, its Laplace transform $F(s)$ is defined as

(a) $\int_0^\infty e^{st}f(t)dt$ (b) $\int_0^\infty e^{-st}f(t)dt$ **(GATE-ME-16)**

(c) $\int_0^\infty e^{ist}f(t)dt$ (d) $\int_0^\infty e^{-ist}f(t)dt$

59. Laplace transform of cos(ωt) is **(GATE-ME-16)**

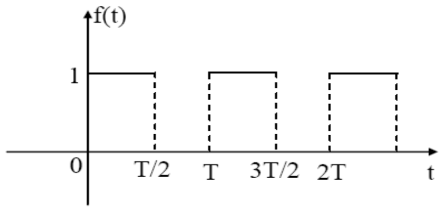
$(A)\frac{s}{s^2+\omega^2}$ $(B)\frac{\omega}{s^2+\omega^2}$ $(C)\frac{s}{s^2-\omega^2}$ $(D)\frac{\omega}{s^2-\omega^2}$

60.Solutions of Laplace's equation having continuous second-order partial derivatives are called **(GATE-ME-16)**

(A) biharmonic functions (B) harmonic functions

(C) conjugate harmonic functions (D) error functions

61.The Laplace transform of the causal periodic square wave of period T shown in the figure below is **(GATE-EC-15)**



(A) $F(s)=\frac{1}{1+e^{-sT/2}}$ (B) $F(s)=\frac{1}{s\left(1+e^{\frac{sT}{2}}\right)}$

(C) $F(s) = \dfrac{1}{s\left(1+e^{-sT}\right)}$ (D) $F(s) = \dfrac{1}{1-e^{-sT}}$

62. The value of $\int_{-\infty}^{+\infty} e^{-t}\delta(2t-2)\,dt$, where $\delta(t)$ is the Dirac delta function is **(GATE-EC-16)**

a) $\dfrac{1}{2e}$ b) $\dfrac{2}{e}$ c) $\dfrac{1}{e^2}$ d) $\dfrac{1}{2e^2}$

63.The Laplace Transform of f(t) = e^{2t} sin(5t) u(t) is **(GATE-EE-16)**

a) $\dfrac{5}{s^2-4s+29}$ b) $\dfrac{5}{s^2+5}$ c) $\dfrac{s-2}{s^2-4s+29}$ d) $\dfrac{5}{s+5}$

64.The solution of the differential equation, for t > 0, y"(t) + 2y'(t) + y(t) = 0 with initial conditions y(0) = 0 and y'(0) = 1, is [u(t) denotes the unit step function],

(GATE-EE-16)

e) $te^{-t}u(t)$ b) $\left(e^{-t}-te^{-t}\right)u(t)$ c) $\left(-e^{-t}+te^{-t}\right)u(t)$ d) $e^{-t}u(t)$

65.Let y(x) be the solution of the differential equation $\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+4y=0$ with initial conditions

$y(0)=0$ and $\left.\dfrac{dy}{dx}\right|_{x=0}=1$. Then the value of y(1) is ____________. **(GATE-EE-16)**

Solutions

LAPLACE TRANSFORM

1Sol. $L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T f(t)e^{-st}.dt, T = 2\pi$

$= \frac{1}{1-e^{-2s\pi}}\int_\pi^{2\pi} e^{-st}.\sin t\ dt$

$= \frac{1}{1-e^{-2s\pi}}\left[\frac{e^{-st}}{s^2+1}(-s\ \sin t - \cos t)\right]_\pi^{2\pi}$

$= \frac{e^{-\pi s}}{(s^2+1)(e^{-\pi s}-1)}$

2Sol. $L\{\cosh mt\} = \frac{s}{s^2-m^2}$

3Sol. (d)

$f(s) = \frac{s+9}{s^2+6s+13} = \frac{(s+3)+6}{(s+3)^2+4}$

$L^{-1}\{f(s)\} = e^{-3t}L^{-t}\left\{\frac{s+6}{s^2+4}\right\}$

(First shifting theorem)

$= e^{-3t}L^{-t}\left\{\frac{s}{s^2+4}+\frac{6}{s^2+4}\right\}$

$= e^{-3t}[\cos 2t + 3\sin 2t]$

4Sol. (b)

Final value $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$\lim_{s\to 0}\frac{s\omega}{s^2\omega^2} = 0$

5Sol. $L^{-1}\{f(s-a)\} = e^{at}f(t)$

6Sol. $L\{e^{at}\cos\omega t\} = \frac{s-a}{(s-a)^2\omega^2}$ (First shifting property)

7Sol. (b)

$$f\left(0^{+}\right)=\underset{x\to\infty}{Lim}\, f\left(t\right)=\underset{s\to 0}{Lim}\, F\left(s\right)=2$$

$$f\left(\infty\right)=f\left(0^{+}\right)=\underset{t\to\alpha}{Lim}\, f\left(t\right)=\underset{s\to 0}{Lim}\, F\left(s\right)=0$$

8Sol. Given $9y^{n}-6y'+y=0$

$$y\left(0\right)=3,\ y'\left(0\right)=1$$

$$\Rightarrow 9L\left\{y^{n}\right\}-6L\left\{y'\right\}+L\left\{y\right\}=0$$

$$\Rightarrow 9\left[s^{2}L\left\{y\right\}-3s-1\right]-6\left[sL\left\{y\right\}-3+L\left\{y\right\}\right]=0$$

$$\Rightarrow L\left\{y\right\}=\frac{9}{3s-1}\Rightarrow y\left(t\right)=3e^{t/3}$$

9Sol. (a)

$$f\left(s\right)=\frac{s+5}{\left(s+1\right)\left(s+3\right)}=\frac{2}{s+1}-\frac{1}{s+3}$$

$$\Rightarrow L^{-1}\left\{\frac{s+5}{\left(s+1\right)\left(s+3\right)}\right\}=2L^{-1}\left\{\frac{1}{s+1}\right\}-L^{-1}\left\{\frac{1}{s+3}\right\}$$

10Sol. (a)

$$L\left\{\cos\alpha\ t\right\}=\frac{s}{s^{2}+\alpha^{2}}=f\left(s\right)$$

$$\Rightarrow L\left\{e^{\alpha t}\cos\alpha t\right\}=f\left(s-\alpha\right)=\frac{s-\alpha}{\left(s-\alpha\right)^{2}+\alpha^{2}}$$ (first shifting theorem)

11Sol. $\left[s^{2}Y\left(s\right)-7s-3\right]-4\left[sY\left(s\right)-3\right]+3Y\left(s\right)=0$

$$\Rightarrow Y\left(s\right)=\frac{7s-9}{\left(s-1\right)\left(s-3\right)}=\frac{1}{s-1}+\frac{6}{s-3}$$

$$\therefore\ y\left(t\right)=e^{t}+6e^{3t}$$

12.Sol. (a)

$$L\left\{u_{a}\left(t\right)\right\}=\int_{0}^{\infty}e^{-as}u_{a}\left(t\right)dt=\frac{e^{-as}}{s}$$

13Sol. (d)

$$L\left\{\left(s+1\right)^{-2}\right\}=te^{-t}$$

14Sol. (b)

15Sol. (d)

$$L\left\{\left(t^2-2t\right)u\left(t-1\right)\right\}$$

$$=L\left\{\left(t-1\right)^2 u\left(t-1\right)-u\left(t-1\right)\right\}=e^{-s}\frac{2}{s^3}-\frac{e^{-s}}{s}$$

16Sol. (b)

By second shifting theorem $L\left\{f\left(t-T\right)\right\}=e^{-sT}f\left(s\right)$

17Sol. (d)

$$L\left\{f\left(t\right)\right\}=\int_0^c e^{-st}.k\ dt+0=k\left[\frac{1-e^{-sc}}{s}\right]$$

18Sol. (c)

$$L\left\{\left(a+bt\right)^2\right\}=a^2L\{1\}+2abL\left(t\right)+b^2L\left\{t^2\right\}$$

$$=\frac{a^2}{s}+\frac{2ab}{s^2}+\frac{2b^2}{s^3}$$

19Sol. (b)

$$h\left(t\right)=\int_0^t f\left(T\right)g\left(t-T\right)dT=f\left(t\right)*g\left(t\right)$$

$$\Rightarrow L\left\{h\left(t\right)\right\}=L\left\{f\left(t\right)*g\left(t\right)\right\}=F\left(s\right)G\left(s\right)=\frac{1}{s+3}$$

20Sol. (d)

$$L\left\{\frac{df}{dt}\right\}=sF\left(s\right)-f\left(0\right)$$

and $L\left\{\int_0^t f\left(t\right)dt\right\}=\dfrac{f\left(s\right)}{s}$

21Sol. (d)

$$L^{-1}\left\{\frac{1}{s^2+2s}\right\}=L^{-1}\left\{\frac{1}{s\left(s+2\right)}\right\}=L^{-1}\left\{\frac{1}{2}\left(\frac{1}{s}-\frac{1}{s+2}\right)\right\}=\frac{1}{2}\left[1-e^{-2t}\right]$$

22Sol. (c)

$$L\{f(t)\} = \int_0^{\pi} e^{-st} \sin t \, dt$$

$$= \left[\frac{e^{-st}}{s^2+1}[-s\sin t - \cos t]\right]_0^{\pi}$$

$$= \frac{e^{-s\pi}}{s^2+1} + \frac{1}{s^2+1} = \frac{1+e^{-\pi s}}{s^2+1}$$

23Sol. $L\{y''(t) + 4y(t)\} = 12L\{t\}$

$$\Rightarrow \left[S^2 L\{y(t)\} - 9\right] + 4L\{y(t)\} = \frac{12}{s^2}$$

$$\Rightarrow (s^2+4)L\{y(t)\} = \frac{12}{s^2} + 9$$

$$\Rightarrow L\{y(t)\} = \frac{12+9s^2}{s^2(s^2+4)} = \frac{3}{s^2} + \frac{6}{s^2+4}$$

$$\Rightarrow y(t) = 3t + 3\sin 2t$$

Ans : (b)

24Sol. (c)

Final value : $\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\, Y(s)$

25Sol. (b)

$$L\{\sin at\} = \frac{a}{s^2+a^2}$$

26Sol. (c)

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s) = 2$$

27Sol. (b)

$$L\{u(t-a)\} = \int_0^{\infty} e^{-st} u(t-a)\, dt = \frac{e^{-as}}{s}$$

28Sol. (a)

Given $F(s) = \dfrac{5s^2+23s+6}{s(s^2+2s+2)}$

$$\Rightarrow f(t) = L^{-1}\{F(s)\} = 3 + e^{-t}[2\cos t + 15\sin t]$$

$\therefore \lim_{t\to\infty} f(t) = 3$ or

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\ F(s) = 3$

29Sol. (a)

$L\{\cos(pt+q)\} = L\{\cos pt\ \cos q - \sin pt\ \sin q\}$

$= \cos q \dfrac{s}{s^2+p^2} - \sin q.\dfrac{p}{s^2+p^2}$

$= \dfrac{scsoq - p\sin q}{s^2+p^2}$

30Sol. (a)

$L\left\{e^{(a+2)t+5}\right\} = e^5 \dfrac{1}{s-(a+2)}$

Where $s > (a+2)$

31Sol. (d)

This is a definition of Dirac delta function

32Sol. (a)

Final value theorem

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\ F(s) = 0$

33Sol. (a)

$L\left\{\int_0^t f(s)\,dt\right\} = \dfrac{f(s)}{s}$

34Sol. (d)

$L\{8t^3\} = 8.L\{t^3\} = 8.\dfrac{6}{s^4} = \dfrac{48}{s^4}$

35Sol. (a)

$L\{\sin\ ht\} = \dfrac{1}{s^2-1}$

36Sol. (b)

$L\{\cos\text{ h }a\ s\} = \dfrac{s}{s^2-a^2}$

37Sol. (b)

$$L\left\{\int_0^t (f/\tau)\,d\tau\right\} = \frac{f(s)}{s}$$

38Sol. (c)

$$L^{-1}\left\{\frac{1}{s^2+s}\right\} = L^{-1}\left\{\frac{1}{s(s+1)}\right\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\} = 1 - e^{-t}$$

39Sol. (d)

$$f(t) = L^{-1}\left[\frac{3s+1}{s^3+4s^2+(k-3)s}\right]$$

$$\underset{t\to\infty}{Lim}\, f(t) = \underset{s\to 0}{Lim}\, s\ F(s) = 1$$

$$\Rightarrow \underset{t\to\infty}{Lim} \frac{3s^2+s}{s^3+4s^2+(k-3)s} = 1 \Rightarrow k = 4$$

40Sol. (a)

$$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\} = L^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right\}$$

(By partial fractions) = $-1 + t + e^{-t}$

41Sol. (c)

$$L\{u((t-\tau)\} = \frac{e^{-\tau s}}{s}$$

42Sol. (a)

$$Z(t) = \int_0^t e^{-u} e^{-2(t-u)}\,du = e^{-2t}\int_0^t e^{u}\,du$$

$$= e^{-t} - e^{-2t}$$

43Sol. (b)

$$\underset{t\to\infty}{Lim}\, f(t) = \underset{s\to 0}{Lim}\, s\ F(s) = \underset{s\to 0}{Lim} \frac{2s^2+s}{s^2+4s+7} = 0$$

44Sol. (d)

$$f(t)=\begin{cases}0 & t<0\\ 1 & 0<t<1\\ 0 & t>1\end{cases}$$

$$g(t)=\begin{cases}0 & 0<t<3\\ 1 & 3<t<5\\ 0 & t>5\end{cases}$$

$$f\left(\frac{t}{2}-\frac{3}{2}\right)=\begin{cases}0 & t<3\\ 1 & 3<t<5=g(t)\\ 0 & t>5\end{cases}$$

45 option (c)

$$x(n)=\left(\frac{1}{3}\right)^{|n|}-\left(\frac{1}{2}\right)^{n}u(n)$$

$$\left(\frac{1}{3}\right)^{|n|}=\left(\frac{1}{3}\right)^{-n}u(-n-1)+\left(\frac{1}{3}\right)^{n}u(n)$$

$$\left(\frac{1}{3}\right)^{|n|}\xrightarrow{ZT}-\frac{z}{z-3}+\frac{z}{z-\frac{1}{3}}$$

$$|z|<3,\ |z|>\frac{1}{3}$$

$$\left(\frac{1}{2}\right)^{n}u(n)=\frac{z}{z-\frac{1}{2}},\ |z|>\frac{1}{2}$$

$$X(z)=-\frac{z}{z-3}+\frac{z}{z-\frac{1}{3}}+\frac{z}{z-\frac{1}{2}}$$

$$ROC=\frac{1}{2}<|z|<3$$

46.option.(d)

Let $F(s)=\dfrac{1}{s^2+s+1}$

$$L\{t\ f(t)\}=-\frac{d}{ds}F(s)=\frac{2s+1}{\left(s^2+s+1\right)^2}$$

47.option(d)

$$y''+2y'+y=\delta(t)$$

$$L\{y''\}+2L\{y'\}+L\{y\}=L\{\delta(t)\}$$

$$\left[s^2\bar{y}-sy(0)-y'(0)\right]+2\left[s\bar{y}-y(0)\right]+\bar{y}=1$$

$$\left(s^2+2s+1\right)\bar{y}+2s+4=1$$

$$\bar{y}=\frac{-3-2s}{(s+1)^2}=\frac{-3}{(s+1)^2}-2\frac{s}{(s+1)^2}$$

$$=\frac{-3}{(s+1)^2}-2\frac{(s+1)-1}{(s+1)^2}$$

$$y(t)=-3te^{-t}-2e^{-t}+2te^{-t}$$

$$y(t)=-te^{-t}-2e^{-t}$$

$$\frac{dy}{dt}=te^{-t}-e^{-t}+2e^{-t}$$

At $t=0,\ \frac{dy}{dt}=0-1+2=1$

48.option.(d)

$$L^{-1}\left\{\frac{1}{s(s+1)}\right\}=L^{-1}\left\{\frac{1}{s}-\frac{1}{s+1}\right\}=1-e^{-t}$$

49.option.(c)

$$\frac{d^2f}{dt^2+f}=0\Rightarrow f^{11}(t)+f(t)=0$$

$$L\left\{f^{11}(t)\right\}+L\left\{f(t)\right\}=0$$

$$\Rightarrow s^2\bar{f}(s)-sf(0)-f^1(0)+\bar{f}(s)=0$$

$$\Rightarrow\left(s^2+1\right)\bar{f}(s)-s(0)-4=0\Rightarrow\bar{f}(s)=\frac{4}{s^2+1}$$

$$\therefore\ f(t)=L^{-1}\left\{\bar{f}(s)\right\}=L^{-1}\left\{\frac{4}{s^2+1}\right\}=4\sin t$$

$$L\left\{f(t)\right\}=4L\{\sin t\}=\frac{4}{s^2+1}$$

50.option.(a)

$$y'(t)+5y(t)=u(t)$$

$$L\{y'(t)\}+5L\{y(t)\}=L\{u(t)\}$$

$$\left[sY(s)-y(0)\right]+5Y(s)=\frac{1}{s}$$

$$(s+5)Y(s)-1=\frac{1}{s}$$

$$(5+s)Y(s)=\frac{1}{s}+1=\frac{1+s}{s}$$

$$\therefore\ Y(s)=\frac{1+s}{s(5+s)}=\frac{1}{5s}+\frac{4}{5(s+5)}$$

$$\Rightarrow y(t)=L^{-1}\left\{\frac{1}{5s}\right\}+\frac{4}{5}L^{-1}\left\{\frac{1}{s+5}\right\}$$

$$y(t)=\frac{1}{5}+\frac{4}{5}e^{-5t}=\frac{1}{5}\left(1+4e^{-5t}\right)$$

51.option.(d)

Let $L\{f(t)\}=\dfrac{1}{\left(s^2+s+1\right)}=F(s)$

$$L\{g(t)\}=L\{t\ f(t)\}$$

$$=-\frac{d}{ds}\left[F(s)\right]$$

$$=-\frac{d}{ds}\left[\frac{1}{s^2+s+1}\right]$$

$$=\frac{(2s+1)}{\left(s^2+s+1\right)^2}$$

52.option.(b)

$$x(t)=L^{-1}\{X(s)\}$$

$$=L^{-1}\left\{\frac{3s+5}{\left(s^2+10s+21\right)}\right\}$$

$$= L^{-1}\left\{\frac{4}{(s+7)} - \frac{1}{(s+3)}\right\}$$

$$= \left(4e^{-7t} - e^{-3t}\right)$$

$$X\left(0^{+}\right) = 4 - 1 = 3$$

53.option.(d)

Given that $L\{\cos 4t\} = \dfrac{s}{\left(s^2+16\right)}$

L $\left\{e^{-2t}\cos 4t\right\} = \dfrac{s+2}{(s+2)^2+16}$ (from first shifting theorem)

54. option(c)

$$L\{f(t)\} = \int_{-\infty}^{\infty} e^{-st}.f(t)dt = \int_{a}^{b} e^{-st}.1dt = \left(\frac{e^{-st}}{-s}\right)_a^b = \frac{-1}{s}(e^{-bs} - e^{-as}) = \frac{e^{-as} - e^{-bs}}{s}$$

55. option(b)

Given $L\{f(t)\} = s^{-3/2}$

$$g(t) = \sqrt{\frac{1}{\pi t}} = \frac{\sqrt{t\pi}}{2t} = \frac{f(t)}{2t}$$

$$L\{g(t)\} = L\left\{\frac{f(t)}{2t}\right\} = \frac{1}{2}\int_{s}^{\infty} f(t)dt = \frac{1}{2}\int_{s}^{\infty} s^{-3/2}ds = \frac{1}{2}\left(\frac{s^{-1/2}}{-1/2}\right)_s^{\infty} = \frac{1}{\sqrt{s}}$$

56. option(b)

$$L\{e^{ijt}\} = L\{\cos 5t + i\sin 5t\} = L\{\cos 5t\} + iL\{\sin 5t\} = \frac{s+5i}{s^2+25}$$

So, option (c) is correct

57. option: (c)

$$f(t) = 2 \ : \ 0 < t < 1$$
$$= 0 \ : \ otherwise$$

$$\therefore L\{f(t)\} = \int_{0}^{t} 2e^{-st}dt = 2\left(\frac{e^{-st}}{s}\right)_0^t = \frac{2-2e^{-s}}{s}$$

58.**Ans: (B)**

Sol: By the definition of Laplace transform of f(t) $\forall t \geq 0$, we have

$$F(s) = L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

59.**Ans: (A)**

Sol: $L(\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$

$\left(\therefore L\{\cos(at)\} = \frac{s}{s^2+a^2}\right)$

60.**Ans: (B)**

Sol: The solution of Laplace's equation having continuous 2nd order partial derivatives is called a harmonic function.

61.**Ans: (B)**

Sol: One period of signal $x_1(t) = u(t) - u(t-T/2)$

$$X_1(s) = \frac{1}{s} - \frac{e^{-sT/2}}{s} = \frac{1-e^{-sT/2}}{s}$$

$$X(s) = \frac{1}{1-e^{-sT}} X_1(s) = \frac{1-e^{-sT/2}}{s(1-e^{-sT})} = \frac{1}{s(1+e^{-sT/2})}$$

62. Ans: (a)

Sol: $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)dt = \int_{-\infty}^{\infty} e^{-t}\delta(2(t-1))dt$

$$= \int_{-\infty}^{\infty} e^{-t}\frac{1}{|2|}\delta(t-1)dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta(t-1)dt$$

$$= \frac{1}{2}e^{-1} = \frac{1}{2e}$$

63. Ans: (a)

Sol: $\sin 5t4(t) \xleftrightarrow{LT} \frac{5}{s^2+5^2}$

$e^{2t}\sin 5t4(t) \xleftrightarrow{LT} \frac{5}{(s-2)^2+25}$ $\left(\therefore e^{at}f(t) \xleftrightarrow{LT} F(s-a)\right)$

$$= \frac{5}{s^2+4-2\times 2(s)+25}$$

$$= \frac{5}{s^2+4-4s+25}$$

$$= \frac{5}{s^2-4s+29}$$

$\left\{\therefore e^{2t}\sin 5t4(t)\right\}\frac{5}{s^2-4s+29}$

64.Ans: (a)

Sol: $\left(s^2+2s+1\right)Y(s)$

So, Natural response $L^{-1}\left[\frac{1}{(s+1)^2}\right]$

$= te^{-t}u(t)$

65.Ans: 7.389

Sol: Given, $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$

$y(0)=0 \qquad y'(0)=1$

$y(0)=0 \qquad y'(0)=1$

$S^2y(s)-sy(0)-y'(0)-4s\ y(s)+4y(0)+4y(s)=0$

$s^2y(s)-1-4sy(s)+4y(s)=0$

$y(s)\left[s^2-4s+\right]=1$

$y(s)=\frac{1}{s^2-4s+4}$

$y(s)=\frac{1}{(s-2)^2}$

$y(t)=t.e^{2t}\left(\therefore \frac{1}{s^{n+1}}\xleftrightarrow{L.T}\frac{t^n}{n?}\right)1$

$y(1)=1.e^2=7.389$