GATEFLIX

FLUID MECHANICS

For

CHEMICAL ENGINEERING

FLUID MECHANICS

SYLLABUS

Fluid statics, Newtonian and non-Newtonian fluids, shell-balances including differential form of Bernoulli equation and energy balance, Macroscopic friction factors, dimensional analysis and similitude, flow through pipeline systems, flow meters, pumps and compressors, elementary boundary layer theory, flow past immersed bodies including packed and fluidized beds, Turbulent flow: fluctuating velocity, universal velocity profile and pressure drop.

ANALYSIS OF GATE PAPERS

Exam Year	Total
2001	05
2002	07
2003	05
2004	07
2005	09
2006	04
2007	05
2008	06
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2018	06

CONTENTS

1 BASICS OF FLUID MECHANICS

1.1 DEFINITION OF FLUID

Fluid mechanics deals with the behaviour of fluids at rest and in motion. A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.
Fluids comprise the liquid and gas (or vapour) phases of the physical forms in which matter exists. The distinction between a fluid and the solid state of matter is clear if you compare fluid and solid behaviour. A solid deforms when a shear stress is applied, but its deformation does not continue to increase with time.

1.2 BASIC EQUATIONS

Analysis of any problem in fluid mechanics necessarily begins, either directly or indirectly, with statements of the basic laws governing the fluid motion. The basic laws, which are applicable to any fluid, are:

1. The conservation of mass
2. Newton's second law of motion
3. The principle of angular momentum
4. The first law of thermodynamics
5. The second law of thermodynamics

Clearly, not all basic laws always are required to solve any one problem. On the other hand, in many problems it is necessary to bring into the analysis additional relations, in the form of equations of state or conservation equations, that describe the behaviour of physical properties of fluid under given conditions.

1.3 SYSTEM AND CONTROL VOLUME

A system is defined as a fixed, identifiable quantity of mass. The system boundaries separate the system from the surroundings.

The boundaries of the system may be fixed or movable; however, there is no mass transfer across the system boundaries. In the familiar piston–cylinder assembly the gas in the cylinder is the system. Heat and work may cross the boundaries of the system, but the quantity of the matter within the system boundaries remains fixed. There is no mass transfer across the system boundaries.
A control volume is an arbitrary volume in the space through which fluid flows. The geometric boundary of the control volume is called the control surface. The control surface may be real or imaginary; it may be at rest or in motion.

2 PROPERTIES OF FLUIDS

2.1 DENSITY

Density is defined as mass per unit volume and denoted by ρ. SI unit of density is Kg/m^3

$$\text{Density}(\rho) = \frac{\text{mass of fluid}}{\text{volume of fluid}}$$

The reciprocal of density is the specific volume (v), which is defined as volume per unit mass.

SI unit of density is m^3/Kg

$$v = \frac{1}{\rho}$$

The density of a substance, in general, depends on temperature and pressure. The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible.

2.2 SPECIFIC GRAVITY/RELATIVE DENSITY

Specific gravity or relative density of a substance is defined as the ratio of the density of a substance to the density of standard substance at a specified temperature (usually water at 4°C, for which water is 1000 kg/m^3).

$$\text{Specific Gravity} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

2.3 VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. Fluid is a substance that deforms continuously under the action of shear stress. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. A property that represents the internal resistance of a fluid to motion is called **viscosity.** The force, a flowing fluid exerts on a body in the flow direction is called the **drag force,** and the magnitude of this force depends, in part, on viscosity.

2.3.1 VISCOUS FORCE IN LIQUIDS AND GASES

1) Molecular momentum transfer:

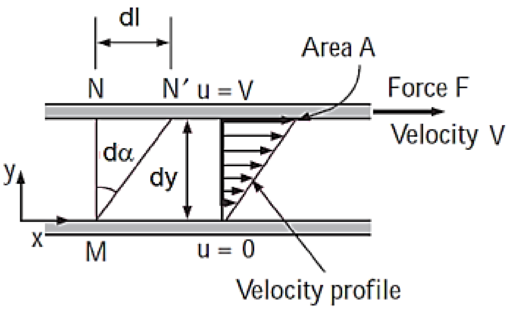
In the flow of liquids and gases, molecules are free to move from one layer to another. When the velocity in the layers are different as in viscous flow, the molecules moving from the layer at lower speed to the layer at higher speed have to be accelerated. Similarly, the molecules moving from the layer at higher velocity to a layer at lower velocity, carry with them a higher value of momentum and these are to be slowed down. Thus, the molecules diffusing across layers transport a net momentum, introducing a shear stress between the layers. The force will be zero if both layers move at the same speed or if the fluid is at rest.

2) Cohesive force:

When cohesive forces exist between atoms or molecules these forces have to be overcome, for relative motion between layers. A shear force is to be exerted to cause fluids to flow.

2.3.2 TYPES OF FLUID

Ideal fluid: Consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a small distance (dy). Now a constant tangential force F is applied to the upper plate while the lower plate is held fixed. After some time upper plate moves continuously under the influence of this force at a constant velocity *V*. The fluid in contact with the upper plate sticks to the plate surface and moves with same velocity as that of the surface. This condition is known as no slip condition.



The shear stress τ_{yx} acting on this fluid layer is

$\tau_{yx} = F / A$

Deformation or shear stain is denoted by dα

Rate of shear stain or deformation is given by $\frac{d\alpha}{dt}$

$$\tan(d\alpha) = d\alpha = \frac{dl}{dy}$$

$$\frac{d\alpha}{dt} = \frac{\left(\frac{dl}{dt}\right)}{dy} = \frac{du}{dy}$$

Where,

u is the velocity is x direction.

$\frac{du}{dy}$ is known as velocity gradient.

1) Newtonian fluid: Fluids for which the rate of deformation is proportional to the shear stress are called **Newtonian fluids.**

$\tau_{yx} \propto$ rate of deformation or rate of shear strain.

$$\tau = \mu \frac{d}{dt}$$

$$\tau = \mu \frac{du}{dy}$$

Where,

μ is constant of proportionality and is known as **Dynamic viscosity.** SI unit is kg/m-s, N. s/m² or Pa / s

Note:

1) Pa is the pressure unit Pascal.
2) A common viscosity unit is poise
   $10\,\text{Poise} = 1\,Ns/m^2$
   $1\,\text{centipoise} = 10^{-2}\,\text{Poise} = 10^{-3}\,Ns/m^2$
3) The viscosity of water at 20°C is 1 centipoise, and thus the unit centipoise serves as a useful reference.

2) Non Newtonian Fluids: Fluids in which shear stress is not directly proportional to deformation rate are known as non-Newtonian fluids. Non Newtonian fluids commonly are classified as having time independent or time dependent behavior. Numerous equations have been proposed to model the observed relations between τ and du/dy for time-independent fluids. $\tau = K\left(\frac{du}{dy}\right)^n$

Where,

n is called the flow behavior index

k the consistency index.

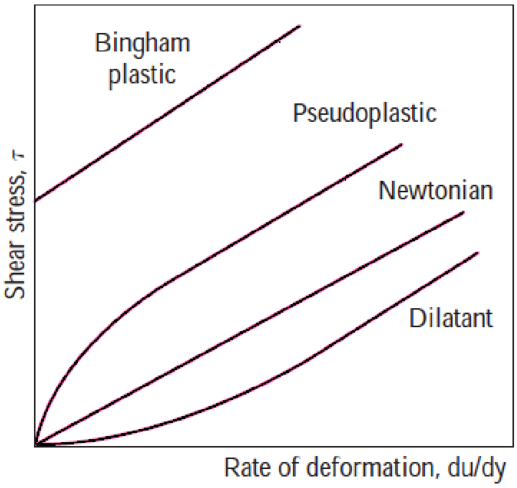
$$\tau = k\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy} = \eta\frac{du}{dy}$$

The term $\eta = k\left(\frac{du}{dy}\right)^{n-1}$ is referred to as the apparent viscosity.

Fluids in which the apparent viscosity decreases with increasing deformation rate (n<1) are called pseudo plastic (or shear thinning) fluids. Most non-Newtonian fluids fall into this group; Examples include polymer solutions, colloidal suspensions, and paper pulp in water. If the apparent viscosity increases with increasing deformation rate (n>1) the fluid is termed dilatants (or shear thickening).Suspension of starch and of sand are examples of dilatants fluids.

A "fluid" that behaves as a solid until a minimum yield stress, τ, is exceeded and subsequently exhibits a linear relation between stress and rte of deformation is referred to as an ideal or Bingham plastic.

Thixotropic fluids show a decrease in η with time under a constant applied shear stress. Rheopectic fluids show an increase in η with time. After deformation some fluids partially return to their original shape when the applied stress is released; such fluids are called viscoelastic.



2.3.3 KINEMATIC VISCOSITY

Kinematic viscosity is defined as the ratio between dynamic viscosity and density denoted by 'ν'

$\nu = \mu/\rho$

The unit in SI system is m^2/s. Stoke is CGS unit of kinematic viscosity given by

1 $(cm^2/s) = 10^{-4}$ m^2/s.

1 centistoke = 10^{-6} m^2/s.

Kinematic viscosity gives the rate of momentum flux or momentum diffusivity. For liquids and gases absolute (dynamic) viscosity is not influenced significantly by pressure. But kinematic viscosity of gases is influenced by pressure due to change in density.

2.3.4 EFFECT OF TEMPERATURE ON VISCOSITY

1) <u>Liquids:</u> In case of liquids the viscosity force is mainly due to cohesive force. The cohesive force decreases. So, viscosity of liquids decreases when temperature increases. The relation of viscosity with temperature is given by

$$\mu = \mu_0\left(\frac{1}{1+\alpha T+7\beta T}\right)$$

2) <u>Gases:</u> In the case of gases, the contribution to viscosity is more due to momentum transfer. As temperature increases, more molecules cross over with higher momentum differences. Hence, in the case of gases, viscosity increases with temperature.

$$\mu = \mu_0 + \alpha T + \beta T^2$$

where,

μ = Viscosity at T^0 in poise

μ_0 = Viscosity at 0^0 in poise

α, β = are constants for liquid and gas

SOLVED EXAMPLES

Example:

'An infinite plate is moved with a velocity of 0.3m/s over a second plate on a layer of liquid for small gap width d=0.3mm, assume a linear velocity distribution the liquid viscosity is 0.65×10^{-3} kg / ms and S.G is 0.88.

a) Calculate kinematic viscosity

b) The shear stress on the lower plate

Solution:

a)

$$V = \frac{\mu}{\rho}$$

$$= 0.65\times10^{-3}\frac{kg}{m.s}\times\frac{m^3}{(0.88\times1000)\,kg}$$

$$= 7.39\times10^{-7}\,m^2/s$$

b)

$$\tau_{lower} = \mu\left(\frac{u}{d}\right)$$

$$= 0.65\times10^{-3}\frac{kg}{m.s}\times0.3\frac{m}{s}\times\frac{1}{0.3\times10^{-3}m}$$

=0.65Pa

Direction of shear stress on lower plate is τ_{lower}

Example:

Calculate the dynamic viscosity of oil, which is used for lubrication b/w a square of size $0.8 \times 0.8\ m^2$ and an inclined plane with angle of inclination 30°, wt = 300 N, slides down the inclined plane with a uniform velocity of 0.3 m/s. the thickness of oil film is 1.5 mm.

Solution:

$$W\sin\theta = F_{oil}$$

$$\frac{F_{oil}}{F_{contact}} = \mu\frac{\Delta u}{\Delta y}$$

$$\frac{F_{oil}}{A} = \mu\frac{u}{y}$$

$$\mu = \frac{F_{oil}}{A}.\frac{u}{y}$$

$$\mu = \frac{W\sin\theta.y}{A.u}$$

$$\mu = \frac{300.\sin30\times1.5\times10^{-3}}{0.64\times0.3} = 0.117\times10\text{Poise}$$

=1.17Poise

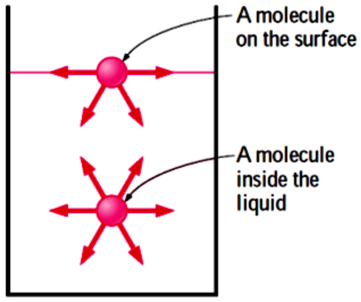
2.4 SURFACE TENSION

Consider two liquid molecules, one at the surface and one deep within the liquid body. The attractive forces applied on the interior molecule by the surrounding molecules balance each other because of symmetry. But the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small. Therefore, there is net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid. This force is balanced by the repulsive forces from the molecules below the surface that are being compressed. The resulting compression effect causes the liquid to minimize its surface area. This is the reason for the tendency of the liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

The surface of the liquid acts like a stretched elastic membrane under tension. The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid. The magnitude of this force per unit length is called surface tension and is usually expressed in the unit N/m.

Surface tension is also defined as the surface energy per unit surface area or work that needs to be done to increase the surface area of the liquid by a unit amount.

Surface tension is a binary property of the liquid & gas or two liquids. The surface tension of air and water at 20°c is about 0.73 N/m.

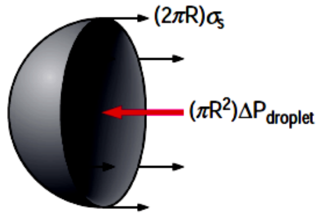


If F is the tensile force on the surface, L is the length of the surface. Surface tension is given by

$$\sigma = \frac{F}{L} \qquad \text{or}$$

$$\sigma = \frac{E_{surface}}{\text{surface Area}}$$

2.4.1 SURFACE TENSION ON LIQUID DROPLET



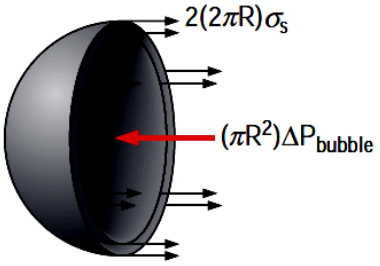
Let σ is surface tension

R is radius of droplet

A_p is area of projection

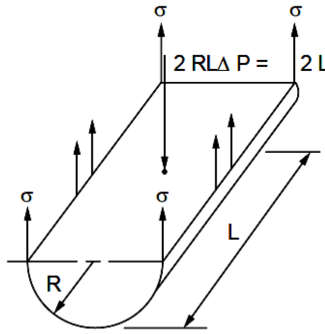
1) Force due to difference in pressure inside & outside the liquid drop
$= \Delta PA_p$
$= \Delta P\pi R^2$ ... (i)
2) Tensile force due to surface tension
$= \sigma \times \text{circumference}$
$= \sigma \times 2\pi R$ ... (ii)
Under equilibrium condition, these 2 forces will be equal
$\therefore \quad \Delta P = \frac{2\sigma}{R}$

2.4.2 SURFACE TENSION ON A SOAP BUBBLE



1) Force due to pressure inside the liquid drop
$= \Delta PA_p$
$= \Delta P\pi R^2$ ... (i)
2) Tensile force due to surface tension
$F_{surface} = 2\pi R.\sigma$
$F_{surface2} = 2\pi(R+t)\sigma$ ... (ii)
Equating the forces, we have
$\Delta P\pi r^2 = \sigma 2\pi(R+t) + \sigma 2\pi R$
Assuming thickness is very small
$T << R$
$\Delta P = \frac{4\sigma}{R}$

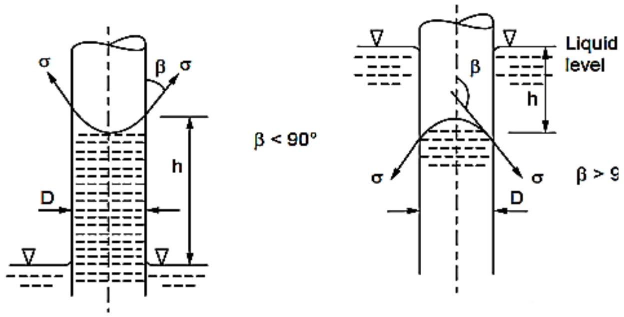
2.4.3SURFACE TENSION ON A LIGHT JET



1) Force due to pressure inside & outside the liquid jet
$= \Delta p.L(2R)$ ... (i)
2) Tensile force due to surface tension
$= 2L.\sigma$ ... (ii)
Equating the forces, we get
$\Delta P = \frac{\sigma}{R}$

2.5 CAPILLARITY

Another consequence of surface tension is the capillary effect, which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries**. The curved free surface of a liquid in a capillary tube is called the **meniscus**. It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges.. The strength of the capillary effect is quantified by the contact (or wetting) angleβ, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. The surface tension force acts along this tangent line toward the solid surface.



1) Force due to surface tension
 $= \sigma.2\pi r$
 $F_y = \sigma\pi r.\cos\beta$
 $F_x = 0$
2) Vertical force is responsible for lifting the liquid in capillary
 $F_y = mg = \rho\pi r^2 h.g$
 Equating the vertical forces, we get
 $2\pi r\sigma\cos\beta = \rho\pi r^2 h.g$
 $\Rightarrow h = \dfrac{2\sigma\cos\beta}{\rho g r}$

Note:
Same expression is used for capillary fall.
Angle, $\beta = 0$ for glass tube & water.
$\beta = 128^o$ for glass tube & mercury.

2.6 THERMO DYNAMIC PROPERTIES

2.6.1 IDEAL GAS EQUATION

$PV = nRT$
Where,
P is pressure in Pa
V is volume in m^3
n is moles of gas
R is universal gas constant (8.314 KJ/mole K)

$PV = \left(\dfrac{M}{M_w}\right)RT$

$\Rightarrow PV = m\left(\dfrac{R}{M_w}\right)T$

$\Rightarrow PV = m(R')T$

$\Rightarrow P = \left(\dfrac{m}{V}\right)R''T$

$\Rightarrow P = \rho R''T$

Where,
R' is characteristic gas constant & for air
$R' = 287\dfrac{J}{kg-K}$
M_w is molecular wt of gas
ρ is density of gas

2.6.2 THERMODYNAMIC PROCESS

a) Isothermal: Constant Temperature
 PV=mRT
 T = const
 PV = const

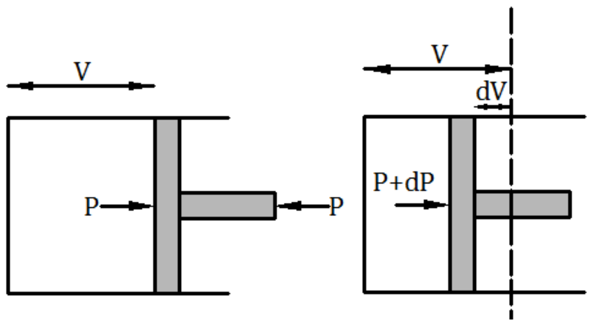
b) Adiabatic Process: No heat transfer takes place
 $\dfrac{P}{\rho^\gamma} = const$
 γ is ratio of specific heat
 $\gamma = \dfrac{C_p}{C_v}$
 γ = 1.4 for air

c) Isobaric process: Constant pressure process
 P = const
 $\dfrac{V}{T} = const$

d) Isochoric process: Constant volume process
 V = const
 $\dfrac{P}{T} = const$

2.7 COMPRESSIBILITY AND BULK MODULUS

It is the measure of volume change under the action of external force.



Let the volume of gas decrease from V to (v-dv), the pressure is increased from P to P+dP

Increase in pressure

$= dp\ kgf/m^2$

Decrease in volume = dv

Volumetric strain $= \frac{-dv}{v}$

Bulk Modulus (k) $= \frac{dp}{-dv/v}$

Compressibility = 1/k

Relationship b/w bulk modulus and pressure (p) for a gas undergoing compression process

a) For Isothermal process:

$\frac{P}{\rho} = const$

for closed system.

Pv = const

by taking log

ln P+ ln V = ln(const)

By differentiating

$\frac{dp}{p} + \frac{dV}{V} = 0$

$\frac{dp}{-dv/v} = P$

$k = p$

b) Adiabatic Process:

$PV^{y} = const$

by taking log both side

In P + γ ln v = ln const

by differentiating

$\frac{dp}{p} + \frac{ydV}{V} = 0$

$\frac{dp}{-dv/v} = \gamma P$

$k = \gamma P$

2.8 VAPOR PRESSURE

At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat}.

Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure** P_{sat}. The **vapour pressure** P_v of a pure substance is defined as the pressure exerted by its vapour in phase equilibrium with its liquid at a given temperature.

P_v is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid (P_v ” P_{sat}).

2.9 CAVITATIONS

The liquid pressure in liquid-flow systems drops below the vapour pressure at some locations, results in vaporization of liquid. For example, water at10°C will convert into vapour and form bubbles at locations (such as the tip regions of impellers or suction sides of pumps) where the pressure drops below 1.23 kPa. The vapour bubbles (called **cavitation bubbles** since they form "cavities" in the liquid) collapse as they are swept away from the low pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation,** and it is an important consideration in the design of hydraulic turbines and pumps.

GATE QUESTIONS

Q.1 A Bingham fluid of viscosity μ = 10 Pa s, and yield stress τ_0 = 10 k Pa, is sheared between flat parallel plates separated by a distance 10^{-3} m. The top plate is moving with a velocity of 1 m/s. The shear stress on the plate is

[GATE–2001]

(A) 10 kPa (B) 20 kpa

(C) 30 Kpa (D) 40 kPa

Q.2 A lubricant 1000 times more viscous than water would have a viscosity (in Pa-s)

[GATE–2003]

(A) 0.01 (B) 0.1

(C) 1 (D) 10

Q.3 The velocity profile for a Bingham plastic fluid flowing (under laminar conditions) in a pipe

[GATE–2003]

(A) Parabolic
(B) Flat
(C) Flat near the wall and parabolic in the middle
(D) Parabolic near the wall and flat in the middle

Q.4 Viscosity if water at 40^0 C lies in range

[GATE –2004]

(A) $1\times10^{-3}-2\times10^{-3}\left(\frac{\text{kg}}{\text{m s}}\right)$

(B) $0.5\times10^{-3}-1\times10^{-3}\left(\frac{\text{kg}}{\text{m s}}\right)$

(C) $1-2\left(\frac{\text{kg}}{\text{m s}}\right)$

(D) $0.5-1\left(\frac{\text{kg}}{\text{m s}}\right)$

Q.5 Match the following types of fluid (in group l) with their respective constitutive relations (in group II), where, τ is the stress and γ is the strain rate,

GROUP-I	GROUP-II
(P) Pseudo plastic Head	(1) $\tau=\mu\,\gamma$ proportional to n^2
(Q) Bingham Plastic	(2) $\tau=\tau_0+K\,\gamma$ proportional to n^3
	(3) $\tau=K\lvert\gamma\rvert^n \quad n<1$
	(4) $\tau=K\lvert\gamma\rvert^n \quad n>1$

[GATE –2005]

(A) P-1, Q-4 (B) P-4, Q-1

(C) P-2, Q-3 (D) P-3, Q-2

Q.6 The apparent viscosity of a fluid is given by $0.007\left|\frac{dV}{dy}\right|^{0.3}$ where $\left(\frac{dV}{dy}\right)$ is the velocity gradient. The fluid is

[GATE–2013]

(A) Bingham plastic
(B) Dilatant
(C) Pseudo plastic
(D) Thixotropic

Q.7 Which of the following statements are CORRECT?
(P) For a Rheopectic fluid, the apparent viscosity increases with time under a constant applied shear stress
(Q) For a pseudo plastic fluid, the apparent viscosity decreases with time under a constant applied shear stress
(R) For a Bingham plastic, the apparent viscosity increases exponentially with the deformation rate
(S) For a dilatant fluid, the apparent viscosity increases with increasing deformation rate

[GATE–2014]

(A) P and Q only
(B) Q and R only
(C) R and S only
(D) P and S only

Q.8 The relation between the stress τ and the strain rate (du_x / dy) for the rapid flow of granular material is given by $\tau = B\left(\frac{du_x}{dy}\right)^2$, where B is a constant. If M, L and T are the mass, length and time dimension respectively, what is the dimension of the constant B?

[GATE –2015]

(A) $ML^{-1}T^{-1}$ (B) $ML^{-1}T^{-2}$
(C) MT^{-1} (D) ML^{-1}

Q. 9 At a shear rate of 10 s^{-1}, the apparent viscosity of a non-Newtonian liquid was found to be 1 Pa s. At a shear rate of 100 s^{-1}, the apparent viscosity of the same liquid was found to be 0.5 Pa s. If the liquid follows power law behavior, the apparent viscosity (in Pa s) at a shear stress of 10 N m^{-2} is______

[GATE–2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9
B	C	D	B	D	B	D	D	0.997

EXPLANATIONS

Q.1 (B)

For Bingham fluids

$$\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)$$

$$= \left(10\times10^3\right) + 10\left(\frac{1}{10^{-3}}\right)$$

$$= 20,000 \text{ Pa}$$

$$\tau = 20 \text{ kPa}$$

Q.2 (C)

$\mu_{water} = 1$ cp $= 0.01$ poise

$= 0.001$ Pa s

$\mu_{Lubricant} = 1000\times0.001$

$= 1$ Pa s

Q.3 (D)

Q.4 (B)

$$\mu_{water} = 1 \text{ cp} = \frac{1}{100} \text{ poise}$$

$$= \frac{1}{100}\ \frac{g}{cm\ s}$$

$$= \frac{1}{100}\left(\frac{g}{cm\ s}\right) \times \frac{100}{1}\left(\frac{cm}{m}\right) \times \frac{1}{1000}\left(\frac{kg}{g}\right)$$

$$= 1\times10^{-3}\left(\frac{kg}{m\ s}\right)$$

So, Option (b) is correct

Q.5 (D)

Q.6 (B)

The Ostwal-de-weele Model

$$\tau = -\mu\left(\frac{dV}{dy}\right)^{n-1}\frac{dV}{dy}$$

where

$$\mu\left(\frac{dV}{dy}\right)^{n-1} = \text{Apparent Viscosity}$$

So By comparing

$n - 1 = 0.3$

So $n = 1.3 > 1$

So given fluid is Dilatant

So Option (b) is correct

Q.7 (D)

Q.8 (D)

$$\tau = \frac{F}{A} = \frac{m\,a}{A} = \frac{kg\left(m/s^2\right)}{m^2}$$

$$= \frac{kg}{m\ s^2} = M\ L^{-1}\ T^{-2}$$

$$\left(\frac{du}{dy}\right)^2 = \left(\frac{m/s}{m}\right)^2 = \left(\frac{1}{s}\right)^2 = T^{-2}$$

$$M\ L^{-1}\ T^{-2} = B\ T^{-2}$$

$$B = M\ L^{-1}$$

So Option (d) is correct.

Q.9 (0.497)

$$\tau = \mu\left(\frac{d\theta}{dt}\right)^n \quad \text{........(1)}$$

$$\tau = \mu\left(\frac{d\theta}{dt}\right)^{n-1}\frac{d\theta}{dt}$$

$$where,\ \mu_{app.} = \mu\left(\frac{d\theta}{dt}\right)^{n-1} \quad \text{.....(2)}$$

$$\frac{(\mu_{app.})_1}{(\mu_{app.})_2} = \frac{\left(\frac{d\theta}{dt}\right)_1^{n-1}}{\left(\frac{d\theta}{dt}\right)_2^{n-1}}$$

$$\frac{1}{0.5} = \frac{10^{n-1}}{100^{n-1}}$$

$$2 = (0.1)^{n-1}$$

$$n = 0.7$$

By Eq. (2)

$$(\mu_{app.})_1 = \mu\left(\frac{d\theta}{dt}\right)^{n-1}$$

$$1 = \mu(10)^{0.7-1}$$

$$\boxed{\mu = 1.99}$$

By Eq(1)

$$\tau = 1.99\left(\frac{d\theta}{dt}\right)^{0.7}$$

$$\frac{d\theta}{dt} = 10$$

By Eq. (2)

$$\mu_{app.} = 1.99(10)^{0.7-1}$$

$$\mu_{app.} = 0.997 \text{ Pa. s}$$

3 PRESSURE & FLUID STATICS

3.1 PRESSURE

Pressure is defined as the normal force exerted by fluid per unit area. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress.

$$P = \frac{F}{A}$$

Since pressure is defined as force per unit area, it has the unit of Newtons per square meter (N/m^2), which is called Pascal (Pa)

1bar = 10^5 Pa = 0.1 MPa = 100 kPa

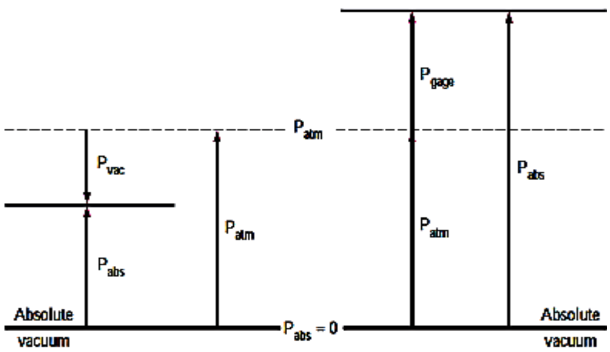
3.1.1 ABSOLUTE, GAUGE, ATMOSPHERIC & VACUUM PRESSURE

1) The pressure values must be stated with respect to a reference level. If the reference level is vacuum (i.e. absolute zero pressure), pressures are termed **absolute pressure**.
2) Most pressure gauges indicate a pressure difference—the difference between the measured pressure and the ambient level (usually atmospheric pressure). Pressure levels measured with respect to atmospheric pressure are termed **gauge pressures**
3) Pressures below atmospheric pressure are called **vacuum pressures**.

   Absolute, gauge, and vacuum pressures are all positive quantities and are related to each other by

$$P_{absolute} = P_{atmospheric} + P_{gauge}$$

$$P_{vacuum} = P_{atmospheric} + P_{absolute}$$



3.1.2 PRESSURE AT A POINT

Pressure is the compressive force per unit area, and it gives the impression of being a vector. However, in fluids under static conditions, pressure is found to be independent of the orientation of the area. This concept is explained by **Pascal's law** which **states that the pressure at a point in a fluid at rest is equal in magnitude in all directions**. Pressure has magnitude but not a specific direction, and thus it is a scalar quantity.

$$P_x = P_y = P_z$$

3.1.3 PRESSURE VARIATION IN A STATIC FLUID (HYDROSTATIC LAW):

For fluids at rest or moving on a straight path at constant velocity, all components of acceleration are zero. In fluids at rest, the pressure remains constant in any horizontal direction (P is independent of x and y) and varies only in the vertical direction.

As a result of gravity, these relations are applicable for both compressible and incompressible fluids.

$$\frac{dp}{dz} = -\rho(g), \quad \frac{dp}{dx} = 0, \quad \frac{dp}{dy} = 0$$

The negative sign is taken because dz is taken positive in upward direction and pressure decrease in upward direction.

For incompressible fluid ρ is constant.

$$\int_{p_o}^{p} dp = -\rho g \int_{z_o}^{z} dz$$

$$P - P_o = -\rho g (z - z_o)$$

$$P - P_o = \rho g h$$

For compressible fluid, ρ varies with pressure, i.e. $\rho = f(P)$

For gases, variation of density with pressure can be expressed by ideal gas equation

$$\rho = \frac{P}{RT}$$

Where,

P is pressure

T is temperature

3.1.4 FLUIDS IN RIGID-BODY MOTION

When fluid is in stationary container, the pressure remains constant along horizontal direction. The pressure varies only along vertical direction. When a fluid is placed in an accelerated container, initially fluid splashes and there is a relative motion between fluid & container boundary. After some time, the liquid comes to rest and attains fixed shape relative to container. The pressure varies in the direction of acceleration.

1) **When container accelerates in vertical direction**

   Case1: Downward acceleration of a_z

   $$\frac{dp}{dz} = -\rho(g - a_z),\ \frac{dp}{dx} = 0,\ \frac{dp}{dy} = 0$$

   Case 2: upward acceleration of a_z

   $$\frac{dp}{dz} = -\rho(g + a_z),\ \frac{dp}{dx} = 0,\ \frac{dp}{dy} = 0$$

2) **When container accelerates in horizontal direction**

   Case1: Acceleration in positive x direction

   $$\frac{dp}{dz} = -\rho(g),\ \frac{dp}{dx} = -\rho a_x,\ \frac{dp}{dy} = 0$$

   Case 2: Acceleration in negative x direction

   $$\frac{dp}{dz} = -\rho(g),\ \frac{dp}{dx} = -\rho(-a_x),\ \frac{dp}{dy} = 0$$

3.2 THE BAROMETER & ATMOSPHERIC PRESSURE

Atmospheric pressure is measured by a device called **barometer;** thus, the atmospheric pressure is often referred to as the barometric pressure. The pressure at point B is equal to the atmospheric pressure, and the pressure at C can be taken to be zero since there is only mercury vapour above point C and the pressure is very low relative to P_{atm} and can be neglected for an excellent approximation. Writing a force balance in the vertical direction gives

$$P_{atm} = \rho gh$$

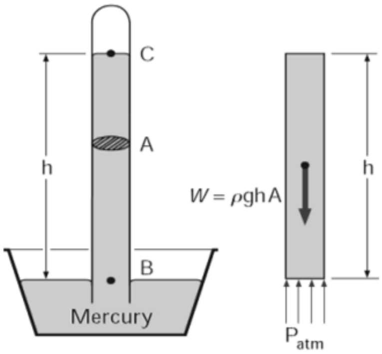
Where,

ρ is the density of mercury,

g is the local gravitational acceleration,

h is the height of the mercury column above the free surface.

Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer. A frequently used pressure unit is the *standard atmosphere*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C $(\rho_{Hg}\ \text{is}\ 13,595\ kg/m^3)$. If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3m would be needed.



3.3 PRINCIPLES OF FLUID STATICS

1) When fluid is at rest, in a continuous fluid, fluid at the same elevation has the same pressure.
2) The pressure at the bottom of a column of fluid is equal to the pressure at the top, plus density multiplied by gravity multiplied by the height of the column of fluid.

   A consequence of the second principle is that when different columns of fluid stack on top of one another, the pressures due to each column simply add up.

3.3.1 HYDRAULIC LIFT

A consequence of the pressure in a fluid remaining constant in the horizontal direction is that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is called **Pascal's law.** The application of Pascal's law in hydraulic lift is shown in fig

$P_1 = P_2$

$F_1 / A_1 = F_2 / A_2$

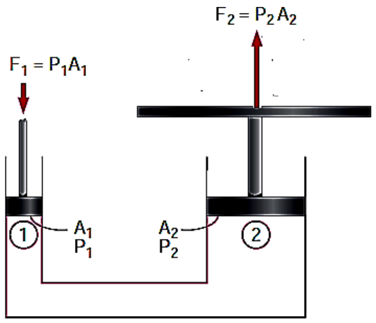


Fig.: Schematic of a hydraulic lift

3.4 PRESSURE MEASUREMENT

Hydrostatic law indicates the pressure difference b/w two points in a static fluid. A device based on this principle is called **manometer**, and it is commonly used to measure small and moderate pressure differences. A manometer mainly consists of a glass or plastic U tube containing one or more fluids such as mercury, water, alcohol, or oil**.** The pressure of a fluid is measured by the following.

1) Manometers
   a. Simple
   - Piezometer
   - U-tube manometer
   - Single column manometer

   b. Differential
   - U-tube differential manometer
   - Inverted U-tube manometer
2) Mechanical gauge

3.4.1 SIMPLE MANOMETER

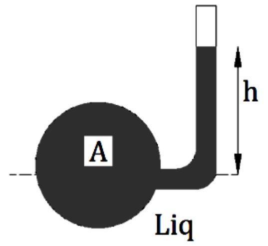
1) **Piezometer:** It is the simplest kind of manometer. It does not have any high density liquid. The tube is connected to the point where pressure is to be measured. The liquid rises in tube to balance the pressure at 'A'.

The gauge pressure P_A is given by

$P_A = \rho g h$

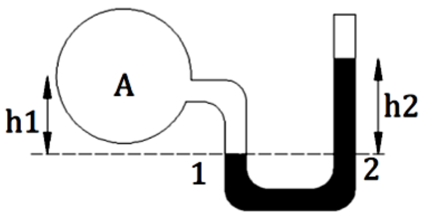
Where

ρ is the density of liquid inside vessel/pipe



Piezometer is used to measure low pressures. The height of column increases if pressure is high. E.g. height of water column is 10.3 m if gauge pressure at 'A' is 1atm ($10^5 N/m^2$).

2) **U-tube manometer:** It has a glass U-tube with liquid having density higher than the density of fluid in the container.



The atmospheric pressure exists and should be taken into account for evaluation of absolute pressure, but while evaluating gauge pressure it is not accounted in equations.

The gauge pressure at 'A' is given by

$P_1 = P_A + \rho_c g h_1$

$P_2 = \rho_m g h_2$

Where,

ρ_C is the density of fluid in container, it can be water or oil

ρ_m is the density of manometric fluid. Usually mercury is chosen as manometric fluid

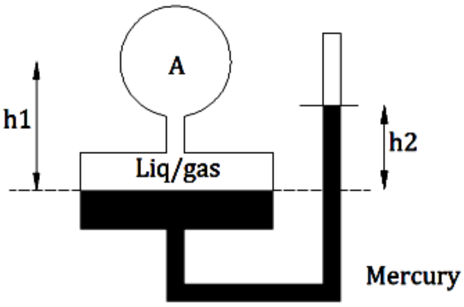
h_2 is difference in mercury level

P_A is pressure in vessel/pipe in gauge

From principle of fluid statics, when fluid is at rest, fluid at the same elevation has the same pressure.

$P_A = \rho_m g h_2 - \rho_w g h_1$

3) Single Column Manometers: In this manometer a large cross-sectional area reservoir is placed in one of the limbs. When pressure is applied, the fluid lowers slightly in the reservoir as compared to the fluid rise in the other limb.



Gauge Pressure at point A is given as

$$P_A = \frac{ah_2}{A}.(\rho_m g - \rho_c g) + (\rho_m g h_2 - \rho_c g h_1)$$

Since A>>α

$$P_A = \rho_m g h_2 - \rho_c g h_1$$

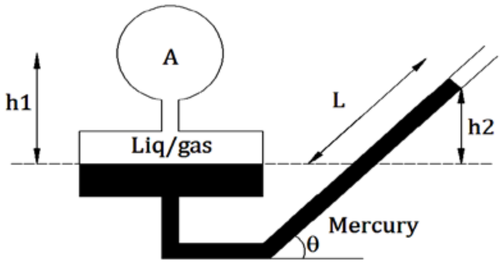
Salient features of Single column manometer:

i) modified form of U-tube manometer

ii) large cross-sectional area (100 times)

iii) due to large area of cross section and small change of pressure, the change in level of reservoir will be very small and can be neglected.

4) Inclined Single Column Manometer: This manometer is more sensitive than straight column. The liquid rises more in the column due to inclination.



$$h_2 = L \sin\theta$$

$$P_A = \frac{ah_2}{A}.(\rho_2 g - \rho_1 g) + (\rho_2 g h_2 - \rho_1 g h_1)$$

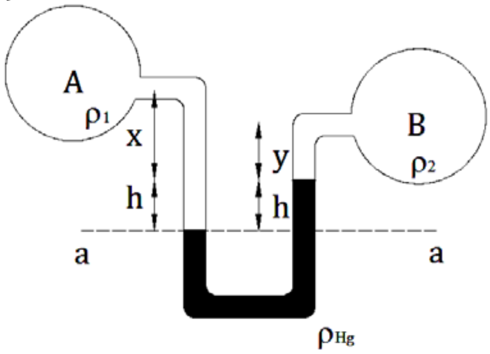
$$P_A = \rho_2 g_2 h_2 - \rho_1 g_1 h_1$$

$$P_A = \rho_2 g_2 L \sin\theta - \rho_1 g h_1$$

3.4.2 DIFFERENTIAL MANOMETERS

Differential Manometers are devices used for measuring the difference of pressure between two points in a pipe or two different pipes. It contains of a U-tube with manometric liquid. The manometric liquid can be of higher density or lower density than pipe liquid.

1) U-tube differential manometer:



Pressure above a-a

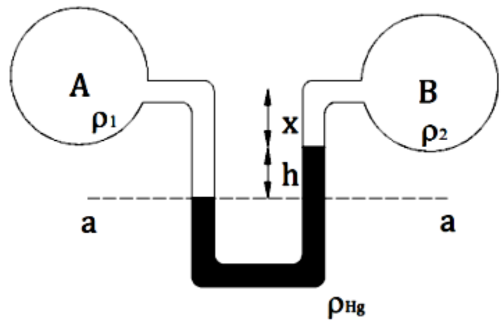
- In the left limb $= \rho_1 g(h+x) + P_A$
- In the right limb $= P_B + \rho_2 g y + \rho_{Hg} g h$

By equating

$$P_A + \rho_1 g(h+x) = P_B + \rho_2 g y + \rho_{Hg} g.H$$

$$P_A - P_B = \rho_2 g y + \rho_{Hg}.g.H - \rho_1 g(h+x)$$

A & B are at same level :

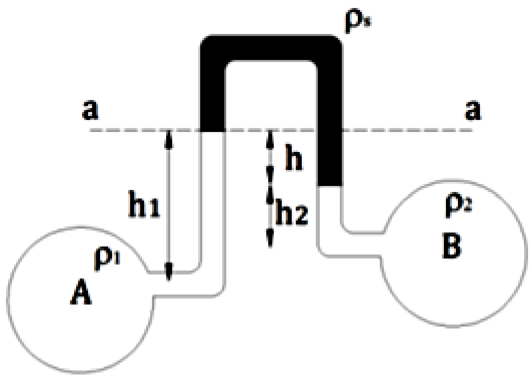


$P_A - P_B = \rho_2 gx + \rho_{Hg} gh - \rho_1 g(h+x)$

If liquid is same, then $\rho_1 gx = \rho_2 gx$

$\therefore P_A - P_B = gh(\rho_{Hg} - \rho_1)$

2) Inverted U-tube differential Manometer:



$P_A - \rho_1 gh_1 = P_B - \rho_2 gh_2 - \rho_s gh$

$P_A - P_B = \rho_1 gh_1 - \rho_2 gh_2 - \rho_s gh$

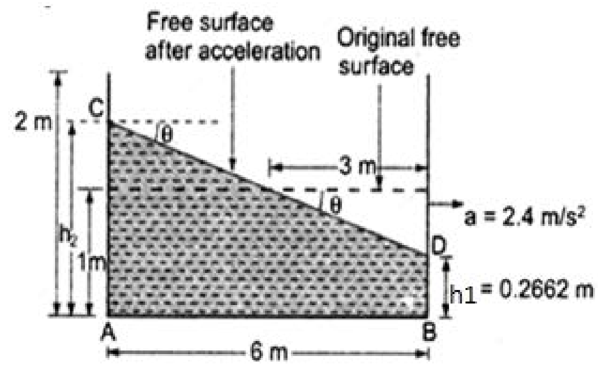
It is used for measuring difference of low pressures

SOLVED EXAMPLES

Example:

A rectangular tank is moving horizontally in the direction of its length with a constant acceleration of $2.4 m/s^2$. The length, width and depth of the tank are 6m, 2.5m and 2m respectively. If the depth of water in the tank is 1m and tank is open at the top then, Calculate:

I. angle of the water surface with the horizontal
II. the maximum and minimum pressure intensities at the bottom



Solution:

Given:

Constant acceleration $a = 2.4 m/s^2$

Length = 6m; Width = 2.5m and depth = 2m,

Depth of water in tank, h = 1m

i) The angle of the water surface to the horizontal

Let θ = the angle of water surface to the horizontal

Using equation, we get

$$\tan\theta = -\frac{a}{g} = -\frac{2.4}{9.81} = -0.2446$$

(the –ve sign shows that the free surface of water is sloping downward as shown in Fig)

$\therefore \quad \tan\theta = -0.2446$ (slope downward)

$\therefore \quad \theta = \tan^{-1} 0.2446 = 13.7446^\circ$ or $13^\circ 44.6'$

ii) The maximum and minimum pressure intensities at the bottom of the tank

From the figure, depth of water at the front end,

$h_1 = 1 - 3\tan\theta = 1 - 3 \times 0.2446 = 0.2662m$

Depth of water at the rear end :

$h_2 = 1 + 3\tan\theta = 1 + 3 \times 0.2446 = 1.7338m$

The pressure intensity will be maximum at the bottom, where depth of water is maximum.

Now, the maximum pressure intensity at the bottom will be at point A and it is given by,

$P_{max} = \rho \times g \times h_2$

$= 1000 \times 9.81 \times 1.7338 N/m^2 = 17008.5 N/m^2$

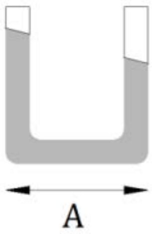
The minimum pressure intensity at the bottom will be at point B and it is given by

$p_{min} = \rho \times g \times h_1$

$= 1000 \times 9.81 \times 0.2662 = 2611.4 N/m^2$

Example:

A U-tube as shown in figure, filled with water to mid level is used to measure the acceleration when fixed on moving equipment. Determine the acceleration a_x as a function of the angle θ and the distance A between legs.



Solution:

This is similar to the formation of free surface with angle θ

$\tan\theta = -a_x/(g + a_y)$

As $a_y = 0$, $\tan\theta = -a_x/g$

The acute angle θ will be given by,

$\theta = \tan^{-1}(a_x/g)$

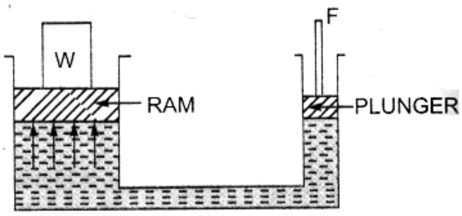
$a_x = g \times \tan\theta$

As $\tan\theta = 2h/A$

$h = A\,a_x/2g$

Example:

A hydraulic press has a ram of 30cm diameter and a plunger of 4.5cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500N.



Solution:

Given:

Dia. of ram $D = 30cm = 0.3m$

Dia. Of plunger, $d = 4.5cm = 0.045m$

Force on plunger, $F = 500N$

Let the weight lifted $= W$

Area of ram, $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$

Area of plunger,

$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.045)^2 = .00159 m^2$

Pressure intensity due to plunger

$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} N/m^2$

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$= \frac{500}{.00159} = 314465.4 N/m^2$

But pressure intensity at ram

$= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{.07068} N/m^2$

$\frac{W}{.07068} = 314465.4$

∴ Weigth

$= 314465.4 \times 0.7068 = 22222N = 22.222kN$

Example:

The diameters of a small piston and a large piston of hydraulic jack are 3 cm and 10cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when:

a) The pistons are at the same level.
b) Small piston is 40cm above the large piston.

The density of the liquid in the jack is given as $1000 kg/m^3$

Solution:

Given:

Dia. of small piston, $d = 3cm$

∴ Area of small piston,

$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 3^2 = 7.068 cm^2$

Dia. of large piston, $D = 10cm$

∴ Area of larger piston,

$$A=\frac{\pi}{4}\times 10^2 = 78.54cm^2$$

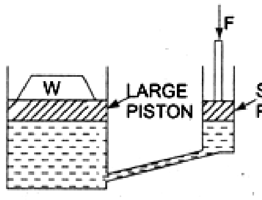
Force on small piston, $F=80N$

Let the load lifted $=W$

a) When the pistons are at the same level

Pressure intensity on small piston

$$\frac{F}{a}=\frac{80}{7.068}N/m^2$$



This is transmitted equally to the large piston.

∴ Pressure intensity on the large piston

$$=\frac{80}{7.068}$$

∴ Force on the large piston

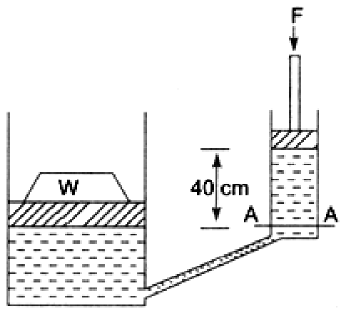
= pressure x area

$$=\frac{80}{7.068}\times 78.54N = 888.96N$$

b) When the small piston is 40cm above the large piston

Pressure intensity on the small piston

$$\frac{F}{a}=\frac{80}{7.068}N/m^2$$



∴ Pressure intensity at section A-A

$=\frac{F}{a}+$ Pressure intensity due to height of 40cm of liquid.

Pressure intensity due to 40cm of liquid

$$=\rho\times g\times h = 1000\times 9.81\times 0.4 N/m^2$$

$$=\frac{1000\times 9.81\times .40}{10^4}N/cm^2 = 0.3924N/cm^2$$

∴ Pressure intensity at section A-A

$$=\frac{80}{7.068}+0.3924$$

$$11.32+0.3924=11.71\ N/cm^2$$

∴ Pressure intensity transmitted to the large piston $=11.71N/cm^2$

∴ Force on the large piston = Pressure × Area of the large piston

$$=11.71\times A = 11.71\times 78.54 = 919.7N$$

Example:

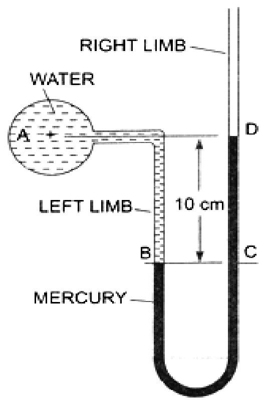
A U-tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to water in the main line, if the difference in level of mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to $9810N/m^2$, calculate the new difference in the level of mercury. Sketch the arrangements in both cases.

Solution:

Given:

Difference in mercury level =10 cm =0.1m

The arrangement is shown in fig (a)



1st Part

Let $P_A=$ (pressure of water in pipe line (i.e., at point A)

The points B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C. But pressure at B = Pressure at A + Pressure due to 10cm (or 0.1 m) of water

$= p_A + \rho \times g \times h$

Where $\rho = 1000 kg/m^3$ and $h = 0.1m$

$= p_A + 1000 \times 9.81 \times 0.1$

$= p_A + 981 N/m^2$ ....(i)

Pressure at C = pressure at D+ Pressure due to 10 cm of mercury

$= 0 + \rho_0 \times g \times h_0$

Where ρ_0 for mercury $= 13.6 \times 1000 kg/m^3$

And $h_0 = 10cm = 0.1m$

∴ Pressure at C

$= 0 + (13.6 \times 1000) \times 9.81 \times .01$

$= 13341.6N$ ....(ii)

But pressure at B is equal to pressure at C. Hence equating the equations (i) and (ii), we get,

$p_A + 981 = 13341.6$

$\therefore p_A = 13341.6 - 981$

$= 12360.6 \frac{N}{m^2}$

2nd Part

Given, $p_A = 9810 N/m^2$

Find new difference of mercury level. The arrangement is shown in following figure. In this case, pressure at A is $9810N/m^2$ which is less than the $12360.6N/m^2$. Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x =Rise of mercury in left limb in cm.

Then fall of mercury in right limb = x cm

The points B, C and D shows the initial conditions whereas points B*, C* and D* show the final conditions.

pressure at B* = Pressure at C*

Or

Pressure at A + Pressure due to (10-x)cm of water

= Pressure at D* + Pressure due to (10-2x) cm of mercury

Or $p_A + \rho_1 \times g \times h_1 = p_D^* + \rho_2 \times g \times h_2$

Or $1910 + 1000 \times 9.81 \times \left(\frac{10 - x}{100}\right)$

$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10 - 2x}{100}\right)$

Dividing by 9.81, we get

1000 + 100 - 10x = 1360 - 272x

Or

272x - 10x = 1360 - 1100

Or

262x = 260

$\therefore \quad x = \frac{260}{262} = 0.992cm$

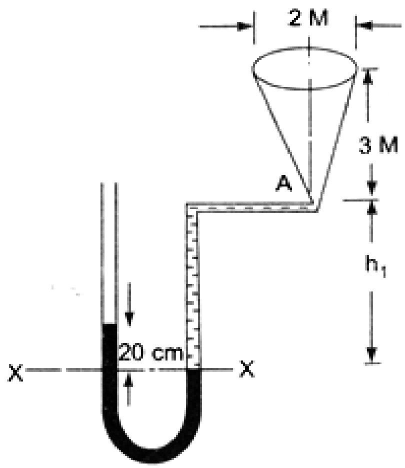
∴ New difference of mercury

$= 10 - 2x cm = 10 - 2 \times 0.992$

$= 8.016cm$

Example:

Fig. shows a conical vessel having its outlet at A to which a U-tube monometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.



Solution:

Vessel is empty:

Given:

Difference of mercury level

$h_2 = 20cm$

Let h_1 = Height of water above X-X

S.G. of mercury, $S_2 = 13.6$

S.G. of water, $S_1 = 1.0$

Density of mercury,

$\rho_2 = 13.6 \times 1000$

Density of water,

$\rho_1 = 1000$

Equating the pressure above datum line X-X, we have,

$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$

Or $13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$

$h_1 = 2.72\text{m}$ of water.

Vessel is full of water:

When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in following figure. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z,

Pressure in left limb =Pressure in right limb

$13.6 \times 1000 \times 9.81 \times \left(0.2 + \frac{2y}{100}\right)$

$= 1000 \times 9.81 \times (3 + h_1 + y/100)$

$13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100)$

Or

$2.72 + 27.2y/100 = 3 + 2.72 + y/100$

Or

$(27.2y - y)/100 = 3.0$

Or

$26.2y = 3 \times 100 = 300$

$\therefore \quad y = \frac{300}{26.2} = 11.45\text{cm}$

The difference of mercury level in two limbs

$= (20 + 2y)\text{cm of mercury}$

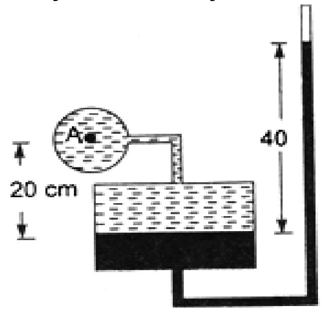
$= 20 + 2 \times 11.45 = 20 + 22.90$

$= 42.90\text{cm of mecury}$

$\therefore$ Reading of monometer = 42.90 cm

Example:

A single column manometer is connected to a pipe containing a liquid of S.G. 0.9 as shown in Fig. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. The specific gravity of mercury is 13.6



Solution:

Given:

Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900\text{kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

$\therefore$ Density, $\rho_2 = 13.6 \times 1000$

$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$

Height of liquid, $h_1 = 20\text{cm} = 0.2\text{m}$

Rise of mercury in right limb,

$h_2 = 40\text{cm} = 0.4\text{m}$

Let $p_A = \text{Pressure in pipe}$

Using equation,

$p_A = \frac{a}{A} h_2 [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g,$

we get,

$= \frac{1}{100} \times 0.4 [13.6 \times 1000 \times 9.81 - 900 \times 9.81]$
$+ 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$

$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$

$= 533.664 + 53366.4 - 1765.8\text{N/m}^2$

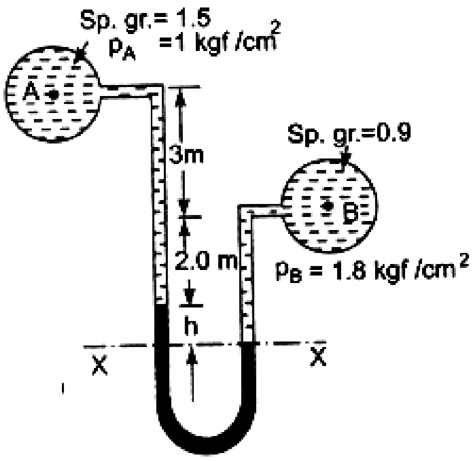
$= 52134\text{N/m}^2$

$= 5.21\text{N/cm}^2$

Example:

A differential manometer is connected at the two points A and B of two pipes as shown in fig. The pipe A contains a liquid of

S.G. = 1.5 while pipe B contains a liquid of S.G. = 0.9. The pressures at A and B are $1 kgf/cm^2$ and $1.80 kgf/cm^2$ respectively. Find the difference in mercury level in the differential manometer.



Solutions:

Given:

S.G. of liquid at A, $S_1 = 1.5 \therefore \rho_1 = 1500$

S.G. of liquid at B, $S_2 = 1.5 \therefore \rho_2 = 1500$

Pressure at A,

$$P_A = 1 kgf/cm^2 = 1 \times 10^4 kgf/m^2$$

$$= 10^4 \times 9.81 N/m^2 (\because 1 kgf = 9.81 N)$$

Pressure at B,

$$P_B = 1.8 kgf/cm^2$$

$$= 1.8 \times 10^4 kgf/m^2$$

$$= 1.8 \times 10^4 \times 9.81 N/m^2 (\because 1 kgf = 9.81 N)$$

Density of mercury $= 13.6 \times 1000 kg/m^3$

Taking X-X as datum line

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + p_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb

$$= 900 \times 9.81 \times (h+2) + p_B$$

$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressures, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

Or

$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

Or

$$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.5h = 2.3$$

$$\therefore h = \frac{2.3}{12.7} = 0.181 m = 18.1 cm$$

3.5 HYDROSTATIC FORCES ON SURFACES

In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the normal stress, which is the pressure, and the variation of pressure is only due to the weight of the fluid. The force exerted on a surface by a fluid at rest is normal to the surface at the point of contact since there is no relative motion between the fluid and the solid surface, and thus no shear forces can act parallel to the surface.

Fluid statics is used to determine the forces acting on floating or submerged bodies and the forces developed by devices like hydraulic presses and car jacks. The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics.

3.5.1 TOTAL PRESSURE

Force is exerted by a static fluid on a surface, either plane or curved when fluid comes in contact with the surfaces. This force always acts normal to the surface.

3.5.2 CENTRE OF PRESSURE

It is defined as the point of application of the total pressure on the surface. The submerged surfaces may be

1) Vertical plane submerged
2) Horizontal plane surface
3) Inclined plane
4) Curved surface

3.5.3 VERTICAL PLANE SURFACE SUBMERGED IN

LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown

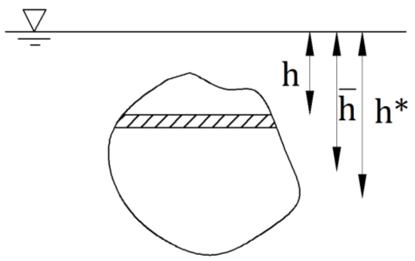
A = Total area of surface

$\bar{h}$ = Distance of C. G. of the area from free surface of liquid

G = Centre of Gravity of plane surface

P = Centre of Pressure

h^* = Distance of centre of pressure from free surface of liquid.



a) Total Pressure

Pressure Intensity at strip $= \rho gh$

Area of strip $dA = b.dh$

Force on strip $dF = \rho.g.h.b.dh$

Total pressure force on the whole surface is

$$\int_s dF = \int \rho ghbdh$$

$$\int_s dF = \rho g \int h.dA$$

$$F = \rho.g.\bar{h}.A$$

$\int h.dA$ is moment of surface area about free surface of liquid is equal moment of C.G. about free surface.

$$\int h.dA = A.\bar{h}$$

b) Centre of Pressure: (h^*)

Principle of Moments: Moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

$F_t.h^* = \sum$moments about free surface of liquid. ...(1)

$$\sum \text{moments} = \int dA.h.\rho gh$$

$$= \rho g \int bh^2 dh$$

$$= \rho gb \int h^2 dh$$

Where,

$\int dA.h^2 = I_o$ is the moment of Inertia of surface about free surface of liquid.

$$\sum \text{moments} = \rho g I_0 \qquad ...(2)$$

$$\therefore \quad F_t.h^* = \rho g I_o$$

$$h^* = \frac{\rho g I_o}{\rho g\bar{h}.A}$$

$$h^* = \frac{I_0}{\bar{h}A}$$

Where,

$\bar{h}$ is the distance of C.G. from free surface

A is the area.

From II axis theorem

$$I_o = I_{C.G.} + A\bar{h}^2$$

$$h^* = \frac{I_{C.G.} + A\bar{h}^2}{\bar{h}.A}$$

1) h^* lies below the C.G. of the surface
2) It is independent of the density of liquid & depends only on surface area.

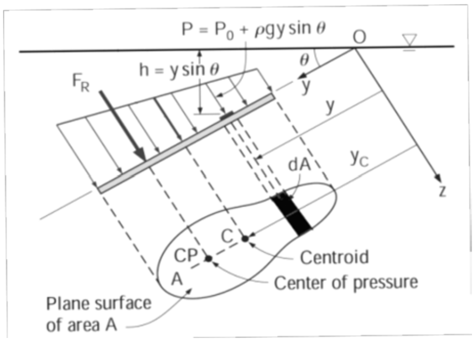
3.5.4 HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

As every point of the surface is at the same depth from free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to P $= \rho gh$ where h is depth of surface

$$F_1 = \rho g\, h \times \text{Area}$$

$$\bar{h} = h = h^*$$

3.5.5 INCLINED PLANE SUBMERGED IN LIQUID



Let

A = Total area of surface

$\bar{h}$ = Distance of C. G. of the area from free surface of liquid

G = Centre of Gravity of plane surface

P = Centre of Pressure

h^* = Distance of centre of pressure from free surface of liquid.

a) Total Pressure

Pressure intensity on the strip $P = \rho gh$

Pressure force dF on the strip $dF = P \times dA = \rho ghdA$

Total pressure force on the whole area,

$F = \int dF = \int \rho ghdA$

From fig. $\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$

$\therefore \; h = y\sin\theta$

$\therefore \; F = \int \rho g\, y\sin\theta dA$

But $\int y\, dA = A\bar{y}$ is the moment of surface at distance 'y'

$\therefore \; F = \rho g\sin\theta A\bar{y}$

$\therefore \; F = \rho g A \bar{h} \quad (\because \bar{h} = \bar{y}\sin\theta)$

Note: the above expression of force is for fluid with no pressure acting on the surface. If pressure acts on the surface

$F = P_0 A + \rho g A\bar{h}$

b) Centre of Pressure

Pressure force on the strip, $dF = \rho gh\, dA = \rho gy\sin\theta dA$

Moment of the force, dF, about axis O-O

$= dF \times y$

$= \rho gy\sin\theta dA.y$

Sum of moments of all such forces about O – O $= \int \rho g\sin\theta y^2 dA$

$= \rho g\sin\theta \int y^2 dA$

Where,

$\int y^2 dA =$ Moment of Inertia of the surface about O – O = I_o

∴ Sum of moments of all force $= \rho g\sin\theta I_o$

$F \times y^* = \rho g\sin\theta I_o$

$y^* = \frac{\rho g\sin\theta I_o}{\rho g A\bar{h}}$

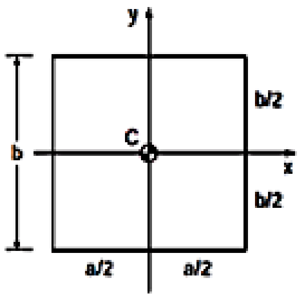
$y^* = \frac{I_o \sin\theta}{A\bar{h}}$

$h^* = \frac{I_0 \sin^2\theta}{A\bar{h}}$

$h^* = \frac{\sin^2\theta}{A\bar{h}}(I_G + A\bar{y}^2)$

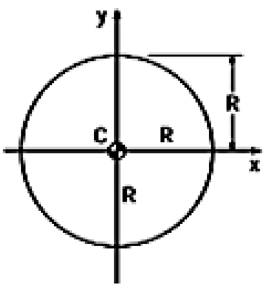
$h^* = \frac{\sin^2\theta}{A\bar{h}}\left(I_G + \frac{A\bar{h}^2}{\sin^2\theta}\right)$

a) Rectangle



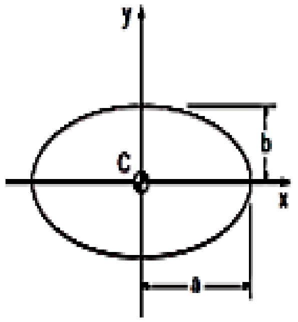
$A = ab, \; I_{xx,C} = \frac{ab^3}{12}$

b) Circle



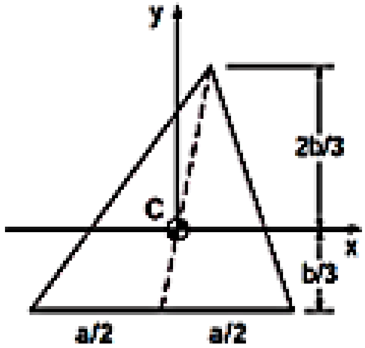
$A = \pi R^2, I_{xx,C} = \frac{\pi R^4}{4}$

c) Triangle



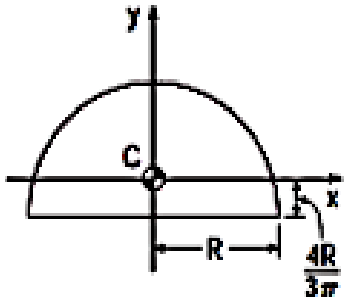
$$A=\pi ab, I_{xx,C}=\frac{\pi ab^3}{4}$$

d) Triangle



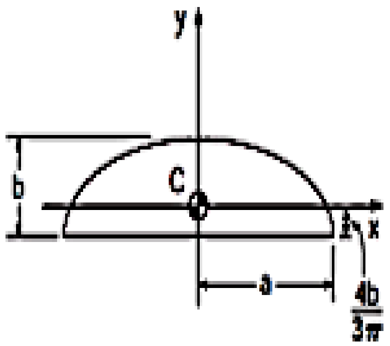
$$A=\frac{ab}{2}, I_{xx,C}=\frac{ab^3}{36}$$

e) Semicircle



$$A=\frac{\pi R^2}{2}, I_{xx,C}=0.109757R^4$$

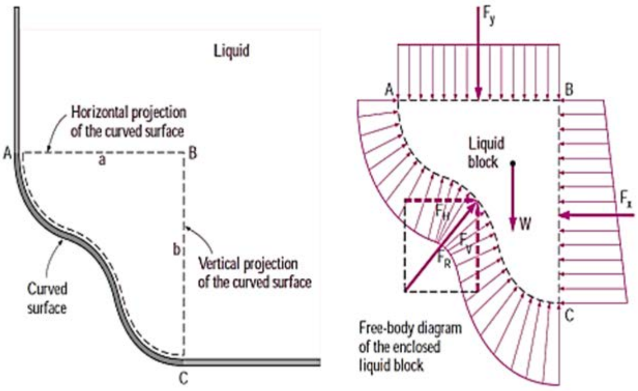
f) Semi ellipse



$$A=\frac{\pi ab^2}{2}, I_{xx,C}=0.109757ab^3$$

3.5.6 CURVED SURFACE SUB-MERGED IN LIQUID

For a submerged curved surface, the determination of the resultant hydrostatic force is more involved since it typically requires the integration of the pressure forces that change direction along the curved surface. The way to determine the resultant hydrostatic force F_R acting on a two-dimensional curved surface is to determine the horizontal and vertical components F_x and F_y separately. This is done by considering the free-body diagram of the liquid block enclosed by the curved surface and the two plane surfaces (one horizontal and one vertical) passing through the two ends of the curved surface. Note that the vertical surface of the liquid block considered is simply the projection of the curved surface on a vertical plane, and the horizontal surface is the projection of the curved surface on a horizontal plane.



The resultant force acting on the curved surface is given by

$$F_R=\sqrt{F_x^{\,2}+F_y^{\,2}}$$

Inclination of resultant with horizontal is given by

$$\tan\theta=\frac{F_y}{F_x}$$

1) The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic

force acting on the vertical projection of the curved surface.

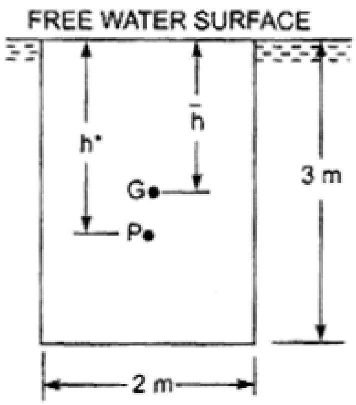
2) The vertical component of the hydrostatic force acting on a curved surface is equal to the weight of liquid supported by the curved surface.

Example:
A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when it's upper edge is horizontal and (a) coincides with water surface, (b) 2.5m below the free water surface.

Solution:
Given:
Width of plane surface, b=2m
Depth of plane surface, d=3m

a) Upper edge coincides with water surface

Total pressure is given by equation as $F = \rho g A\bar{h}$



Where,

$\rho = 1000 kg/m^3,\ g = 9.81 m/s^2$

$A = 3\times 2 = 6m^2,\ \bar{h} = \frac{1}{2}\times 3 = 1.5m$

$\therefore F = 1000\times 9.81\times 6\times 1.5$

$= 88290N$

Depth of centre of pressure is given by equation as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

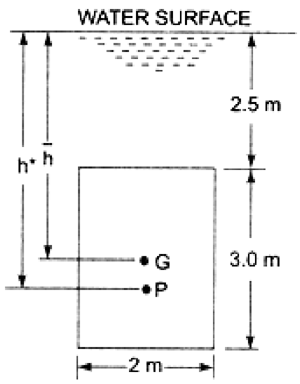
Where,

I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5m^4$$

$$\therefore h^* = \frac{4.5}{6\times 1.5} + 1.5 = 0.5 + 1.5 = 2.0m$$

b) Upper edge is 2.5, below water surface



Total pressure (F) is given by

$F = \rho g A\bar{h}$

Where,

$\bar{h}$ =Distance of C.G. from free surface of water

$= 2.5 + \frac{3}{2} = 4.0m$

$\therefore\ F = 1000\times 9.81\times 6\times 4.0$

$= 235440N$

Center of pressure is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where

$I_G = 4.5, A = 6.0, \bar{h} = 4.0$

$$h^* = \frac{4.5}{6.0\times 4.0} + 4.0$$

$= 0.1875 + 4.0 = 4.1875 = 4.1875m.$

Example:
A circular opening, 3m diameter, in a vertical side of a tank is closed by disc of 3m diameter which can rotate a horizontal diameter.
Calculate:

i) The force on the disc, and
ii) The torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4m.

Solution:

Given:

Dia. of opening $d = 3m$

Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 m^2$

Depth of C.G. $\bar{h} = 4m$

i) Force on the disc is given by equation as

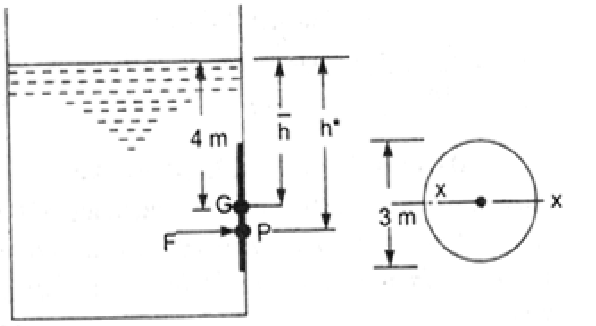
$F = \rho g A\bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$
$= 277368 N = 277.368 kN$

ii) To find the torque required to maintain the disc in equilibrium, first calculate the point application of force acting on the disc, i.e., center of pressure of the force F. The depth of centre of pressure (h*) is given by equation as

$$h* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0$$

$$\left\{ \because I_G = \frac{\pi}{64} d^4 \right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14 m$$



The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

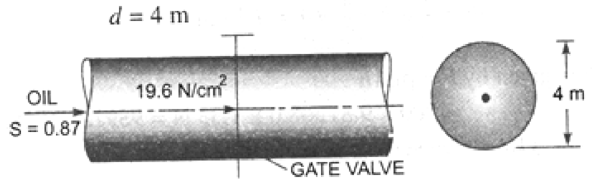
$= F(h^* - \bar{h}) = 277368(4.14 - 4.0) = 38831 Nm$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Example:

A pipe line which is 4m in diameter contains a gate valve. The pressure at the centre of the pipe is $19.6 N/cm^2$. If the pipe is filled with oil of S.G. 0.87; find the force exerted by the oil upon the gate and position of centre of pressure.

Solution:



Given:

Dia. of pipe, $d = 4m$

∴ Area,

$A = \frac{\pi}{4} \times 4^2 = 4p m^2$

∴Density of oil $\rho_0 = 0.87 \times 1000 = 870 kg/m^3$

∴ Weight density of oil,

$w_0 = \rho_0 \times g = 870 \times 9.81 N/m^3$

Pressure at the centre of pipe,

$p = 19.6 N/cm^2 = 19.6 \times 10^4 N/m^3$

∴ Pressure head at the centre

$= \frac{p}{w_0} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988 m$

∴ The height of equivalent free oil surface from the centre of pipe =22.988m

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988 m$

$F = \rho g A\bar{h}$

Where $\rho =$ density of oil $= 870 kg/m^3$

$F = 870 \times 9.81 \times 4\pi \times 22.988 = 2465500 N = 2.465 MN$

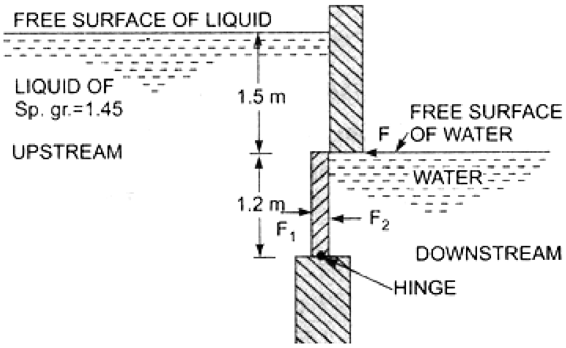
(ii)Position of centre of pressure (h*) is given as

$$h* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16h} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$= 0.043 + 22.988 = 23.031 m$

Or, centre of pressure is below the centre of the pipe by a distance of 0.043m

Example:
A vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2 m high. On the upstream of the gate, the liquid of S.G. 1.45 lies upto a height of 1.5m above the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.



Solution:
Given:
Width of gate, $b = 2m$
Depth of gate, $d = 1.2m$
∴ Area,
$A = b \times d = 2 \times 1.2 = 2.4 m^2$
Sp. gr. of liquid $= 1.45$
∴ Density of liquid,
$\rho_1 = 1.45 \times 1000 = 1450 kg/m^3$
Let $F_1 =$ Force exerted by the fluid of sp. gr 1.45 on gate
$F_2 =$ Force exerted by water on the gate.
The force
$F_1 =$ is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$
Where
$\rho_1 = 1.45 \times 1000 = 1450 kg/m^2$
$\bar{h}_1 =$ Depth of C.G. of gate from free surface of liquid
$$= 1.5 + \frac{1.2}{2} = 2.1m$$
$\therefore F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691 N$
Similarly, $F_2 = \rho_2 g . A\bar{h}_2$
Where $\rho_2 = 1000 kg/m^3$

$\bar{h}_2 =$ Depth of C.G. of gate from free surface of water
$$= \frac{1}{2} \times 1.2 = 0.6m$$
$\therefore F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126 N$

(i)Resultant force on the gate
$= F_1 - F_2 = 71691 - 14126 = 57565N$

(ii) Position of centre of pressure of resultant force.

The force F_1 will be acting at a depth of h_1* from free surface of liquid, given by the relation
$$h* = \frac{I_G}{A\bar{h}} + \bar{h}_1$$
where
$$I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 m^4$$
∴
$$h_1* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571m$$

∴ Distance of F_1 from hinge
$= (1.5 + 1.2) - h_1* = 2.7 - 2.1571 = 0.5429m$
The force F_2 will be acting at a depth of h_2* from free surface of water and is given by
$$h_2* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$
Where
$I_G = 0.288 m^4, \bar{h}_2 = 0.6m, A = 2.4m^2$
$$h_2* = \frac{.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8m$$
Distance of F_2 from hinge
$= 1.2 - 0.8 = 0.4m$
The resultant force 57565N will be acting at a distance given by
$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$
$$= \frac{38921 - 5650.4}{57565} \text{m above hinge}$$
$= 0.578m$ above the hinge

(iii)Force at the top of gate which is capable of opening the gate.

Let F is the force required on the top of the gate to open it as shown in fig.

Taking the moments of F, F_1 and F_2 about the hinge, we get

$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$

Or

$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

$= 27725.5N.$

Example:

A tank contains water up to a height of 0.5m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water up to 1m height. Calculate:

i) total pressure on one side of the tank,

ii) the position of centre of pressure from one side of the tank, which is 2m wide

Solution:

Given:

Depth of water $= 0.5m$

Depth of liquid $= 1m$

Sp. gr of liquid $= 0.8$

Density of liquid,

$\rho_1 = 0.8 \times 1000 = 800 kg/m^3$

Density of water,

$\rho_2 = 1000 kg/m^3$

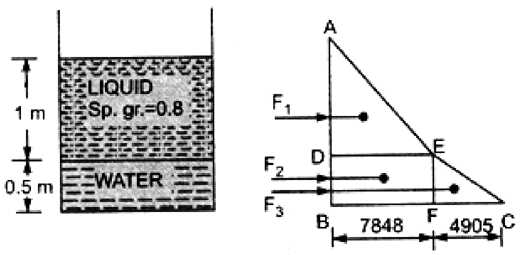
Width of tank $= 2m$

(i) Total pressure on one side is calculated by drawing pressure diagram, which is shown in fig .

Intensity of pressure on top, $p_A = 0$

Intensity of pressure on D (or DE), $p_D = \rho_1 g.h_1$

$= 800 \times 9.81 \times 1.0 = 7848 N/m^2$



Intensity of pressure on base (or BC), $p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = \frac{12753N}{m^2}$$

Now Force,

$F_1 =$ Area of $\Delta ADE \times$ Width of tank

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848N$$

Force,

$F_2 =$ Area of rectangle DBFE $\times$ Width of tank

$= 0.5 \times 7848 \times 2 = 7848N$

$F_3 =$ Area of $\Delta EFC \times$ Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5N$$

$\therefore$ Total force $F = F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5 = 18148.5N$

(ii) Centre of pressure (h*). Taking the moments of all forces about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 \left(AD + \frac{1}{2} BD \right) + F_3 \left[AD + \frac{2}{3} BD \right]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2} \right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5 \right)$$

$= 5232 + 9810 + 3270 = 18312$

$$\therefore \ h^* = \frac{18312}{18148.5} = 1.009 m \text{ from top}$$

Example:

A circular plate 3.0m diameter is immersed in water in such a way that their greatest and least depths below the free surface are 4m and 1.5m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.

Solution:

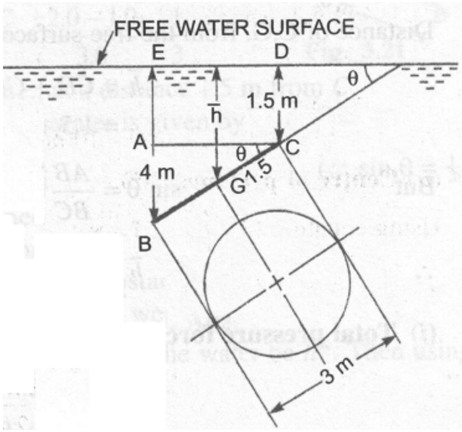
Given

Dia. of plate, $d = 3.0m$

$\therefore$ Area,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3.0)^2 = 7.0685 m^2$$

Distance $DC = 1.5m, BE = 4m$



Distance of C.G. from free surface

$=\bar{h}=CD+GC\sin\theta=1.5+1.5\sin\theta$

But

$$\sin\theta=\frac{AB}{BC}=\frac{BE-AE}{BC}=\frac{4.0-DC}{3.0}=\frac{4.0-1.5}{3.0}$$

$$=\frac{2.5}{3.0}=0.8333$$

$\therefore\ \bar{h}=1.5+1.5\times .8333=1.5+1.249=2.749\,m$

i) Total Pressure (F)

$F=\rho gA\bar{h}$

$=1000\times 9.81\times 7.0685\times 2.749=190621\,N$

ii) Centre of pressure (h*)

Using equation, we have

$$h^*=\frac{I_G\sin^2\theta}{A\bar{h}}+\bar{h}$$

Where

$$I_G=\frac{\pi}{64}(d)^4=\frac{\pi}{64}(3)^4=3.976\,m^4$$

$$h^*=\frac{3.976\times(.8333)\times .8333}{7.0685\times 2.749}+2.749=0.1420+2.749$$

$=2.891\,m.$

Example:

If in the above problem, the given circular plate is having a concentric circular hole of diameter 1.5m, then calculate the total pressure and position of the centre of pressure on one face of the plate.

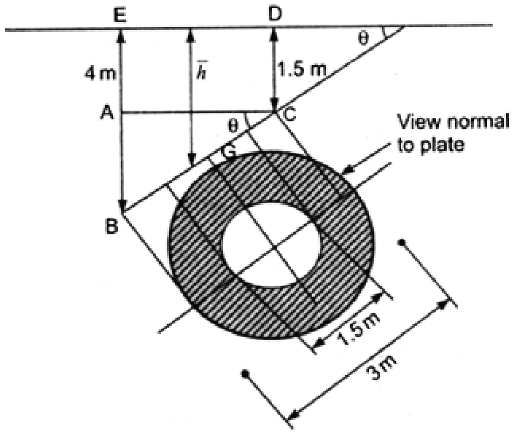
Solution:

Given: [referring to given figure]

Dia. of plate, $d=3.0\,m$

$\therefore$ Area of solid plate

$$=\frac{\pi}{4}d^2=\frac{\pi}{4}(3)^2=7.0685\,m^2$$



Dia. of hole in the plate, $d_0=1.5\,m$

$$\therefore \text{Area of hole}=\frac{\pi}{4}d_0^2=\frac{\pi}{4}(1.5)^2=1.7671\,m^2$$

$\therefore$ Area of the given plate A

= Area of solid plate − Area of hole

$=7.0685-1.7671=5.3014\,m^2$

Distance of $CD=1.5, BE=4\,m$

Distance of C.G. from the free surface,

$\bar{h}=CD+GC\sin\theta$

$=1.5+1.5\sin\theta$

But

$$\sin\theta=\frac{AB}{BC}=\frac{BE-AE}{BC}=\frac{4-1.5}{3}=\frac{2.5}{3}$$

$$\therefore\ \bar{h}=1.5+1.5\times\frac{2.5}{3}=1.5+1.25=2.75\,m$$

i) Total pressure force (F)

$F=\rho gA\bar{h}$

$=1000\times 9.81\times 5.3014\times 2.75$

$=143018\,N=143.018\,kN$

ii) Position of centre of pressure (h*)

Using equation, we have

$$h^*=\frac{I_G\sin^2\theta}{A\bar{h}}+\bar{h}$$

where

$$I_G=\frac{\pi}{64}\left[d^4-d_0^4\right]=\frac{\pi}{64}\left[3^4-1.5^4\right]m^4$$

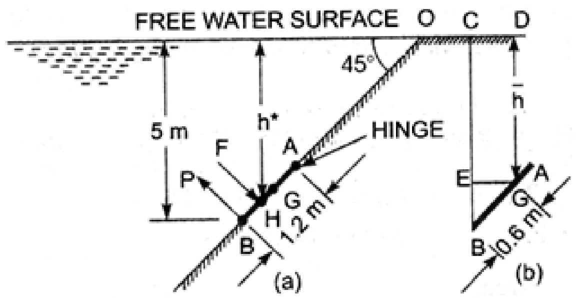
$$A=\frac{\pi}{4}\left[d^2-d_0^2\right]=\frac{\pi}{4}\left[3^2-1.5^2\right]m^2$$

$$\sin\theta=\frac{2.5}{3}\ \text{and}\ \bar{h}=2.75$$

$$\therefore \quad h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right] \times \left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right] \times 2.75} + 2.75$$

$$= 0.177 + 2.75 = 2.927\,m$$

Example:
An inclined rectangular sluice gate AB, 1.2m x 5m size as shown in fig is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.



Solution:
Given:

$A = \text{Area of gate} = 1.2 \times 5.0 = 6.0\,m^2$

Depth of C.G. of the gate from free surface of the water $= \bar{h}$

$= DG = BC - BE$

$= 5.0 - BG \sin 45°$

$$5.0 - 0.6 \times \frac{1}{\sqrt{2}} = 4.576\,m$$

The total pressure force (F) acting on the gate,

$F = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 6.0 \times 4.576$

$= 269343\,N$

This force is acting at H, where the depth of h from free surface is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

Where, $I_G = \text{M.O.I. of gate}$

$$= \frac{bd^3}{12} = \frac{5.0 \times 1.2^3}{12} = 0.72\,m$$

$\therefore$ Depth of centre of pressure

$$h^* = \frac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = .013 + 4.576 = 4.589\,m$$

But from fig, $\frac{h^*}{OH} = \sin 45°$

$\therefore$

$$OH = \frac{h^*}{\sin 45°} = \frac{4.589}{\frac{1}{\sqrt{2}}} = 4.589 \times \sqrt{2} = 6.489\,m$$

Distance, $BO = \frac{5}{\sin 45°} = 5 \times \sqrt{2} = 7.071\,m$

Distance, $BH = BO - OH = 0.071 - 6.489 = 0.582\,m$

$\therefore$ Distance,

$AH = AB - BH = 1.2 - 0.582 = 0.618\,m$

Taking the moments about the hinge A

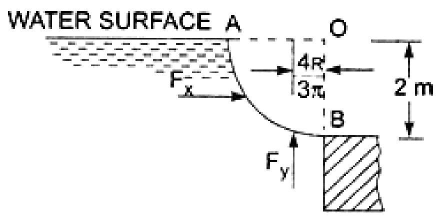
$P \times AB = F \times (AH)$

Where P is the force normal to the gate applied at B

$\therefore P \times 1.2 = 269343 \times 0.618$

$$\therefore \quad P = \frac{269343 \times 0.618}{1.2} = 138708\,N$$

Example:
Fig shows a quadrant shaped gate of radius 2m. Find the resultant force due to water per meter length of the gate. Find also the angle at which the total force will act.



Solution:
Given:
Radius of gate =2m
Width of gate =1m
Horizontal Force,

F_x = Force on the projected area of the curved surface on vertical plane

= Force on $BO = \rho g A \bar{h}$

Where,

$$A = \text{Area of BO} = 2 \times 1 = 2\,m^2, \bar{h} = \frac{1}{2} \times 2 = 1\,m$$

$$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620\,N$$

This will act at depth of $\frac{2}{3}\times 2=\frac{4}{3}$m from free surface of liquid,

Vertical Force,

F_y = Weight of water (imagined) supported by AB

$=\rho g\times$ Areaof AOB$\times 1.0$

$=1000\times 9.81\times \frac{\pi}{4}(2)^4\times 1.0=30819N$

This will act a distance of

$\frac{4R}{3\pi}=\frac{4\times 0.2}{3\pi}=0.848$ m from OB.

$\therefore$ Resultant force, F is given by

$F=\sqrt{F_x^2+F_y^2}$

$=\sqrt{19620^2+30819^2}$

$=36534.4N.$

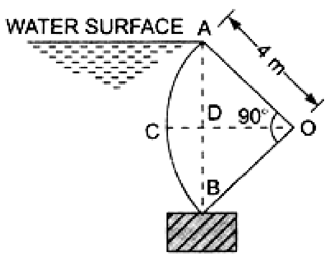
The angle made by the resultant with horizontal is given by

$\tan\theta=\frac{F_y}{F_x}=\frac{30819}{19620}=1.5708$

$\therefore\ \theta=\tan^{-1}1.5708=57°31'$

Example:

Find the horizontal and vertical component of water pressure acting on the face of a sector gate of 90° with radius 4m as shown in fig. Take width of gate as unity.



Solution:

Given:

Radius of gate, R=4m

Horizontal component of force acting on the gate is

F_x = Force on area of gate projected on vertical plane

= Force on area ADB

$=\rho g A\bar{h}$

Where $A=AB\times$ Width of gate

$=2\times AD\times 1$ $\quad(\because AB=2AD)$

$=2\times 4\times \sin 45°=8\times .707=5.656m^2$

$\{\because AD=4\sin 45°\}$

$\bar{h}=\frac{AB}{2}=\frac{5.656}{2}=2.828m$

$\therefore\ F_x=1000\times 9.81\times 5.656\times 2.828N=156911N$

Vertical component

F_y = Weight of water supported or enclosed by the curved surface

= Weight of water in portion ACBDA

$=\rho g\times$ Area of ACBDA$\times$ Width of gate

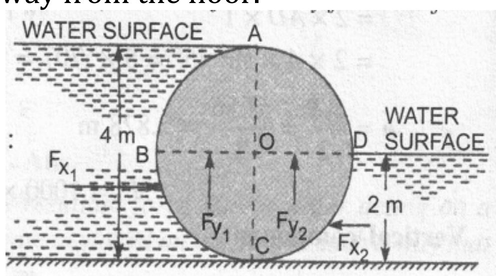
$=1000\times 9.81\times[\textit{Area of sector ACBOA} - \textit{Area of }\Delta ABO]\times 1$

$=9810\times\left[\frac{\pi}{4}R^2-\frac{AO\times BO}{2}\right]$ $\quad[\because\ \Delta AOB$ is a right angled$]$

$=9810\times\left[\frac{\pi}{4}4^2-\frac{4\times 4}{2}\right]=44796N$

Example:

A cylindrical gate of 4m diameter & 2m long has water on its both sides as shown in Fig. Determine the magnitude, location and direction of the resultant force exerted by the water on the gate. Find also the least weight of the cylinder so that it may not be lifted away from the floor.



Solution:

Given:

Dia. of gate =4m

Radius =2m

(i) The force acting on the left sides of the cylinder are

The horizontal component, F_{x_1}

Where F_{x_1} = Force of water on area projected on vertical plane

= Force on area AOC

$=\rho g A\bar{h}$

Where A = AC × width = 4×2= 8cm^2

$\bar{h} = \frac{1}{2} \times 4 = 2m$

$F_{x_1} = 1000 \times 9.81 \times 8 \times 2$

$= 156960N$

F_{y_1} = Weight of water enclosed by ABCOA

$= 1000 \times 9.81 \times \left[\frac{\pi}{2} R^2\right] \times 2.0 = 9810 \times \frac{\pi}{2} \times 2^2 \times 2.0 = 123276N$

Right Side of the Cylinder

$F_{x_2} = \rho g A_2 \bar{h}_2 =$ Force on vertical area CO

$= 1000 \times 9.81 \times (2 \times 2) \times \frac{2}{2} = 39240N$

F_{y_2} = Weight of water enclosed by DOCD

$= \rho g \times \left[\frac{\pi}{4} R^2\right] \times \text{Width of gate}$

$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times 2 = 61638N$

$\therefore$ Resultant force in the direction of x,

$F_x = F_{x_1} - F_{x_2} = 156960 - 39240 = 117720N$

Resultant force in the direction of y,

$F_y = F_{y_1} + F_{y_2} = 123276 + 61638 = 184914N$

i) Resultant force, F is given as

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(117720)^2 + (184914)^2} = 219206N$

ii) Direction of resultant force is given by

$\tan\theta = \frac{F_y}{F_x} = \frac{184914}{117720} = 1.5707$

$\therefore \quad \theta = 57°31'$

iii) Location of the resultant force

Force, F_{x_1} acts at a distance of $\frac{2 \times 4}{3} = 2.67m$ from the top surface of water on left side, while F_{x_2} acts at a distance of $\frac{2}{3} \times 2 = 1.33m$ from free surface on the right side of the cylinder .The resultant force F_x in the direction of x will act at a distance of y from the bottom as

$F_x \times y = F_{x_1}[4 - 2.67] - F_{x_2}[2 - 1.33]$

Or

$117720 \times y = 156960 \times 1.33 - 39240 \times .67$

$= 208756.8 - 26290.8 = 182466$

$\therefore \quad y = \frac{182466}{117720} = 1.55m$ from the bottom

Force F_{y_1} acts at a distance $\frac{4R}{3\pi}$ from AOC or at a distance $\frac{4 \times 2.0}{3\pi} = 0.8488m$ from AOC towards left of AOC.

Also F_{y_2} acts at a distance $\frac{4R}{3\pi} = 0.8488m$ from AOC towards the right of AOC. The resultant force F_y will act at a distance x from AOC which is given by

$F_y \times x = F_{y_1} \times .8488 - F_{y_2} \times .8488$

Or $184914 \times x = 123276 \times .8488 - 61638 \times .8488$

$= .8488[123276 - 61638] = 52318.4$

$\therefore x = \frac{52318.4}{184914} = 0.2829m$ from AOC

iv) Least weight of cylinder. The resultant force in the upward direction is

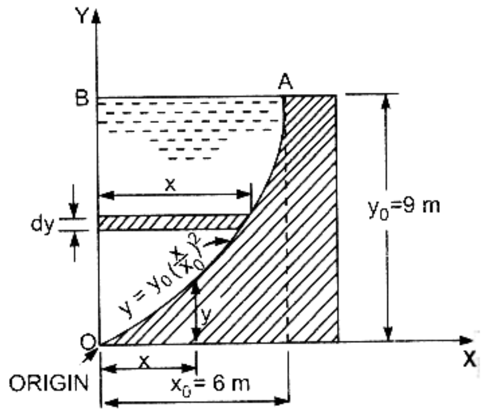
$F_y = 184914N$

Thus the weight of cylinder should not be less than the upward force F_y.

Hence, weight of cylinder should be at least 184914N

Example:

A dam has a parabolic shape $y = y_0\left(\frac{x}{x_0}\right)^2$ as shown in fig. below having $x_0 = 6m$ and $y_0 = 9m$. The fluid is water with density $= 1000 kg/m^3$. Compute the horizontal, vertical and the resultant thrust exerted by water per meter length of the dam.



Solution:

Given:

Equation of the curve OA is

$$y = y_0\left(\frac{x}{x_0}\right)^2 = 9\left(\frac{x}{6}\right)^2 = 9\times\frac{x^2}{36} = \frac{x^2}{4}$$

Or $\quad x^2 = 4y$

$\therefore \quad x = \sqrt{4y} = 2y^{1/2}$

Width of dam, $b = 1\text{m}$

i) Horizontal thrust exerted by water

$F_x =$ Force exerted by water on vertical surface OB, i.e., the surface obtained by projecting the curved surface on vertical plane

$= \rho g A\bar{h}$

$$= 1000\times 9.81\times(9\times 1)\times\frac{9}{2} = 397305\text{N}$$

ii) Vertical thrust exerted by water

$F_y =$ Weight of water supported by curved surface OA upto free surface of water

= Weight of water in the portion ABO

$= \rho g \times$ Area of OAB $\times$ Width of dam

$$= 1000\times 9.81\times\left[\int_0^9 x\times dy\right]\times 1.0$$

$$= 1000\times 9.81\times\left[\int_0^9 2y^{1/2}\times dy\right]\times 1.0$$

$(\because x = 2y^{1/2})$

$$= 19620\times\left[\frac{y^{3/2}}{(3/2)}\right]_0^9 = 19620\times\frac{2}{3}\left[9^{3/2}\right]$$

$$= 19620\times\frac{2}{3}\times 27 = 353160\text{N}$$

iii) Resultant thrust exerted by water

$$F = \sqrt{F_X{}^2 + F_Y{}^2} = \sqrt{397305^2 + 353160^2} = 531574\text{N}$$

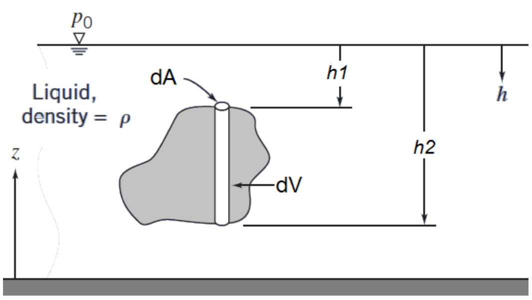
iv) Direction of resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{353160}{397305} = 0.888$$

$$\theta = \tan^{-1} 0.888 = 41.63^o$$

3.6 BUOYANCY & FLOATATION

When a body is immersed in a fluid, an upward force is exerted on the body; this upward force is known as the buoyant force. This force is because of difference in pressure.



Force acting on the element because of difference in pressure on the top and bottom.

$dF = \rho g(h_2 - h_1)dA$

$F_B = \int \rho g(h_2 - h_1)dA$

$F_B = \rho g V$

Where,

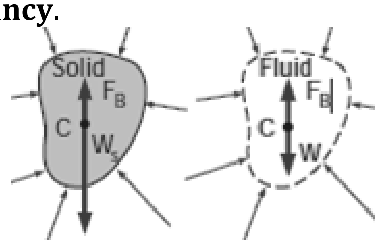
ρ is the density of fluid

V is the volume of body immersed in fluid or volume of fluid displaced by that body.

The relation ρgV is simply the weight of the liquid whose volume is equal to the immersed volume of the body. Thus the buoyant force acting on the body is equal to the weight of the liquid displaced by the body. Note that the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body

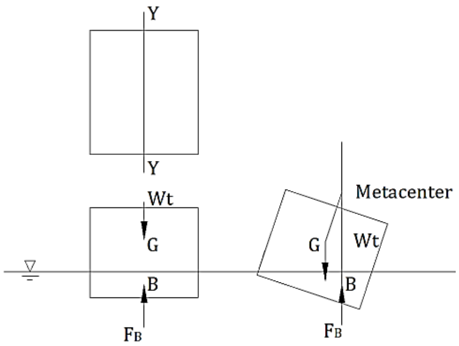
3.6.1 CENTRE OF BUOYANCY

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume. The centroid of displaced fluid is known as **centre of buoyancy**.



3.6.2 META CENTRE

It is defined as the point about which a body starts oscillating when the body is tilled by a small angle. The Meta centre may also be defined as the intersection point of line of action of buoyant force and normal to the body when the body is tilted by an angle.



Meta centric height = GM

$$= \frac{I(\text{Moment of Inertia about yy of the plan})}{\text{Volume of fluid displaced}} - BG$$

B.G is the distance between CG & CB points.

3.6.3 OSCILLATION OF A FLOATING BODY

When body floats in the fluid and it is given a disturbance in clockwise direction or anti clock wise direction. The body oscillates about its metacenter. The time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{k^2}{GM.g}}$$

Where,

GM is Meta centric ht

K is radius of gyration

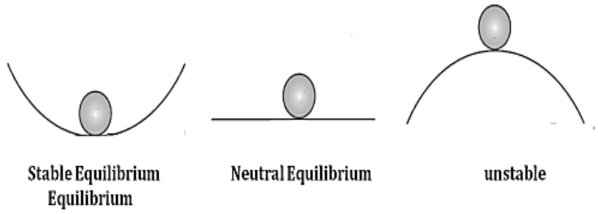
3.6.4 CONDITIONS OF EQUILIBRIUM OF SUBMERGED & FLOATING BODIES

There are 3 types of equilibrium conditions

i) Stable Equilibrium

ii) Neutral equilibrium

iii) Unstable equilibrium



i) Stable equilibrium:

Any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position.

ii) Neutral equilibrium:

If someone moves the ball to the right or left, it will stay at its new location. It has no tendency to move back to its original location, nor does it continue to move.

iii) Unstable Equilibrium:

It is a situation, in which the ball may be at rest at the moment, but any disturbance, even an infinitesimal one, causes the ball to roll off the hill—it does not return to its original position; rather it moves away from it.

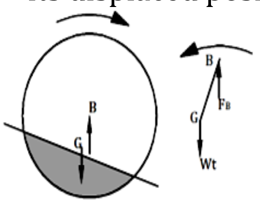
3.6.4.1 STABILITY IN SUBMERGED BODIES

1) **Stable Equilibrium:** When $W = F_B$ and point B is above G. A small displacement in clockwise direction, gives couple due

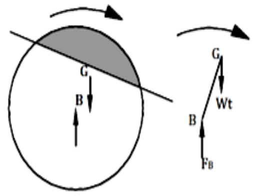
to F_B & weight in anticlockwise direction. Thus, the body will return to its original position. Hence, equilibrium is stable.

2) **Unstable Equilibrium:** If $W = F_B$ and point B is below point 'G'. A small displacement to the body, in the clock wise direction, gives couple due to $W \,\&\, F_B$ also in the clockwise direction. Thus, body will move away from its original position. Hence, equilibrium is unstable

3) **Neutral Equilibrium**: If $F_B = W$ and B & G are at the same point, the displacement of body does not result in any couple of Wt & F_B. Body remains at its displaced position



Stable equilibrium



Unstable equilibrium

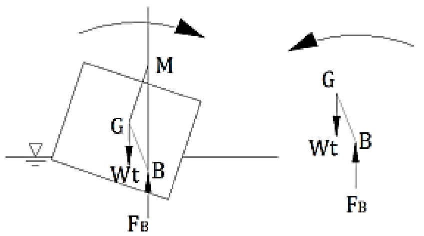
3.6.42 STABILITY IN FLOATING BODY

The stability of floating body is determined from position of metacentre(M). In case of floating body, the weight of body is equal to the buoyant force.

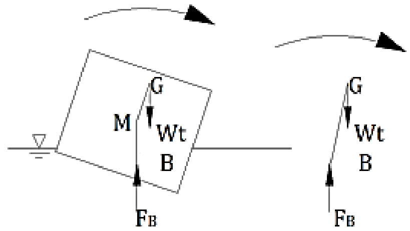
1) **Stable Equilibrium:** When M is above G, because of a small displacement to the body in the clock wise direction, the couple between Wt & F_B causes rotation in anti-clockwise direction.

2) **Unstable Equilibrium:** When M is below G, because of small displacement to the body in the clock wise direction, the couple between Wt & F_B causes rotation in clockwise direction.

3) **Neutral:** If M lies at the C.G. of body, the displacement of body does not result in any couple of Wt & F_B. Body remains at its displaced position.



Stable equilibrium



Unstable equilibrium

Example:

Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5m, when it floats horizontally in water. The density of wooden block is $650 kg/m^3$ and its length is 6.0m.

Solution:

Given:

Width = 2.5m

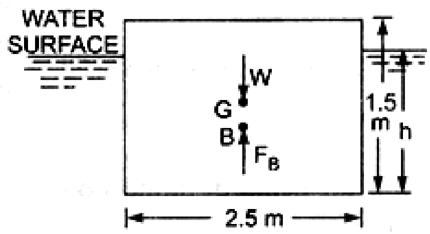
Depth = 1.5m

Length = 6.0m

Volume of the block

$= 2.5 \times 1.5 \times 6.0 = 22.50 m^3$

Density of wood, $\rho = 65 kg/m^3$



$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 N = 143471 N$

For equilibrium, the weight of water displaced = Weight of wooden block

$= 143471 N$

$\therefore$ Volume of water displaced

$$=\frac{\text{Weight of water displaced}}{\text{Weight density of water}}=\frac{143471}{1000\times 9.81}=14.625m^3$$

Position of center of Buoyancy:
Volume of wooden block in water
= Volume of water displaced
Or
$2.5\times h\times 6.0=14.625m^3$,
Where,
h is depth of wooden block in water

$$\therefore\ h=\frac{14.625}{2.5\times 6.0}=0.975m$$

$\therefore$ Centre of Buoyancy

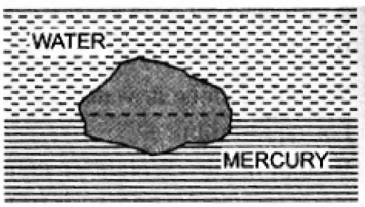
$$=\frac{0.975}{2}=0.4875m \text{ from base}$$

Example:
Find the density of a metallic body which floats at the interface of mercury of S.G. 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.
Solution:
Let the volume of the body $=Vm^3$
Then volume of body sub-merged in mercury

$$=\frac{40}{100}V=0.4Vm^3$$

Volume of body sub-merged in water

$$=\frac{60}{100}V=0.6Vm^3$$



For the equilibrium of the body
Total buoyant force (upward force) =Weight of the body
But total buoyant force= Force of buoyancy due to water + Force of buoyancy due to mercury
Force of buoyancy due to water = Weight of water displaced by body
= Density of water × Volume of mercury displaced
$=1000\times g\times$Volume of body in water
$= 1000\times g\times 0.6\times VN$
And, force of buoyancy due to mercury
= Weight of mercury displaced by body
= g × Density of water× Volume of mercury displaced
$=g\times 13.6\times 1000\times$ volume of body in mercury
$= g\times 13.6\times 1000\times 0.4VN$
Weight of the body
=Density x g x Volume of body
$\therefore$ For equilibrium, we have
Total buoyant force =Weight of the body

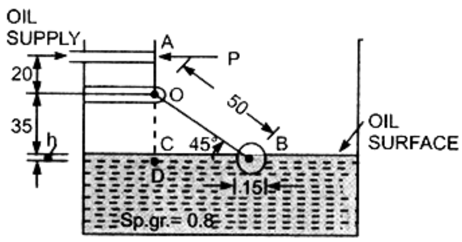
$$1000\times g\times 0.6\times V+13.6\times 1000\times g\times .4V=\rho\times g\times V$$

Or

$$\rho=600+13600\times .4=600+54400=6040.00kg/m^3$$

$\therefore$ Density of the body $=6040.00kg/m^3$

Example:
A float valve regulates the flow of oil of S.G. 0.8 into a cistern. The spherical float is 15 cm in diameter. AOB is a weightless link carrying the float at one end, and a valve at the other end which closes the pipe through which oil flows into the cistern. The link is mounted on a frictionless hinge at O and the angle AOB is 135°. The length of OA is 20cm, and the distance between the centre of the float and the hinge 50 cm. When of the flow is stopped AO will be vertical. The valve is to be pressed on to the seat with a force of 9.81 N to completely stop the flow of oil into the cistern. It was observed that the flow of oil is stopped when the free surface of oil in the cistern is 35 cm below the hinge. Determine the weight of the float.



Solution:
Given:
Sp. gr. of oil =0.8
$\therefore$ Density of oil

$= \rho_0 = 0.8 \times 1000 = 800 kg/m^3$

Dia. of float, D=15cm

$\angle AOB = 135°$

$OA = 20cm$

Force, P=9.81N

OB=50 cm

Let the weight be W.

When the flow of oil is stopped, the centre of float is shown in Fig. The level of oil is also shown. The centre of float is below the level of oil, by a depth 'h'

From ΔBOD,

$$\sin 45° = \frac{OD}{OB} = \frac{OC+CD}{OB} = \frac{35+h}{50}$$

$50 \times \sin 45° = 35 + h$ Or

$$h = 50 \times \frac{1}{\sqrt{2}} - 35 = 35.355 - 35 = 0.355cm = .00355m$$

The weight of float is acting through B, but the upward buoyant force is acting through the centre of weight of oil displaced

Volume of oil displaced :

$$= \frac{2}{3}\pi r^3 + h \times \pi r^2 \left\{ r = \frac{D}{2} = \frac{15}{2} = 7.5cm \right\}$$

$$= \frac{2}{3} \times \pi \times (0.75)^3 + .00355 \times \pi \times (0.75)^2 = 0.000945 m^3$$

=Weight of oil displaced

$= \rho_0 \times g \times \text{Volume of oil}$

$= 800 \times 9.81 \times 0.00945 = 7.416N$

The buoyant force and weight of the float passes through the same vertical line, passing through B.

Let the weight of float is W. Then net vertical force on float

=Buoyant force –Weight of float = (7.416-W)

Taking moments about the hinges O, we get

$$P \times 20 = (7.416 - W) \times BD = (7.416 - W) \times 50 \times \cos 45°$$

Or $9.81 \times 20 = (7.416 - W) \times 35.355$

$\therefore$

$$W = 7.416 - \frac{20 \times 9.81}{35.355} = 7.416 - 5.55 = 1.866N$$

Example:

A rectangular pontoon is 5m long, 3m wide and 1.20m high. The depth of immersion of the pontoon is 0.80m in sea water. If the centre of gravity is 0.6m above the bottom of the pontoon, determine the Meta – centric height. The density for sea water $= 1025 kg/m^3$

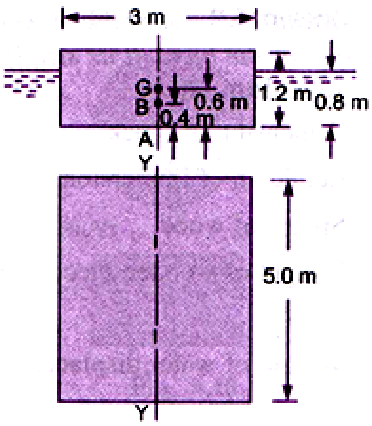
Solution:

Given:

Dimension of pontoon $= 5m \times 3m \times 1.20m$

Depth of immersion =0.8m

Distance AG=0.6m



PLAN AT WATER SURFACE

Distance $AB = \frac{1}{2} \times \text{Depth of immersion}$

$$= \frac{1}{2} \times .8 = 0.4m$$

Density for sea water $= 1025 kg/m^3$

Meta-centre height GM, given by equation as

$$GM = \frac{I}{\forall} - BG$$

Where I = Moment of Inertia of the plan of the pontoon about Y-Y axis

$$= \frac{1}{12} \times 5 \times 3^3 m^4 = \frac{45}{4} m^4$$

$\forall$ = Volume of the body sub- merged in water

$= 3 \times 0.8 \times 5.0 = 12.0 m^3$

$BG = AG = AB = 0.6 - 0.4 = 0.2m$

$$GM=\frac{45}{4}\times\frac{1}{12.0}-0.2=\frac{45}{48}-0.2=0.9375-0.2=0.7375\,m$$

Example:
A solid cylinder of diameter 4.0 m has a height of 4.0m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder = 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable.

Solution:
Given: D=4m
Height, h=4m
S.G. = 0.6
Depth of cylinder in water = S.G x h
$=0.6\times4.0=2.4\,m$
$\therefore$ Distance of centre of buoyancy (B) from A

$$AB=\frac{2.4}{2}=1.2\,m$$

Distance of centre of gravity (G) from A

$$AG=\frac{h}{2}=\frac{4.0}{2}=2.0\,m$$

$\therefore BG=AG-AB=2.0-1.2=0.8\,m$
Now the meta-centric height GM is given by

$$GM=\frac{I}{\forall}-BG$$

Where
I=M. O. I. of the plan of the body about Y-Y axis

$$=\frac{\pi}{64}D^4=\frac{\pi}{64}\times(4.0)^4$$

$\forall=$ Volume of cylinder in water

$$=\frac{\pi}{4.0}\times D^2\times\text{Depth of cylinder in water}=\frac{\pi}{4}\times4^2\times2.4\,m^3$$

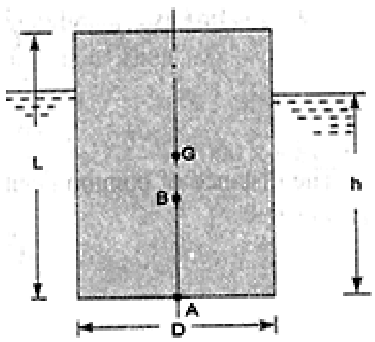
$$\therefore\frac{1}{\forall}=\frac{\frac{\pi}{64}\times4^4}{\frac{\pi}{4}\times4^2\times2.4}=\frac{1}{16}\times\frac{4^2}{2.4}=\frac{1}{2.4}=0.4167\,m$$

$$GM=\frac{1}{\forall}-BG=0.4167-0.8=-0.3833\,m$$

-ve sign means that the meta-centre (M) is below the centre of gravity (G). Thus the cylinder is in unstable equilibrium.

Example:
A wooden cylinder of S.G. 0.6 and circular cross-section is required to float in oil (S.G. 0.90). Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameter.

Solution:
Given:
Dia of cylinder =D
Height of cylinder =L
Sp. Gr. Of cylinder $S_1=0.6$
Sp. Gr of oil $S_2=0.9$
Let the depth of cylinder immersed in oil=h



For the principle of buoyancy
Weight of cylinder = wt. of oil displaced

$$\frac{\pi}{4}\times D^2\times L\times0.6\times1000\times9.81=\frac{\pi}{4}\times D^2\times h\times0.9\times1000\times9.81$$

Or $\quad L\times0.6=h\times0.9$

$$\therefore\quad h=\frac{0.6\times L}{0.9}=\frac{2}{3}L$$

The distance of centre of gravity G from A,

$$AG=\frac{L}{2}$$

The distance of centre of buoyancy B from A

$$AB=\frac{h}{2}=\frac{1}{2}\left[\frac{2}{3}L\right]=\frac{L}{3}$$

$$\therefore BG=AG-AB=\frac{L}{2}-\frac{L}{3}=\frac{3L-2L}{6}=\frac{L}{6}$$

The meta-centric height GM is given by

$$GM=\frac{I}{\forall}-BG$$

Where $I = \frac{\pi}{64} D^4$ and $\forall$ = Volume of cylinder in oil $= \frac{\pi}{4} D^2 \times h$

$$\therefore \frac{I}{\forall} = \left(\frac{\pi}{64} D^4 / \frac{\pi}{4} D^2 h \right) = \frac{1}{16} \frac{D^2}{h} = \frac{D^2}{16 \times \frac{2}{3} L} = \frac{3D^2}{32L}$$

$$\left\{ \because \; h = \frac{2}{3} L \right\}$$

$$\therefore \; GM = \frac{3D^2}{32L} - \frac{L}{6}$$

For stable equilibrium, GM should be +ve or,

$$GM > 0 \text{ or } \frac{3D^2}{32L} - \frac{L}{6} > 0$$

Or

$$\frac{3D^2}{32L} > \frac{L}{6} \text{ or } \frac{3 \times 6}{32} > \frac{L^2}{D^2}$$

Or

$$\frac{L^2}{D^2} < \frac{18}{32} \text{ or } \frac{9}{16}$$

$$\therefore \quad \frac{L}{D} < \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\therefore \quad \frac{L}{D} < 3/4$$

Example:
The time period of rolling of a ship of weight 29430kN in sea water is 10seconds. The centre of buoyancy of the ship is 1.5 m below the centre of gravity. Find the radius of gyration of the ship if the moment of inertia of the ship at the water line about fore and aft axis is1000m^4. The specific weight of sea water as $10100 N/m^3$

Solution:
Given:
Time period T=10sec
Distance between centre of buoyancy and centre of gravity, BG=1.5m
Moment of Inertia, $I = 10000 m^4$
Weight $W = 29430 kN = 29430 \times 1000 N$
Let the radius of gyration =K

First calculating the meta–centric height, which is given as

$$GM = BM - BG = \frac{I}{\forall} - BG$$

Where I= Moment of Inertia
And $\forall =$ Volume of water displaced

$$= \frac{\text{Weight of ship}}{\text{Sp.weight of sea water}} = \frac{29430 \times 1000}{10104} = 2912.6 m^3$$

$$\therefore GM = \frac{10000}{2912.6} - 1.5 = 3.433 - 1.5 = 1.933 m$$

Using equation, $T = 2\pi \sqrt{\frac{K^2}{GM \times g}}$

We get

$$10 = 2\pi \sqrt{\frac{K^2}{1.933 \times 9.81}} = \frac{2\pi K}{\sqrt{1.933 \times 9.81}}$$

Or

$$K = \frac{10 \times \sqrt{1.933 \times 9.81}}{2\pi} = 6.93 m$$

GATE QUESTIONS

Q.1 What is the force required (in Newton) to hold a spherical balloon stationary in water at a depth of H from the air-water interface? The balloon is of radius 0.1 m and is filled with air.

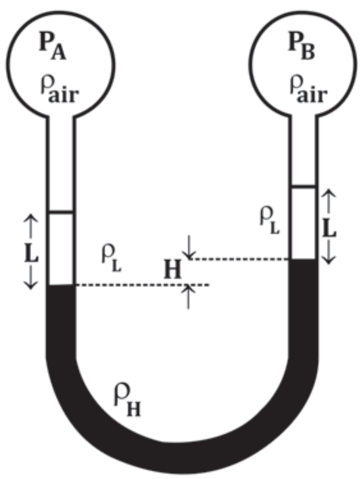
[GATE –2004]

(A) $\frac{4\pi g}{3}$ (B) $\frac{0.1\pi g h}{4}$

(C) $\frac{0.1\pi g h}{8}$ (D) $\frac{0.04\pi g h}{3}$

Q.2 For the manometer setup shown in the figure, the pressure difference $P_A - P_B$ is given by

[GATE –2004]



(A) $(\rho_H - \rho_{air}) g H$

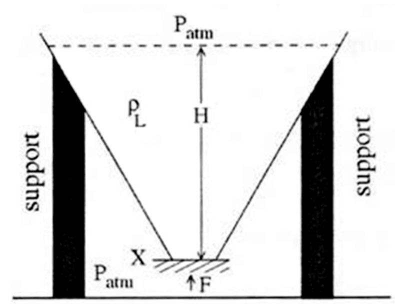
(B) $(\rho_H - \rho_L) g H$

(C) $(\rho_H - \rho_L) g H - (\rho_L - \rho_{air}) g (L - H)$

(D) $(\rho_H - \rho_L) g L + (\rho_L - \rho_{air}) g H$

Q.3 A conical tank with a bottom opening of cross-sectional area A is filled with water and is mounted on supports, as shown in the figure. What is the force F with which plate X must be pushed up to prevent water from leaking? Assume that the density of air is negligible as compared to the density of water ρ_L

[GATE –2004]



(A) $\rho_L V g$

(B) $(\rho_L g H) A$

(C) $\frac{\rho_L V g}{2}$

(D) $\frac{\rho_L V g}{3}$

Q.4 A dam of width 50 m is used to hold water in a reservoir. If the water height is 10 m from the bottom of the dam, what is the total force F acting on the dam due to the water? Assume g = 10 m/s^2, and the fluid density is 1000 kg/m^3

[GATE –2005]

(A) $F = 12.5 \times 10^6$ N

(B) $F = 25 \times 10^6$ N

(C) $F = 50 \times 10^6$ N

(D) $F = 5 \times 10^6$ N

Common Data for Q.5 and Q.6

A balloon of mass 0.01 kg is charged with hydrogen to a pressure of 102kPa and released from the ground level. During its rise the hydrogen is permitted to escape from the balloon in order to maintain a constant differential pressure of 2 kPa under which condition the diameter of the balloon remains at 0.4 m. As this balloon rises it is assumed that the temperature in and around the balloon remains constant at 273 K. Further, the inertia of the balloon and the air resistance due to the rising balloon may be neglected. Assume that the density of air at 273 K is 1.2733 kg/m^3, the average molecular weight of air is 28.9, the atmospheric pressure is 100 kPa and the acceleration due to gravity is 10 m/s^2

Q.5 Select the correct value of the upward thrust (in N) expressed in terms of the outside pressure P which is expressed in Pa.

[GATE –2005]

(A) $10.06\times10^{-7}P - 0.0122$

(B) $3.97\times10^{-6}P - 0.01006$

(C) $15.03\times10^{-7}P - 0.0534$

(D) $8.08\times10^{-6}P - 0.1362$

Q.6 Select the value of the outside pressure P in Pa for which there will be no force on the balloon?

[GATE –2005]

(A) 25340 (B) 35530

(C) 12130 (D) 16860

Q.7 The magnitude of the force (in N) required to hold a body of volume 0.05 m^3 and mass 40 kg in water (density 1000 kg/m^3) at a depth of 0.1 m is (g : 9.81 m/s^2)

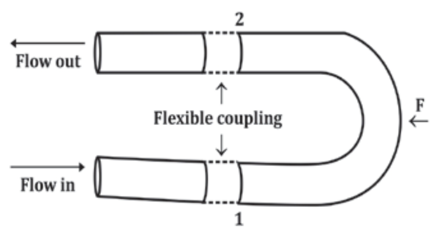
(A) zero (B) 98.1

(C) 490.5 (D) 882.9

[GATE –2006]

Q.8 The figure shows the idealized view of a return elbow or U bend, which is connected to two pipes by flexible hoses that transmit no force. Water with density 1000 kg/m^3 flows at velocity of 10 m/s through the pipe, which has a uniform ID of 0.1m. The gauge pressure at points 1 and 2 are 304 kPa and 253 kPa respectively. The horizontal force F required to keep the elbow in position is

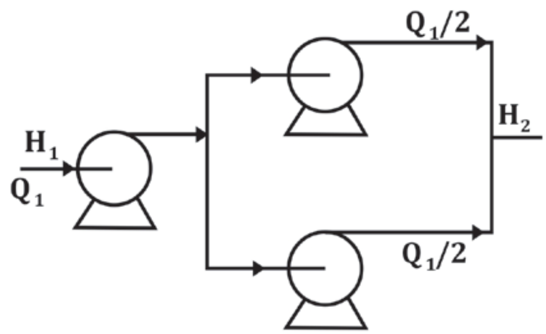
[GATE –2007]



(A) 1574 N (B) 1970 N

(C) 5942 N (D) 7533 N

Q.9 The figure shows a series-parallel configuration of three identical centrifugal pumps. The head increase ΔH across a single such pump varies with flowrate Q according to $\Delta H = a - bQ^2$. The expression for the total head increase $\Delta H = H_2 - H_1$ in terms of a and b and the total flowrate Q_1 for this configuration is given by



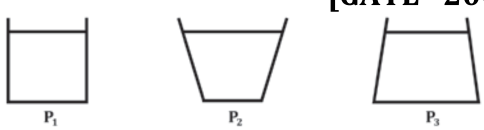
[GATE –2007]

(A) $2a - \frac{5}{4}bQ_1^{\ 2}$ (B) $2a - bQ_1^{\ 2}$

(C) $2a - 2bQ_1^{\ 2}$ (D) $a - bQ_1^{\ 2}$

Q.10 Three containers are filled with water up to the same height as shown. The pressures at the bottom of the containers are denoted as P_1, P_2 and P_3. Which ONE of the following relationships is true?

[GATE –2008]



(A) $P_3 > P_1 > P_2$

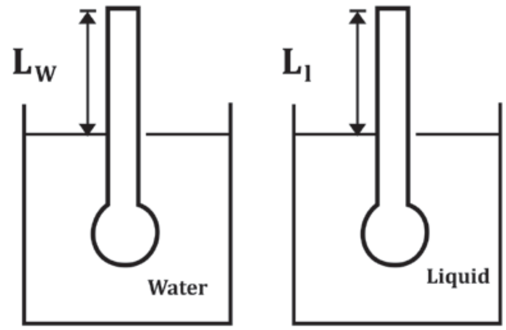
(B) $P_2 > P_1 > P_3$

(C) $P_1 > P_2 = P_3$

(D) $P_1 = P_2 = P_3$

Q.11 A hydrometer, with stem cross-sectional area of 2.82×10^{-5} m^2, is immersed in a very large vessel containing water as shown in the figure. The immersed volume is 15×10^{-6} mg and the length of the stem about water surface is L_w. If the entire volume of water is replaced by a liquid with specific gravity 1.5 if the length of the stem above the liquid surface is L_l, then the difference $L_l - L_w$ is

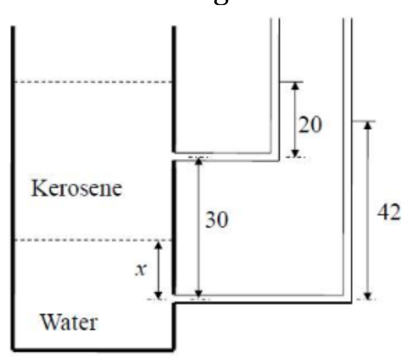
[GATE –2010]



(A) -177 mm (B) 177 mm

(C) -266 mm (D) 266 mm

Q.12 A vertical cylindrical vessel has a layer of kerosene (of density 800 kg/m^3) over a layer of water (of density 1000 kg/m^3). L-shaped glass tubes are connected to the column 30 cm apart. The interface between the two layers lies between the two points at which the L-tubes are connected. The levels (in cm) to which the liquids rise in the respective tubes are shown in the figure below.



The distance (x in cm, rounded off to the first decimal place) of the interface from the point at which the lower L-tube is connected is _____

[GATE–2016]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12
A	A	B	B	B	A	B	B	A	D	B	10

EXPLANATIONS

Q.1 (A)

Force Required = Buoyancy Force

$$F = \rho_f \; V_{fd} \; g$$

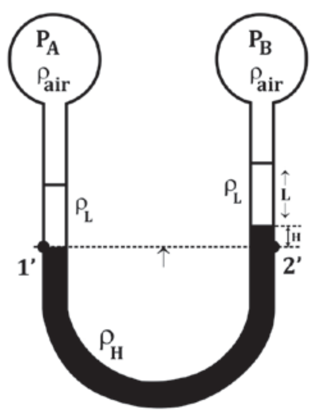
$$= (1000)\left(\frac{4}{3}\pi r^3\right) g$$

$$= (1000)\left(\frac{4}{3}\pi (0.1)^3\right) g$$

$$= \frac{4}{3}\pi g$$

So Option (a) is correct

Q.2 (A)



Balancing pressure at 1' and 2'

$$P_A + \rho_{air} \; g \, H + \rho_L \; g \, L = P_B + \rho_L \; g \, L + \rho_H \; g \, H$$

$$P_A - P_B = (\rho_H - \rho_{air}) g \, H$$

So Option (a) is correct

Q.3 (B)

$$\begin{pmatrix}\text{Force acting in} \\ \text{upward direction}\end{pmatrix} = \begin{pmatrix}\text{Hydrostatic} \\ \text{Force}\end{pmatrix} \times \text{Area}$$

$$F = (\rho_L \; g \, H) A$$

So Option (b) is correct

Q.4 (B)

$$F = \int_0^{10} (h \, g \, \rho)(50 \times dh)$$

$$= 50 \times \rho \, g \int_0^{10} h \, dh$$

$$= 50 \times 1000 \times 10 \left(\frac{h^2}{2}\right)_0^{10}$$

$$= 25 \times 10^6 \text{ N}$$

Q.5 (B)

Let outside pressure = P Pa

Pressure inside balloon = P + 2000 Pa

$$\text{Upward thrust} = \left[\frac{4}{3}\pi (0.2)^3 \rho_{air} \; g\right] - \left[\frac{4}{3}\pi \, (0.2)^3 \rho_{H_2} \; g\right] - [0.01 \, g]$$

$$= \frac{4}{3}\pi (0.2)^3 \left[\frac{28.9 \times 10^{-3} P}{8.314 \times 273}\right] 10 - \frac{4}{3}\pi (0.2)^3 \left[\frac{2 \times 10^{-3}(P+2000)}{8.314 \times 273}\right] 10 - (0.01)(10)$$

$$= 3.97 \times 10^{-6} \; P - 0.1006 \quad N$$

So Option (b) is correct

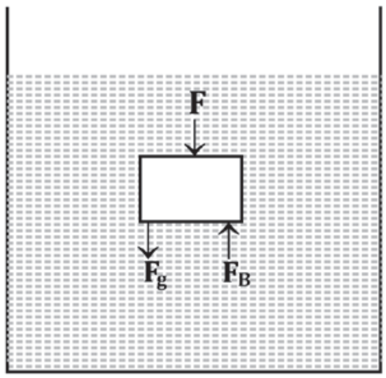
Q.6 (A)

$3.97 \times 10^{-6} \; P - 0.1006 = 0$

$P = 25340$ Pa

So Option (a) is correct

Q.7 (B)



Force Balance on Body

$F + F_g = F_B$

$F + m\,g = \rho_F\, V_{fd}\, g$

$F = (1000 \times 0.05 \times 9.8) - (40 \times 9.81)$

$F = 98.1$ N

So Option (b) Is correct

Q.8 (B)

$P_1 A_1 - P_2 A_2 - F =$ Change in Momentum

$P_1 A_1 - P_2 A_2 - F = \rho\, Q((-V_2) - (V_1))$

$P_1 A_1 - P_2 A_2 + \rho\, Q\, 2\, V = F$

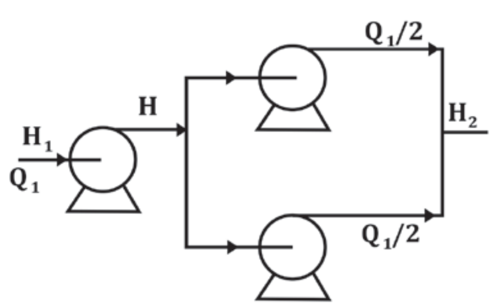
$A(P_1 - P_2 + \rho\, V\, 2\, V) = F$

$F = \frac{\pi}{4}(0.1)^2 (304 \times 10^3 - 253 \times 10^3 + 2 \times 1000 \times 10^2$

$F = 1970$ N

So Option (b) is correct

Q.9 (A)



$H - H_1 = a - bQ_1{}^2$

$H = H_1 + a - bQ_1{}^2$

$H_2 - H = a - b\left(\frac{Q_1}{2}\right)^2$

$H_2 - (H_1 + a - bQ_1{}^2) = a - b\left(\frac{Q_1}{2}\right)^2$

$H_2 - H_1 = 2a - bQ_1{}^2\left(1 + \frac{1}{4}\right)$

$H_2 - H_1 = 2a - \frac{5}{4}bQ_1{}^2$

So, Option (a) is correct

Q.10 (D)

Q.11 (B)

Given

Case I

Volume $= 15 \times 10^{-6}$

$(L - L_w)A = 15 \times 10^{-6}$

$(L - L_w) = \frac{15 \times 10^{-6}}{2.82 \times 10^{-5}} = 0.532$...........(1)

$W_t =$ Weight of fluid displaced

$= (L - L_w) A \times 1000$

$= 15 \times 10^{-6} \times 1000 = 15 \times 10^{-3}$

Similarly in Case II

$W = (L - L_L) A \times 1500$

$\frac{15 \times 10^{-3}}{1500 \times 2.82 \times 10^{-2}} = (L - L_L) = 0.345$............(2)

Now,

Eq(1) – Eq(2)

$(L - L_W) - (L - L_L) = 0.532 - 0.345$

$L_L - L_W = 178$ mm

So Option (b) is correct

Q.12 (10)

Balancing Pressure at point 1 and 2

$P_2 = P_1 + (30-x)\rho_1 g + x\rho_2 g$

$(0.42\times100\times9.8) = (0.2\times800\times9.8) +$

$((30-x)\times800\times9.8) =$

$(x\times1000\times9.8)$

$420 = 160 + 240 - 800x + 1000x$

$x = 0.1 \text{ m} = 10 \text{ cm}$

4 KINEMATICS

4.1 INTRODUCTION

Fluid kinematics deals with describing the motion of fluids without necessarily considering the forces and moments that cause the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known, then the pressure distribution & hence forces acting on the fluid can be determined.

4.2 METHODS OF DESCRIBING FLUID MOTION

1) **Lagrangian Method:** A single fluid particle is followed during its motion and its velocity, acceleration, density etc. are described.

2) **Eulerian Method:** The velocity, acc, density etc. are described as a point in flow field. It is commonly used in fluid mechanics.

4.3 TYPES OF FLUID FLOW

4.3.1 STEADY & UNSTEADY FLOW

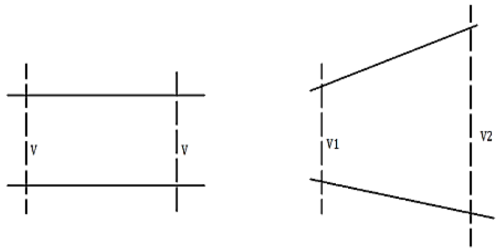
Steady flow is defined as that type of flow in which the fluid characteristics like velocity etc. at a point do not change with time.

$$\left(\frac{dP}{dt}\right)_{(x_o,y_o,z_o)} = 0$$

$$P = f(x,y,z)$$

Where,
P is flow parameter (velocity, acceleration, density etc)

Example:
Fluid flowing in pipe of uniform cross section or varying cross section with constant mass flow rate. The velocity of fluid may vary with position but remains constant w.r.t time.



Unsteady flow is that type of flow in which the velocity, pressure, density at a point changes with respect to time.

$$\left(\frac{dP}{dt}\right)_{(x_o,y_o,z_o)} \neq 0$$

$$P = f(x,y,z,t)$$

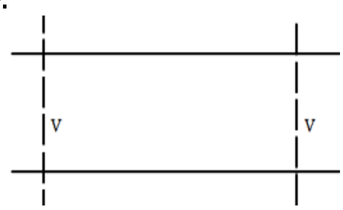
Example:
Fluid flowing in pipe of uniform cross section or varying cross section with variable mass flow rate

4.3.2 UNIFORM FLOW & NON UNIFORM FLOW

Uniform flow is defined as that type of flow in which the velocity at any given time does not change w.r.t. space.

$$\left(\frac{dP}{ds}\right)_t = 0$$

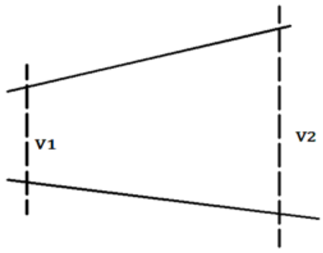
Example:
Fluid flowing through uniform cross section. The velocity remains constant w.r.t. space. The mass flow rate can vary w.r.t. time.



$$V_1 = V_2$$

Non Uniform flow is that type of flow in which velocity changes with respect to space at given instant.

Example:
Fluid flowing through non uniform cross section. The velocity varies w.r.t space. The mass flow rate can vary



$V_1 \neq V_2$

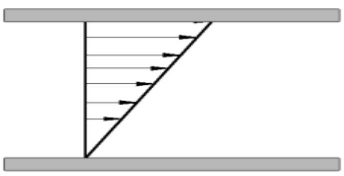
1) **Steady Uniform flow:** Flow at constant rate through a duct of uniform cross-section.
2) **Steady non-uniform flow:** Flow at constant rate through a duct of non-uniform cross-section (tapering pipe).
3) **Unsteady Uniform flow:** Flow at varying rates through a long straight pipe of uniform cross-section. (Again the region close to the walls is ignored.
4) **Unsteady non-uniform flow:** Flow at varying rates through a duct of non-uniform cross-section

4.3.3 ONE-, TWO- AND THREE-DIMENSIONAL FLOWS

1) **One dimensional:** One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space coordinate only.

Example:

i) Fluid flowing between two parallel plates



Velocity is along 'x' direction

$U = f(y)$

ii) Velocity flow field given by

$\bar{V} = ax\hat{i} + bx\hat{j}$

Here velocity has two components but depends only on one dimension. Therefore one dimensional flow

2) **Two dimensional:** Two dimensional flows is that type of flow in which the flow parameter such as velocity is a function of time and two space coordinates.
Velocity flow field given by
$\bar{V} = ax\hat{i} - by\hat{j}$

3) **Three dimensional:** Three dimensional flows is that type of flow in which the flow parameter such as velocity is a function of time and three spaces coordinate.
Velocity flow field given by
$\bar{V} = ax\hat{i} + by\hat{j} + cz\hat{k}$
$\bar{V} = axy\hat{i} - byz\hat{j}$

4.3.4 LAMINAR AND TURBULENT FLOWS

Viscous flow regimes are classified as laminar or turbulent on the basis of flow structure. In the laminar regime, flow structure is characterized by smooth motion in lamina or layers. Flow structure in the turbulent regime is characterized by random, three-dimensional motions of fluid particles in addition to the mean motion.
In laminar flow there is no mixing between adjacent fluid layers. A thin filament of dye injected into a laminar flow appears as a single line; there is no dispersion of dye throughout the flow. A dye filament injected into turbulent flow disperses quickly throughout the flow field; the line of dye breaks up into entangled threads of dye. This behaviour of turbulent flow is due to the velocity fluctuations present; the mixing of fluid particles from adjacent layers of fluid results in rapid dispersion of

the dye. If one measures the x component of velocity at a fixed locations in a pipe for both laminar and turbulent steady flow, the traces of velocity versus time appears. In the turbulent flow the flow velocity trace indicates random fluctuations of the instantaneous velocity,

$u = \bar{u} + u'$

In one dimensional laminar flow, the shear stress is related to the velocity gradient by the simple relation

$T_{yx} = \mu du / dy$

In turbulent flow there is no universal relationship between the stress field and the mean velocity field. Thus in turbulent flows we must rely heavily on semi empirical theories and on experimental data.

The type of flow is determined by Reynold's No.

$$Re = \frac{\rho VL}{\mu}$$

Where,

ρ = Density

L = Characterstic length

V = Velocity

μ = Dynamic Viscosity

4.3.5 COMPRESSIBLE & INCOMPRESSIBLE FLOWS

Flows in which variations in density are negligible are termed incompressible; when density variations within a flow are not negligible, the flow is called compressible. Gas flowing with negligible heat transfer may also be considered incompressible provided that the flow speed is small relative to the speed of sound.

The ratio of the flow speed V, to the local speed of sound C, in the gas is defined as the Mach number.

$$M = \frac{V}{C}$$

Thus gas flows with M<0.3 can be treated as incompressible.

4.3.6 INTERNAL AND EXTERNAL FLOWS

Flows completely bounded by solid surfaces are called internal or duct flows. Flows over bodies immersed in an unbounded fluid are termed external flows. Both internal and external flows may be laminar or turbulent, compressible or incompressible.

4.4 CONTINUITY EQUATION IN THREE-DIMENSIONS

Continuity equation is based on mass conservation principle.

(mass flow rate in) - (mass flow rate out) = (rate of change of mass in control volume)

$$m_{in} - m_{out} = \frac{dm}{dt}$$

In Cartesian coordinate system

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

Where,

$u = f_1(x,y,z)$

$v = f_2(x,y,z)$

$w = f_3(x,y,z)$

This equation is applicable to

1. Steady & unsteady
2. Compressible, incompressible.
3. Uniform, non-uniform

For steady flow $\frac{\partial \rho}{\partial t} = 0$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

For incompressible & steady flow

$$\frac{\partial}{\partial x}(u) + \frac{\partial}{\partial y}(v) + \frac{\partial}{\partial z}(w) = 0$$

For two dimensional, incompressible & steady flow

$$\frac{\partial}{\partial x}(u) + \frac{\partial}{\partial y}(v) = 0$$

4.5 CONTINUITY EQUATION IN ONE DIMENSION

For steady state condition

$(mass_{in}) = (mass_{out})$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For incompressible fluid

$$\rho_1 = \rho_2$$

$$A_1 V_1 = A_2 V_2$$

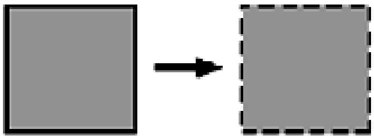
Continuity equation is valid for incompressible steady state.

4.6 MOTION OF FLUID ELEMENT

a) linear motion
b) Rotation motion
c) linear deformation
d) angular deformation

4.6.1 LINEAR MOTION

In pure translation motion the fluid particle retains its shape. It does not deform.



4.6.2 VELOCITY & ACCELERATION

The velocity in flow field is given by

$$\bar{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

Where,

$$u = f_1(x,y,z)$$

$$v = f_2(x,y,z)$$

$$w = f_3(x,y,z)$$

The magnitude of resultant velocity is given by

$$|\bar{V}| = \sqrt{u^2 + v^2 + w^2}$$

The acceleration is given by

$$a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Where,

$$a_x = \frac{du}{dt},\ a_y = \frac{dv}{dt},\ a_z = \frac{dw}{dt}$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\left(\frac{\partial x}{\partial t}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial y}{\partial t}\right) + \frac{\partial u}{\partial z}.\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

$$a_x = \frac{u.\partial u}{\partial x} + \frac{v.\partial u}{\partial y} + \frac{w.\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{u.\partial v}{\partial x} + \frac{v.\partial v}{\partial y} + \frac{w.\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{u.\partial w}{\partial x} + \frac{v.\partial w}{\partial y} + \frac{w.\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady state,

$$\frac{dv}{dt} = 0$$

$$\frac{\partial u}{\partial t} = 0,\ \frac{\partial v}{\partial t} = 0,\ \frac{\partial w}{\partial t} = 0$$

Hence acceleration

$$a_x = \frac{u.\partial u}{\partial x} + \frac{v.\partial u}{\partial y} + \frac{w.\partial u}{\partial z}$$

4.6.3 LOCAL ACCELERATION & CONVECTIVE ACCELERATION

Local acceleration is defined as the rate of change of velocity with respect to time at a given point in a flow field.

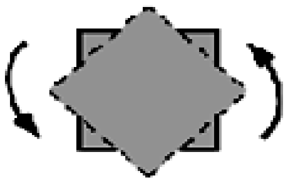
$$\frac{\partial u}{\partial t},\ \frac{\partial v}{\partial t},\ \frac{\partial w}{\partial t}$$

4.6.4 CONVECTIVE ACCELERATION

Convective acceleration is defined as change of velocity with change in position of fluid particle in a fluid flow.

$$\left(\frac{u.\partial u}{\partial x} + \frac{u.\partial u}{\partial y} + \frac{u.\partial v}{\partial z}\right)$$

4.6.5 FLUID ROTATION



Rotation of fluid particle is a vector quantity given by

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

Where,

ω_x is rotation about x axis,

ω_y is rotation about y axis &

ω_z is rotation about z axis

In vector notation

$$\omega = \frac{1}{2}(\nabla \times \vec{V})$$

Rotational Components are given by

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

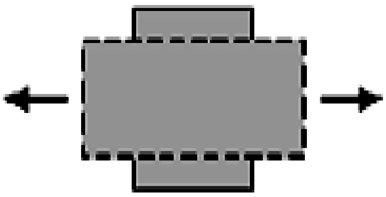
$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

For irrotational fluid flow,

$$\omega_x = \omega_y = \omega_z = 0$$

4.6.6 LINEAR STRAIN RATE

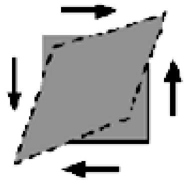
It is defined as the rate of increase in length per unit length. Mathematically, the linear strain rate of a fluid element depends on the initial orientation or direction of the line segment upon which we measure the linear strain



Linear strain is given by

$$\varepsilon_{xx} = \frac{\partial u}{\partial x},\ \varepsilon_{yy} = \frac{\partial v}{\partial y},\ \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

4.6.7 ANGULAR DEFORMATION/SHEAR STRAIN



Rate of shear strain is given by

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

4.7 FLOW PATTERNS

Fluid Mechanics is a subject with visualizations. Patterns of flow can be visualized in several ways. Basic types of line patterns used to visualize flow are streamline, path line, streak line and time line.

4.7.1 STREAM LINE

Stream lines are lines drawn in the flow field so that the tangent at any point gives the direction of velocity of particle at that point. Since the streamlines are tangent to the velocity vector at every point in the flow field, there can be no flow across a streamline.

Differential equation of streamline

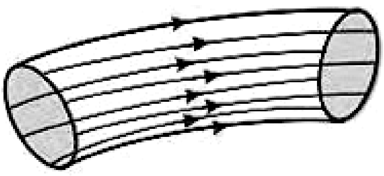
$$\frac{ds}{dt} = V$$

$$dt = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

4.7.11 STREAM TUBE

A bundle of neighboring streamlines may be imagined to form a passage through which the fluid flows. This passage is known as a stream-tube.



Properties of Stream tube:

1) The stream-tube is bounded on all sides by streamlines.
2) Fluid velocity does not exist across a streamline; no fluid may enter or leave a stream-tube except through its ends.
3) The entire flow in a flow field may be imagined to be composed of flows through stream-tubes arranged in some arbitrary positions

4.7.2 PATH LINE

A path line means the path or line actually described by a single fluid particle as it moves during a period of time. The path line indicates direction of the velocity of the same fluid particle at successive instant of time.

4.7.3 STREAK LINE

Let at any instant these particles arrive at points Q, R and S.Q, R and S represent the end points of the trajectories of these three particles at the instant. The curve joining the points S, R, Q and the fixed point P will define the streak line at that instant. The fixed point P will also lie on the line, since at any instant; there will be always a particle of some identity at that point.

In the steady flow, the velocity at each point in the flow field remains constant with time and, consequently, the streamlines do not vary from one instant to the next. This implies that a particle located on a given streamline will remain on the same streamline. Furthermore, consecutive particles passing through a fixed point in space will be in the same streamline and subsequently, will remain on this streamline. **Thus in a steady flow, path line, streak lines, and streamlines are identical lines in the flow field**

4.8 STREAM FUNCTION (Ψ)

It is defined as scalar function of space & time, such that its partial derivative w.r.t. to any direction gives the velocity perpendicular to that direction. It is defined for two dimensional flows.

$$U = \frac{\partial \Psi}{\partial y}, \quad V = -\frac{\partial \Psi}{\partial x}$$

Properties of stream function (Ψ) :

1) If stream function exists, it is possible case of continuous, incompressible steady flow.
2) If stream function (Ψ) satisfies Laplace Equation i.e.,

$$\frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial x^2} = 0$$

Then, it is possible case of irrotational flow.

4.8.1 CONSTANT STREAM FUNCTION

$$\Psi = \text{const}$$

$$d\Psi = 0$$

$$d\Psi = \frac{\partial \Psi}{\partial x}dx + \frac{\partial \Psi}{\partial y}dy$$

$$0 = vdx - udy$$

$$\frac{dy}{dx} = \frac{v}{u}$$

4.9 VELOCITY POTENTIAL FUNCTION

It is defined as a scalar function of space & time, such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

$\varphi = f(x, y, z)$ for steady flow.

$$u = -\frac{d\varphi}{dx}, \; v = -\frac{d\varphi}{dy}, \; w = -\frac{d\varphi}{dz}$$

Properties of Potential Function:

1. If velocity potential (φ) exists, the flow should be irrotational.
2. If (φ) satisfy the Laplace equation, the flow is continuous, incompressible, and steady.

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0$$

4.10 EQUIPOTENTIAL LINE

It is defined as line along which the velocity potential is constant.

φ = const

$$d\,\varphi=0$$

$\varphi=f(x,y)$ for steady, 2D flow

$$d\varphi=\frac{d\varphi}{dx}dx+\frac{d\varphi}{dy}dy$$

$$0=udx+vdy$$

$$\frac{dy}{dx}=-\frac{u}{v}$$

4.10.1 RELATION B/W STREAM FUNCTION & VELOCITY POTENTIAL FUNCTION

$$u=\frac{\partial\psi}{\partial x}=-\frac{\partial\varphi}{\partial y}$$

$$v=\frac{\partial\psi}{\partial y}=\frac{\partial\varphi}{\partial x}$$

Hence,

$$\frac{\partial\varphi}{\partial x}=-\frac{\partial\Psi}{\partial y}$$

$$\frac{\partial\varphi}{\partial y}=\frac{\partial\psi}{\partial x}$$

Stream line and equipotential line are orthogonal to each other

$$\left(\frac{dy}{dx}\right)_{\text{equipotential}}\cdot\left(\frac{dy}{dx}\right)_{\text{stream line}}=-1$$

Example:

The following cases represents the two velocity components, determine the third component of velocity such that they satisfy the continuity equation:

i) $u=x^2+y^2+z^2, v=xy^2-yz^2+xy$

ii) $v=2y^2, w=2xyz$

Solution:

The continuity equation for incompressible fluid is given by equation as

$$\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}+\frac{\delta w}{\delta z}=0$$

Case 1:

$u=x^2+y^2+z^2$

$\therefore\quad \frac{\delta u}{\delta x}=2x$

$v=xy^2-yz^2+xy$

$\therefore\quad \frac{\delta v}{\delta y}=2xy-z^2+x$

Substituting the values of $\frac{\delta u}{\delta x}$ and $\frac{\delta v}{\delta y}$ in continuity equation

$$2x+2xy-z^2+x+\frac{\delta w}{\delta z}=0$$

Or

$$\frac{\delta w}{\delta z}=-3x-2xy+z^2 \text{ or}$$

$$\delta w=\left(-3x-2xy+z^2\right)\delta z$$

Integration of both sides gives,

$$\int\delta w=\int\left(-3x-2xy+z^2\right)\delta z$$

$$w=(-3xz-2xyz+\frac{z^3}{3})+C$$

Where constant of integration cannot be function of z, but can be a function of x and y that is f(x,y).

$$w=\left(-3xz-2xyz+\frac{z^3}{3}\right)+f(x,y)$$

Case 2:

$v=2y^2\ \therefore\ \frac{\delta v}{\delta y}=4y$

$w=2xyz\ \therefore\ \frac{\delta w}{\delta z}=2xy$

Substituting the values of $\frac{\delta v}{\delta y}$ and $\frac{\delta w}{\delta z}$ in continuity equation, we get

$$\frac{\delta u}{\delta x}+4y+2xy=0$$

or

$$\frac{\delta u}{\delta x}=-4y-2xy$$

or

$\delta u = (-4y - 2xy)\delta x$

Integrating we get,

$u = -4xy - 2y\frac{x^2}{2} + f(y,z)$

$= -4xy - x^2y + f(y,z)$

Example:

A fluid flow field is given by

$V = x^2y\hat{i} + y^2z\hat{j} - (2xyz + yz^2)\hat{k}$

Prove that it is a case of possible steady incompressible fluid flow. Calculate the velocity and acceleration at the point (2,1,3).

Solution:

For the given fluid flow field

$u = x^2y \qquad \therefore \frac{\partial u}{\partial x} = 2xy$

$v = y^2z \qquad \therefore \frac{\partial v}{\partial y} = 2yz$

$w = -2xyz - yz^2 \quad \therefore \frac{\partial w}{\partial z} = -2xy - 2yz$

For a case of possible steady incompressible fluid flow, the continuity equation should be satisfied

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Substituting the value of $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$, we get,

$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}$

$= 2xy + 2yz - 2xy - 2yz = 0$

Hence, the velocity field

$V = x^2yi + y^2zj - (2xyz + yz^2)k$

is a possible case of fluid flow

Velocity at (2, 1, 3)

Substituting the values x = 2, y = 1 and z = 3 in velocity field, we get

$V = x^2y\hat{i} + y^2z\hat{j} - (2xyz + yz^2)\hat{k}$

$= (2^2 \times 1)\hat{i} + (1^2 \times 3)\hat{j} - (2\times 2\times 1\times 3 + 1\times 3^2)\hat{k}$

$= 4\hat{i} + 3\hat{j} - 21\hat{k}$

And resultant velocity

$= \sqrt{4^2 + 3^2 + (-21)^2}$

$= \sqrt{16+9+441} = \sqrt{466} = 21.587$ units

Acceleration (2, 1, 3)

The acceleration components a_x, a_y, and a_z for steady flow are

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$

$u = x^2y$

$\frac{\partial u}{\partial x} = 2xy,\ \frac{\partial u}{\partial y} = x^2,\ \frac{\partial u}{\partial z} = 0$

$v = y^2z$

$\frac{\partial v}{\partial x} = 0,\ \frac{\partial v}{\partial y} = 2yz,\ \frac{\partial v}{\partial z} = y^2$

$w = -2xyz - yz^2,$

$\frac{\partial w}{\partial x} = -2xyz,\ \frac{\partial w}{\partial y} = -2xz - z^2,$

$\frac{\partial w}{\partial z} = -2xy - 2yz$

Substituting these values in acceleration components, we get acceleration at (2, 1, 3)

$a_x = x^2y(2xy) + y^2z(x^2) - (2xyz + yz^2)(0)$

$= 2x^3y^2 + x^2y^2z$

$= 2(2^3)(1^2) + (2)^2(1)^2 \text{ x } 3$

$= 2\times 8 + 12$

$= 16 + 12$

$= 28$ units

$a_y = x^2y(0) + y^2z(2yz) - (2xyz + yz^2)(y^2)$

$= 2y^3z^2 - 2xy^3z - y^3z^2$

$=18-12-9$
$=-3\text{units}$

$$a_z = x^2y(-2yz)+y^2z(-2xz-z^2)$$
$$-(2xyz+yz^2)(-2xy-2yz)$$
$$=-2x^2y^2z-2xy^2z^2-y^2z^3$$
$$+[4x^2y^2z+2xy^2z^2+4xy^2z^2+2y^2z^3]$$
$$=(-2\times2^2\times1^2\times3)-(2\times2\times1^2\times3^2)-1^2\times3^3$$
$$+\{(4\times2^2\times1^2\times3)+(2\times2\times1^2\times3^2)$$
$$+(4\times2\times1^2\times3^2)+(2\times1^2\times3^3)\}$$
$$=-24-36-27+(48+36+72+54)$$
$$=123$$

∴ Acceleration= $a_x\hat{i}+a_y\hat{j}+a_z\hat{k}=28\hat{i}-3\hat{j}+123\hat{k}$

Or

magnitude of resultant acceleration

$$\sqrt{28^2+(-3)^2+123^2}=\sqrt{784+9+15129}=\sqrt{15922}$$
$$=126.18\text{units}$$

Example:

The velocity potential function is given by $\phi=5(x^2-y^2)$. Calculate the velocity component at the point $(4,5)$.

Solution:

$$\phi=5(x^2-y^2)$$
$$\frac{\delta\phi}{\delta x}=10x$$
$$\frac{\delta\phi}{\delta y}=-10y$$

But the velocity components u and v are given by equation as

$$u=-\frac{\delta\phi}{\delta x}=-10x$$
$$v=-\frac{\delta\phi}{\delta y}=-(-10y)=10y$$

The velocity components at the point (4, 5), i.e., at $x=4, y=5$ are

$$u=-10\times4=-40\text{units}$$
$$v=10\times5=50\text{units}$$

Example:

The stream function for a two-dimensional flow is given by $\psi=2xy$. Calculate the velocity at the point $P(2,\ 3)$. Also, find the velocity potential function ϕ

Solution:

Given:

$\psi=2xy$

The velocity components u and v in terms of ψ are

$$u=-\frac{\delta\psi}{\delta y}=-\frac{\delta}{\delta y}(2xy)=-2x$$
$$v=\frac{\delta\psi}{\delta x}=\frac{\delta}{\delta x}(2xy)=2y$$

At the point $P(2,\ 3)$., we get

$$u=-2\times2=-4\text{units}$$
$$v=2\times3=6$$

∴ Resultant velocity @P

$$=\sqrt{u^2+v^2}=\sqrt{4^2+6^2}=\sqrt{52}=7.21\text{units/sec}$$

Velocity potential function ϕ

We know

$$\frac{\delta\phi}{\delta x}=-u=-(-2x)=2x \qquad .....(i)$$
$$\frac{\delta\phi}{\delta y}=-v=-2y \qquad .....(ii)$$

Integrating equation (i), we get

$$\int d\phi=\int 2x\,dx$$
$$\phi=\frac{2x^2}{2}+C=x^2+C \qquad .....(iii)$$

Where C is a constant which is independent of x but can be a function of y

Differentiating equation (iii) w.r.t. 'y', we get $\frac{\delta\phi}{\delta y}=\frac{\delta C}{\delta y}$

But from (ii), $\frac{\delta\phi}{\delta y}=-2y$

$$\therefore \frac{\delta C}{\delta y}=-2y$$

Integrating this equation, we get

$$C=\int -2y\ dy=-\frac{2y^2}{2}=-y^2$$

Substituting this value of C in equation (iii),

we get $\phi = x^2 - y^2$

Example:
In a two dimensional incompressible flow, the fluid velocity components are given by $u = x - 4y$ and $v = -y - 4x$. Show that velocity potential exists and determine its form. Find also the stream function.
Solution:
Given: $u = x - 4y \text{ and } v = -y - 4x$

$$\therefore \frac{\partial u}{\partial x} = 1 \text{ and } \frac{\partial v}{\partial y} = -1$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

Hence flow is continuous and velocity potential exists.
Let ϕ =Velocity potential.
Let velocity components in terms of velocity potential is given by

$$\frac{\delta\phi}{\delta x} = -u = -(x - 4y) = -x + 4y \text{....(i)}$$

$$\frac{\delta\phi}{\delta y} = -v = -(y - 4x) = y + 4x \text{ ....(ii)}$$

Integrating equation (i), we get

$$\phi = -\frac{x^2}{2} + 4xy + C \text{ .....(iii)}$$

Where C is constant of integration, and is independent of x. This constant can be a function of y.
Differentiating the above equation, i.e., equation (iii) w.r.t. 'y', we get

$$\frac{\delta\phi}{\delta y} = 0 + 4x + \frac{\delta C}{\delta y}$$

But from equation (iii), we have

$$\frac{\delta\phi}{\delta y} = y + 4x$$

Equating the two values of $\frac{\delta\phi}{\delta y}$, we get

$$\frac{\delta C}{\delta y} = y$$

Integrating the above equation, we get

$$C = \frac{y^2}{2} + C_1$$

Where C_1 is a constant of integration, which is independent of x and y.

Taking it equal to zero, we get $c = \frac{y^2}{2}$

Substituting the value of C in equation (iii), we get

$$\phi = -\frac{x^2}{2} + 4xy + \frac{y^2}{2}$$

Value of Stream functions
Let ψ =stream function
The velocity components in terms of stream function are

$$\frac{\delta\psi}{\delta x} = v = -y - 4x \qquad \text{.....(iv)}$$

and

$$\frac{\delta\psi}{\delta y} = -u = -(x - 4y) = -x + 4y \qquad \text{......(v)}$$

Integrating equation (iv) w.r.t. x, we get

$$\psi = -yx - \frac{4x^2}{2} + k \qquad \text{.......(vi)}$$

Where k is a constant of integration which is independent of x but can be function of y
Differentiating equation (vi) w.r.t. y, we get

$$\frac{\delta\psi}{\delta y} = -x - 0 + \frac{\delta k}{\delta y}$$

But from equation (v), we have

$$\frac{\delta\psi}{\delta y} = -x + 4y$$

Equating the two values of $\frac{\delta\psi}{\delta y}$, we get

$$\frac{\delta k}{\delta y} = 4y$$

Integrating the above equation, we get

$$k = \frac{4y^2}{2} = 2y^2$$

Substituting the values of k in equation (vi), we get

$$\psi = -yx - 2x^2 + 2y^2$$

Example:

A fluid flow is given by $V = 8x^3\hat{i} - 10x^2y\hat{j}$ Find the shear strain rate and state whether the flow is rotational or irrotational.

Solution:

Given:

$V = 8x^3\hat{i} - 10x^2y\hat{j}$

$u = 8x^3, \; \frac{\delta u}{\delta x} = 24x^2, \; \frac{\delta u}{\delta y} = 0$

$v = -10x^2y, \; \frac{\delta v}{\delta x} = -20xy, \; \frac{\delta v}{\delta y} = -10x^2$

(i) Shear strain rate is given by equation as

$$= \frac{1}{2}\left(\frac{\delta v}{\delta x} + \frac{\delta u}{\delta y}\right) = \frac{1}{2}(-20xy + 0) = -10xy$$

(ii) Rotation in x-y plane is given by equation or

$$\omega_z = \frac{1}{2}\left(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}\right) = \frac{1}{2}(-20xy - 0) = -10xy$$

As rotation $\omega_z \neq 0$. Hence flow is rotational.

Example:

The velocity components in a two-dimensional flow are

$u = \frac{y^3}{3} + 2x - x^2y$ and $v = xy^2 - 2y - \frac{x^3}{3}$

Show that these components represent a possible case of an irrotational flow.

Solution:

Given $u = \frac{y^3}{3} + 2x - x^2y$

$$\frac{\delta u}{\delta x} = 2 - 2xy$$

$$\frac{\delta u}{\delta y} = \frac{3y^2}{3} - x^2 = y^2 - x^2$$

Also $v = xy^2 - 2y - \frac{x^3}{3}$

$$\therefore \frac{\delta v}{\delta y} = 2xy - 2$$

i) For a two-dimensional flow, continuity equation is $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$

Substituting the value of $\frac{\delta u}{\delta x}$ and $\frac{\delta v}{\delta y}$, we get

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 2 - 2xy + 2xy - 2 = 0$$

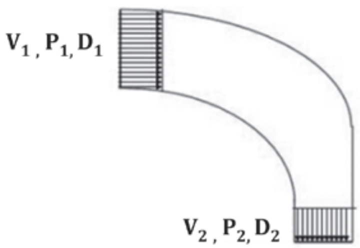
$\therefore$ It is a possible case of a fluid flow.

ii) Rotation, ω_z is given by

$$\omega_z = \frac{1}{2}\left(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}\right) = \frac{1}{2}[(y^2 - x^2) - (y^2 - x^2)] = 0$$

GATE QUESTIONS

Q.1 The inlet velocity of water ($\rho = 1000 kg/m^3$) in a right-angled bend- reducer is V_1=1 m/s, as shown below. The inlet diameter is D_1=0.8 m and the outlet diameter is D_2=0.4 m. The flow is turbulent and the velocity profiles at the inlet and outlet are flat (plug flow). Gravitational forces are negligible.



(a) Find the pressure drop (P_1-P_2) across the bend assuming negligible friction losses.

(b) If the actual pressure drop is (P_1-P_2) = 8.25 kPa, find the friction loss factor (K_f) based on the velocity V_1

[GATE–2001]

Q.2 The stream function in a xy-plane is given below: $\psi = \frac{1}{2}x^2y^3$ The velocity vector for this stream function is:

[GATE –2010]

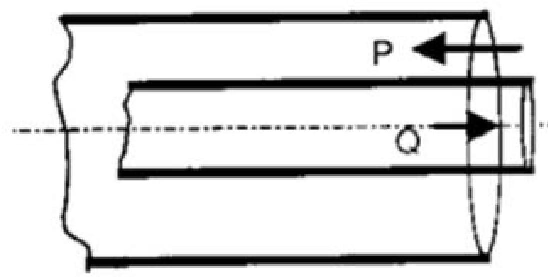
(A) $xy^3\,\hat{i} - \frac{3}{2}x^2y^2\,\hat{j}$

(B) $\frac{3}{2}x^2y^2\,\hat{i} - xy^3\,\hat{j}$

(C) $\frac{3}{2}x^2y^2\,\hat{i} + xy^3\,\hat{j}$

(D) $xy^3\,\hat{i} + \frac{3}{2}x^2y^2\,\hat{j}$

Q.3 Two liquids (P and Q) having same viscosity are flowing through a double pipe heat exchanger as shown in the schematic below.



Densities of P and Q are 1000 and 800 kg/m^3 respectively. The average velocities of the liquids P and Q are 1 and 2.5 m/s respectively. The inner diameters of the pipes are 0.31 and 0.1 m. Both pipes are 5 mm thick. The ratio of the Reynolds numbers Re_p to Re_q is

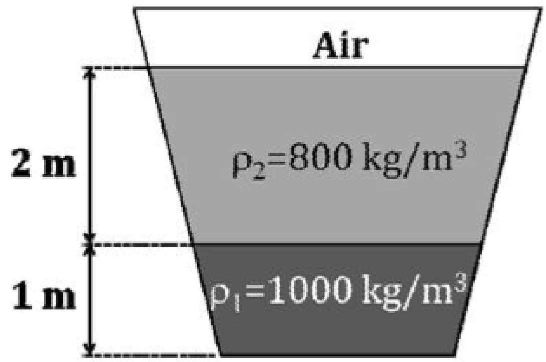
[GATE–2011]

(A) 2.5 (B) 1.5

(C) 1 (D) 4

Q.4 An open tank contains two immiscible liquids of densities (800 kg/m^3 and 1000 kg/m^3) as shown in the figure. If g 10 m/s^2, under static conditions, the gauge pressure at the bottom of the tank in Pa is______

[GATE–2013]



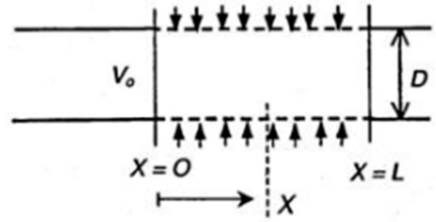
Q.5 Two packed towers are designed for the same mass velocity of the gas. The first has liquid and gas flow rates of 30 kg/s and 1.2 kg/s, respectively, while the corresponding flow rates in the second tower are 67.5 kg/s and 1.8 kg/s. The ratio of the design diameter of the wider tower to that of the narrower tower

[GATE-2018]

(A) 2 (B) 1.8
(C) 1.5 (D) 1.225

Q.6 A pipe has a porous section of length L as shown in the figure. Velocity at the start of this section is V_o. If fluid leaks into the pipe through the porous section at a volumetric rate per unit area q $(x/L)^2$, what will be the axial velocity in the pipe at any x? Assume incompressible one-dimensional flow i.e,, no gradients in the radial direction.

[GATE–2003]



(A) $V_x = V_a + q\dfrac{x^3}{L^2 D}$

(B) $V_x = V_a + \dfrac{1}{3}q\dfrac{x^3}{L^2}$

(C) $V_x = V_a + 2q\dfrac{x^3}{LD}$

(D) $V_x = V_a + \dfrac{4}{3}q\dfrac{x^3}{L^2 D}$

Q.7 An incompressible fluid is flowing through a contraction section of length L and has a 1-Dimentional Steady state velocity distribution, $u = u_0\left(1+\dfrac{2x}{L}\right)$. If u_0 = 2 m/s and L = 3 m, the convective acceleration (in m/s^2) of the fluid at L is

[GATE–2014]

(A) 2 (B) 5
(C) 8 (D) 10

ANSWER KEY:

1	2	3	4	5	6	7
A	B	C	26000	D	D	C

EXPLANATIONS

Q.1 (A)

By Continuity Equation

$$A_1V_1 = A_2V_2$$

$$V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 \Rightarrow \left(\frac{0.8}{0.4}\right)^2$$

$$V_2 = 4 \text{ m/s}$$

Applying Bernoulli Equation

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^{\,2} - V_1^{\,2}}{2}$$

$$\frac{\Delta P}{1000} = \frac{16-1}{2}$$

$$\Delta P = 7500 \text{ Pa} = 7.5 \text{ kPa}$$

(B)

Given: $\Delta P = 8.25$ kPa

Applying Bernoulli Equation

$$\frac{\Delta P}{\rho} = \frac{V_2^{\,2} - V_1^{\,2}}{2} + h_f$$

$$\frac{8.25}{1000} - \frac{16-1}{2} = h_f$$

$$h_f = 0.75$$

Now friction loss factor based on V_1

$$h_f = k_f\left(\frac{V_1^{\,2}}{2}\right)$$

$$k_f = \frac{0.75\times 2}{1} = 1.50$$

Q.2 (B)

$$\widehat{V} = v_x\,\hat{i} + v_y\,\hat{j}$$

$$= \frac{\partial\psi}{\partial y}\,\hat{i} - \frac{\partial\psi}{\partial x}\,\hat{j}$$

$$\psi = \frac{1}{2}x^2y^3$$

$$\frac{\partial\psi}{\partial y} = \frac{3}{2}x^2y^2$$

$$\frac{\partial\psi}{\partial x} = xy^3$$

$$\widehat{V} = \frac{3}{2}x^2y^2\,\hat{i} - xy^3\,\hat{j}$$

So Option (b) is correct

Q.3 (C)

$$D_P = D_{Op} - D_q = 0.31 - 0.1$$

$$D_P = 0.2 \text{ m}$$

$$\frac{Re_p}{Re_q} = \frac{\rho_p\ V_p\ D_p}{\rho_q\ V_q\ D_q}$$

$$= \frac{1000\times 1\times 0.2}{800\times 2.5\times 0.1} = 1$$

So Option (c) is correct

Q.4 (26000 Pa)

$$\left(\begin{matrix}\text{Presure at}\\ \text{bottom}\end{matrix}\right) = (P_{atm}) + (P_1) + (P_2)$$

$$(P_{gauge}) + (P_{atm}) = (P_{atm}) + (P_1) + (P_2)$$

$$P_{gauge} = \rho_1 g\, h_1 + \rho_2\, g\, h_2$$

$$P_{gauge} = (800\times 2\times 10) + (1000\times 1\times 10)$$

$$P_{gauge} = 26000 \text{ Pa or } \left(\frac{kg}{m\, s^2}\right)$$

Q.5 (D)

By continuity Equation

$$\dot{m} = \rho A V$$

$$\frac{\dot{m}}{\rho A} = V$$

$$\frac{\dot{m}_1}{A_1} = \frac{\dot{m}_2}{A_2} \qquad [\because \rho_1 = \rho_2]$$

$$\frac{d_2}{d_1} = \sqrt{\frac{\dot{m}_2}{\dot{m}_1}} = \sqrt{\frac{1.8}{1.2}} = \sqrt{\frac{3}{2}}$$

$$\frac{d_2}{d_1} = 1.224$$

So Option (d) is correct

Q.6 (D)

For incompressible flow, mass balance provide

$$\frac{\pi}{4} D^2 V_{x|x} + q\left(\frac{x}{L}\right)^2 \pi D\, dx = \frac{\pi}{4} D^2 V_{x|x+dx}$$

$$\lim_{dx \to 0} \frac{\frac{\pi}{4} D^2 V_{x|x+dx} - \frac{\pi}{4} D^2 V_{x|x}}{dx} = \pi q D \left(\frac{x}{L}\right)^2 D$$

$$\frac{d}{dx}\left(\frac{\pi}{4} D^2 V_x\right) = \pi q D \left(\frac{x}{L}\right)^2$$

$$\frac{dV_x}{dx} = \frac{4\, q\, x^2}{D\, L^2}$$

Integrate

$$V_x = \frac{4q}{D\, L^2}\left(\frac{x^3}{3}\right) + c$$

Substituting $V_x = V_0$ at x = 0,

C = V_0

$$V_x = V_0 + \frac{4q}{D\, L^2}\left(\frac{x^3}{3}\right)$$

Q.7 (C)

Acceleration

$$a = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t}$$

$$= (u)\frac{\partial (u)}{\partial x}$$

$$a = u_0\left(1 + \frac{2x}{L}\right)\frac{\partial\left(u_0\left(1 + \frac{2x}{L}\right)\right)}{\partial x}$$

$$= u_0\left(1 + \frac{2x}{L}\right) u_0\left(\frac{2}{L}\right)$$

$$= u_0^{\,2}\left(\frac{2}{L}\right)\left(1 + \frac{2x}{L}\right)$$

$$= 2^2\left(\frac{2}{3}\right)(1 + 2)$$

$$= 8\ m\ /\ s$$

So Option (c) is correct

5 BERNOULLI'S EQUATION & ITS APPLICATION

5.1 INTRODUCTION

Consider a small element of fluid in flow field. The energy in the element as it moves in the flow field is conserved. This principle of conservation of energy is used in the determination of flow parameters like pressure, velocity and potential energy at various locations in a flow. The concept is used in the analysis of flow of ideal as well as real fluids. Energy can neither be created nor destroyed. It is possible that one form of energy is converted to another form.

5.2 EULER'S EQUATION

Euler's equation is obtained from the conservation of momentum for a fluid particle moving along a streamline. The forces due to gravity & pressure are taken into consideration.

$$\frac{dP}{\rho} + VdV + gdz = 0$$

This equation is known as Euler's equation of motion. The assumptions involved are:

1) Steady flow
2) Motion along a stream line
3) Ideal fluid (frictionless)

In the case of incompressible flow, this equation can be integrated to obtain Bernoulli Equation.

5.3 BERNOULLI'S EQUATION

The Bernoulli's equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow.

By integrating Euler's equation for incompressible flow

$$\int \frac{dP}{\rho} + \int VdV + \int gdz = \int 0$$

$$\frac{P}{\rho} + \frac{V^2}{2} + zg = const$$

Multiplying the Bernoulli's equation by the density (ρ). Each term in this equation has units of pressure.

$$P + \frac{\rho V^2}{2} + \rho zg = const$$

- P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

  $\frac{P}{\rho g}$ is known as pressure head

- $\frac{\rho V^2}{2}$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to stop isentropically.

  $\frac{\rho V^2}{2}$ is known as dynamic head

- ρgz is the **hydrostatic pressure,** which is not pressure in a real sense since its value depends on the reference level selected. It accounts for the elevation effects, i.e., of fluid weight on pressure.

  Z is gradient or datum head.

  The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure.** Therefore, the Bernoulli's equation states that the total pressure along a streamline is constant. The sum of the static and dynamic pressures is called the **stagnation pressure,** and it is expressed as

  Stagnation pressure = static pressure + dynamic pressure.

Stagnation pressure $= P + \frac{\rho v^2}{2}$

5.4 APLLICATION OF BERNOULLI'S EQUATION

Venturimeter and Orifice meters are the obstruction type meters commonly used for the measurement of flow through pipes. In each case the meter acts as an obstacle placed in the path of the flowing fluid causing local changes in pressure and velocity

i) Venturimeter
ii) Orifice plate
iii) Pitot tube

5.4.1 VENTURIMETER

A venturimeter consist of a short converging part, throat & diverging part. The liquid undergoes gradual contraction & expansion, therefore it has lesser losses.

By applying Bernoulli's equations at 1 & 2

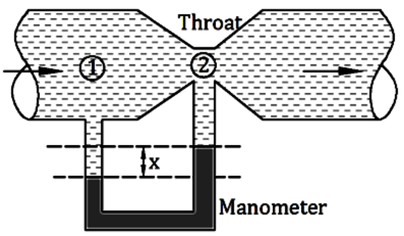
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Points 1 & 2 are at same horizontal level,

$\therefore\ z_1 = z_2$

$$h = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \frac{V_2^2 - V_1^2}{2g}$$

$$Q = a_1 v_1 = a_2 v_2$$



$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

$$Q_{theo} = a_2 v_2 = a_2 \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

In case of ideal fluid, the above equation is valid. To modify the equation for real fluid, coefficient of discharge (C_d) is multiplied to theoretical flow. It accounts for viscous loss, expansion loss & boundary roughness. It is defined as

$$c_d = \frac{\text{actual discharge}}{\text{theoretical discharge}}$$

$$Q_{act} = c_d a_2 v_2 = c_d a_2 \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

$$h = \frac{P_A - P_B}{\rho g} = \left(\frac{\rho_m}{\rho} - 1\right) x$$

Where,

x = the difference in mercury level

ρ_m = Density of heavy liquid

ρ = Density of flowing fluid

Case1:

Manometric fluid is lighter then liquid flowing in pipe (in case of inverted manometer)

$$h = x\left(1 - \frac{\rho_m}{\rho}\right)$$

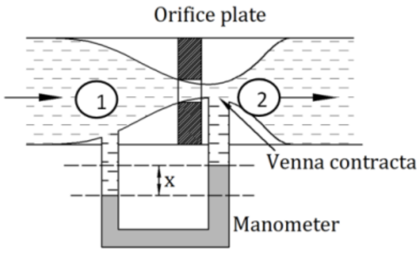
Case2:

Inclined/vertical venturimeter with differential U-tube Manometer

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \left(\frac{\rho_m}{\rho} - 1\right) x$$

5.4.2 ORIFICE METER OR ORIFICE PLATE

It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. The orifice diameter is kept generally 0.35 times the diameter of pipe. Due to sudden expansion & contraction, loss is high in orificemeter.



A differential Manometer is connected to measure the pressure difference at section (1), which is at distance of about 1.5-2.0 times the pipe diameter and section (2), which is at a distance of about half diameter of the orifice on the downstream side.

By applying Bernoulli's equations at 1 & 2

$$\frac{P_1}{\rho g}+\frac{V_1^{\,2}}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^{\,2}}{2g}+z_2$$

Points 1 & 2 are at same horizontal level,

$\therefore z_1 = z_2$

$$h=\left(\frac{P_1}{\rho g}-\frac{P_2}{\rho g}\right)+\frac{V_2^{\,2}-V_1^{\,2}}{2g}$$

$$Q = a_1 v_1 = a_2 v_2$$

As the liquid comes out of orifice it contracts further and the area just outside the orifice is lower compared to the area of the orifice. This section is called as vena-contracta. Area of jet at the vena-contracta is less than the area of the orifice itself due to convergence of stream lines. The coefficient of contraction C_c is defined as follows.

$$c_c=\frac{\text{area of jet at vena contracta}(a_2)}{\text{area of orifice}(a_o)}$$

The value of coefficient of contraction varies from 0.61 to 0.69 depending on the shape and size of the orifice

$$a_2 = c_c . a_0$$

$$Q = a_1 v_1 = a_2 v_2 = c_c . a_0 v_2$$

$$v_2=\frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_2}{a_1}\right)^2 c_c^{\,2}}}$$

Due to the viscous effects, the actual flow velocity through the orifice will always be less than the theoretical possible velocity. The velocity coefficient C_v is defined as follows.

$$C_v=\frac{\text{Actual of jet at venna contacta}}{\text{Therotcial veocity of jet at orifice}}$$

The value of coefficient of contraction varies from 0.61 to 0.69 depending on the shape and size of the orifice.

$$V_{act_2}=c_v\frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

$$Q_{act}=a_2 v_{act_2}=c_c a_0 c_v\frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

$$Q_{act}=c_d a_0\frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Where,

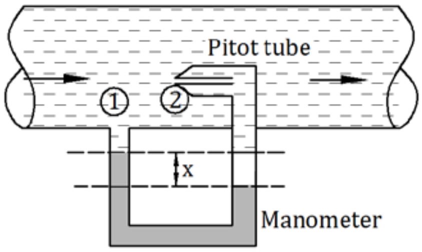
$C_d = C_c\ C_v$

Average value of C_d for orifices is 0.62.

5.4.3 PITOT TUBE

It is a device used for measuring flow velocity at any point in a pipe.

Principle: If velocity of flow at any point becomes zero, the pressure is increased due to conversion of kinetic energy to pressure energy.



By applying Bernoulli's equations at 1 & 2

Stagnation pressure = static pressure + Dynamic pressure

$$\frac{P_1}{\rho g}+\frac{V_1^{\,2}}{2g}=\frac{P_2}{\rho g}$$

$$\frac{V_1^{\,2}}{2g}=\frac{P_2}{\rho g}-\frac{P_1}{\rho g}=h$$

$$V_{act}=c_v\sqrt{2gh}$$

$$h=\frac{P_2-P_1}{\rho g}=\left(\frac{\rho_m}{\rho}-1\right)x$$

Where,

C_v is velocity coefficient

5.5 BERNOULIS EQUATION FOR REAL FLUIDS

$$\frac{P_1}{\rho g}+\frac{V_1^{\,2}}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^{\,2}}{2g}+z_2+h_f$$

Where,

h_f is head loss due to viscous force & minor losses.

5.6 FREE LIQUID JETS

Free liquid jet is defined as the jet of water coming out from the nozzle in atmosphere. The path travelled by the free jet is parabolic, given by equation of parabola

$$y=x\tan\theta-\frac{1}{2}gx^2\sec^2\theta$$

1. *Max height*

$$(U\sin\theta)^2-(0)^2=2gy_{max}$$

$$y_{max}=\frac{U^2\sin^2\theta}{2g}$$

2. *Time of flight*

$$y=U\sin\theta g t-\frac{1}{2}gt^2$$

For full flight, y = 0

$$U\sin\theta g t-\frac{1}{2}gt^2=0$$

$$t=\frac{2U\sin\theta}{g}$$

3. *Max Distance (Range)*

$$R=U\cos\theta g t$$

$$=\frac{2U\cos\theta g U\sin\theta}{g}$$

$$=\frac{U^2\sin 2\theta}{g}$$

4. *Value of θ for max range*

$$R=\frac{U^2\sin 2\theta}{g}$$

$$\sin 2\theta=1$$

$$2\theta=90^o \text{ Or } \theta=45^o$$

Example:

Water is flowing through a pipe of 5cm diameter under a pressure of $29.43N/cm^2$ (gauge) and with mean velocity of $2m/s$. Find the total head or total energy per unit weight of the water at a cross–section, which is 5m above the datum line.

Solution:

Given:

Diameter of pipe= 5 cm = 0.05m

Pressure,

$P=29.43N/cm^2=29.43\times10^4N/m^2$

Velocity, $V=2.0m/s$

Datum head, $Z=5m$

Total head

=Pressure head+kinetic head+datum head

Pressure head $=\frac{P}{\rho g}=\frac{29.43\times10^4}{1000\times9.81}=30m$

Kinetic head $=\frac{V^2}{2g}=\frac{2\times2}{2\times9.81}=0.204m$

$\therefore$ Total head $=\frac{P}{\rho g}+\frac{V^2}{2g}+z=30+0.204+5$

=35.204m

Example:

An oil of S.G. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10cm. The oil–mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d=0.98$

Solution:

Given:

S.G. Of oil, $S_0=0.8$

S.G. Of mercury, $S_h = 13.6$

Reading of differential manometer, x =25cm

$\therefore$ Difference of pressure head,

$$h = x\left(\frac{S_h}{S_o} - 1\right) = 25\left(\frac{13.6}{0.8} - 1\right) \text{cm of oil}$$

=25(17-1) =400 cm of oil.

Dia at inlet, $d_1 = 20cm$

$$\therefore\ a_1 = \frac{\pi}{4} \times d_1^{\ 2} = \frac{\pi}{4} \times 20^2 = 314.16cm^2$$

$d_2 = 10cm$

$$\therefore\ a_2 = \frac{\pi}{4} \times 10^2 = 78.54cm^2$$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation

$$\text{Or } Q = c_d \frac{a_1 a_2}{\sqrt{a_1^{\ 2} - a_2^{\ 2}}} \times \sqrt{2gh}$$

$$= 0.98 \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.7}{\sqrt{98696 - 6168}} = 70422.4cm^3/s$$

$= 70.422 litres/sec$

Example:

A $30cm \times 15cm$ venturimeter is inserted in vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20cm. Find the discharge. Take $C_d = 0.98$

Solution:

Given:

Dia at inlet, $d_1 = 30cm$

$$\therefore a_1 = \frac{\pi}{4}(30)^2 = 706.85cm^2$$

Dia at throat, $d_2 = 15cm$

$$\therefore a_2 = \frac{\pi}{4}(15)^2 = 176.7cm^2$$

$$h = x\left(\frac{S_h}{S_o} - 1\right) = 20\left(\frac{13.6}{1} - 1\right)$$

$= 20 \times 12.6 = 252cm$ of water

$C_d = 0.98$

$$\text{Discharge, } Q = c_d \frac{a_1 a_2}{\sqrt{a_1^{\ 2} - a_2^{\ 2}}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756cm^3/s = 125.756 lit/s$

Example:

In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16cm and 8cm respectively. A is 2 meters above B. The pressure gauge readings have shown that the pressure at B is greater than at A by $0.981N/cm^2$. Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.

Solution:

Given:

S.G. of oil, $S_0 = 0.8$

$$\therefore\ \text{Density, } \rho = 0.8 \times 1000 = 800 \frac{kg}{m^3}$$

Dia. at A, $d_a = 16cm = 0.16m$

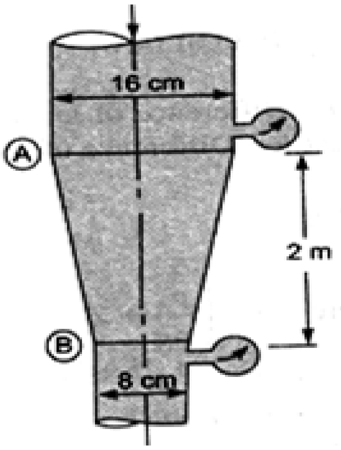
$\therefore$ Area at A,

$$a_1 = \frac{\pi}{4}(0.16)^2 = 0.0201m^2$$

Dia at B, $d_b = 8cm = 0.08m$

$\therefore$ Area at B,

$$a_2 = \frac{\pi}{4}(0.08)^2 = 0.005026m^2$$



i) Difference of pressure,

$p_B - p_A = 0.981\,N/cm^2$

$= 0.981\times10^4\,N/m^2 = 9810\,N/m^2$

Difference of pressure head

$\therefore \dfrac{p_B - p_A}{\rho g} = \dfrac{9810}{800\times9.81} = 1.25$

Applying Bernoulli's equation at A and B and taking the reference as line passing through section B, we get,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

Or $\dfrac{P_A}{\rho g} - \dfrac{P_B}{\rho g} + z_A - z_B = \dfrac{V_B^2}{2g} - \dfrac{V_A^2}{2g}$

Or $\left(\dfrac{P_A - P_B}{\rho g}\right) + 2.0 - 0.0 = \dfrac{V_B^2}{2g} - \dfrac{V_A^2}{2g}$

Or

$$-1.25 + 2.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g} \left\{ Q\, \frac{P_A - P_B}{\rho g} = 1.25 \right\}$$

$$0.75 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g} \text{ -----} (i)$$

Now applying continuity equation at A and B, we get

$V_A \times A_1 = V_B \times A_2$

Or $V_B = \dfrac{V_A \times A_1}{A_2}$

Substituting the value of V_B in equation (i), we get

$$0.75 = \frac{16V_A^2}{2g} - \frac{V_A^2}{2g} = \frac{15V_A^2}{2g}$$

$$\therefore \quad V_A = \sqrt{\frac{0.75\times2\times9.81}{15}} = 0.99\,m/s$$

$\therefore$ Rate of flow, $Q = V \times A$

$= 0.99\times0.0201 = 0.01989\,m^3/s$

ii) Difference of level of mercury in the U-tube.

Let h=Difference of mercury level.

Where

Then $h = x\left(\dfrac{S_h}{S_o} - 1\right)$

Where

$h = \dfrac{P_A}{\rho g} - \dfrac{P_B}{\rho g} + z_A - z_B$

$= -1.25 + 2.0 - 0 \quad \left(Q\, \dfrac{P_B - P_A}{\rho g} = 1.25\right)$

$= 0.75$

$\therefore \quad 0.75 = x\left(\dfrac{13.6}{0.8} - 1\right) = x \times 16$

$\therefore x = \dfrac{0.75}{16} = 0.04687\,m$

$= 4.687\,cm$

Example:

A pitot- static tube placed in the centre of a 300mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if pressure difference between the two orifices is 60 mm of water .Take the coefficient of pitot tube as $C_v = 0.98$

Solution:

Given:

Dia of pipe, d=300mm=0.30m

Diff of pressure head,

$h = 60mm$ of water $= 0.06m$ of water

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80\times$ centre line velocity

Centre line velocity is given by equation

$= C_v\sqrt{2gh} = 0.98\times\sqrt{2\times9.81\times0.06} = 1.063\,m/s$

$\therefore\ \overline{V} = 0.80\times1.063 = 0.8504\,m/s$

Discharge, $Q = \text{area of pipe} \times \overline{V}$

$Q = \frac{\pi}{4}(0.30)^2 \times 0.8504 = 0.06 m^3/s$

Example:

A pitot–tube is inserted in a pipe of 300mm diameter. The static pressure in pipe is 100mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, recorded by the pitot–tube is $0.981 N/cm^2$. Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$.

Solution:

Given:

Dia of pipe d=300mm = 0.30m

$\therefore$ Area, $A = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$

Static pressure head = 100mm of mercury (vacuum)

$= -\frac{100}{1000} \times 13.6 = -1.36 m$ of water

Stagnation pressure

$= .981 N/cm^2 = 981 \times 10^4 N/m^2$

$\therefore$ Stagnation pressure head

$= \frac{.981 \times 10^3}{\rho g} = \frac{0.981 \times 10^3}{1000 \times 9.81} = 1m$

h = Stagnation pressure head –static pressure head

$= 1.0 - (-1.36) = 1.0 + 1.36 = 2.36 m$ of water

$\therefore$ Velocity at centre $V = C_v\sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668 m/s$

Mean velocity,

$\overline{V} = 0.85 \times 6.668 = 5.6678 m/s$

$\therefore$ Flow rate ofwater $= \overline{V} \times \text{area of pipe}$

$= 5.6678 \times 0.07068 = 0.4006 m^3/s$

Example:

A pipe of dia 400 mm carries water at velocity 25 m/s. The pressure at the points A & B are given as $29.43 N/cm^2$ and $22.263 N/cm^2$, while the datum head at A and B are 28m and 30m. Find the loss of head b/w A & B.

Solution:

D = 400 m = 0.4 m

$V_A = V = 25 m/s$

Total energy at A

$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$

Total energy at B

$= \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$

$H_2 = E_A - E_B$

$\left(\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A\right) - \left(\frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B\right)$

$\frac{P_A}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30, \quad \frac{V_A^2}{2g} = 31.85$

$\frac{P_B}{\rho g} = \frac{22.563 \times 10^4}{1000 \times 9.81} = 23, \quad \frac{V_B^2}{2g} = 31.85$

$E_A - E_A = (30 + 28) - (23 + 30) = 5m$

Example:

A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, what will be the diameter of the jet at a point 4.5 m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s.

Solution:

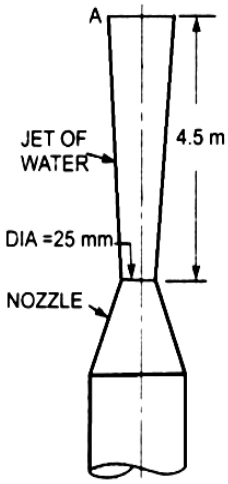
Given:

Diameter of nozzle, $d_1 = 25mm = 0.025m$

Velocity of jet at nozzle $V_1 = 12 m/s$

Height of point A $h = 4.5m$

Let the velocity of the jet at a height of $4.5 m = V_2$



Considering the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy.)

Initial velocity,

$u = V_1 = 12 m/s$

Final velocity,

$V = V_2$

Value of $g = 9.81 m/s^2$ and h=4.5m

Using, $V^2 - U^2 = 2gh$, we get

$V_2^2 - 12^2 = 2 \times (-9.81) \times 4.5$

$\therefore V_2 = \sqrt{12^2 - (2 \times 9.81 \times 4.5)}$

$= \sqrt{144 - 88.29} = 7.46 m/s$

Now, applying continuity equation to the outlet of nozzle and at point A, we get

$a_1 v_1 = a_2 v_2$

Or

$$A_2 = \frac{A_1 V_1}{V_2} = \frac{\frac{\pi}{4} D_1^2 \times V_1}{V_2}$$

$$= \frac{\pi \times 0.025^2 \times 12}{4 \times 7.46} = 0.0007896$$

Let D_2 = Diameter of jet at point A

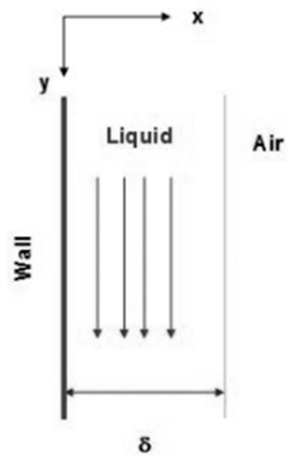
Then,

$$A_2 = \frac{\pi}{4} D_2^2 = 0.0007896$$

$$D_2 = \sqrt{\frac{0.0007896 \times 4}{\pi}} = 0.0317 m = 31.7 mm$$

GATE QUESTIONS

Q.1 Consider the flow in a liquid film of constant thickness (Q) along a vertical wall as shown in the figure below.



Assuming laminar, one-dimensional, fully-developed flow, the y-direction Navier Stokes equation reduces to $\mu \frac{d^2V_y}{dx^2} + \rho g = 0$ Where V_y is the velocity in y direction, μ is the viscosity and ρ is the density of the liquid.

[GATE–2002]

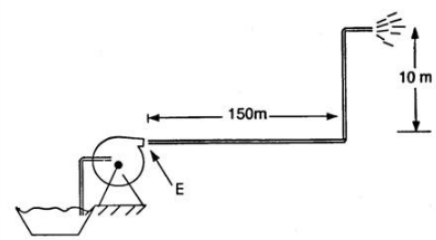
(a) State the boundary conditions to be used for the solution of velocity profile?

(b) Solve for the velocity profile

(c) If Q is the volumetric flow rate per unit width of the wall, how is it related to the film thickness (δ)

Q.2 A centrifugal pump is used to pump water through a horizontal distance of 150 m and then raised to an overhead tank 10 m above. The pipe is smooth with an I.D. of 50 mm. What head (m of water) must the pump generate at its exit (E) to deliver water at a flow rate of 0.001 m^3 is? The Fanning friction factor, f is 0.0062

[GATE–2003]

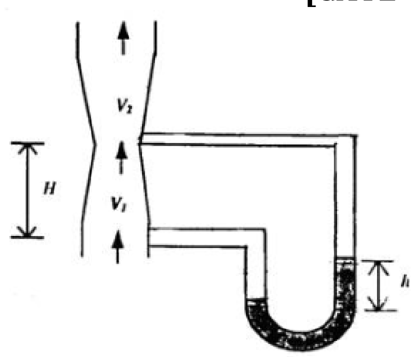


(A) 10 m (B) 11 m

(C) 12 m (D) 20 m

Q.3 The pressure differential across a vertical venturimeter (shown in Figure) is measured with the help of a mercury manometer to estimate flow rate of water flowing through it. The expression for the velocity of water at the throat is

[GATE–2003]



(A) $\dfrac{V_2^{\,2}-V_1^{\,2}}{2g}=h\dfrac{\rho_m}{\rho_f}$

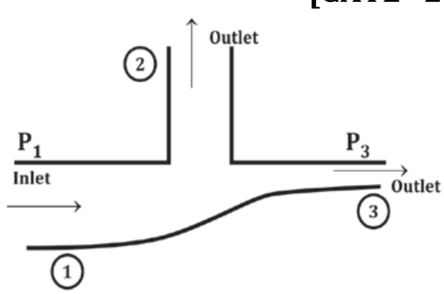
(B) $\dfrac{V_2^{\,2}-V_1^{\,2}}{2g}=h\dfrac{(\rho_m-\rho_w)}{\rho_f}$

(C) $\dfrac{V_2^{\,2}-V_1^{\,2}}{2g}=H+h\dfrac{(\rho_m-\rho_w)}{\rho_f}$

(D) $\dfrac{V_2^{\,2}}{2g}=H+h\dfrac{(\rho_m-\rho_w)}{\rho_f}$

Q. 4 A pipeline system carries crude oil of density 800 kg/m^3. The volumetric flow rate at point 1 is 0.28 m^3/s. The cross sectional areas of the branches 1, 2 and 3 are 0.012, 0.008 and 0.004 m^2 respectively. All the three branches are in a horizontal plane and the friction is negligible. If the pressures at the points 1 and 3 are 270 kPa and 240 kPa respectively, then the pressure at point 2 is

[GATE –2007]

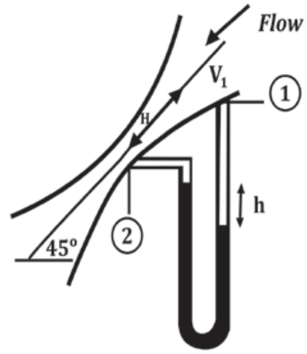


(A) 202 kPa (B) 240 kPa

(C) 284 kPa (D) 355 kPa

Q.5 The pressure differential across a venturimeter, inclined at 45^0 to the vertical (as shown in the figure) is measured with the help of a manometer to estimate the flowrate of a fluid flowing through it. If the density of the flowing fluid is ρ and the density of the manometer fluid is ρ_m, the velocity of the fluid at the throat can be obtained from the expression

[GATE –2007]



(A) $\dfrac{V_2^{\,2}-V_2^{\,2}}{2g}=\dfrac{h(\rho_m-\rho)}{\rho}+H\sin45^0$

(B) $\dfrac{V_2^{\,2}-V_2^{\,2}}{2g}=\dfrac{h\,\rho_m}{\rho}+H\sin45^0$

(C) $\dfrac{V_2^{\,2}-V_2^{\,2}}{2g}=\dfrac{h\,\rho_m}{\rho}$

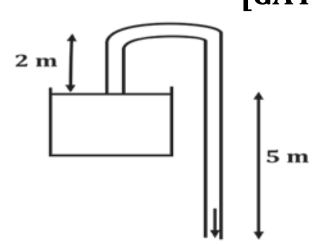
(D) $\dfrac{V_2^{\,2}-V_2^{\,2}}{2g}=\dfrac{h(\rho_m-\rho)}{\rho}$

Common Data for Q.6 and Q.7

A siphon tube having a diameter of 2 cm draws water from a large open reservoir and discharges into the open atmosphere as shown in the figure.

Assume incompressible fluid and neglect frictional losses. (g = 9.8 m/s^2)

[GATE –2008]



Q.6 The velocity (in m/s) at the discharge point is

[GATE –2008]

(A) 9.9 (B) 11.7

(C) 98 (D) 136.9

Q.7 The Volumetric Flow rate (in L/s) of water at the discharge is

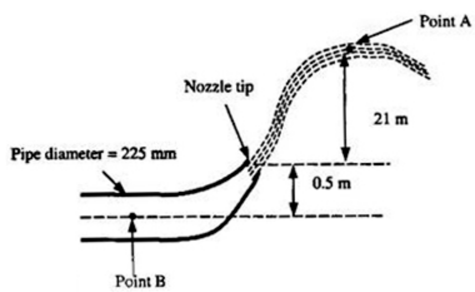
[GATE –2008]

(A) 3.11 (B) 3.67

(C) 30.77 (D) 42.99

Common Data for Q.8 and Q.9

A free jet of water is emerging from a nozzle (diameter =75 mm) attached to a pipe (diameter 225 mm) as shown below.



The velocity of water at point A is 18 m/s. Neglect friction in the pipe and nozzle. Use g = 9.81 m/s^2 and density of water-1000 kg/m^3.

Q.8 The velocity of water at the tip of nozzle (in m/s) is

[GATE –2009]

(A) 13.4 (B) 18

(C) 23.2 (D) 27.1

Q.9 The gauge pressure in (kPa) at point B is

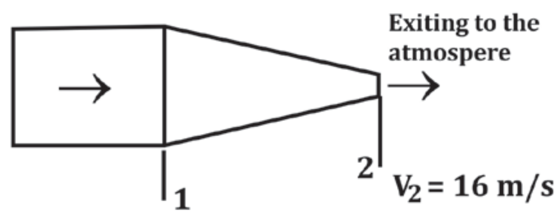
[GATE –2009]

(A) 80 (B) 100

(C) 239.3 (D) 367.6

Q.10 Water (density 1000 kg/m^3) is flowing through a nozzle, as shown below and exiting to the atmosphere. The relationship between the diameters of the nozzle at locations 1 and 2 is $D_1 = 4 D_2$. The average velocity of the stream at location 2 is 16 m/s and the frictional loss between location 1 and location 2 is 10000 Pa. Assuming steady state and turbulent flow, the gauge pressure in Pa, at location 1 is______.

[GATE–2013]



Q.11 In a steady and incompressible flow of a fluid (density 1.25 kg m^{-3}), the difference between stagnation and static pressures at the same location in the flow is 30 mm of mercury (density 13600 kg m^{-3}). Considering gravitational acceleration as 10 m s^{-2}, the fluid speed (in m s^{-1}) is____

[GATE–2014]

(A) 70.8 (B) 80.8

(C) 90.8 (D) 100.8

Q.12 A centrifugal pump delivers water at the rate of 0.22 m^3/s from a reservoir at ground level to another

reservoir at a height H, through a vertical pipe of 0.2 m diameter. Both the reservoirs are open to atmosphere. The power input to the pump is 90 kW and it operates with an efficiency of 75%.
Data:
Fanning friction factor for pipe now is
f = 0.004. Neglect other head losses. Take gravitational acceleration g = 9.8 m/s^2 and density of water is 1000 kg/m^3.
The height H, in meters, to which the water can be delivered (up to one decimal place) is ______

[GATE–2015]

Q.13 Water (density 1000 kg m^{-3}) is pumped at a rate of 36 m^3/h, from a tank 2 m below the pump, to an overhead pressurized vessel 10 m above the pump. The pressure values at the point of suction from the bottom tank and at the discharge point to the overhead vessel are 120 kPa and 240 kPa, respectively. All pipes in the system have the same diameter. Take acceleration due to gravity, g = 10 m s^{-2}. Neglecting frictional losses, what is the power (in kW) required to deliver the fluid?

[GATE–2016]

(A) 1.2 (B) 2.4

(C) 3.6 (D) 4.8

Q.14 The initial water level in a tank is 4 m. When the valve located at the bottom is opened, the rate of change of water level (L) with respect to time (t) is, $\frac{dL}{dt} = -k\sqrt{t}$

Where k = 0.6m s$^{-3/2}$. The level of water (in m) in the tank at time 0.5 s after opening the valve is_____ (rounded off to second decimal place).

[GATE–2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12
-	B	B	C	D	A	A	D	D	137500	B	26.07
13	**14**										
B	3.82										

EXPLANATIONS

Q.1 (A)

Navier-Stoke Equation

$$\rho \frac{Dv}{Dt} = -\nabla P + \rho g + \mu \nabla^2 v$$

For steady state $\frac{\partial^2 V_y}{\partial x^2} = 0$

$$\rho\left[\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right] =$$

$$\mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial P}{\partial y} + \rho g_y$$

$$0 = \mu\left[\frac{\partial^2 v_y}{\partial x^2}\right] + \rho g$$

at x = 0 $\Rightarrow v_y = 0$ (No slip)

at x = $\delta \Rightarrow \tau_{yx} = 0$ (Free Liq. Surface)

$$\frac{dv_y}{dx} = 0$$

(B)

$$\mu\left(\frac{d^2 v_y}{dx^2}\right) = -\rho g$$

$$\frac{dv_y}{dx} = \frac{-\rho g}{\mu}\int dx + c_1$$

$$\frac{dv_y}{dx} = \frac{-\rho g}{\mu} x + c_1$$

at x = δ, $\frac{dv_y}{dx} = 0$

$$0 = -\frac{\rho\, g\, \delta}{\mu} + c_1$$

$$c_1 = \frac{\rho\, g\, \delta}{\mu}$$

$$\frac{dv_y}{dx} = \frac{\rho\, g}{\mu}(\delta - x)$$

$$\int dv_y = \frac{\rho\, g}{\mu}\int(\delta - x)\,dx + c_2$$

$$v_y = \frac{\rho\, g}{\mu}\left(\delta - \frac{x^2}{2}\right) + c_2$$

$$x = 0,\ v_y = 0$$

$$0 = \frac{\rho\, g}{\mu}(\delta - 0) + c_2$$

$$c_2 = 0$$

$$v_y = \frac{\rho\, g}{\mu}\left(\delta - \frac{x^2}{2}\right)$$

(C)

$$q = \iint_A v_y\ dA = \int_0^W \int_0^\delta v_y\ dx\ dz$$

$$= W\int_0^\delta v_y\ dx$$

$$= W\int_0^\delta \frac{\rho\, g}{\mu}\left(\delta - \frac{x^2}{2}\right) dx$$

$$= \frac{W\rho\, g}{\mu}\left(\frac{\delta x^2}{2} - \frac{x^3}{3}\right)_0^\delta$$

$$= \frac{W\rho\, g}{\mu}\left(\frac{\delta^3}{2} - \frac{\delta^3}{3}\right)$$

$$q = \frac{W\rho\, g\, \delta^3}{3\,\mu}$$

Q.2 (B)

$Q = AV$

$0.001 = V\frac{\pi}{4}(0.05)^2$

$V = 0.509 \text{ m/s}$

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A + \begin{pmatrix}\text{Pump}\\ \text{I/P}\end{pmatrix} = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \frac{4fLV^2}{2Dg}$$

$$0+0+0+\begin{pmatrix}\text{Pump}\\ \text{I/P}\end{pmatrix} = 0+0+10 + \frac{4\times0.0062\times(150+10)\times(0.509)^2}{2\times9.8\times0.05}$$

$$\begin{pmatrix}\text{Pump}\\ \text{I/P}\end{pmatrix} = 11.05 \text{ m Head}$$

So Option (b) is correct Option

Q.3 (B)

Applying Bernoulli Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{V_2^2 - V_1^2}{2g} = \frac{P_1 - P_2}{\rho g} + (z_1 - z_2)$$

$$= \frac{P_1 - P_2}{\rho g} - H$$

$$P_1 - P_2 = \rho_w g H + g h(\rho_m - \rho_w)$$

$$\frac{V_2^2 - V_1^2}{2g} = \frac{\rho_w g H + g h(\rho_m - \rho_w) - \rho_w g H}{\rho g}$$

$$\frac{V_2^2 - V_1^2}{2g} = \frac{h(\rho_m - \rho_w)}{\rho_w}$$

So Option (b) is correct

Q.4 (C)

Given

$P_1 = 270 \text{ kPa}$, $P_3 = 240 \text{ kPa}$

$A_1 = 0.012 \text{ m}^2$, $A_2 = 0.008 \text{ m}^2$

$A_3 = 0.004 \text{ m}^2$

$Q_1 = 0.28 \text{ m}^3/\text{s}$

$P_2 = ?$

$$V_1 = \frac{Q_1}{A_1} = \frac{0.28}{0.012} = 23.33 \text{ m/s}$$

Applying Bernoulli Equation between points 1 and 3

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + 0 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + 0$$

$$\frac{(270-240)10^3}{800} + \frac{(23.33)^2}{2} = \frac{V_3^2}{2}$$

$V_3 = 24.88 \text{ m/s}$

Now Applying continuity equation

$A_1V_1 = A_2V_2 + A_3V_3$

$0.28 = (0.008)V_2 + (0.004)24.88$

$V_2 = 22.55 \text{ m/s}$

Applying Bernoulli Equation between points 1 and 2

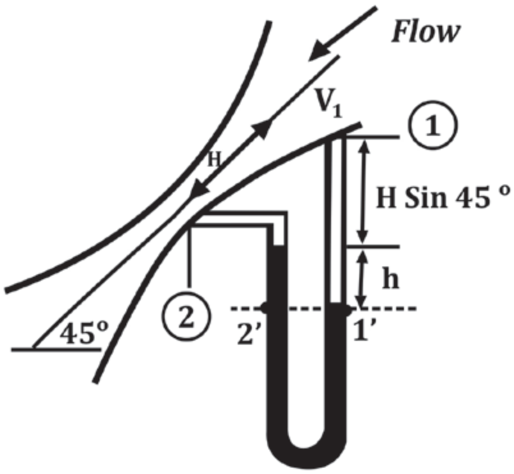
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + 0 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + 0$$

$$\frac{(270)10^3}{800} + \frac{(23.33)^2 - (22.55)^2}{2} = \frac{P_2}{800}$$

$P_2 = 284 \text{ kPa}$

So Option (c) is correct

Q.5 (D)



$P_1' = P_2'$

$P_1 + \rho g H \sin 45 + \rho g h = P_2 + \rho_m g h$

$P_1 - P_2 = (\rho_m - \rho) g h - \rho g H \sin 45$

Applying Bernoulli Equation at point 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + H \sin 45$$

$$\frac{(\rho_m - \rho) g h}{\rho g} - \frac{\rho g H \sin 45}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + H \sin 45$$

$$\frac{(\rho_m - \rho) h}{\rho} = \frac{V_2^2 - V_1^2}{2g}$$

So Option (d) is correct

Q.6 (A)

Applying Bernoulli Equation a point 1 and point 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$2g\left(Z_1 - Z_2\right) = V_2^2$$

$$V_2 = \sqrt{2 \times 9.81 \times 5}$$

$$V_2 = 9.9 \text{ m/s}$$

So Option (a) is correct

Q.7 (A)

$Q = AV$

$$Q = \left(\frac{\pi}{4} D^2\right) V_2$$

$$Q = \frac{\pi}{4}(0.02)^2 \times 9.9$$

$Q = 3.1086$ L/s

So Option (a) is correct

Q.8 (D)

Applying Bernoulli Equation at Point A and C

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C$$

$$\frac{18^2}{2g} + + 21.5 = \frac{V_C^2}{2g} + 0.5$$

$$\frac{18^2}{2g} + 21 = \frac{V_C^2}{2g}$$

$$V_C = 27.129 \text{ m/s}$$

So Option (d) is correct

Q.9 (D)

By Continuity Equation,

$A_B V_B = A_C V_C$

$V_B = 3$ m / s

Applying Bernoulli Equation at Point B and Point C

$$\frac{P_B}{\rho g} + \frac{V_B^{\,2}}{2g} + Z_B = \frac{P_C}{\rho g} + \frac{V_C^{\,2}}{2g} + Z_C$$

$$\frac{P_B - P_C}{\rho g} = \frac{27.12^2 - 3^2}{2(9.81)} + 0.5$$

$$\frac{P_B - P_C}{1000 \times 9.8} = \frac{725.349}{2(9.81)} + 0.5$$

$$P_B - P_C = 367.6 \text{ kPa}$$

So Option (d) is correct

Q.10 (137500 Pa)

By Continuity Equation

$A_1 V_1 = A_2 V_2$

$(D_1)^2 V_1 = (D_2)^2 V_2$

$V_1 = 1$ m / s

Applying Bernoulli Equation at Point 1 and Point 2

$$\frac{P_1}{\rho g} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\,2}}{2g} + z_2$$

$$\frac{P_1 - P_{atm}}{1000} = \frac{16^2 - 1^2}{2} + 10$$

$$\Delta P_{gauge} = 137500 \text{ Pa}$$

Q.11 (B)

Applying Bernoulli Equation

$$\frac{P_1}{\rho g} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\,2}}{2g} + z_2$$

$$\frac{P_{Stag.} - P_1}{1000} = \frac{V_1^{\,2}}{2}$$

$$\Delta P = (h \rho g) = (0.030)(13600)10 = 4080 \ \frac{kg}{m\,s^2}$$

$$V_1 = \sqrt{\frac{(4080)(2)}{1.25}} = 80.80 \text{ m / s}$$

So Option (b) is correct

Q.12 (26.07 m)

Applying Bernoulli Equation

$$\frac{P_1}{\rho} + \frac{V_1^{\,2}}{2} + g\,z_1 + (\eta P) = \frac{P_2}{\rho} + \frac{V_2^{\,2}}{2} + g\,z_2 + h_{fs}$$

Now, Q = AV

$$0.22 = \left(\frac{\pi}{4} D^2\right) V \Rightarrow V = 7.002 \text{ m / s}$$

$$h_{fs} = \frac{4 f \ H V^2}{2D} = \frac{4(0.004)(H)(7.0028)^2}{2(0.2)}$$

So from Bernoulli Equation

$$\left(\frac{\eta P}{\rho\, Q}\right) = gH + \frac{4(0.004)(H)(7.0028)^2}{2(0.2)}$$

$$\left(\frac{0.75 \times 90}{1000 \times 0.22}\right) = 9.8\text{ H} + 1.96\text{ H}$$

H = 26.07 m

Q.13 (B)

Applying Bernoulli Equation

$$\frac{P_1}{\rho} + \frac{V_1^{\,2}}{2} + g\,z_1 + \left(\begin{array}{c}\text{Head Developed}\\ \text{by pump (H)}\end{array}\right) = \frac{P_2}{\rho} + \frac{V_2^{\,2}}{2} + g\,z_2$$

$$H = \frac{P_2 - P_1}{\rho} + 12$$

$$H = \frac{120 \times 1000}{1000 \times 10} + 12 = 24$$

Power = ρ g Q H

$$= 1000 \times 10 \times \frac{36}{3600} \times 24$$

Power=2.4 kW

So Option (b) is correct

Q.14 (3.82)

given, $\frac{dL}{dt} = -k\sqrt{t}$

$$\int_0^L dL = -0.6\int_0^{0.5} t^{1/2}\ dt$$

$$L - 4 = -0.6\left[t^{3/2}\right]_0^{3/2}\left(\frac{2}{3}\right)$$

$$L - 4 = -0.6\left[\frac{1}{2}\right]_0^{3/2}\left(\frac{2}{3}\right)$$

L = 3.82 m

6 FLOW THROUGH CONDUITS/PIPES

6.1 INTERNAL FLOW

Fluids are conveyed (transported) through closed conduits in numerous industrial processes. It is found necessary to design the pipe system to carry a specified quantity of fluid between specified locations with minimum head loss.

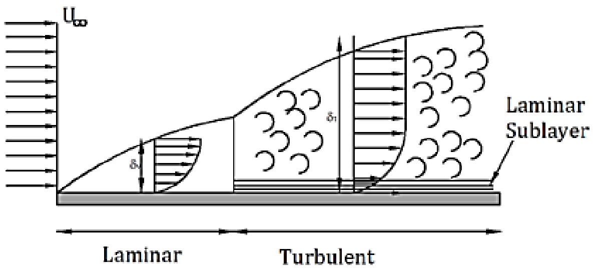
The flow may be laminar with fluid flowing in an orderly way, with layers not mixing macroscopically. The momentum transfer and consequent shear induced is at the molecular level by pure diffusion. Such flow is encountered with every viscous fluids.

The flow turns turbulent under certain conditions with macroscopic mixing of fluid layers in the flow. At any location, the velocity varies about a mean value.

The flow is controlled by (i) pressure gradient (ii) the pipe diameter or hydraulic mean diameter (iii) the fluid properties like viscosity and density and (iv) the pipe roughness.

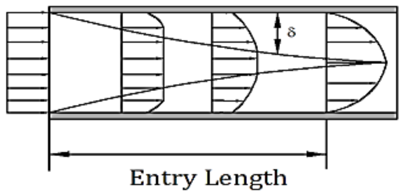
6.1.1 BOUNDARY LAYER

When fluid flows over surface, the molecules near the surface are brought to rest due to the viscosity of the fluid. The adjacent layers also slow down, but to a lower and lower extent. This slowing down is found limited to a thin layer near the surface. The fluid beyond this layer is not affected by the presence of the surface. The fluid layer near the surface in which velocity of fluid is less than free stream velocity is known as **boundary layer**.



6.1.2 ENTRY LENGTH

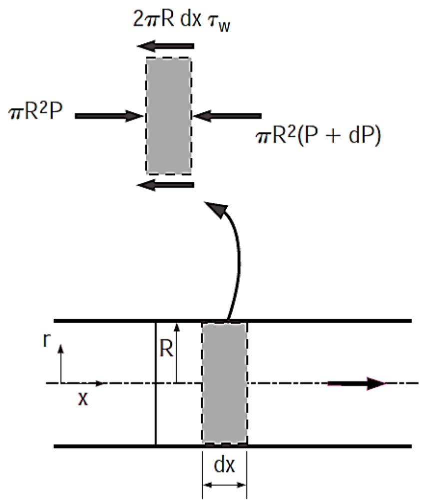
In internal flow, the boundary layer develops all over the circumference. At some distance from the entrance, the boundary layers merge. The velocity profile beyond this point remains unchanged. The distance upto this point is known as entry length.



6.2 LAMINAR FLOW/VISCOUS FLOW

6.2.1 ANALYSIS OF FULLY DEVELOPED LAMINAR FLOW IN CIRCULAR DUCT

i) Shear stress



At low velocity the fluid moves in layers. The shear stress in laminar flow is given by

$$\tau_w = \mu \frac{du}{dy}$$

force balance on fluid element gives

$$Pg\pi r^2 - \tau \times 2\pi r dx - (P + dP)\pi r^2 = 0$$

$$\tau_w 2\pi r = -\frac{dP}{dx}\pi r^2$$

$$\tau_w = -\frac{dP}{dx}\times\frac{r}{2}$$

ii) Velocity distribution

$$\tau_w = \mu\frac{du}{dy}$$

$y = R - r$ (y is measured from the pipe wall)

$dy = -dr$

$$\therefore \tau_w = \mu\frac{du}{dr}$$

$$\therefore -\mu\frac{du}{dr} = -\frac{dP}{dx}\times\frac{r}{2}$$

$$\therefore \int du = \frac{1}{\mu}\int\left(\frac{dP}{dx}\right)\frac{r}{2}dr$$

$$\therefore U = \frac{1}{\mu}\left(\frac{dP}{dx}\right)\frac{1}{4}r^2 + C$$

On applying boundary conditions:

1) For r = R, U = 0

$$0 = \frac{1}{4\mu}\left(\frac{dP}{dx}\right)R^2 + C$$

$$C = -\frac{1}{4\mu}\left(\frac{dP}{dx}\right)R^2$$

$$U = -\frac{1}{4\mu}\left(\frac{dP}{dx}\right)\left[R^2 - r^2\right]$$

2) for, r = 0, U = U_{max}

$$U_{max} = -\frac{1}{4\mu}\left(\frac{dP}{dx}\right)R^2$$

$$U = U_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

iii) Average velocity

$dQ = U dA$

$dA = 2\pi r dr$

$$dQ = U_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]\times dA$$

$$\int dQ = \int U_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]\times 2\pi r dr$$

$$Q = \frac{\pi}{8\mu}\left(\frac{-dP}{dx}\right)R^4$$

$$\bar{U} = \frac{Q}{A} = \frac{\frac{\pi}{8\mu}\left(\frac{-dP}{dx}\right)R^4}{\pi R^2} = -\frac{1}{8\mu}\left(\frac{dP}{dx}\right)R^2$$

$$\frac{U_{max}}{U} = 2$$

iv) Pressure drop for length (l)

Average velocity is given by

$$\bar{U} = \frac{1}{8\mu}\left(\frac{dP}{dx}\right)R^2$$

By rearranging above equation

$$-\int_{P_2}^{P_1} dP = \int_0^L \frac{8\mu\bar{U}}{R^2}$$

Pressure drop across length 'L' is

$$P_1 - P_2 = \frac{32\mu\bar{U}L}{D^2}$$

$$h_f = \frac{P_1 - P_2}{\rho g} = \frac{32\mu\bar{U}L}{\rho g D^2}$$

The above equation is **Hagen Poiseuille equation**

6.2.2 FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES

i) $U = -\frac{1}{2\mu}\left(\frac{dP}{dx}\right)\left[ty - y^2\right]$

Where,

t is the distance between parallel plates

Velocity is max at y = t/2

$$U_{max} = -\frac{1}{8\mu}\left(\frac{dP}{dx}\right)t^2$$

$$U = 4U_{max}\times\left[\frac{y}{t} - \frac{y^2}{t^2}\right]$$

ii) Shear stress distribution

$$\tau = -\frac{1}{2}\left(\frac{dP}{dx}\right)\left[t - 2y\right]$$

at centre, shear stress will be zero.

iii) $\bar{U} = \text{Avg. velocity} = \frac{-1}{12\mu}\left(\frac{dP}{dx}\right)t^2$

$$\frac{U_{max}}{U} = \frac{3}{2}$$

iv) Pressure head for a given length

$$P_1 - P_2 = \frac{12\mu\bar{U}L}{t^2}$$

6.3 TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects the wall shear stress. Turbulent flow is characterized by random and rapid fluctuations of swirling regions of fluid, called eddy, throughout the flow. These fluctuations provide an additional mechanism for momentum and energy transfer. In laminar flow, fluid particles flow in an orderly manner along pathlines, and momentum and energy are transferred across streamlines by molecular diffusion. In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, thus greatly enhancing mass, momentum, and heat transfer. As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients

In turbulent flow, instantaneous values of the velocity fluctuate about an average value, which suggests that the velocity can be expressed as the sum of an average value $\overline{u}$ and a fluctuating component u',

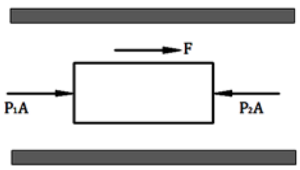
$$u = \overline{u} + u'$$

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to wall is zero. The friction resistance for turbulent flow is.

1) $\propto V^n$ where n varies from 1.5-2.0
2) $\propto \rho$
3) $\propto A$ Area of contact
4) $\propto P$ Pressure
5) $\propto$ Nature of surface

6.3.1 EXPRESSION FOR COEFFICIENT OF FRICTION IN TERMS OF SHEAR STRESS

Consider a fluid element of length 'L' & diameter'd'



$$P_1A - P_2A - F = 0$$

$$(P_1 - P_2)A = F$$

F = force due to shear stress.

$$F = \tau_0(\pi dL)$$

$$(P_1 - P_2)A = \tau_0(\pi dL)$$

$$\frac{P_1 - P_2}{\rho g} = \frac{4fLV^2}{2gd} = \frac{\tau_0(\pi dL)}{\rho g}$$

$$A = \frac{\pi}{4}d^2$$

$$f = \frac{2\tau_o}{\rho V^2}$$

6.3.2 SHEAR STRESS IN TURBULENT FLOW

Shear stress in turbulent flow is sum of shear stress due to viscous flow & turbulent flow

$$\tau = \tau_v + \tau_t$$

Viscous/laminar shear stress is given by

$$\tau_V = \mu\frac{du}{dy}$$

J. Boussinesq expressed turbulent shear in terms of average velocity gradient

$$\tau_t = \eta\frac{d\overline{u}}{dy}$$

Where,

η is Eddy viscosity,

$\overline{du}$ is average velocity.

6.3.3 REYNOLD'S EXPRESSION FOR TURBULENT SHEAR STRESS

Reynold expressed turbulent shear stress between two layers of a fluid at a small distance in terms of velocity fluctuations

$$\tau_t = \rho u'v'$$

Where,

u' and v' are fluctuating velocity components in the direction of x & y due to turbulence.

τ_t will also be varying, Hence to find the shear stress time avg on both side is considered $\overline{\tau} = \rho \overline{u'v'}$

6.3.4 PRANDTL MIXING LENGTH THEORY FOR TURBULENT SHEAR STRESS

The turbulent shear stress can only be calculated if u' and v' is known. According to Prandtl, the mixing length 'l', is that distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the lumps are mixed in the other layer in such a way that momentum of particle in the direction of motion is same.

Prandtll expressed velocity fluctuations in terms of mixing length

$$U' = l\frac{du}{dy} \ \& \ V' = l\frac{du}{dy}$$

$$\tau_t = \rho l^2 \left(\frac{du}{dy}\right)^2$$

Total shear stress is given by

$$\overline{\tau} = \mu\frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2$$

6.3.5 VELOCITY DISTRIBUTION IN TURBULENT FLOW IN PIPES

$$\overline{\tau} = \mu\frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2$$

Viscous shear stress is negligible except near the boundary.

Prandtl assumed that mixing length is proportional to 'y'

$l \propto y$

$l = ky$

Where,

K is Karman factor = 0.4

$$\tau_t = \rho(ky)^2 \left(\frac{du}{dy}\right)^2 = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2$$

by rearranging the above equation

$$\frac{du}{dy} = \sqrt{\frac{\tau}{\rho k^2 y^2}}$$

$$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau}{\rho}}$$

$$U = (\ln y)\sqrt{\frac{\tau}{\rho}} g \frac{1}{k} + C$$

$U^* = \sqrt{\frac{\tau}{\rho}}$ (U* is known as Shear velocity)

Since y is distance from the surface of pipe

$U = U_{max}$ at y=R

$$U_{max} = \frac{U^*}{k}\ln R + C$$

$$C = U_{max} - \frac{U^*}{k}\ln R$$

$$U = \frac{U^*}{k}\ln\left(\frac{y}{R}\right) + U_{max}$$

$$U = \frac{U^*}{0.4}\ln\left(\frac{y}{R}\right) + U_{max}$$

$$U = U_{max} + 2.5U^* \ln\left(\frac{y}{R}\right)$$

This is Prandtl's universal velocity distribution equation for turbulent flow in pipes.

$(U_{max} - U)$ is known as velocity defect.

6.3.6 HYDRO DYNAMICALLY SMOOTH & ROUGH BOUNDARIES

For turbulent flow analysis along a boundary, the flow is divided in two portions. Viscous portion near the surface is known as laminar sub layer. The boundaries are considered as smooth or rough on the basis of ratio of average irregularities 'k' and laminar sub layer thickness 'δ'

if $k < \delta$ Smooth

if $k > \delta$ Rough

if $\frac{k}{\delta} < 0.25$ boundary is smooth if $\frac{k}{\delta} > 6$ boundary is rough

6.3.7 VELOCITY DISTRIBUTION FOR TURBULENT FLOW IN SMOOTH PIPES

$$U = \frac{U^*}{k} \ln y + C$$

at $y = 0$, U=-∞

It means at some finite distance from wall, the velocity will be equal to zero. Let y' be the distance from wall where velocity is zero

$y = y'$, U=0

$$U = 0 = \frac{U^*}{k} \ln y' + C$$

$$C = \frac{-U^*}{k} \ln y'$$

$$U = \frac{U^*}{k} \ln\left(\frac{y}{y'}\right)$$

For smooth boundary

$y' \propto \delta'$

Where,

δ' is thickness of laminar sub layer

From experiments, y'& δ' is given by

$$y' = \frac{\delta'}{107}$$

$$\delta' = \frac{11.6\nu}{U^*}$$

$$U = \frac{U^*}{k} \ln\left(\frac{y.100}{\delta}\right)$$

6.3.8 REYNOLDS' NO IN TERMS OF ROUGHNESS

$$Re = \frac{U^* k}{\nu}$$

Where,

k is average height of irregularities

ν is kinematic viscosity

U^* = Shear velocity $= \sqrt{\frac{\tau}{\rho}}$

i) $\frac{U^* k}{\nu} < 4$, boundary is smooth

ii) $4 < \frac{U^* k}{\nu} < 100$, boundary is in transition

iii) $\frac{U^* k}{\nu} > 100$, the boundary is rough.

6.3.9 VELOCITY IN ROUGH PIPES

(Nikurddse's Experiment) show that

$$y' = \frac{k}{30}$$

$$u = \frac{u^*}{k} \text{In}(y / y')$$

$$u = 2.5u^* \text{In}\left(\frac{y}{k/30}\right)$$

$$u = 2.5u^* \text{In}\left(\frac{30y}{k}\right)$$

Example:

A Pipe carrying water has avg irregularities of 0.15 mm. What type of boundary it is? (Shear stress developed is $4.9N/m^2, \nu = 0.01 \text{strokes}$)

Solution:

$k = 0.15 \times 10^{-3} m$

$\tau_0 = 4.9 N/m^2$

$\nu = 0.01 \text{strokes} = 0.01' cm^2/s = \Delta 10^{-6} m^2/s$.

$\rho = 1000 Kg/m^3$

$$\delta = \frac{11.6\nu}{u*}$$

$$\delta = \frac{11.6 \times 10^{-6}}{u*}$$

$$u^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 m/s$$

Roughness Reynold No.:

$$\frac{u^* k}{v} = \frac{0.07 \times 15 \times 10^3}{10^{-6}}$$

6.4 LOSS OF ENERGY IN FLUID FLOW

6.4.1 MAJOR LOSS

Darcy-Weisbach Formula:

$$h_f = \frac{4f.L.V^2}{d.2g}$$

h_f = Loss of head due to viscosity.

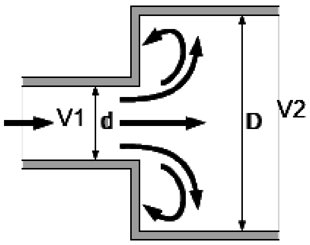
$f = \frac{16}{Re}$ for Re<2000 (Viscous flow).

$= \frac{0.079}{Re^{\frac{1}{4}}}$ for Re varying from 4000 to 10^6.

(for turbulent flow in smooth pipes).
Where,
L=Length,
V = mean velocity of flow,
d = diameter of pipe.

6.4.2 MINOR ENERGY LOSS

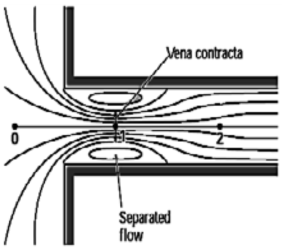
1) **Loss of head due to sudden enlargement:** Due to sudden change of diameter the liquid flowing from the smallest pipe is not able to follow the abrupt change of boundary. Thus the flow separates from the boundary and turbulent eddies are formed.

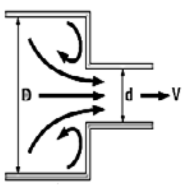


Head loss is 'm' is given by

$$h_e = \frac{(v_1 - v_2)^2}{2g}$$

2) **Loss due to sudden contraction:** As the liquid comes out of orifice it contract further and the area just outside the orifice is lower compared to the area of the orifice. This section is called as vena-contracta. The liquid expands from vena contracta due to which the loss of energy takes place.





Energy loss is given by

$$h_e = \frac{v^2.k}{2g}$$

Where,

$$k = \left[\frac{1}{C_c} - 1\right]^2$$

C_c is coefficient of contraction

if $C_c = 0.62,\ k = 0.375$

$C_c = 0.62$, then k=0.375

$$\therefore h_e = \frac{v^2.k}{2g} = 0.375\frac{v_2^2}{2g}$$

if C_c is not given

$$h_e = 0.5\frac{v_2^2}{2g}$$

3) **Loss of head at entrance of a pipe:**

$$h_i = 0.5\frac{v^2}{2g}$$

4) **Loss of head due to exit:**

$$h_o = \frac{v^2}{2g}$$

5) **Loss of Head due to an obstruction in a pipe:** Due to sudden enlargement of the area of flow beyond the obstruction, head loss takes place.
After vena contracta, sudden enlargement takes place.
a =max area of obstruction
A = area of pipe
V = velocity of liquid.

$$\text{Head loss} = \frac{v^2}{2g}\left[\frac{A}{c_c(A-a)} - 1\right]^2$$

6) **Loss of head due to bend in pipe:**

$$h_b = \frac{k.v^2}{2g}$$

The value of 'k' depends on

- Angle of bend
- Radius of curvature of bend
- Diameter of pipe

Example:

At a sudden enlargement of a water main from 240 mm to 480 mm dia the hydraulic gradient rises by 10 mm. Estimate the rate of flow.

Solution:

Hydraulic Gradient

$$\left(\frac{P_1}{\rho g}+z_1\right)-\left(\frac{P_2}{\rho g}+z_2\right)$$

$$\frac{P_1}{\rho g}+z_1+\frac{v_1^2}{2g}=\frac{P_2}{\rho g}+z_2+\frac{v_2^2}{2g}+\frac{(v_1-v_2)^2}{2g}$$

$$d_1^2 v_1 = d_2^2 v_2$$

$d_1 = 240mm$

$d_2 = 480mm$

$v_1=4v_2$

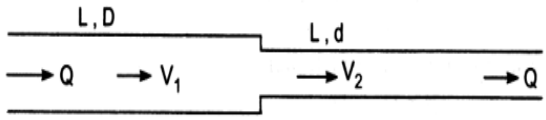
$v_2=0.1808=0.181m/s$

$Q=A_2\, v_2= 0.03275m^3/s = 32.75lit/s$

6.5 FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES

6.5.1 EQUIVALENT PIPE

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.



If no minor loss is assumed

$$H=Z_A-Z_B=h_{f_{total}}$$

$$H=\frac{4f_1L_1V_1^2}{2gd_1}+\frac{4f_2L_2V_2^2}{2gd_2}+\frac{4f_3L_3V_3^2}{2gd_3}$$

$$Q=\frac{\pi d^2}{4}.V_1=\frac{\pi d_2^2}{4}V_2=\frac{\pi d_3^2}{4}V_3$$

For friction coefficient,

$$H=\frac{4\times16fQ^2}{\pi^2.2g}\left[\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}\right]$$

$$H=\frac{4fLV_1^2}{2g\,d}=\frac{4\times16Q^2f}{\pi^2 2g}\left[\frac{L_{eq}}{d_{eq}^5}\right]$$

$$\frac{L_{eq}}{d_{eq}^5}=\left[\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}\right]$$

6.5.2 POWER TRANSMISSION THROUGH PIPES

Total head available at outlet of pipe

$=H-h_f$ (Minor loss neglected)

$$=H-\frac{4f\,lv^2}{d.2g}$$

Weight of water flowing

$$Q=\frac{\pi d^2}{4}.v$$

$$W=\rho gQ=\rho g\frac{\pi d^2}{4}.v$$

Power transmitted

$=W\times$ head available at outlet

$$P=\left(H-\frac{4f\,lv^2}{2gd}\right)\frac{\rho g\,\pi d^2}{4}v$$

6.6 FLOWS THROUGH NOZZLES

Total head at inlet of pipe

= total head energy + losses

But, total head at outlet of nozzle $=\frac{v^2}{2g}$

By Bernoulli's equation for real fluids Assuming minor loss to be negligible

$$H-h_f=\frac{v^2}{2g}$$

$$h_f=\frac{4f\,lv^2}{2gd}$$

By continuity equation

$AV=av$

$$H=\frac{v^2}{2g}+\frac{4f\,L}{2gD}\left(\frac{a}{A}v\right)^2$$

$$v=\sqrt{\frac{2gh}{\left(1+\frac{4f\,L}{D}\times\frac{a^2}{A^2}\right)}}$$

GATEFLIX

6.6.1 POWER TRANSMITTED THROUGH NOZZLE

K.E of the jet at the outlet of Nozzle $=\frac{1}{2}mv^2$

Mass flow rate $=\rho av$

$$K.E.=\frac{1}{2}\rho av^3$$

$$\eta=\frac{\text{power of outlet of Nozzle}}{\text{Power at the inlet of pipe}}=\frac{1/2\rho av^3}{\rho g\,Q.H}$$

Example:

An oil of viscosity $0.1Ns/m^2$ and relative density 0.9 is flowing through a circular pipe of diameter 50mm and of length 300m. The rate of flow of fluid through the pipe is 3.5 lit/s. Find the pressure drop in a length of 300m and also the shear stress at the pipe wall.

Solution:

Given:

Viscosity, $\mu=0.1Ns/m^2$

Relative density $=0.9$

density of oil $=0.9\times1000=900kg/m^3$

$D=50mm=.05m$

$L=300m$

$$Q=3.5liters/S=\frac{3.5}{1000}=.0035m^3/s$$

(i) Pressure drop

$$p_1-p_2=\frac{32\mu\bar{u}L}{D^2},\ \text{where}\ \bar{u}=\frac{Q}{Area}=\frac{.0035}{\frac{\pi}{4}D^2}=\frac{.0035}{\frac{\pi}{4}(.05)^2}=1.782m/s$$

The Reynolds number (R_e) is given by

$$R_e=\frac{\rho VD}{\mu}$$

Where

$\rho=900kg/m^3$, averagevelocity $=\bar{u}=1.782m/s$

$$\therefore\quad R_e=900\times\frac{1.782\times.05}{0.1}=801.9$$

As Reynolds number is less than 2000, the flow is viscous/laminar

$$\therefore p_1-p_2=\frac{32\times0.1\times1.782\times300}{0.05^2}=684288N/mm^2$$

$$=68.43N/m^2$$

ii) Shear Stress at the pipe wall (τ_0)

The shear stress at any radius r is given by the equation

i.e., $$\tau=\frac{-\delta p}{\delta x}\frac{r}{2}$$

$\therefore$ Shear stress at pipe wall, where r=R is given by

$$\tau=\frac{-\delta p}{\delta x}\frac{R}{2}$$

Now $$\frac{\delta p}{\delta x}=\frac{-(p_2-p_1)}{x_2-x_1}=\frac{p_1-p_2}{x_2-x_1}=\frac{p_1-p_2}{L}$$

$$=\frac{684288}{300}\frac{N/m^2}{m}=2280.96N/m^3$$

And $$R=\frac{D}{2}=\frac{.05}{2}=.025m$$

$$\tau_0=2280.96\times\frac{.025}{2}\frac{N}{m^2}=28.512N/m^2$$

Example:

A laminar flow is taking place in a pipe of diameter 200mm. The maximum velocity is 1.5m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4cm from the wall of the pipe.

Solution:

Given:

Dia of pipe, D=200mm=0.20m

$U_{max}=1.5m/s$

(i)**Mean velocity ,** $\bar{u}$

Ratio of

$$\frac{U_{max}}{\bar{u}}=2.0\ \text{or}\ \frac{1.5}{\bar{u}}=2.0\quad\therefore\bar{u}=\frac{1.5}{2.0}=0.75m/s$$

(ii)**Radius at which $\bar{u}$ occurs**

The velocity, u, at any radius 'r' is given by

$$u=-\frac{1}{4\mu}\frac{\delta p}{\delta x}\left[R^2-r^2\right]=-\frac{1}{4\mu}\frac{\delta p}{\delta \mu}R^2\left[1-\frac{r^2}{R^2}\right]$$

But from equation U_{max} is given by

$$U_{max}=-\frac{1}{4\mu}\frac{\delta p}{\delta x}R^2\qquad\therefore u=U_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\qquad .....(i)$$

Now the radius r at which

$u=\bar{u}=0.75m/s$

$$\therefore 0.75 = 1.5\left[1-\left(\frac{r}{D/2}\right)^2\right]$$

$$\therefore 0.75 = 1.5\left[1-\left(\frac{r}{0.1}\right)^2\right]$$

$$\therefore \frac{0.75}{1.5} = 1-\left(\frac{r}{0.1}\right)^2$$

$$\therefore \left(\frac{r}{0.1}\right)^2 = 1-\frac{0.75}{1.50} = 1-\frac{1}{2} = \frac{1}{2}$$

$$\therefore \frac{r}{0.1} = \sqrt{\frac{1}{2}} = \sqrt{0.5}$$

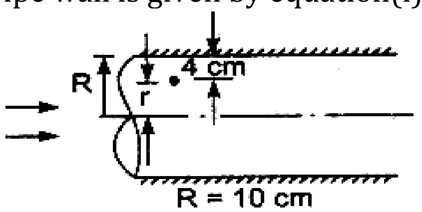
$$\therefore r = 0.1\times\sqrt{.5} = 0.1\times 707 = .0707\text{m}$$

$$= 70.7\text{mm}$$

(iii) **Velocity at 4cm from the wall**

$\therefore\ r = R-4.0 = 10-4.0 = 6.0\text{cm} = 0.06\text{m}$

The velocity at a radius =0.06m Or 4cm from pipe wall is given by equation(i)

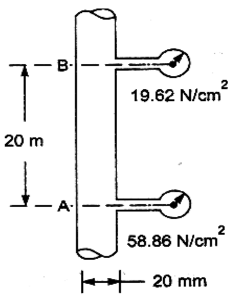


$$= U_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 1.5\left[1-\left(\frac{.06}{.1}\right)^2\right]$$

$$= 1.5[1.0-.36] = 1.5\times .64 = 0.96\text{m/s}$$

Example:

Crude oil of $\mu = 1.5$ poise and relative density 0.9 flows through a 20mm diameter vertical pipe. The pressure gauges fixed 20m apart read 58.86N/cm^{2}and 19.62N/cm^2 as shown in Fig. Find the direction and rate of flow through the pipe.



Solution:

Given:

$$\mu = 1.5\text{poise} = \frac{1.5}{10} = 0.15\text{Ns/m}^2$$

Relative density $= 0.9$

$\therefore$ Density of oil

$\therefore$ $= 0.9\times 1000 = 900\text{kg/m}^3$

Dia. Of pipe, $D = 20\text{mm} = 0.02\text{m}\ \ L = 20\text{m}$

$P_A = 58.86\text{N/cm}^2 = 58.86\times 10^4\ \text{N/m}^2$

$P_B = 19.62\text{N/cm}^2 = 19.62\times 10^4\ \text{N/m}^2$

Solution:

(i)Direction of flow.

To find the direction of flow, the total energy $\left(\frac{p}{\rho g}+\frac{v^2}{2g}+Z\right)$ at the lower end A and at the upper end B is to be calculated. The direction of flow will be given from the higher energy to the lower energy. As here the diameter of the pipe is same and hence kinetic energy at A and B will be same. Hence to find the direction of flow, calculating $\left(\frac{p}{\rho g}+Z\right)$ at A and B.

Taking the level at A as datum.

The value of $\left(\frac{p_A}{\rho g}+Z\right)$ at A

$$= \frac{6\times 10^4\times 9.81}{900\times 9.81}+0 = 66.67\text{m}$$

The value of $\left(\frac{p}{\rho g}+Z\right)$ at B

$$= \frac{2\times 10^4\times 9.81}{900\times 9.81}+20 = 22.22+20 = 42.22\text{m}$$

As the value of $\left(\frac{p}{\rho g}+Z\right)$ is higher at A. Hence, flow takes place from A to B.

(ii) Rate of flow. The loss of pressure head for viscous flow through circular pipe is given by

$$h_f = \frac{32\mu \bar{u} L}{\rho g D^2}$$

For a vertical pipe $h_f =$ Loss of peizometric head

$$= \left(\frac{p_A}{\rho g} + Z_A\right) - \left(\frac{p_B}{\rho g} + Z_B\right) = 66.67 - 42.22 = 24.45\text{m}$$

$$\therefore \quad 24.45 = \frac{32 \times 0.15 \times \bar{u} \times 20.0}{900 \times 9.81 \times (.02)^2}$$

Or

$$\bar{u} = \frac{24.45 \times 900 \times 9.81 \times .0004}{32 \times 0.15 \times 20.0} = 0.889\ ;\ 0.9\text{m/s}$$

The Reynolds number should be calculated. If Reynolds number is less than 2000, the flow will be laminar and the above expression for loss of pressure head for laminar flow can be used.

Now Reynolds number $= \frac{\rho VD}{\mu}$

$$= 900 \times \frac{0.9 \times .02}{0.15} = 108$$

As Reynolds number is less than 2000, the flow is laminar.

$\therefore$ Rate of flow = avg. velocity * area

$$= \bar{u} \times \frac{\pi}{4} D^2 = 0.9 \times \frac{\pi}{4} \times (.02)^2\ \text{m}^3/\text{s} = 2.827 \times 10^{-4}\ \text{m}^3/\text{s}$$

$$= 0.2827\ \text{litres/s}$$

Example:

A shaft having a diameter of 50mm rotates centrally in a journal bearing having a diameter of 50.15mm and length 100mm. The annular space between the shaft and the bearing is filled with oil having viscosity of 0.9 poise. Determine the power absorbed in the bearing when the speed of rotation is 60 rpm.

Solution:

Given:

Dia. of shaft, $D = 50\text{mm}$ or $.05\text{m}$

Dia of bearing $D_1 = 50.15\text{mm}$ or 0.05015m

Length , $L = 100\text{mm}$ or 0.1m

$$\mu \text{ of oil} = 0.9 \text{ poise} = \frac{0.9}{10} \frac{\text{Ns}}{\text{m}^2}$$

$N = 600\text{r.p.m.}$

Power = ?

$\therefore$ Thickness of oil film, $t = \frac{D_1 - D}{2} = \frac{50.15 - 50}{2}$

$$= \frac{0.15}{2} = 0.075\text{mm} = 0.075 \times 10^{-3}\ \text{m}$$

Tangential speed of shaft,

$$V = \frac{\pi DN}{60} = \frac{\pi \times 0.05 \times 600}{60} = 0.5 \times \pi\ \text{m/s}$$

$\therefore$ Shear stress

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{t} = \frac{0.9}{10} \times \frac{0.5 \times \pi}{0.075 \times 10^{-3}} = 1883.52\text{N/m}^2$$

$\therefore$ Shear force (F)

$$= \tau \times \text{Area} = 1883.52 \times \pi D \times L$$

$$= 1883.52 \times \pi \times .05 \times 0.1 = 29.586\text{N}$$

Resistance torque

$$T = F \times \frac{D}{2} = 29.586 \times \frac{.05}{2} = 0.7387\text{Nm}$$

$$\text{Power, } P = T \times \omega = T \times \frac{2\pi N}{60} = \frac{2\pi NT}{60} \text{watts}$$

$$\text{Power} = \frac{2\pi NT}{60} = \frac{2\pi \times 600 \times 0.7387}{60} = 46.41\text{W}.$$

Example:

An oil of S.G. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 liters/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000m. Take $\nu = .29$ stokes.

Solution:

Given:

S.G. of oil $S = 0.7$

Dia. of pipe $d = 300\text{mm} = 0.3\text{m}$

Discharge $Q = 500\ \text{litres/s} = 0.5\text{m}^3/\text{s}$

Length of pipe $L = 1000\text{m}$

Velocity,

$$V = \frac{Q}{\text{Area}} = \frac{0.5}{\frac{\pi}{4} d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073\text{m/s}$$

Reynolds number,

$$Re = \frac{vd}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times 10^4$$

$\therefore$ Co-efficient of friction,

$$f=\frac{0.79}{R_e^{1/4}}=\frac{0.79}{\left(7.316\times10^4\right)^{1/4}}=0.0048$$

$\therefore$ Head lost due to friction,

$$h_f=\frac{4\times f\times L\times V^2}{d\times 2g}=\frac{4\times0.0048\times1000\times7.073^2}{0.3\times2\times9.81}=163.18m$$

Power required

$$=\frac{\rho g\,.\,Q\,.\,h_f}{1000}kW$$

Where

$\rho=$ density of oil $=0.7\times1000=700kg/m^3$

$\therefore$ Power required

$$=\frac{700\times9.81\times0.5\times163.18}{1000}=560.28kW$$

Example:

Water is flowing through a pipe of diameter 200mm with a velocity of 3m/s. Find the head lost due to friction for a length of 5m if the co-efficient of friction is given by $f=0.02+\frac{.09}{R_e^{\,0.3}}$ where R_e is Reynolds number. The kinematic viscosity of water = 0.1 Stokes.

Solution:

Given:

Dia. of pipe, $d=200mm=0.20m$

Velocity, $V=3m/s$

Length, $L=5m$

Kinematics viscosity,

$v=0.01stoke=.01\times10^{-4}m^2/s$

Reynolds number,

$$Re=\frac{vd}{\nu}=\frac{3\times0.2}{0.01\times10^{-4}}=6\times10^5$$

Value of f,

$$f=0.02+\frac{0.09}{Re^{0.3}}=0.02+\frac{0.09}{\left(6\times10^5\right)^{0.3}}$$

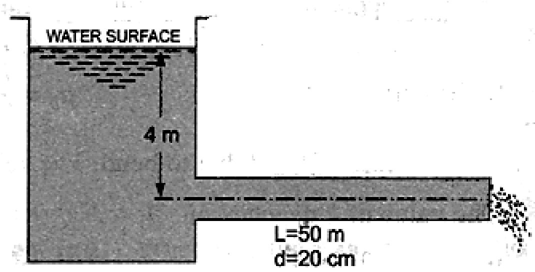
$$=0.02+\frac{0.09}{54.13}=0.02166$$

Head lost due to friction:

$$h_f=\frac{4\times f\times L\times V^2}{d\times 2g}=\frac{4.0\times0.02166\times5.0\times3^2}{0.20\times2.0\times9.81}=0.993\text{ m of water}$$

Example:

Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take $f=.099$ in the formula $h_f=\frac{4\times f\times L\times V^2}{d\times 2g}$.



Solution:

Given

Dia. of pipe $d=20cm=0.20m$

Length of pipe, $L=50m$

Height of water $H=4m$

Co-efficient of friction $f=.009$

Let the velocity of water in pipe $=V\ m/s$

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have[Taking point 1 on the top and 2 at the outlet of pipe].

$$\frac{P_1}{\rho g}+\frac{V_1^{\,2}}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^{\,2}}{2g}+z_2+\text{all losses}$$

Consider datum line passing through the centre of pipe

$$0+0+4.0=0+\frac{V_2^{\,2}}{2g}+0+(h_i+h_f)$$

Or $$4.0=\frac{V_2^{\,2}}{2g}+h_i+h_f$$

But the velocity in pipe= V ,

$\therefore V=V_2$

$$4.0=\frac{V^2}{2g}+h_i+h_f$$

From equation

$$h_i=0.5\frac{V^2}{2g}\ \text{ and }\ h_f=\frac{4\times f\times L\times V^2}{d\times 2g}$$

Substituting these values, we have

$$4.0=\frac{V^2}{2g}+0.5\frac{V^2}{2g}+\frac{4\times f\times L\times V^2}{d\times 2g}$$

$$\therefore 4=\frac{v^2}{2g}\left[1.0+0.5+\frac{4\times 0.009\times 50}{0.2}\right]$$

$$\therefore 4=\frac{v^2}{2g}\left[1.0+0.5+9.0\right]$$

$$\therefore 4=10.5\frac{v^2}{2g}$$

$$\therefore\ V=\sqrt{\frac{4\times 2\times 9.81}{10.5}}=2.734\ m/sec$$

$\therefore$ Rate of flow,

$$Q=A\times V=\frac{\pi}{4}\times(0.2)^2\times 2.734=0.08589\ m^3/s$$

$$=85.89\ litres/s.$$

Example:
A syphon of diameter 200 mm connects two reservoirs having a difference in elevation of 15m. The total length of the syphon is 600m and the summit is 4 m above the water level in the upper reservoir. If the separation takes place at 2.8m of water absolute, find the maximum length of syphon from upper reservoir to the summit. Take f=0.004 and atmospheric pressure$=10.3m$ of water.

Solution:
Given

Dia. of syphon $d=200m=0.2m$

Difference of level in two reservoirs $=15m$

Total length of pipe $=600m$

Height of summit from upper reservoirs $=4m$

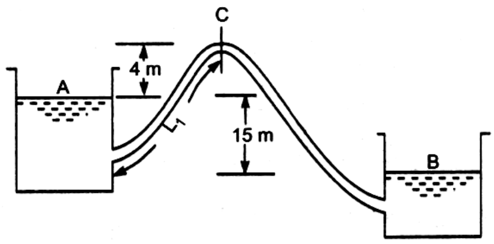
Pressure head at summit

$$\frac{p_c}{\rho g}=2.8m \text{ of water absolute}$$

Atmospheric pressure head

$$\frac{p_c}{\rho g}=10.3m \text{ of water absolute}$$

Co-efficient of friction, $f=.004$



Applying Bernoulli's equation to points A and C and taking the datum line passing through, A and C

$$\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+z_A=\frac{p_C}{\rho g}+\frac{V_C^2}{2g}+z_C\ +\text{ Losses of head}$$

due to friction between A and C

Substituting the values of pressures in terms of absolute, we have

$$10.3+0+0=2.8+\frac{V^2}{2g}+4.0+h_{f1}$$

$$[\because V_C=\text{velocity in pipe}=V]$$

$$\therefore\ h_{f1}=10.3-2.8-4.0-\frac{V^2}{2g}=3.5-\frac{V^2}{2g}\ .....(i)$$

Applying Bernoulli's equation to points A and B and taking the datum line passing through B,

$$\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+z_A=\frac{p_B}{\rho g}+\frac{V_B^2}{2g}+z_B\ +\text{ Losses of head}$$

due to friction between A and B

But , $\frac{p_A}{\rho g}=\frac{p_B}{\rho g}=\text{atmospheric pressure}$

And, $V_A=0,\ V_B=0,\ z_A=15,\ z_B=0$

$$\therefore\ 0+0+15=0+0+0+h_f$$

$$h_f=15 \text{ or } \frac{4\times f\times L\times V^2}{d\times 2g}=15$$

Or $\frac{4\times .004\times 600\times V^2}{0.2\times 2\times 9.81}=15$

Or $V=\sqrt{\frac{15\times 0.2\times 2\times 9.81}{4\times .004\times 600}}=2.47m/s$

Substituting this value of V in equation (i) we get

$$h_{f1}=3.5-\frac{2.47^2}{2\times 9.81}=3.5-0.311=3.189m\ .....(ii)$$

But

$$h_{f2}=\frac{4\times f\times L_1\times V^2}{d\times 2g}\qquad ....(iii)$$

Where L_1 =inlet leg of syphon or length of syphon from upper reservoir to the summit.

$$h_{f1} = \frac{4 \times .004 \times L_1 \times (2.47)^2}{0.2 \times 2 \times 9.81} = 0.0248 \times L_1$$

Substituting this value in equation (ii),

$$0.0248 L_1 = 3.189$$

$$\therefore L_1 = \frac{3.189}{0.0248} = 128.58\text{m}$$

Example:

Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400m, 200m 300m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water levels is 16m. If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then considering them.

Solution:

Given:

Difference of water levels, $H = 16\text{m}$

Length & dia. of pipe1,

L1 = 400m and d1 = 400 mm = 0.4 m

Length & dia of pipe 2

L_2 = 200m and d_2 = 200m = 0.2m

Length & dia of pipe 3

L_3 = 200m and d_3 = 300m = 0.3m

Also, $f_1 = f_2 = f_3 = 0.005$

(i) Discharge through the compound pipe first neglecting minor losses.

Let V_1, V_2, V_3 are the velocities in the 1^{st} 2^{nd} and 3^{rd} pipe respectively.

From continuity, we have

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$\therefore V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^{\ 2}}{\frac{\pi}{4}d_2^{\ 2}} \times V_1 = \frac{d_1^{\ 2}}{d_2^{\ 2}} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

And

$$V_3 = \frac{A_1V_1}{A_3} = \frac{\frac{\pi}{4}d_1^{\ 2}}{\frac{\pi}{4}d_3^{\ 2}} \times V_1 = \frac{d_1^{\ 2}}{d_3^{\ 2}} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 1.77V_1$$

Now using equation of

$$H = \frac{4 \times f_1 \times L_1 \times V_1^{\ 2}}{d_1 \times 2g} + \frac{4 \times f_2 \times L_2 \times V_2^{\ 2}}{d_2 \times 2g} + \frac{4 \times f_3 \times L_3 \times V_3^{\ 2}}{d_3 \times 2g}$$

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81}$$

$$+ \frac{4 \times 0.005 \times 00 \times (1.77V_1)^2}{0.3 \times 2 \times 9.81}$$

$$16 = \frac{V_1^{\ 2}}{2 \times 9.81}(20 + 320 + 63.14) = \frac{V_1^{\ 2}}{2 \times 9.81} \times 403.14$$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882\text{m/s}$$

$\therefore$ Discharge,

$$Q = A_1 \times V_1 = \frac{\pi}{4}(0.4)^2 \times 0.882 = 0.1108\ \text{m}^3/\text{s}$$

(ii) Discharge through the compound pipe considering minor losses also

Minor losses are:

(a) At inlet, $h_i = \frac{0.5V_1^2}{2g}$

(b) Between 1st and 2nd pipe, due to contraction,

$$h_C = \frac{0.5V_2^{\ 2}}{2g} = \frac{0.5(4V_1)^2}{2g} \qquad (\because V_2 = 4V_1)$$

$$= \frac{0.5 \times 16 \times V_1^{\ 2}}{2g} = 8 \times \frac{V_1^{\ 2}}{2g}$$

(c) Between 2nd and 3rd pipe, due to sudden enlargement,

$$h_i = \frac{0.5V_1^2}{2g}$$

$$h_e = \frac{(V_2 - V_3)^2}{2g} = \frac{(4V_1 - 1.77V_1)^2}{2g} \qquad (\because V_3 = 1.77V_1)$$

$$= (2.23)^2 \times \frac{V_1^{\ 2}}{2g} = 4.973\frac{V_1^{\ 2}}{2g}$$

(d)At the outlet of 3rd pipe,

$$h_o = \frac{V_3^{\ 2}}{2g} = \frac{(1.77V_1)2}{2g} = 1.77^2 \times \frac{V_1^{\ 2}}{2g} = 3.1329\frac{V_1^{\ 2}}{2g}$$

The major losses are

$$= \frac{4 \times f_1 \times L_1 \times V_1^2}{d_1 \times 2g} + \frac{4 \times f_2 \times L_2 \times V_2^2}{d_2 \times 2g} + \frac{4 \times f_3 \times L_3 \times V_3^2}{d_3 \times 2g}$$

$$= \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81}$$

$$+ \frac{4 \times 0.005 \times 300 \times (1.77V_1)^2}{0.3 \times 2 \times 9.81}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

∴ Sum of minor losses and major losses

$$= \left[\frac{0.5V_1^2}{2g} + 8 \times \frac{V_1^2}{2g} + 4.973\frac{V_1^2}{2g} + 3.1329\frac{V_1^2}{2g}\right] + 403.14\frac{V_1^2}{2g}$$

$$= 419.746\frac{V_1^2}{2g}$$

But total loss must equal to H (or 16m)

$$\therefore \quad 419.746 \times \frac{V_1^2}{2g} = 16$$

$$\therefore V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.746}} = 0.864 m/s$$

∴ Discharge

$$Q = A_1 V_1 = \frac{p}{4} \times (0.4)^2 \times 0.864 = 0.1085 m^3/s$$

Example:

Two pipes have a length L each. One of them has diameter D, and the other has diameter d. If the pipes are arranged in parallel, the loss of head, when a total quantity of water Q flows through them is h, but, if the pipes are arranged in series and the same quantity Q flows through them, the loss of head is H. If $d = \frac{D}{2}$, find the ratio of H to h, neglecting secondary losses and assuming the pipe co-efficient has a constant value.

Solution:

Given:

Length & dia. of pipe1

$L_1 = L$ and $d_1 = D$

Length & dia. of pipe2

$L_2 = L$ and $d_2 = d$

Total Discharge =Q

Head loss when pipes are arranged in parallel = h

Head loss when pipes are arranged in series = H

$d = \frac{D}{2}$ and f is constant

1st case:

When pipes are connected parallel

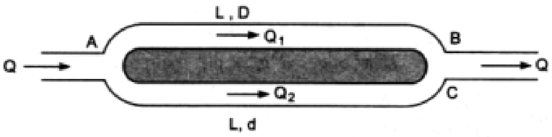
$Q = Q_1 + Q_2$ ...(i)

Loss od head in each pipe =h

For pipe AB, $\frac{4fL_1V_1^2}{d_1 \times 2g} = h,$

$$\text{where } V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\frac{\pi}{4}D^2} = \frac{4Q_1}{\pi D^2}$$

$d_1 = D$



$$\therefore \quad \frac{4fL\left(\frac{4Q_1}{\pi D^2}\right)^2}{D \times 2g} = h \text{ or } \frac{32fLQ_1^2}{\pi^2 D^5 \times g} = h \quad .....(ii)$$

For pipe AC, $\frac{32fLQ_2^2}{\pi^2 d^5 \times g} = h$

$$\therefore \quad \frac{32fLQ_1^2}{\pi^2 D^5 \times g} = \frac{32fLQ_2^2}{\pi^2 d^5 \times g} \text{ or } \frac{Q_1^5}{D^5} = \frac{Q_2^5}{d^5}$$

$$\text{or } \left(\frac{Q_1}{Q_2}\right)^2 = \frac{D^5}{d^5} = \frac{(2d)^5}{d^5} = 2^5 = 32 \qquad [\therefore D = 2d]$$

$$\therefore \quad \frac{Q_1}{Q_2} = \sqrt{32} = 5.657 \text{ or } Q_1 = 5.657Q_2$$

Substituting the value of Q1 in equation (i), we get

$Q = 5.657Q_2 + Q_2 = 6.657Q_2$

$$\therefore Q_2 = \frac{Q}{6.657} = 0.15Q \quad .....(iv)$$

From (i)

$$\therefore \quad Q_1 = Q - Q_2 = Q - 0.15Q = 0.85Q \quad .....(v)$$

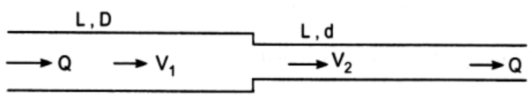
2nd case:

When pipes are connected in series

Total loss = sum of head losses on two pipes

$$H = \frac{4f.L.V_1^2}{d_1 \times 2g} + \frac{4f.L.V_2^2}{d_2 \times 2g}$$

where $V_1 = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2}$, $V_2 = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$



$$\therefore \quad H = \frac{4f.L.\left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} + \frac{4f.L.\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g}$$

$$\text{or} \quad H = \frac{32fLQ^2}{D^5 \pi^2 \times g} + \frac{32fLQ^2}{d^5 \pi^2 \times g} \qquad \text{....(vi)}$$

From equation (ii), $\frac{32fl}{\pi^2 D^5 \times g} = \frac{h}{Q_1^2}$

and frome quation (iii), $\frac{32fl}{\pi^2 d^5 \times g} = \frac{h}{Q_2^2}$

Substituting these values in equation (vi), where

$$H = Q^2 \times \frac{h}{Q_1^2} + Q^2 \times \frac{h}{Q_2^2} = \frac{Q^2}{Q_1^2}h + \frac{Q^2}{Q_2^2}h = h\left[\frac{Q^2}{Q_1^2} + \frac{Q^2}{Q_2^2}\right]$$

$$\frac{H}{h} = \frac{Q^2}{Q_1^2} + \frac{Q^2}{Q_2^2}$$

But from equations (iv) and (v)

$Q_1 = .85Q$ and $Q_2 = 0.15Q$

$$\frac{H}{h} = \frac{Q^2}{.85^2 Q^2} + \frac{Q^2}{.15^2 Q^2} = \frac{1}{.85^2} + \frac{1}{.15^2}$$

$$= 1.384 + 44.444 = 45.828$$

GATE QUESTIONS

Q.1 Applying a pressure drop across a capillary results in a volumetric flow rate Q under laminar flow conditions. The flow rate for the same pressure drop in a capillary of the same length but half the radius is

[GATE–2001]

(A) Q/2 (B) Q/4

(C) Q/8 (D) Q /16

Q.2 For turbulent flow of an incompressible fluid through a pipe, the flow rate Q is proportional to $(\Delta P)^n$, where ΔP is the pressure drop. The value of exponent n is

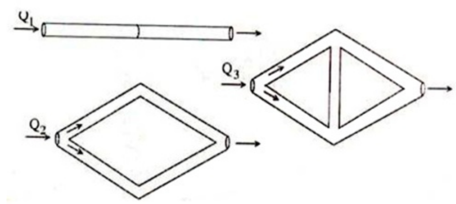
[GATE–2002]

(A) 1 (B) 0

(C) < 1 (D) > 1

Q.3 Three piping networks, as shown in the figure, are placed horizontally. They are made using identical pipe segments and are subjected to the same pressure drop across them. Assuming no pressure losses at junctions, the flow rates across the three networks are related as Q_1: Q_2: Q_3.

[GATE –2004]

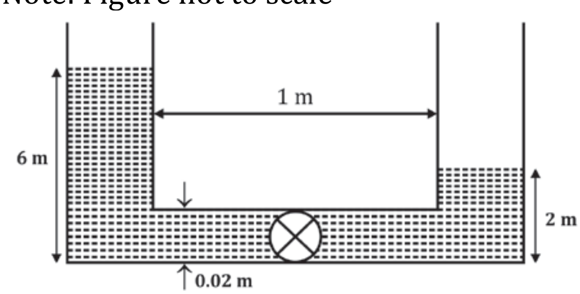


(A) $1:\sqrt{3}:2$ (B) 1:2:3

(C) 1:2:2 (D) $1:\sqrt{2}:\sqrt{2}$

Common Data for Q.4 and Q.5

Two tanks, A and B, of cross sectional area 1 m^2 each, contain a fluid of density 1000 kg/m^3 and viscosity 1 kg/(m.s). The tanks are connected by a pipe of diameter 0.02 m and length 1 m, and a check valve, at the bottom. Assume that the flow is laminar, and there is no friction in the check valve. In the initial state, the height of the fluid in the tank A and 6m and the height of the fluid in tank B is 2 m (as shown in the figure below). The check valve is opened, and the fluid flows from tank A to tank B till the levels in the two tanks are equal in the final state. Assume g=10 m/s^2 in the calculations.
Note: Figure not to scale



Q.4 What is the average fluid velocity in the pipe as soon as the valve is opened?

[GATE –2005]

(A)0.25 m/s (B)0.5 m/s
(C)1 m/s (D)2 m/s

Q.5 What is the total energy loss between the initial and final states due to the fluid flow?

[GATE –2005]

(A) 2×10^4 J (B) 16×10^4 J

(C) 8×10^4 J (D) 4×10^4 J

Q.6 A liquid is pumped at the flow rate Q through a pipe of length L. The pressure drop of the fluid across the pipe is ΔP, now a leak develops at the mid-point of the length of the pipe and the fluid leaks at the rate of Q/2. Assuming that the friction factor in the pipe remains unchanged, the new pressure drop across the pipe for the same inlet flow rate (Q) will be

[GATE –2006]

(A) $\left(\frac{1}{2}\right)\Delta P$ (B) $\left(\frac{5}{8}\right)\Delta P$

(C) $\left(\frac{3}{4}\right)\Delta P$ (D) ΔP

Q.7 In a laminar flow through a pipe of radius R, the fraction of the total fluid flowing through a circular cross-section of radius R/2 centered at the pipe axis is

(A) 3/8 (B) 7/16
(C) 1/2 (D) 3/4

[GATE –2006]

Q.8 Losses for flow through valves and fittings are expressed in terms of

[GATE –2008]

(A) Drag coefficient
(B) Equivalent length of a straight pipe
(C) Shape Factor
(D) Roughness Factor

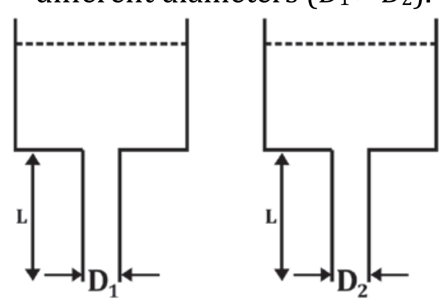
Q.9 Given a pipe of diameter D, the entrance length necessary to achieve fully developed laminar flow is proportional to (N_{Re} is Reynolds Number)

[GATE –2008]

(A) $D\,N_{Re}$ (B) $\frac{D}{N_{Re}}$

(C) $D\,N_{Re}{}^2$ (D) $\frac{D}{N_{Re}{}^2}$

Q.10 Two identical reservoirs, open at the top, are drained through pipes attached to the bottom of the tanks as shown below. The two drain pipes are of the same length, but of different diameters ($D_1 > D_2$).



Assuming the flow to be steady and laminar in both drain pipes, if the volumetric flow rate in the larger pipe is 16 times of that in the smaller pipe, the ratio D_1/D_2 is

[GATE –2009]

(A) 2 (B) 4

(C) 8 (D) 16

Q.11 In Hagen-Poiseuille flow through a cylindrical tube, the radial profile of shear stress is

[GATE –2010]

(A) Constant

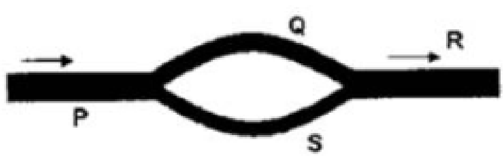
(B) Cubic

(C) Parabolic

(D) Linear

Q.12 A liquid is flowing through the following piping network. The length of pipe sections P, Q, R and S shown in the schematic are equal. The diameters of the sections P and R are equal and the diameter of the section Q is twice that of S. The flow is steady and laminar. Neglecting curvature and entrance effects, the ratio of the volumetric flow rate in the pipe section Q to that in S is

[GATE–2011]



(A) 16 (B) 8

(C) 2 (D) 1

Q.13 Water is flowing under laminar conditions in a pipe of length L. If the diameter of the pipe is doubled, for a constant volumetric flow rate, the pressure drop across the pipe

[GATE–2012]

(A) Decreases 2 times

(B) Decreases 16 times

(C) Increases 2 times

(D) Increases 16 times

Q.14 For uniform laminar flow (in the x-direction) past a flat plate at high Reynolds number, the local boundary layer thickness (δ) varies with the distance along the plate (x) as

[GATE–2012]

(A) $\delta \propto x^{\frac{1}{4}}$ (B) $\delta \propto x^{\frac{1}{3}}$

(C) $\delta \propto x^{\frac{1}{2}}$ (D) $\delta \propto x$

Q.15 For a Newtonian fluid flowing in a circular pipe under steady state conditions in fully developed laminar flow, the Fanning friction factor is

[GATE–2013]

(A) $0.046\ Re^{-0.2}$

(B) $0.0014 + \frac{0.125}{Re^{0.32}}$

(C) $\frac{16}{Re}$

(D) $\frac{24}{Re}$

Q.16 In case of a pressure driven laminar flow of a Newtonian fluid of viscosity (μ) through a horizontal circular pipe, the velocity of the fluid is proportional to

[GATE–2014]

(A) μ (B) $\mu^{0.5}$

(C) μ^{-1} (D) $\mu^{-0.5}$

Q.17 Two different liquids are flowing through different pipes of the same diameter. In the first pipe, the flow is laminar with centerline velocity $V_{max,1}$, whereas in the second pipe, the flow is turbulent. For turbulent flow, the average velocity is 0.82 times the centerline velocity, $V_{max,2}$. For equal volumetric flow rates in both the pipes, the ratio $V_{max,1} / V_{max,2}$ (up to two decimal places) is

[GATE–2015]

Q.18 For uniform laminar flow over a flat plate, the thickness of the boundary layer δ, at a distance x from the leading edge of the plate follows the relation

[GATE–2015]

(A) $\delta(x) \propto x^{-1}$

(B) $\delta(x) \propto x$

(C) $\delta(x) \propto x^{1/2}$

(D) $\delta(x) \propto x^{-1/2}$

Q.19 For Fanning friction factor f (for flow in pipes) and drag coefficient C_D (for flow over immersed bodies) which of the following statements are true?

P. f accounts only for the skin friction.

Q. C_D accounts only for the form friction.

R. C_D accounts for both skin friction and form friction.

S. Both f and C_D depend on the Reynolds number.

T. For laminar flow through a pipe f doubles on doubling the volumetric flow rate.

[GATE–2015]

(A) R, S, T (B) P, Q, S

(C) P, R, S (D) P, Q, S, T

Q.20 For a flow through a smooth pipe, the Fanning friction factor (f) is given by $f = mRe^{-0.2}$ in the turbulent flow regime, where Re is the Reynolds number and m is a constant. Water flowing through a section of this pipe with a velocity 1 m/s results in a frictional pressure drop of 10 kPa. What will be the pressure drop across this section (in kPa), when the velocity of water is 2 m/s?

[GATE–2016]

(A) 11.5 (B) 20

(C) 34.8 (D) 40

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
D	C	C	B	D	B	B	B	A	A	D	A	B	C
15	16	17	18	19	20								
C	C	1.64	C	C	C								

EXPLANATIONS

Q.1 (D)

In laminar flow, Hagen Poiseulle Equation is used

$$\Delta P = \frac{32\mu L v}{D^2} = \frac{8\mu L v}{R^2}$$

$$Q = AV = (\pi R^2)\left(\frac{\Delta P R^2}{8\mu L}\right)$$

$$Q \propto R^4$$

$$\frac{Q_1}{Q_2} = \left(\frac{R_1}{R_2}\right)^4 = 2^4$$

$$Q_2 = \frac{Q_1}{16}$$

So Option (d) is correct

Q.2 (C)

Pressure drop is given by

$$\Delta P = \frac{2 f L v^2}{D}$$

f for turbulent flow = 0.0014

$$v^2 = \frac{Q^2}{A^2}$$

$$\Delta P = \frac{2 f L}{D}\left(\frac{Q^2}{A^2}\right)$$

$$\Delta P \propto Q^2$$

$$Q \propto \Delta P^{0.5}$$

So, n = 0.5 < 1

So Option (c) is correct

Q.3 (C)

Pressure Drop in pipe is given by

$$\Delta P = \frac{4 f L V^2}{2D}$$

So, $\Delta P = K (L Q^2)$

For Case-I

$$\Delta P_1 = K (2L Q_1^2)$$

$$Q_1 = \frac{1}{\sqrt{2}}\left(\frac{\Delta P_1}{V}\right)^{\frac{1}{2}} \quad \text{.........(1)}$$

$$\Delta P_2 = K\left[2L\left(\frac{Q_2}{2}\right)^2\right]$$

$$Q_2 = \sqrt{2}\left(\frac{\Delta P_2}{V}\right)^{\frac{1}{2}}$$

Similarly Q_3

$$Q_3 = \sqrt{2}\left(\frac{\Delta P_3}{V}\right)^{\frac{1}{2}}$$

$$Q_1 : Q_2 : Q_3 = \frac{1}{\sqrt{2}} : \sqrt{2} : \sqrt{2}$$

$$= 1 : 2 : 2$$

So Option (b) is correct

Q.4 (B)

In laminar flow, Hagen poiseuille equation

$$\Delta P = \frac{32\mu L v}{D^2} \Rightarrow v = \frac{\Delta P\, D^2}{32\mu L}$$

$$\Delta P = (6-2)\,1000 \times 10$$

$$= 4 \times 10^4 \text{ Pa}$$

$$v = \frac{(4\times10^4)(0.02)^2}{32\times1\times1} = 0.05\frac{\text{m}}{\text{s}}$$

So Option (b) is correct

Q.5 (D)

$$h_{fs} = \frac{\Delta P}{\rho} = \frac{(6-4)\,g\,\rho}{\rho}$$

$$= 2\times10\ \frac{\text{m}^2}{\text{s}^2}\left(\begin{array}{l}\text{Energy loss per}\\ \text{unit mass}\end{array}\right)$$

Total mass flow $= \rho\, v = 1000\times2\times1$

$= 2000$ kg

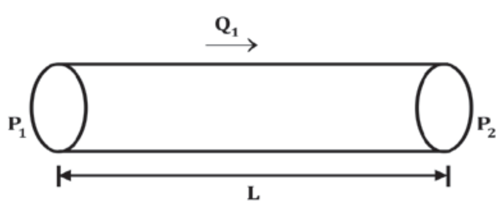
Total Energy loss $= 2000\times20$

$= 4\times10^4$ J

So, Option (d) is correct Option

Q.6 (B)

CASE-1



Pressure drop through pipe

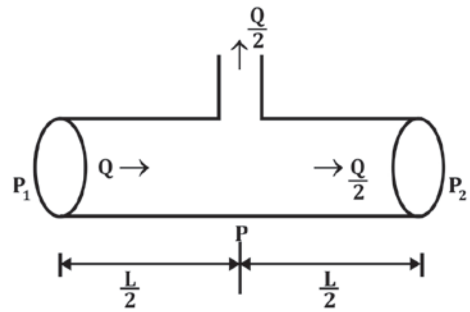
$$\Delta P = \frac{2f\,L\,u^2\rho}{D}$$

but, $u = \frac{Q}{A}$

So, $\Delta P = \frac{2f\,L\,Q^2\rho}{D\,A^2}$

$$\Delta P_1 = K\left(L\,Q^2\right)$$

CASE – 2



$$P_1 - P = K\left(\frac{L}{2}Q^2\right)\text{......................(1)}$$

$$P - P_2 = K\left(\frac{L}{2}\left(\frac{Q}{2}\right)^2\right)\text{................(2)}$$

$(1)+(2)$

$$P_1 - P_2 = K\left(\frac{L}{2}Q^2 + \frac{L}{8}Q^2\right)$$

$$\Delta P_2 = K\frac{5}{8}L\,Q^2 = \frac{5}{8}\Delta P_1$$

So Option (b) is correct

Q.7 (B)

Fraction of fluid flowing through a circular cross section of radius $\frac{R}{2}$

$$= \frac{\int_0^{R/2} u\,(2\pi r)\,dr}{\int_0^{R} u\,(2\pi r)\,dr} = \frac{\int_0^{R/2} u_{max}\left[1-\left(\frac{r}{R}\right)^2\right](2\pi r)\,dr}{\int_0^{R} u_{max}\left[1-\left(\frac{r}{R}\right)^2\right](2\pi r)\,dr}$$

$$= \frac{\int_0^{R/2}\left[r-\frac{r^3}{R^2}\right]dr}{\int_0^{R}\left[r-\frac{r^3}{R^2}\right]dr} = \frac{\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^{R/2}}{\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^{R}}$$

$$= \frac{\frac{R^2}{8} - \frac{R^2}{64}}{\frac{R^2}{2} - \frac{R^2}{4}} = \frac{7}{16}$$

So, Option (b) is correct

Q.8 (B)

Q.9 (A)

Transition Length for laminar flow is given by

$$\frac{x_t}{D} = 0.05\ N_{Re}$$

$$x_t \propto D\ N_{Re}$$

So Option (a) is correct

Q.10 (A)

For laminar flow Pressure drop is given by Hagen Poiseulle Equation

$$\Delta P = \frac{32\ \mu\ L\ V}{D^2}$$

$$\Delta P = \frac{128\ \mu\ L\ Q}{\pi\ D^4}$$

$$Q \propto D^4$$

$$\frac{Q_1}{Q_2} = 16 = \left(\frac{D_1}{D_2}\right)^2$$

$$\frac{D_1}{D_2} = 2$$

So option (a) is correct

Q.11 (D)

$$\Delta P = \frac{32\ \mu\ L\ V}{D^2}$$

$$\tau = \frac{\Delta P}{2L} r$$

$$\tau \propto r$$

So Option (d) is correct

Q.12 (A)

In laminar flow, Hagen Poiseulle Equation is used

$$v = \frac{\Delta P\ D^2}{32 \mu\ L}$$

$$q = \left(\frac{\pi}{4} D^2\right) v$$

$$q = \frac{\pi\ \Delta P\ D^2}{128\ \mu\ L}$$

$$(\Delta P)_Q = (\Delta P)_S$$

$$\frac{q_Q}{q_S} = \left(\frac{D_Q}{D_S}\right)^4 = 2^4 = 16$$

So Option (a) is correct

Q.13 (B)

In laminar flow, Hagen Poiseulle Equation is used

$$\Delta P = \frac{32 \mu\ L\ v}{D^2}$$

$$\Delta P = \frac{128\ \mu\ L\ Q}{D^4\ \pi}$$

$$\Delta P \propto \frac{1}{D^4}$$

$$\frac{\Delta P_2}{\Delta P_1} = \left(\frac{D_1}{D_2}\right)^4 = \frac{1}{16}$$

$$16\ \Delta P_2 = \Delta P_1$$

So Option (b) is correct

Q.14 (C)

For Boundary layer thickness across flat plate

$$\delta = 5\sqrt{\frac{\mu\ x}{\rho\ V_\infty}}$$

$$\delta \propto x^{\frac{1}{2}}$$

So Option (c) is correct

Q.15 (C)

Q.16 (C)

In laminar flow, Hagen Poiseulle Equation is used

$$\Delta P = \frac{32\,\mu\, L\, v}{D^2}$$

$$v \propto \frac{1}{\mu}$$

So Option (c) is correct

Q.17 (1.64)

For Laminar Flow

$$V_{avg,1} = 0.5\ V_{max,1}$$

For Turbulent Flow

$$V_{avg,2} = 0.82\ V_{max,2}$$

For same Flow rate

$$Q_1 = Q_2$$

$$\left(\frac{\pi}{4} D^2\right) V_{avg,1} = \left(\frac{\pi}{4} D^2\right) V_{avg,2}$$

$$0.5\ V_{max,1} = 0.82\ V_{max,2}$$

$$\frac{V_{max,2}}{V_{max,1}} = \frac{0.82}{0.5} = 1.64$$

Q.18 (C)

Blasius Equation

$$\delta = \frac{5x}{\sqrt{Re}} = \frac{5x}{\sqrt{\dfrac{\rho\ v\ x}{\mu}}}$$

So, $\delta \propto \sqrt{x}$

So option (c) is correct

Q.19 (C)

Q.20 (C)

Given: $f = mRe^{-0.2}$

$V_1 = 1\ m/s,\ \Delta P_1 = 10\ kPa$

$V_2 = 2\ m/s,\ \Delta P_2 = \_\_\_\_$

Pressure drop due to friction is given by

$$\Delta P_f = \frac{4\,f\ L\,v^2}{2\ D} = \frac{4m\left(\dfrac{\rho v d}{\mu}\right)^{-0.2} v^2}{2\ D}$$

$$\Delta P_f \propto v^{1.8}$$

$$\frac{\Delta P_{f2}}{\Delta P_{f1}} = \left(\frac{v_2}{v_1}\right)^{1.8}$$

$$\Delta P_{f2} = \left(2^{1.8}\right)10 = 34.8\ kPa$$

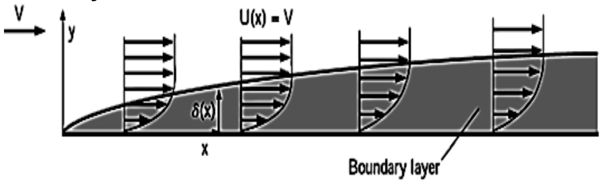
So option (c) is correct

7 EXTERNAL FLOW

7.1 BOUNDARY LAYER FORMATION

The variation of velocity from zero to free stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary.

Boundary layer is a very thin layer of the fluid, in the immediate neighbourhood of the solid boundaries where the variation of velocity is from zero to free stream velocity.



The velocity gradient $= \frac{du}{dy}$

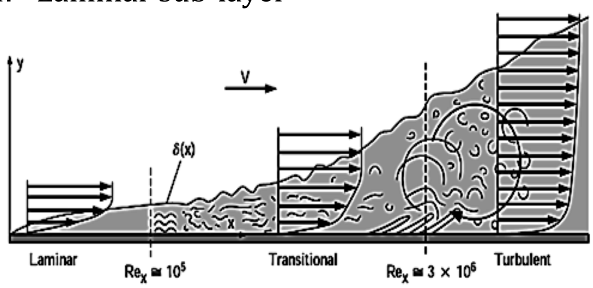
Fluid exerts a shear stress on the wall in the direction of motion. The value of shear stress is given by $\tau = \frac{\mu du}{dy}$

The velocity gradient is set up in the fluid near the surface of the plate. This velocity gradient develops shear resistance which retards the fluid. Downstream the leading edge, the boundary layer region increases because the retarded fluid is further retarded.

7.2 REGIONS OF BOUNDARY LAYER

Boundary layer can be divided in three major regions:

1. Laminar
2. Transition
3. Turbulent
4. Laminar sub-layer



7.2.1 LAMINAR BOUNDARY LAYER

In laminar region the fluid flows in streamline. The viscous force is higher than inertia force.

The Reynold's No. for flat plate is given by

$$Re_x = \frac{U_\infty \times x}{\nu}$$

Where,

U_∞ = free stream velocity

X = distance from leading edge

ν = kinematic viscosity of fluid

For laminar flow

$R_{ex} < 5\times10^5$

7.2.2 TURBULENT BOUNDARY LAYER

When the length of plate is more than the critical length 'x_{cr}', calculated from equation $\left(\frac{Re\times\nu\times5\times10^5}{U_\infty}\right)$

then, transition from laminar to turbulent takes place.

7.2.3 LAMINAR SUBLAYER

It is region in the turbulent zone adjacent to the solid surface of the plate the velocity variation is influenced only by viscous effects. Though velocity distribution is parabolic for small thickness, we can reasonably assume that velocity variation is linear here

$$\tau_o = \mu\left(\frac{du}{dy}\right) \Rightarrow \mu\left(\frac{\Delta u}{\Delta y}\right)$$

τ_o is constant in laminar sub-layer.

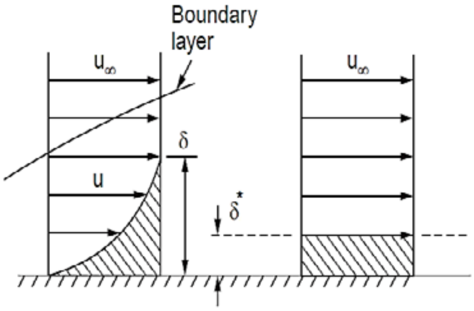
7.3 BOUNDARY LAYER THICKNESS

It is defined as the distance from the boundary of the solid measured in the y-direction to the point, where the velocity of fluid is approximately equal to 0.99 times free stream velocity (U_{∞})

S_{lam}, S_{tur}, S' laminar sub layer

7.3.1 DISPLACEMENT THICKNESS $(\delta)^*$

It is defined as displacement of surface in the direction normal to the surface to compensate for the reduction in the flow rate, due to boundary layer formation.



The plate is displaced by distance $(\delta)^*$ given by

$$\delta^* = \int_0^{\delta}\left(1-\frac{u}{U_{\infty}}\right)dy$$

7.3.2 MOMENTUM THICKNESS

It is defined as the distance, measured normal to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.

$$\theta = \int_0^{\delta}\frac{u}{U_{\infty}}\left(1-\frac{u}{U_{\infty}}\right)dy$$

7.3.3 ENERGY THICKNESS

It is defined as the distance measured normal to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in K.E. of the flowing fluid.

$$\delta^{**} = \int_0^{\delta}\frac{u}{U_{\infty}}\left(1-\frac{u^2}{U_{\infty}^2}\right)dy$$

7.4 DRAG FORCE ON FLAT PLATE DUE TO BOUNDARY LAYER

7.4.1 VON KARMAN MOMENTUM INTEGRAL EQUATION

$$\frac{\tau_0}{\rho U_{\infty}^2} = \frac{d\theta}{dx}$$

$$\tau_0 = \left(\frac{d\theta}{dx}\right)\rho U_{\infty}{}^2$$

τ_0 is shear stress.

θ is momentum thickness

The above equation is valid for

1. Zero pressure gradient.
2. Laminar & turbulent flow
3. Incompressible steady flow.

7.4.2 LOCAL COEFFICIENT OF DRAG

$$c_d^* = \frac{\tau_0}{\left(\frac{1}{2}\rho U_{\infty}^2\right)}$$

Local shear stress in laminar flow is obtained by

$$\tau_0 = \mu\frac{du}{dy}$$

Average coefficient of drag

$$c_d = \frac{F_{avg}}{\left(\frac{1}{2}\rho U_{\infty}^2\right)}$$

$$F_{avg} = \int_0^L \tau_o.b.dx$$

b is the width of plate

L is the length up to which average force is to be evaluated

7.5 BOUNDARY CONDITION FOR THE VELOCITY PROFILE

1) at $y = 0, U = 0, \frac{du}{dy}$ has some finite value.

2) at $y = \delta, u = U_{\infty}$,

3) at $y = \delta, \frac{du}{dy} = 0$,

Assume velocity profile to be

$$\frac{u}{U_\infty} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$$

Using boundary conditions, value of coefficients obtained are

$a = 0$, $b = 2$, $c = -1$,

The velocity profile for laminar boundary layer flows is given by

$$\frac{u}{U_\infty} = 2(y/\delta) - (y/\delta)^2$$

Equation involved

$$\frac{\tau_0}{\rho U^2} = \frac{do}{dx}$$

U is free stream velocity

$$\tau_o = \frac{\mu du}{dy}$$

$$\frac{du}{dy} = \frac{d}{dy}\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]$$

$$\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2Y}{\delta^2}\right] \text{ at y=0}$$

$$\frac{du}{dy} = \frac{2U}{\delta},\ \tau = \mu\frac{2u}{\delta} \ldots (1)$$

$$\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right]dy$$

$$\theta = \int_0^\delta \left[2\left(\frac{Y}{\delta}\right) - \left(\frac{Y}{\delta}\right)^2\right]\left[1 - 2\left(\frac{Y}{\delta}\right) + \left(\frac{Y}{\delta}\right)^2\right]dy$$

$$\theta = \frac{2\delta}{15}$$

$$\frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{2\delta}{15}\right) \quad \ldots (2)$$

$$\mu\frac{2U}{\delta} = \frac{d}{dx}\left(\frac{2\delta}{15}\right)\rho U^2$$

$$\rho d\delta = \frac{15\mu dx}{\delta U}$$

$$\frac{\delta^2}{2} = \frac{15\mu}{\rho U}x + C$$

at $x = 0, \delta = 0, C = 0$ at

$$\delta = \sqrt{\frac{30\mu x^2}{\rho U x}}$$

$$\frac{\rho u x}{\mu} = R_e$$

$$\delta = \sqrt{30}.\sqrt{\frac{x^2}{R_{e_x}}}$$

Shear stress in terms of Reynolds No

$$\tau_o = \frac{2\mu U}{\delta}$$

$$\tau_o = \frac{2\mu U}{5.48\frac{x}{\sqrt{Re_x}}} = 0.365\frac{\mu U}{x}\sqrt{\frac{\rho U x}{\mu}}$$

Coefficient of drag

$$c_D = \frac{F_0}{\left(\frac{1}{2}\rho A U^2\right)}$$

$$F_D = \int_o^L \tau_o.b.dx$$

Where b is thickness of plate

$$F_D = \int_0^L 0.365\frac{\mu U}{x}\sqrt{\frac{\rho U x}{\mu}}\,\text{ф}dx$$

$$F_D = 0.365\mu U\sqrt{\frac{\rho U}{\mu}}\,\text{ф}\int_0^L \frac{1}{\sqrt{x}}dx$$

$$F_D = 0.73b\mu U\sqrt{\frac{\rho U L}{\mu}}$$

$$C_d = \frac{0.73b\mu U\sqrt{\frac{\rho U L}{\mu}}}{\frac{1}{2}\rho A U^2}$$

$$C_d = \frac{1.46\mu}{\rho U}\sqrt{\frac{\rho U L}{\mu}}$$

$$C_d = 1.46\sqrt{\frac{\mu}{\rho U L}}$$

$$C_d = \frac{1.46}{\sqrt{Re_L}}$$

7.6 ANALYSIS OF TURBULENT BOUNDARY LAYER

7.6.1 TURBULENT BOUNDARY LAYER ON FLAT PLATE

Blasius, on basis of his experiments gave expression for velocity profile in turbulent flow over flat plate

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^n$$

Where $n = 1/7, R_e < 10^7$

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

Since shear stress in turbulent flow is given by

$$\tau_o = 0.0225\rho u^2 \left[\frac{\mu}{\rho\delta u}\right]^{1/4}$$

7.6.2 BOUNDARY LAYER THICKNESS IN TURBULENT FLOW

$$\delta = 0.37\left(\frac{\mu}{\rho U_\infty}\right)^{1/5} gx^{4/5}$$

$$\delta = \frac{0.37x}{\left(Re_x\right)^{1/5}}$$

7.6.3 COEFFICIENT OF DRAG

If $R_e > 5\times10^5$ & less then10^7

$$\delta = \frac{0.37x}{\left(R_{e_x}\right)^{1/5}},$$

$$C_D = \frac{0.072}{\left(R_{e_L}\right)^{1/5}}$$

Where,

x = distance from leading edge

R_{e_x} = Reynold's No. for length x

R_{e_L} = Reynold's No at the length 'L' of plate

For $R_e > 10^7, R_e > 10^9$

$$C_D = \frac{0.455}{\left(\log 10 R_{e_L}\right)^{2.58}}$$

Example:

Determine the thickness of the boundary layer at the trailing edge of smooth plate of length 4m & width 1.5 m, when the plate is moving with a velocity of 4 m/s. In stationary air, kinematic viscosity $\nu = 1.5\times10^{-5} m^2/s$.

Solution:

$$Re_L = \frac{UL}{\nu} = \frac{4\times4}{1.5\times10^{-5}} = 10.66\times10^5$$

Turbulent

$$\delta = \frac{0.37x}{\left(Re_x\right)^{1/5}} = 92.19mm$$

7.7 LIFT

A component in the direction of force is called drag F_D

A component normal to the direction of force is called Lift F_L

It is expressed by dimensionless quantities like *Lift Coefficient* C_L *and Drag Coefficient* C_D

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}$$ where, F_D is the Drag Force

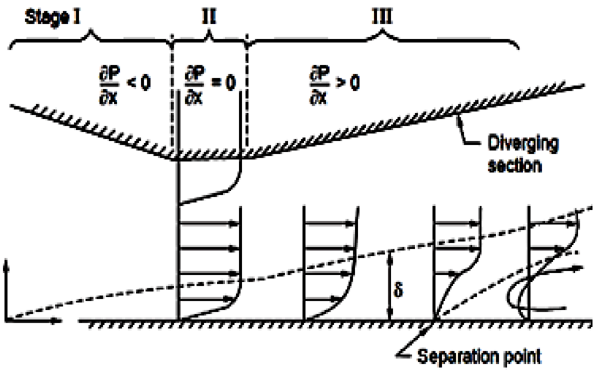
$$C_L = \frac{F_L}{\frac{1}{2}\rho U^2 A}$$ where, F_L is the Lift Force

U is the free stream Velocity

$$F_L = W = \frac{1}{2}\rho U^2 C_L A$$

7.8 BOUNDARY LAYER SEPARATION

Separation of flow is said to occur when the direction of the flow velocity near the surface is opposite to the direction of the free stream velocity, which means (du/dy) ≤ 0. **If (dp/dx) increases to the extent that it can overcome the shear near the surface, then separation will occur**. Such a pressure gradient is called adverse pressure gradient. In the case of incompressible flow in a nozzle a favourable pressure gradient exists & Separation will not occur in such flows. In the case of diverging section of a diffuser, separation can occur if the rate of area increase is large.



7.8.1 LOCATION OF SEPARATION POINT

$\left(\frac{\partial u}{\partial y}\right)_{y=0} < 0$ Separated flow

$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$ Verge of separation

$\left(\frac{\partial u}{\partial y}\right)_{y=0} > 0$ No separation

Example:

Find the displacement thickness, the momentum thickness and energy for the velocity distribution in the boundary layer given by $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Solution:

Given:

Velocity distribution, $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(i) Displacement thickness δ^* is given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Substituting the value of, $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ we have

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \int_0^\delta \left\{1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right\} dy = \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3}$$

(ii) Momentum thickness θ is given by

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy = \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$= \left[\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}\right] = \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$= \frac{15\delta - 25\delta + 15\delta - 3\delta}{15} = \frac{30\delta - 28\delta}{15} = \frac{2\delta}{15}$$

(iii) Energy thickness δ^{**} is given by

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]^2\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left[\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right]\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{4y^2}{\delta^2} - \frac{y^4}{\delta^4} + \frac{4y^3}{\delta^3}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{8y^3}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^3} + \frac{12y^4}{\delta^4} - \frac{6y^5}{\delta^5} + \frac{y^6}{\delta^6}\right) dy$$

$$= \left[\frac{2y}{2\delta} - \frac{y^3}{3\delta^2} - \frac{8y^4}{4\delta^3} + \frac{12y^5}{5\delta^4} - \frac{6y^6}{6\delta^5} + \frac{y^7}{7\delta^6}\right]_0^\delta$$

$$= \frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta^2} - \frac{2\delta^4}{\delta^3} + \frac{12\delta^5}{5\delta^4} - \frac{\delta^6}{\delta^5} + \frac{\delta^7}{7\delta^6} = \delta - \frac{\delta}{3} - 2\delta + \frac{12}{5}\delta - \delta + \frac{\delta}{7}$$

$$= -2\delta - \frac{\delta}{3} + \frac{12}{5}\delta + \frac{\delta}{7} = \frac{-210\delta - 35\delta + 252\delta + 15\delta}{105}$$

$$= \frac{-245\delta + 267\delta}{105} = \frac{22\delta}{105}$$

Example:

Air is flowing over a smooth plate with a velocity of 10 m/s. The length of the plate is 1.2m and width is 0.8m. If laminar boundary layer exists up to a value of $R_e = 2\times10^5$, find the maximum distance from the leading edge upto which laminar boundary layer exists. Find the maximum thickness of laminar boundary layer if the velocity profile is given by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Take kinematics viscosity for air = 0.15 stokes.

Solution:

Given:

Velocity of air, $U = 10\,m/s$

Length of plate, $L = 1.2\,m$

Width of plate, $b = 0.8\,m$

Reynolds number upto which laminar boundary exists = 2×10^5

ν for air $= 0.15$ stokes $= 0.15\times10^{-4}\ m^2/s$

Reynolds number, $Re_L = \frac{\rho U x}{\mu} = \frac{Ux}{\nu}$

If $R_{e_x} = 2\times10^5$, then x denotes the distance from leading edge upto which laminar boundary layer exists

$$\therefore\ 2\times10^5 = \frac{10\times x}{0.15\times10^{-4}}$$

$$\therefore\ x = \frac{2\times10^5\times0.15\times10^{-4}}{10} = 0.30\,m = 300\,mm$$

Maximum thickness of the laminar boundary for the velocity profile,

$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ Is given by

$$\delta = \frac{5.48\ \times\ x}{\sqrt{R_{e_x}}} = \frac{5.48\times0.30}{\sqrt{2\times10^5}} = 0.00367\,m = 3.67\,mm$$

Example:

Determine the thickness of the boundary layer at the trailing edge of smooth plate of length 4m and of width 1.5m, when the plate is moving with a velocity of 4m/s in stationary air. Take kinematics viscosity of air as $1.5\times10^{-5}\,m^2/s$

Solution:

Given:

Length of plate, $L = 4\,m$

Width of plate, $b = 1.5\,m$

Velocity of plate, $U = 4\,m/s$

Kinematics viscosity, $\nu = 1.5\times10^{-5}\,m^2/s$

Reynolds number,

$$Re_L = \frac{U\times L}{\nu} = \frac{4\times4}{1.5\times10^{-5}} = 10.66\times10^5$$

As the Reynolds number is more than 5×10^5 and hence the boundary layer at the trailing edge is turbulent.

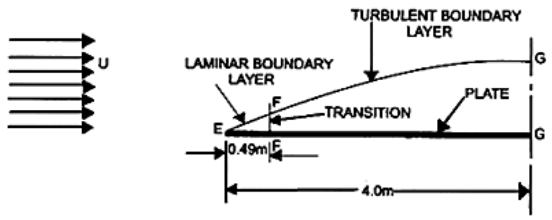
The boundary layer thickness for turbulent boundary layer is given by

$$\delta = \frac{0.37x}{\left(R_{e_x}\right)^{1/5}} \quad \left(\text{Here } x = L \text{ and } R_{e_x} = R_{e_L}\right)$$

$$= \frac{0.37\times4.0}{\left(10.66\times10^5\right)^{1/5}} = 0.0921\,m = 92.1\,mm$$

Example:

Water is flowing over a thin smooth plate of length 4m and width 2m at a velocity of 1.0m/s. If the boundary layer flow changes from laminar to turbulent at a Reynolds number 5×10^5, find (i) the distance from leading edge upto which boundary layer is laminar, (ii) the thickness of boundary layer at the transition point, and (iii) the drag force on one side of the plate. Take viscosity of water $\mu = 9.81\times10^{-4}\,Ns/m^2$.



Solution:

Given:

Length of plate, $L = 4\,m$

Width of plate, $b = 2\,m$

Velocity of flow, $U = 1.0\,m/s$

Reynolds number for laminar boundary layer $= 5\times10^5$

Viscosity of water, $\mu = 9.81\times10^{-4}\ Ns/m^2$

i) Let the distance from leading edge upto which laminar boundary layer exists $= x$

$$\therefore 5\times10^5 = \frac{\rho U x}{\mu} = \frac{1000\times1\times x}{9.81\times10^{-4}}$$

$$\therefore \quad x = \frac{5\times10^5\times9.81\times10^{-4}}{1000}$$

$= 0.4900m = 490mm$

ii) Thickness of boundary layer at the point where the boundary layer changes from laminar to turbulent i.e., at Reynolds number $= 5\times10^5$, is given by Blasius's solution as

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.49}{\sqrt{5\times10^5}} = 3.46mm$$

$\left(\text{Here } x = 49cm = 0.49m,\ R_{e_x} = 5\times10^5\right)$

iii) Drag force on the plate on one side

= Drag due to laminar boundary layer

+ Drag due to turbulent boundary

a) Drag due to laminar boundary layer (i.e., from E to F)

$$F_{EF} = \frac{1}{2}\rho AU^2\times C_D \qquad ......(i)$$

Where C_D is given by Blasius solution for laminar boundary layer as

$$C_D = \frac{1.328}{\sqrt{R_{e_x}}} = \frac{1.328}{\sqrt{5\times10^5}} \quad \left(\text{for } EF, R_{e_x} = 5\times10^5\right)$$

$= 0.001878$

A=Area of plate upto laminar boundary layer

$= 0.49\times b = 0.49\times2 = 0.98\,m^2$

Substituting the value of C_D and A in equation(i), we get

$$F_{EF} = \frac{1}{2}\times1000\times0.98\times1.0^2\times.001878 = 0.92N$$

....(ii)

b) Drag force due to turbulent boundary layer from F to G =Drag force due to turbulent boundary layer from E to G – Drag force due to turbulent flow from E to F

$= \left(F_{EG}\right)_{turb} - \left(F_{EF}\right)_{turb}$

Now, $\left(F_{FG}\right)_{turb} = \frac{1}{2}\rho AU^2\times C_D$

Where C_D is

$$C_D = \frac{0.072}{\left(R_{e_L}\right)^{1/5}}$$

But

$$R_{e_L} = \frac{\rho UL}{\mu} = 1000\times\frac{1.0\times4.0}{9.81\times10^{-4}} = 40.77\times10^5$$

$$\therefore \quad C_D = \frac{0.072}{\left(40.77\times105\right)^{1/5}} = 0.00343$$

$$\therefore \quad \left(F_{EG}\right)_{turb} = \frac{1}{2}\rho AU^2\times C_D$$

$$= \frac{1}{2}\times1000\times(4\times2)\times1^2\times.00343 = 13.72N$$

Also $\quad \left(F_{EF}\right)_{turb} = \frac{1}{2}\rho A_{EF}\times U^2\times C_D$

Where A_{EF} =Area of plate upto

$EF = EF\times b = 0.49\times2 = 0.98\,m^2$

And

$$C_D = \frac{0.072}{\left(R_{EF}\right)^{1/5}} = \frac{0.072}{\left(5\times10^5\right)^{1/5}} = .00522$$

$$\left(F_{EF}\right)_{turb} = \frac{1}{2}\times1000\times0.98\times1^2\times.00522 = 2.557N$$

$\therefore$ Drag force due to turbulent boundary layer from F to G

$=\left(F_{EG}\right)_{turb}-\left(F_{EF}\right)_{turb}=13.72-2.557=11.163N$

∴ Drag force on the plate on one side
=Drag force due to laminar boundary layer upto F + Drag force due to turbulent boundary layer from F to G

$=0.92+11.163=12.083N$

Example:
For the following velocity profiles, determine whether the flow has separated or on the verge of separation or will attach with the surface.

i) $\frac{u}{U}=\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$

ii) $\frac{u}{U}=2\left(\frac{y}{\delta}\right)^2-\left(\frac{y}{\delta}\right)^3$

iii) $\frac{u}{U}=-2\left(\frac{y}{\delta}\right)+\left(\frac{y}{\delta}\right)^2$

Solution:
Given
1st velocity profile

$\frac{u}{U}=\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$ or $u=\frac{3U}{2}\left(\frac{y}{\delta}\right)-\frac{U}{2}\left(\frac{y}{\delta}\right)^3$

Differentiating w.r.t., y, the above equation becomes,

$$\frac{\delta u}{\delta y}=\frac{3U}{2}\times\frac{1}{\delta}-\frac{U}{2}\times 3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

At $y=0$,

$$\left(\frac{\delta u}{\delta y}\right)_{y=0}=\frac{3U}{2\delta}-\frac{3U}{2}\left(\frac{0}{\delta}\right)^2\times\frac{1}{\delta}=\frac{3U}{2\delta}$$

As $\left(\frac{\delta u}{\delta y}\right)_{y=0}$ is positive. Hence flow will not separate or flow will remain attached with the surface.

2nd velocity profile

$$\frac{u}{U}=2\left(\frac{y}{\delta}\right)^2-\left(\frac{y}{\delta}\right)^3$$

$$\therefore u=2U\left(\frac{y}{\delta}\right)^2-U\left(\frac{y}{\delta}\right)^3$$

$$\therefore \frac{\delta u}{\delta y}=2U\times 2\left(\frac{y}{\delta}\right)\times\frac{1}{\delta}-U\times 3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

At $y=0$, $\left(\frac{\delta u}{\delta y}\right)_{y=0}$

$$=2U\times 2\left(\frac{0}{\delta}\right)\times\frac{1}{\delta}-U\times 3\left(\frac{0}{\delta}\right)^2\times\frac{1}{\delta}=0$$

As $\left(\frac{\delta u}{\delta y}\right)_{y=0}=0$, the flow is on verge of separation.

3rd velocity profile

$$\frac{u}{U}=-2\left(\frac{y}{\delta}\right)+\left(\frac{y}{\delta}\right)^2$$

$$\therefore u=-2U\left(\frac{y}{\delta}\right)+U\left(\frac{y}{\delta}\right)^2$$

$$\frac{\delta u}{\delta y}=-2U\left(\frac{1}{\delta}\right)+2U\left(\frac{y}{\delta}\right)\times\frac{1}{\delta}$$

At y=0,

$$\left(\frac{\delta u}{\delta y}\right)_{y=0}=-\frac{2U}{\delta}+2U\left(\frac{0}{\delta}\right)\times\frac{1}{\delta}=-\frac{2U}{\delta}$$

As $\left(\frac{\delta u}{\delta y}\right)_{y=0}$ is negative the flow has separated.

GATE QUESTIONS

Q.1 For flow past a flat plate, if x is the distance along the plate in the direction of flow, the boundary layer thickness is proportional to

[GATE–2002]

(A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$

(C) x (D) $\frac{1}{x}$

Q.2 The following table provides four sets of fanning friction factor data, for different values of Reynolds number (Re) and roughness factor $\left(\frac{k}{D}\right)$

[GATE–2017]

	Re	10^2	10^3	10^5	10^5
	$\left(\frac{k}{D}\right)$	0	0.001	0	0.001
Set I	f	0.16	0.016	$16x10^{-5}$	$16x10^{-5}$
Set II	f	0.016	0.16	0.0055	0.0045
Set III	f	0.16	0.016	0.0045	0.0055
Set IV	f	0.0045	0.0055	0.016	0.16

Which of the above set of friction factor data is correct

(A) Set I (B) Set II

(C) Set III (D) Set IV

ANSWER KEY:

1	2
A	C

EXPLANATIONS

Q.1 (A)

$$\delta = \frac{5\,x}{\sqrt{\left(\frac{\rho v x}{\mu}\right)}}$$

$$\delta \propto \sqrt{x}$$

So Option (a) is correct

Q.2 (C)

Laminar

$$f = \frac{16}{Re}, \text{So when}$$

$$Re = 10^2 \Rightarrow f = 0.16$$

$$Re = 10^3 \Rightarrow f = 0.016$$

Turbulent

$$f = 0.0014 + \frac{0.125}{Re^{0.32}}$$

$$Re = 10^5 \Rightarrow f = 0.0045$$

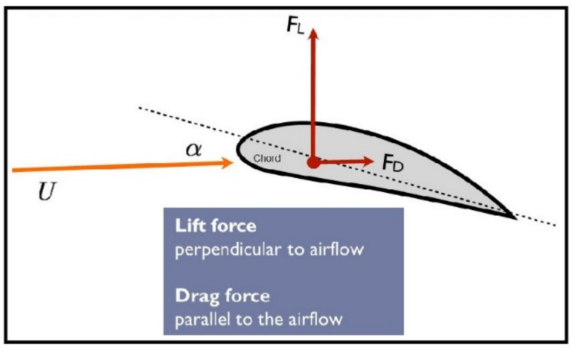
8 FLOW PAST IMMERSED BODIES

8.1 LIFT

A force acting in opposite to the relative motion of any object is called drag F_D

A component normal to the direction of applied force is called Lift F_L

It is expressed by dimensionless quantities like *Lift Coefficient* C_L *and Drag Coefficient* C_D



$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}$$ where, F_D is the Drag Force

$$C_L = \frac{F_L}{\frac{1}{2}\rho U^2 A}$$ where, F_L is the Lift Force

U is the free stream Velocity

$$F_L = W = \frac{1}{2}\rho U^2 C_L A$$

8.2 STOKE'S LAW

It is found that for low Reynold's Number i.e. less than 1, the total Drag force is given by

$$F_D = 3\pi\mu D U$$

But the Total drag force is also given by

$$F_D = \frac{1}{2}\rho U^2 C_D A$$

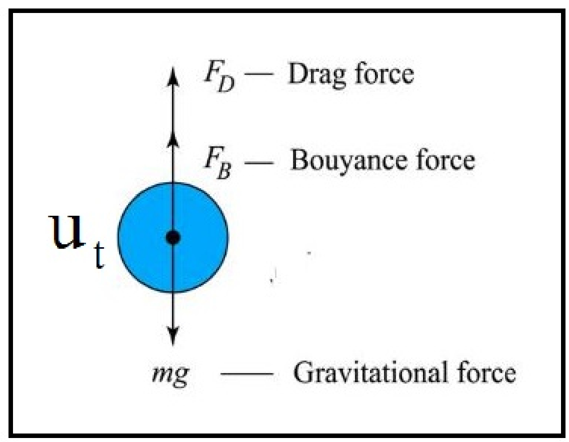
So, $3\pi\mu DU = \frac{1}{2}\rho U^2 C_D A$

On solving the equation, we get

$$C_D = \frac{24}{Re}$$ **(Stoke's Law)**

8.3 TERMINAL VELOCITY

When the body is moving freely under the influence of gravitational force. By the time the acceleration attained by the body keeps reducing. **The maximum velocity attained by the particle when the acceleration is zero is called Terminal Settling Velocity.**



From Fig,

$$\text{Net Force} = F_g - F_B - F_D$$

$$F = mg - \frac{m_p}{\rho_p}\rho\, g - C_D \frac{1}{2}\rho\, U^2 A$$

$$m\, a = m_p\, g - \frac{m_p}{\rho_p}\rho\, g - C_D \frac{1}{2}\rho\, U^2 A$$

Acceleration $(a) = 0$, $U = u_t$

$$\frac{g(\rho_p - \rho)}{\rho_p} = C_D \frac{1}{2}\rho\, U^2 A$$

$$u_t = \sqrt{\frac{2\,m\,g\,(\rho_p - \rho)}{\rho_p\, C_D\, \rho\, A_p}}$$

Where, A_p = Projected Area

If the particle is considered as Spherical

$$A_p = \frac{\pi}{4} D_p^{\,2}$$

Then the Equation Becomes

$$u_t = \sqrt{\frac{4\,g\,D_p\,(\rho_p - \rho)}{3\,C_D\,\rho}}$$

Where, u_t = Terminal Settling Velocity

D_p =Diameter of Spherical Partical

ρ_p =Density of particle

ρ= Density of fluid

CASE-I: - For low Re Number (Re<1) or for Laminar flow, The Terminal Settling Velocity is given by Stoke's Law or (Stoke's Regime)

Here, $C_D = 3\pi\mu D_p U_0$

On solving we get, $C_D = \dfrac{24}{Re}$

So, $u_t = \sqrt{\dfrac{4\,g\,D_p\,(\rho_p - \rho) \times \rho u_t D_p}{3\,\rho \times 24 \times \mu}}$

$$u_t = \frac{g\,D_p^{\,2}\,(\rho_p - \rho)}{18\,\mu}$$

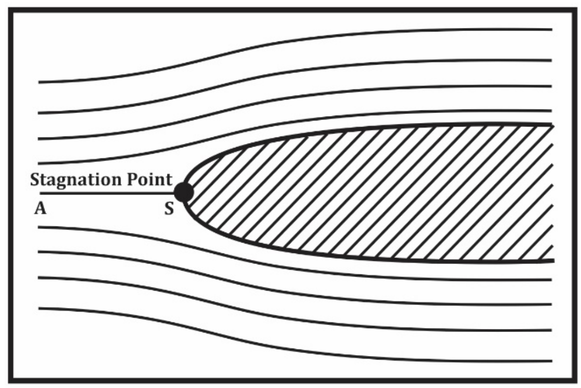
CASE-I: - For high Re Number (Re>1000) or for Turbulent flow, The Terminal Settling Velocity is given by Newton's Law or (Newton's Regime)

$$C_D = 0.44$$

$$u_t = 1.75\sqrt{\frac{g\,D_p\,(\rho_p - \rho)}{\rho}}$$

8.4 STAGNATION POINT

The streamlines in the fluid flowing past the object in the figure shown below show that the fluid stream in the plane of the section is split by the object into two parts, one passing over the top of the object and other under the bottom. Streamline AS divide the two parts and terminates at a definite point S at the nose of the body. This point is called **Stagnation point.**



Stagnation pressure is the static pressure at a stagnation point in a fluid flow. At a stagnation point the fluid velocity is zero and all kinetic energy has been converted into pressure energy.

Applying Bernoulli Equation at points A and S

$$\frac{P_A}{\rho} + \frac{V_A^{\,2}}{2} + 0 = \frac{P_S}{\rho} + \frac{V_S^{\,2}}{2} + 0$$

But, $V_S = 0$

$$\frac{P_S - P_A}{\rho} = \frac{V_A^{\,2}}{2}$$

GATE QUESTIONS

Q.1 The diameter of a drop of liquid fuel changes with time, due to combustion, according to the relationship, $D = D_0\left(1-\frac{t}{t_b}\right)$, while burning, the drop falls at its terminal velocity under Stokes' flow regime. The distance it will travel before complete combustion, is given by

[GATE –2010]

(A) $\frac{D_0^2\ \Delta\rho\ t_b\ g}{18\ \mu}$

(B) $\frac{D_0^2\ \Delta\rho\ t_b\ g}{36\ \mu}$

(C) $\frac{D_0^2\ \Delta\rho\ t_b\ g}{54\ \mu}$

(D) $\frac{D_0^2\ \Delta\rho\ t_b\ g}{108\ \mu}$

Q.2 The drag coefficient for a bacterium moving in water at 1 mm/s, will be of the following order of magnitude (assume size of the bacterium to be 1 micron and kinematic viscosity of water to be 10^{-6} m^2/s)

[GATE–2012]

(A) 24000 (B) 24

(C) 0.24 (D) 0.44

ANSWER KEY:

1	2
C	A

EXPLANATIONS

Q.1 (C)

By Stokes Law

$$u_t = \frac{g\,D_p^{\,2}\,\Delta\rho}{18\,\mu}$$

Integrate

$$\int u_t\;dt = \int \frac{g\,\Delta\rho}{18\,\mu}\left(1-\frac{t}{t_b}\right)^2 dt$$

$$\int u_t\;dt = \int \frac{g\,\Delta\rho}{18\,\mu}\left(1+\frac{t^2}{t_b^{\,2}}-\frac{2t}{t_b}\right) dt$$

$$u_{t_b} = \frac{g\,D_0^{\,2}\,t_b\,\Delta\rho}{54\,\mu}$$

So Option (c) is correct

Q.2 (A)

$d_p = 10^{-6}$ m

v = 10^{-6} m^2 /s

x = 10^{-3} m /s

$$\text{Re} = \frac{v\,d}{\nu} = \frac{10^{-3}\times 10^{-6}}{10^{-6}} = 10^{-3} < 1$$

For low Re < 1

$$C_D = \frac{24}{\text{Re}} = 24000$$

So Option (a) is correct

9 FLOW THROUGH POROUS MEDIUM

9.1 DECRIPTION OF POROUS MEDIUM

Porous medium means a solid, or a collection of solid particles, with sufficient open space in or around the particles to enable a fluid to pass through or around them.

One of the important term for porous medium is porosity (ε).

$$\varepsilon = \frac{\text{Total Volume} - \text{Volume of Solid}}{\text{Total Volume}}$$

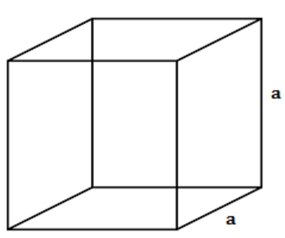
$$\varepsilon = \frac{A_{Void}}{A}$$

9.2 HYDRAULIC DIAMETER

In flow through porous medium the fluid follows an undefined path. One method of describing the flow in porous medium in non-circular conduit is termed as "Hydraulic Diameter (D_H)".

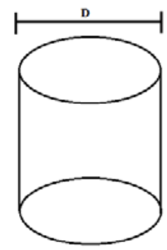
$$D_H = \frac{4A}{P}$$

(a) Cube



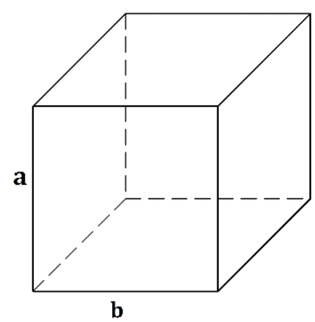
$$D_H = \frac{4 \times a^2}{4a} = a$$

(b)Cylinder



$$D_H = \frac{4\left(\frac{\pi}{4}D^2\right)}{\pi D} = D$$

(c)Cuboid



$$D_H = \frac{4(ab)}{2(a+b)} = \frac{2(ab)}{(a+b)}$$

9.3 FRICTION IN FLOW THROUGH BEDS OF SOLIDS

With the help of Hagen-Poiseuille equation and experimental data, there are many expression to determine the pressure drop in bed per unit length.

9.3.1 Kozeny carman equation

The epression is given as

$$\frac{\Delta P}{L} = \frac{150\ \mu\ V_0\ (1-\varepsilon)^2}{(\phi\ d_p)^2\ \varepsilon^3}$$

The above equation is applicable for low Reynold number.

It gives viscous loss

9.3.2 Burke Plummer Equation

The expression is given as

$$\frac{\Delta P}{L} = \frac{1.75\,\rho_f V_0^{\,2}(1-\varepsilon)}{\phi\, d_p\, \varepsilon^3}$$

The above Equation is applicable for High Reynold number.

It given Kinetic Energy loss.

9.3.3 Ergun Equation

The Ergun equation include both the losses i.e. viscous loss and kinetic energy loss. It gives pressure drop for overall range of bed.

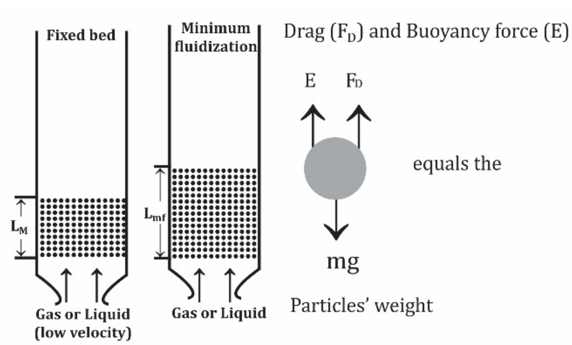
It is given as

$$\frac{\Delta P}{L} = \frac{150\,\mu\, V_0\,(1-\varepsilon)^2}{(\phi\, d_p)^2\,\varepsilon^3} + \frac{1.75\,\rho_f V_0^{\,2}(1-\varepsilon)}{\phi\, d_p\, \varepsilon^3}$$

9.4 FLUIDIZATION

When a bed of solid particle is passed with fluid the pressure drop increases as the fluid velocity increases. When the force exerted by fluid counter balance the weight of bed the particles start to rise.

Fluidization is a process in which solids are caused to behave like a fluid by blowing gas or liquid upwards through the solid-filled reactor



Fluidized bed are widely used for conducting gas solid reactions (coal combustion), gas solid catalytic reactions (catalytic cracking of petroleum), etc. Several applications also utilize liquid fluidized beds (bioreactors).

Some of the important design parameters for such systems are: the minimum fluidization velocity (V_{mf}), bed expansion of fluidization

(L), and pressure variation in the bed (ΔP).

Once the bed is fluidized, the pressure drop across the bed remains constant but the bed height increases as the velocity of fluid is increased.

For liquid, the state of fluidization past the minimum fluidization stage is called homogeneous/ smooth / particulate / non–bubbling fluidized bed, as the bed expands smoothly. At higher velocity, there is a carry-over of particles.

For gases, the particulate or homogeneous fluidization occurs only for small (fine) particles. For large particles, bubbles are formed. At even higher velocity, vigorous Fluidization occurs, with turbulent motion of solid clusters and bubbles. Such state is called "Fast Fluidized Bed". There may be carryover/entrainment of particles with the outgoing gas

Pressure drop in laminar flow is given by Hagen–Poiseuille Equation

$$\Delta P = \frac{32\,\mu\,L\,V}{D^2}$$

$$\text{Sphericity}(\phi_s) = \frac{6\,V_P}{S_P\,D_P}$$

Equivalent Diameter is given by

$$D_{Equi.} = \frac{2}{3}\phi_s D_P\left(\frac{\varepsilon}{1-\varepsilon}\right)$$

$$\varepsilon = \text{Porosity of Bed}$$

$$\text{So, } \Delta P = \frac{32\,\mu\,L\,V}{\left(\frac{2}{3}\phi_s D_P\left(\frac{\varepsilon}{1-\varepsilon}\right)\right)^2}$$

On Solving we get

$$\frac{\Delta P}{L} = \frac{150\,\mu\,V_0\,(1-\varepsilon)^2}{\phi_s^{\,2}\,D_P^{\,2}\,\varepsilon^3}$$

The above equation is called 'Kozeny Carman Equation' and it is used to find pressure drop across fluidized bed in laminar flow in pipe and tube.
It gives viscous loss and is used for low Reynold's Number.

Now, Pressure drop in turbulent flow is given by

$$\Delta P = \frac{2\,\rho\,f\,L\,V^2}{D}$$

$$\Delta P = \frac{2\,\rho\,f\,L\,V^2}{\frac{2}{3}\phi_s D_P\left(\frac{\varepsilon}{1-\varepsilon}\right)}$$

On solving we get

$$\frac{\Delta P}{L} = \frac{1.75\,\rho\,V_0^{\,2}\,(1-\varepsilon)}{\phi_s\,D_P\,\varepsilon^3}$$

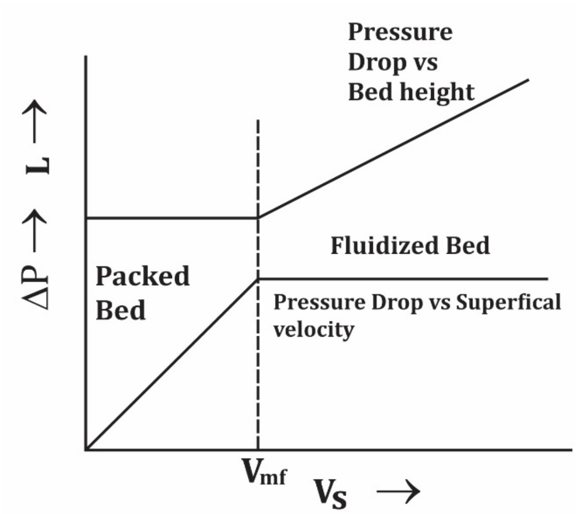
The above equation is called 'Burke Plummer Equation' and it is used to find pressure drop across fluidized bed in turbulent flow in pipe and tube.
It gives Kinetic Energy loss and is used for High Reynold's Number.

In Laminar flow there is some pressure drop due turbulent behavior and vice versa.
The Ergun Equation gives pressure drop for both laminar and turbulent flow in fluidized bed.

$$\frac{\Delta P}{L} = \frac{150\,\mu\,V_0\,(1-\varepsilon)^2}{\phi_s^{\,2}\,D_P^{\,2}\,\varepsilon^3} + \frac{1.75\,\rho\,V_0^{\,2}\,(1-\varepsilon)}{\phi_s\,D_P\,\varepsilon^3}$$

9.4.1 Minimum Fluidization Velocity

At the certain velocity, the force exerted by fluid counter balance the weight of bed the particles start to rise, this velocity is called Minimum Fluidization Velocity (V_{mf}).



At incipient fluidization,

$$\frac{\Delta P}{L} = (\rho_p - \rho)(1-\varepsilon)\ g$$

where, ΔP = Pressure Drop across Bed

L = Height of Bed

ε = Porosity of Bed

ρ_p = Density of Particle

ρ = Density of Fluid

To find Minimum fluidization Velocity,

$$(\rho_p - \rho)(1-\varepsilon)\ g = \frac{150\ \mu\ V_{mf}\ (1-\varepsilon)^2}{\phi_s^{\ 2}\ D_P^{\ 2}\ \varepsilon^3} + \frac{1.75\ \rho\ V_{mf}^{\ 2}\ (1-\varepsilon)}{\phi_s\ D_P\ \varepsilon^3}$$

9.4.2 Superficial fluid Velocity

It is hypothetical velocity in an empty channel

$$V_0 = \varepsilon\ V$$

V = Average fluid Velocity

GATE QUESTIONS

Q.1 A gas (density= 1.5 kg/m^3, viscosity= 2 x 10^{-5} kg/m s) flowing through a packed bed (particle size 0.5 cm, porosity= 0.5) at a superficial velocity of 2 m/s causes a pressure drop of 8400 Pa / m. The pressure drop for another gas, with density of 1.5 kg/m^3 and viscosity of 3 x 10^{-5} kg/m s, flowing at 3 m/s will be

[GATE–2002]

(A) 8400 Pa/m

(B) 18900 Pa/m

(C) 12600 Pa/m

(D) 16800 Pa/m

Q.2 Under fully turbulent flow conditions, the frictional pressure drop across a packed bed varies with the superficial velocity (V) of the fluid as

[GATE –2009]

(A) V^{-1} (B) V

(C) $V^{3/2}$ (D) V^2

Q.3 The height of a fluidized bed at incipient fluidization is 0.075 m, and the Corresponding voidage is 0.38. If the voidage of the bed increases to 0.5, then the height of the bed would be

[GATE –2010]

(A) 0.058 m (B) 0.061 m

(C) 0.075 m (D) 0.093 m

Common Data for Q.4 and Q.5

For a liquid flowing through a packed bed the pressure drop per unit length of the bed $\frac{\Delta P}{L}$ is

$$\frac{\Delta P}{L} = \frac{150\,\mu\,V_0\,(1-\varepsilon)^2}{(\phi\,d_p)^2\,\varepsilon^3} + \frac{1.75\,\rho_f V_0^{\,2}(1-\varepsilon)}{\phi\,d_p\,\varepsilon^3}$$

Where V_0 is the Superficial Velocity, ε is the porosity, d_p is the average particle size, ϕ is the particle sphericity, ρ_f is the liquid density, μ is the liquid viscosity.

Given data:

$d_p = 1\times10^{-3}$ m , $\phi = 0.8$, $\rho_f = 1000$ kg/m^3,

$\mu = 1\times10^{-3}$ kg m^{-1}s^{-1}

Particle Density $\rho_p = 2500$ kg / m^3

and acceleration due to gravity = 9.8 m / s^2

Q.4 When V_0 is 0.005 m/s and $\varepsilon = 0.5$, which ONE of the following is the CORRECT value for the ratio of the viscous loss to the kinetic energy loss?

[GATE–2011]

(A) 0.09 (B) 1.07

(C) 10.71 (D) 93

Q.5 On further increasing V_0, incipient fluidization is achieved. Assuming that the porosity of the bed remains unaltered, the pressure drop per unit length (in Pa/m) under incipient fluidization condition is

[GATE –2011]

(A) 3675 (B) 7350

(C) 14700 (D) 73501

Q.6 A bed of spherical glass beads (density 3000 kg/m^3, diameter 1 mm, bed porosity 0.5) is to be fluidized by a liquid of density 1000 kg/m^3 and viscosity 0.1 Pa s. Assume that the Reynolds number based on particle diameter is very small compared to one. If g = 10 m/s^2, then the minimum velocity (in m/s) required to fluidize the bed is

[GATE–2012]

(A) 3.33×10^{-4}

(B) 3.33×10^{-1}

(C) 3

(D) 30

Q.7 A cylindrical packed bed of height 1 m is filled with equal sized spherical particles. The particles are nonporous and have a density of 1500 kg/m^3. The void fraction of the bed is 0.45. The bed is fluidized using air (density 1 kg/m^3). If the acceleration due to gravity is 9.8 m/s^2, the pressure drop (in Pa) across the bed at incipient fluidization (up to one decimal place) is_____

[GATE–2015]

ANSWER KEY:

1	2	3	4	5	6	7
B	D	D	C	B	A	8080

EXPLANATIONS

Q.1 (B)

Case-I

$\Delta P_1 = 8400\ Pa/m$

$\rho_{f1} = 1.5\ kg/m^3$

$\mu_1 = 2\times10^{-5}\ kg/m\,s$

$V_{0,1} = 2\ m/s$

$\rho_P = 0.005\ kg/m^3$

$\varepsilon = 0.5$

Case-II

$\Delta P_2 = \_\_\_\_$

$\rho_{f1} = 1.5\ kg/m^3$

$\mu_2 = 3\times10^{-5}\ kg/m\,s$

$V_{0,2} = 3\ m/s$

$$Re_1 = \frac{\rho\, v\, d}{\mu(1-\varepsilon)} = \frac{(1.5)(2)(0.005)}{(2\times10^{-5})(0.5)}$$

$$= 1500 > 1000$$

$$Re_2 = \frac{\rho\, v\, d}{\mu(1-\varepsilon)} = \frac{(1.5)(3)(0.005)}{(3\times10^{-5})(0.5)}$$

$$= 1500 > 1000$$

So, Burke Plummer Equation is used

$$\frac{\Delta P}{L} = \frac{1.75\,\rho_f V_0^{\,2}(1-\varepsilon)}{\phi\, d_p\, \varepsilon^3}$$

$$\frac{\Delta P_2}{\Delta P_1} = \left(\frac{V_{0,2}}{V_{0,1}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\Delta P_2 = \frac{9}{4}\times 8400$$

$$\Delta P_2 = 18900\ \frac{Pa}{m}$$

So Option (b) is correct

Q.2 (D)

In turbulent flow

$$\Delta P = \frac{1.75\rho V^2 \Delta L}{D_P}\frac{1-\varepsilon}{\varepsilon^3}$$

$$\Delta P \propto V^2$$

So Option (d) is correct

Q.3 (D)

At Incipient Fluidization

$$\frac{\Delta P}{L} = (\rho_p - \rho)(1-\varepsilon)g$$

$$L \propto \frac{1}{(1-\varepsilon)}$$

$$\frac{L_1}{L_2} \propto \frac{(1-\varepsilon_2)}{(1-\varepsilon_1)}$$

$$L_2 = 0.093\ m$$

So Option (d) is correct

Q.4 (C)

$$\frac{\text{Viscous Loss}}{\text{K.E. Loss}} = \frac{\dfrac{150\,\mu\, V_0\,(1-\varepsilon)^2}{(\phi\, d_p)^2\,\varepsilon^3}}{\dfrac{1.75\,\rho_f V_0^{\,2}(1-\varepsilon)}{\phi\, d_p\,\varepsilon^3}}$$

$$= \frac{150\,\mu\,(1-\varepsilon)}{1.75\,\rho_f V_0\ \phi\, d_p}$$

$$= \frac{150\times(1\times10^{-3})\times(1-0.5)}{1.75\times1000\times0.005\times0.8\times(1\times10^{-3})} = 10.71$$

So Option (c) is correct

Q.5 (B)

At Incipient fluidization V_0,

$$\frac{\Delta P}{L} = \left(\rho_P - \rho_f\right)(1-\varepsilon)\ g$$

$$= (2500-1000)(1-0.5)\ 9.8$$

$$= 7350\ \frac{Pa}{m}$$

So Option (b) is correct

Q.6 (A)

Incipient fluidization for low Re number s given by

$$\frac{\Delta P}{L} = \left(\rho_p - \rho_f\right)(1-\varepsilon)\ g = \frac{150\ \mu\ V_{mf}\ (1-\varepsilon)^2}{\left(\phi\ d_p\right)^2 \varepsilon^3}$$

$$V_{mf} = \frac{(2000)(10)\left(10^{-3}\right)\left(0.5^3\right)}{(0.5)(150)(0.1)}$$

$$= 3.33\times10^{-4}\ m/s$$

So Option (a) is correct

Q.7 (8080 Pa)

Given

$L = 1$ m

$\rho_P = 1500\ kg/m^3$

$\varepsilon = 0.5$

$\rho_f = 1\ kg/m^3$

ΔP at Incipient Fluidization

$$\frac{\Delta P}{L} = \left(\rho_P - \rho_f\right)(1-\varepsilon)\ g$$

$$= (1500-1)(1-0.45)9.8$$

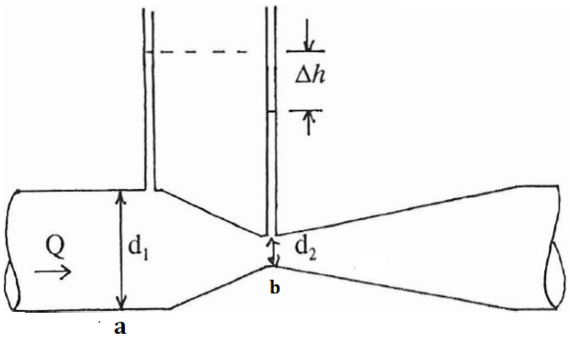
$$= 8080\ Pa$$

10 TRANSPORTATION OF FLUIDS

10.1 FLOW METERS

Flow meters are the equipment used for measuring flow in a channel

(a) Venturimeter



After applying Bernoulli Equation at point 'a' and 'b' we get

$$V_b = \frac{1}{\sqrt{\alpha_a - \alpha_b\ \beta^4}}\sqrt{\frac{2\left(P_a - P_b\right)}{\rho}}$$

where, $\beta = \frac{D_b}{D_a}$

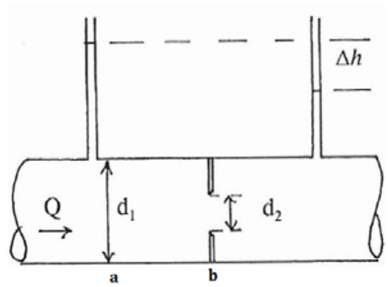
So, $Q = V_b\left(\frac{\pi}{4}D_b{}^2\right)C_V$

$$Q = \frac{C_V\left(\frac{\pi}{4}D_b{}^2\right)}{\sqrt{\alpha_a - \alpha_b\ \beta^4}}\sqrt{\frac{2\ \Delta P}{\rho}}$$

Where, C_V = Venturi Coefficient

=0.9 to 0.95

(b) Orifice Meter

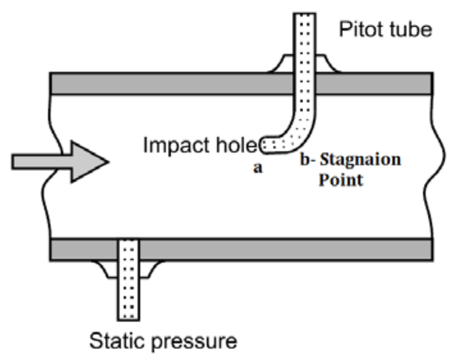


After applying Bernoulli Equation at point 'a' and 'b' we get

$$Q = \frac{C_O\left(\frac{\pi}{4}D_b{}^2\right)}{\sqrt{\alpha_a - \alpha_b\ \beta^4}}\sqrt{\frac{2\ \Delta P}{\rho}}$$

Where, C_O = Orifice Coefficient

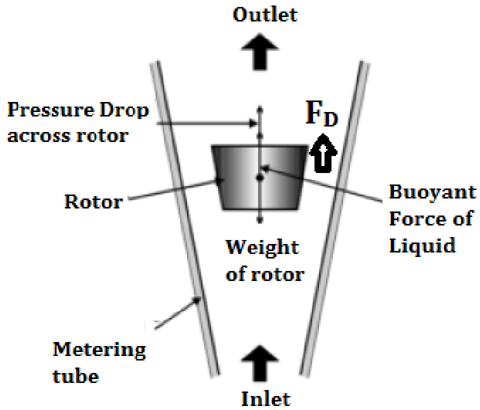
(c) Pitot tube



After applying Bernoulli Equation at point 'a' and 'b' we get

$$V_a = \frac{1}{\sqrt{\alpha_a}}\sqrt{\frac{2\ \Delta P}{\rho}}$$

(d) Rotameter



$$F_D = C_D A_P \left(\frac{\rho u^2}{2}\right)$$

Where, u = Relative Velocity

10.2 CAVITATION AND SUCTION LIFT IN PUMP

From energy consideration, whether the suction pressure is below atmospheric pressure or well above it as long as the fluid remains liquid. If the suction pressure is slightly greater than the vapour pressure, some liquid may flash to vapour inside the pump. The process is called cavitation.

If the suction pressure is actually less than the vapour pressure, there will be vapourization in the suction line and no liquid can be drawn into the pump.

10.3 NPSH

To avoid cavitation, the pressure must exceed the vapor pressure by a certain value is called NPSH (Net Positive Suction Head).

$$\text{NPSH} = \left[\frac{P_a' - P_V}{\rho g}\right] - \frac{h_{fs}}{g} - z_a$$

Where, P_V = Vapour pressure,

$$\frac{h_{fs}}{g} - z_a = \text{Suction lift}$$

10.4 POWER CONSUMPTION

An Important consideration in design of an agitated vessel is the power required to drive the impeller

It is given by

$$P = \frac{k^2\pi^2}{2} N_Q \times \rho\, n^3 D^5$$

where, D=Impeller Diameter

N_Q = Flow number

$$N_Q = \frac{q}{n\,D^3}$$

GATE QUESTIONS

Q.1 The operation of a rotameter is based on

[GATE–2001]

(A) Variable flow area
(B) Rotation of a turbine
(C) Pressure drop across a nozzle
(D) Pressure at a stagnation point

Q.2 With increasing flow rate, the hydraulic efficiency of a centrifugal pump

[GATE–2002]

(A) Monotonically decreases
(B) Decreases and then increases
(C) remains constant
(D) Increases and then decreases

Q.3 The equivalent diameter for flow through a rectangular duct of width B and height H is

[GATE –2004]

(A) $\frac{HB}{2(H+B)}$

(B) $\frac{HB}{(H+B)}$

(C) $\frac{2HB}{(H+B)}$

(D) $\frac{4HB}{(H+B)}$

Q.4 Match the following for a centrifugal pump with impeller speed n.

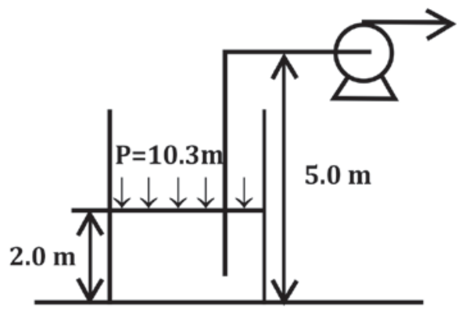
(P) Capacity	(1) proportional to n
(Q) Head	(2) proportional to n^2
	(3) proportional to n^3

[GATE –2006]

(A) P-2, Q-1 (B) P-1, Q-3

(C) P-2, Q-3 (D) P-1, Q-2

Q.5 A storage vessel exposed to atmosphere (absolute pressure: 10.3 m of water) has a diameter of 3 m and is initially filled with water to a height of 2 m. The pump draws water from the vessel and is located at an elevation of 5 m above the bottom of the vessel. The frictional head loss in the suction pipe is 2 m of water. If the vapour pressure of the liquid at the temperature of operation is 3 m of water, then the available NPSH is



[GATE –2010]

(A) 2.3 m (B) 5.3 m

(C) 6.3 m (D) 8.3 m

Q.6 Match the pumps in Group I with the corresponding fluids in Group ll.

[GATE–2011]

GROUP I	GROUP II
(P) Gear Pump	(1) Highly viscous liquid
(Q) Peristatic Pump	(2) Aqueous Sterile liquid
	(3) Slurry

(A) P-III; Q-I (B) P-II; Q-I

(C) P-III; Q-II (D) P-I; Q-II

Q.7 The local velocity of a fluid along a streamline can be measured by

[GATE–2012]

(A) Pitot tube
(B) Orifice
(C) Venturi meter
(D) Rotameter

Q.8 Slurries are most conveniently pumped by a

[GATE–2014]

(A) Syringe pump
(B) Vacuum pump
(C) Diaphragm pump
(D) Gear pump

Q.9 Match the following

GROUP I	GROUP II
(P) Turbulence	(1) Reciprocating Pump
(Q) NPSH	(2) packed Bed
(R) Ergun Equation	(3) fluctuating Velocity
(S) Rotameter	(4) Impeller
(T) power Number	(5) Vena Contracta

[GATE–2014]

(A) P-III, R-II, T-IV

(B) Q-V, R-II, S-III

(C) P-III, R-IV, T-II

(D) Q-III, S-V, T-IV

Q.10 The characteristics curve (Head-Capacity relationship) of a centrifugal pump is represented by the equation $\Delta H_{PUMP} = 43.8 - 0.19\,Q$, where ΔH_{PUMP} is the head developed by the pump (in m) and Q is the flowrate (in m^3/h) through the pump. The pump is to be used for pumping water through a horizontal pipeline. The frictional head loss ΔH_{Piping} (in m) is related to the water flowrate Q_L (in m^3/h) by the equation $\Delta H_{Piping} = 0.0135\,Q_L^2 + 0.045\,Q_L$. The flowrate (in m^3/h, rounded off to the first decimal place) of water pumped through the above pipeline is_____

[GATE–2016]

Q.11 In a venture meter, ΔP_1 and ΔP_2 are the pressure drops corresponding to volumetric flowrates Q_1 and Q_2. If $Q_2/Q_1 = 2$, then $\Delta P_2 / \Delta P_1$ equals

[GATE–2017]

(A) 2 (B) 4

(C) 0.5 (D) 0.25

Q.12 Pitot tube is used to measure

[GATE-2018]

(A) Liquid level in a tank
(B) Flow velocity at a point
(C) Angular deformation
(D) Vorticity

Q.13 A venturimeter is installed to measure the flow rate of water in a 178 mm diameter (ID) pipe. The throat diameter is 102 mm. The differential pressure measured using a manometer is 154.3 kN/m^2. The data given are: discharge coefficient 0.98; water density 1000 kg/m^3.
The volumetric flow rate of water (in m/s) is ______

[GATE-2018]

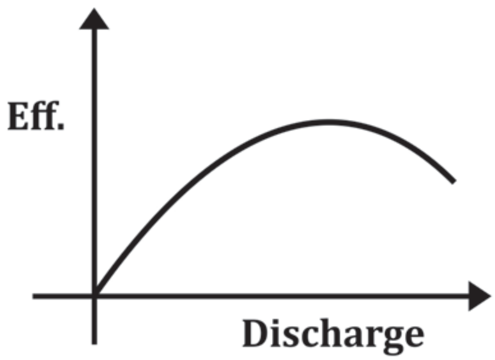
ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
A	D	C	D	A	D	A	C	A	48.9	4	B	0.148

EXPLANATIONS

Q.1 (A)

Q.2 (D)



Q.3 (C)

Equivalent Diameter = $4 \times$ Hydraulic Radius

$$= 4\times\left[\frac{\text{c/s Area}}{\text{Wetted Parameter}}\right]$$

$$= 4\times\left[\frac{BH}{2(B+H)}\right]$$

$$= \frac{2BH}{(B+H)}$$

So Option (c) is correct

Q.4 (D)

Capacity is proportional to n

Head is proportional to n^2

Q.5 (A)

Given : $h_{atm} = 10.3$ m

$h_v = 3$ m

$h_f = 2$ m

$h_l = 5 - 2 = 3$ m

NPSH= $h_{atm} - h_v - h_f - h_l$

$= 10.3 - 3 - 2 - 3$

$= 2.3$ m

So Option (a) is correct

Q.6 (D)

Q.7 (A)

Q.8 (C)

Q.9 (A)

Q.10 (48.92)

$\Delta H_{PUMP} = 43.8 - 0.19\ Q$

$\Delta H_{Piping} = 0.0135\ Q_L^{\ 2} + 0.045\ Q_L$

The pump head to compensate for head loss due to friction

$43.8 - 0.19\ Q = 0.0135\ Q^2 + 0.045\ Q$

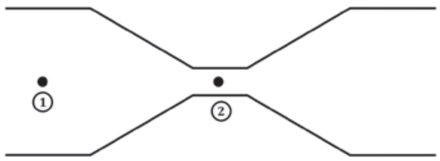
$0.0135\ Q^2 + 0.235\ Q - 43.8 = 0$

$$Q^2 + \frac{0.235}{0.0135}\ Q - \frac{43.8}{0.0135} = 0$$

$$Q = \frac{-(17.407) \pm \sqrt{(17.407)^2 - 4(1)(-3244.44)}}{2(1)}$$

$$Q = \frac{-(17.407) \pm 115.24}{2} = 48.92\ m^3/h$$

Q.11 (4)



$$V = \frac{1}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}}\sqrt{\frac{2\,\Delta P}{\rho}}$$

$$Q = A\,V$$

$$Q = \frac{\pi}{4}D_2^{\,2}\left(\frac{1}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}}\sqrt{\frac{2\,\Delta P}{\rho}}\right)$$

$$Q \propto \sqrt{\Delta P}$$

$$\frac{\Delta P_2}{\Delta P_1} = \left(\frac{Q_2}{Q_1}\right)^2 = 2^2 = 4$$

Q.12 (B)

Q.13 (0.148)

Given: $C_D = 0.98$

Let $a_1 = \frac{\pi}{4}(0.178)^2 = 0.02488\ \text{m}^2$

$a_2 = \frac{\pi}{4}(0.102)^2 = 0.00817\ \text{m}^2$

Now, $P = \rho g h$

$$h = \frac{(154.3\times10^3)}{10^3\times9.81} = 15.72\ \text{m}$$

$$\text{Vol. Flow rate}(Q) = \frac{C_D a_1 a_2\sqrt{2gh}}{\sqrt{a_1^{\,2}-a_2^{\,2}}}$$

$$Q = \frac{0.98\times(0.02488)(0.00817)\times\sqrt{2\times9.81\times15.72}}{10^3\times9.81}$$

$$Q = 0.148\ \frac{\text{m}^3}{\text{s}}$$