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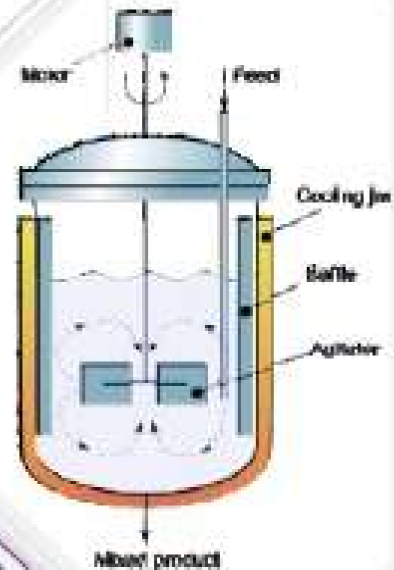

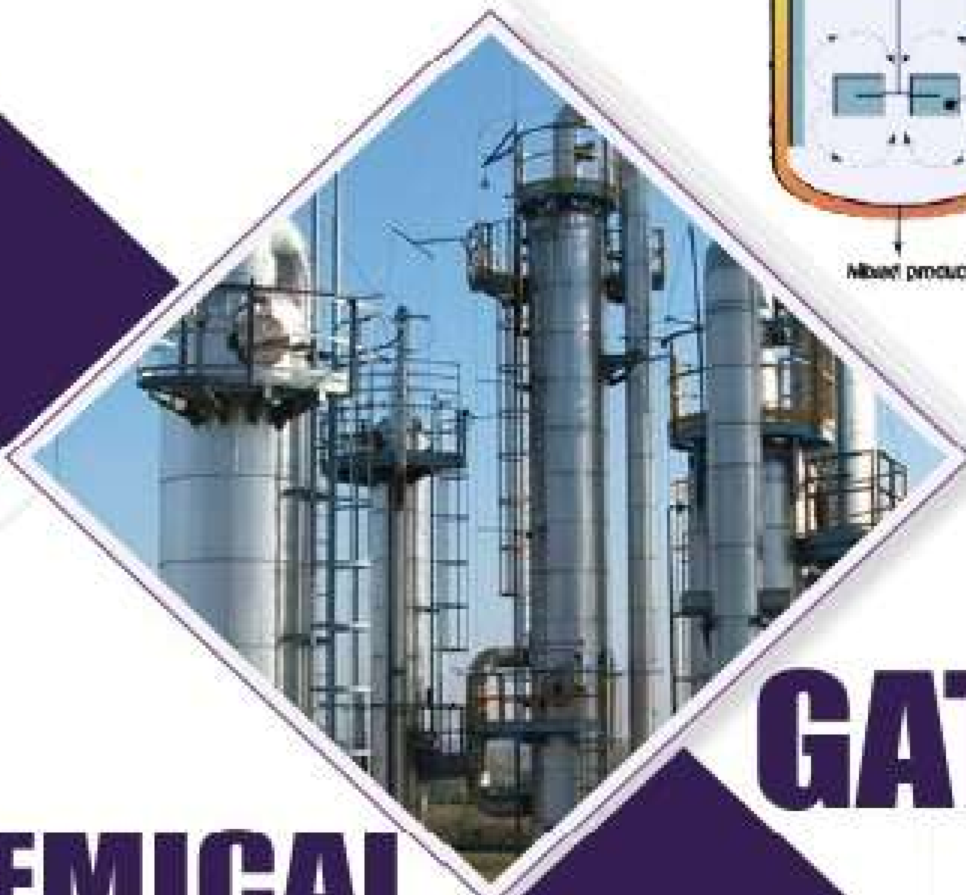

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CHAPTER 1
CHEMICAL REACTION ENGINEERING

1.1 CONCISE NOTES

1.1.1 REACTION KINETICS

The rate of reaction can be expressed as the rate of disappearance of component A as $-{dN_A}/{dt}$ The rate of change of A (in no. of moles of A). The negative sign indicates that disappearance of reactant A during reaction.

The rate of reaction can be expressed in various forms as follows:

i. Based on unit volume of reacting fluid

$$-r_A = -\frac{1}{V}\frac{dN_A}{dt}$$

ii. Based on unit mass of solid in fluid solid system

$$-r'_A = -\frac{1}{W}\frac{dN_A}{dt}$$

iii. Based on unit surface of solid in gas-solid system

$$-r''_A = -\frac{1}{S}\frac{dN_A}{dt}$$

The reaction rate is an intensive quantity & depends on concentration & temperature. From above rate equations, we've

$$(-r_A)V = (-r'_A)W = (-r''_A)S$$

1.1.2 RATE CONSTANT

The rate constant *k* depends on the reaction temperature and can be calculated by following theories

i.	Arrhenius Law	$k \propto e^{-E/RT}$
ii.	Collision Theory	$k \propto T^{1/2}e^{-E/RT}$
iii.	Transition State Theory	$k \propto Te^{-E/RT}$

1.1.3 REACTOR DESIGN

The reactor design equations for ideal reactors

1.1.3.1 FOR CONSTANT VOLUME REACTORS

i. Batch Reactor

$$t = C_{Ao}\int_0^{XA}\frac{-dX_A}{(-r_A)}$$

ii. Mixed Flow Reactor

$$\tau = \frac{C_{Ao}.X_A}{(-r_A)}$$

iii. Plug Flow Reactor

$$\tau = C_{Ao}\int_0^{X_A} \frac{-dX_A}{(-r_A)}$$

1.1.3.2 FOR VARIABLE VOLUME REACTORS

i. Batch Reactor

$$t = C_{Ao}\int_0^{X_A} \frac{-dX_A}{(1+\varepsilon_A X_A).(-r_A)}$$

ii. Mixed Flow Reactor

$$\tau = \frac{C_{Ao}.X_A}{(1+\varepsilon_A X_A).(-r_A)}$$

iii. Plug Flow Reactor

$$\tau = C_{Ao}\int_0^{X_A} \frac{-dX_A}{(1+\varepsilon_A X_A).(-r_A)}$$

NOTE: For variable volume systems, the volume varies linearly with conversion in terms of conversion as follows:

$$V = V_0(1+\varepsilon_A X_A)$$

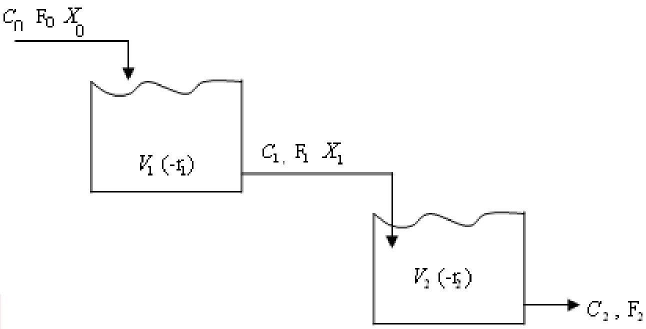
Where ε_A is the volume fraction.

1.1.4 CSTR IN SERIES

For 1st Order Reaction,

$$C_2 = \frac{C_0}{\left((1+\tau_1 K\right)(1+\tau_2 K)}$$

$$For,\ equal\ size\ CSTR,\ \tau_1 = \tau_2 = \tau\ (say)$$



$$C_2 = \frac{C_0}{(1+\tau K)^2}$$

$$For\ N\ CSTR\ in\ series$$

$$C_N = \frac{C_0}{(1+\tau k)^N}$$

$$X_N = 1 - \frac{1}{(1+\tau k)^N}$$

$$\tau_{total} = N\tau$$

- N number of CSTR in series behaves as PFR.

1.1.5 MULTIPLE REACTION SYSTEMS

1.1.5.1 PARALLEL REACTIONS

$$A \xrightarrow{k_1} R(desired)$$

$$A \xrightarrow{k_2} R(undesired)$$

$$r_R = \frac{dC_R}{dt} = k_1 C_A^{n_1}$$

$$r_S = \frac{dC_S}{dt} = k_2 C_A^{n_2}$$

$$Selectivity$$

$$\boxed{S_{R/S} = \frac{r_R}{r_S} = \frac{k_1}{k_2} C_A^{n1-n2}}$$

For $n_1 > n_2 \Rightarrow$ keep C_A high

For $n_1 < n_2 \Rightarrow$ keep C_A Low

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1.1.5.2 SERIES REACTIONS

$$A \xrightarrow[first]{k_1} R \xrightarrow[first]{k_1} S$$

- **FOR PFR / BATCH REACTOR**

$$-r_A = k_1 C_A$$

$$\frac{-dC_A}{dt} = k_1 C_A$$

On Solving, we get

$$C_A = C_{AO}\ e^{-k_1 t}$$

and

$$\frac{dC_R}{dt} = k_1 C_A - k_2 C_R$$

On Solving, we get

$$\frac{C_R}{C_{AO}} = \frac{k_1}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right)$$

hence the $C_{R\,\max}$ *and* $t_{\max}$ *will be*

$$\tau_{\max} = \frac{\ln(k_2 / k_1)}{k_2 - k_1} \quad \text{and} \quad \frac{C_{R\,\max}}{C_{A0}} = \left(\frac{k_1}{k_2}\right)^{\frac{k_1}{(k_2 - k_1)}}$$

- **FOR CSTR**

$$-r_A = k_1 C_A$$

$$r_R = k_1 C_A - k_2 C_R$$

We know that

$$C_A = \frac{C_{AO}}{1 + k_1 \tau_m}$$

Material balance on R

Input + Generation of R = Output

$$(r_R)V = vC_R$$

$$(k_1 C_A - k_2 C_R)V = vC_R$$

$$k_1 C_A \tau_m - k_2 C_R \tau_m = C_R \qquad \because \frac{V}{v} = \tau_m$$

$$(1 + k_2 \tau_m) C_R = k_1 C_A \tau_m$$

$$C_R = \frac{k_1 C_A \tau_m}{(1 + k_2 \tau_m)}$$

$$C_R = \frac{k_1 C_{A0} \tau_m}{(1 + k_1 \tau_m (1 + k_2 \tau_m)}$$

to maximize R,

$$\frac{dC_R}{d\tau_m} = 0$$

we obtain,

$$\boxed{\tau_{m\max} = \frac{1}{\sqrt{k_1 . k_2}}}$$

$$\boxed{\frac{C_{R\max}}{C_{Ao}} = \frac{1}{\left[\left(\frac{k_2}{k_1}\right)^{1/2} + 1\right]^2}}$$

1.1.6 RESIDENCE TIME DISTRIBUTION

1.1.6.1 RESIDENCE TIME

The time the atoms have spent in the reactor is called the *residence time* of the atoms in the reactor. All molecules take different path so residence time is not same for all molecules. The distribution of residence times for all molecules in leaving the vessel is called *exit age distribution* (E) or RTD.

1.1.6.2 RTD MEASUREMENT

The RTD is determined experimentally by injecting a tracer, into the reactor at some time *t = 0* and then measuring the tracer concentration, *C*, in the effluent stream as a function of time.

The two most used methods of injection are *pulse input* and *step input*.

i. PULSE INPUT EXPERIMENT

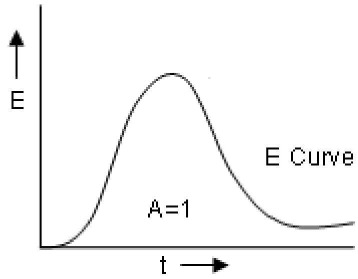
In pulse input the amount of tracer M is suddenly injected in 1 shot into the feed stream entering the reactor at time t = 0.

Let dM is amount of material living at time between t & t + d t

dM = C.v.dt

Let fraction of material that has residence time between t & t + Δ t,

$$E = \frac{C.v}{M} = \frac{C}{M/v}$$



$$\int_0^M dM = \int_0^\infty C.v.dt$$

$$M = v\int_0^\infty C.dt$$

$$\frac{M}{v} = \int_0^\infty C.dt$$

$$\therefore \frac{C}{E} = \int_0^\infty C.dt \Rightarrow \boxed{E = \frac{C}{\int_0^\infty C.dt}} \; (time)^{-1}$$

- **E CURVE FROM C CURVE:-**

From the data of *C v/s t.* Plot C curve & find the area under the curve then find E for each time & plot *E v/s t* which is the E curve.

If we consider time interval t_1 to t_2 fraction of material leaving the vessel that has resided in the vessel between times t_1 & t_2 = $\int_{t_1}^{t_2} E\,dt$

Fraction of all the material that has resided for a time t in the vessel between $t = 0$ *to* $t = \infty$ *is* 1.

$$\boxed{\int_0^\infty E.\,dt = 1}$$

Age for an element refers to the time spent by the element of the exit stream of vessel (i.e. time it has resided in the vessel).

The fraction of the exit stream of age t_1 is $\int_0^{t_2} E.\,dt$

The fraction of material older then t_1 *is*

$$\int_{t_1}^{\infty} E dt = \int_{0}^{\infty} E.dt - \int_{0}^{t_1} E.\,dt$$

$$\int_{t_1}^{\infty} E dt = 1 - \int_{0}^{t_1} E.\,dt$$

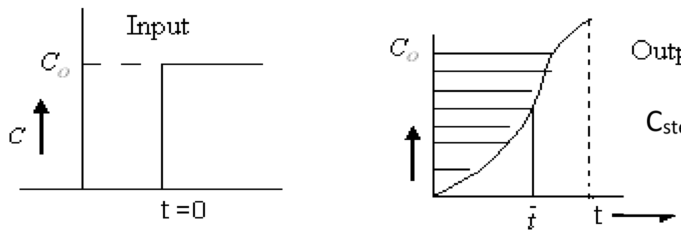
ii. STEP INPUT EXPERIMENT

Consider a constant rate of tracer addition to a feed that is initiated at time t = 0. Before this time no tracer is added to the reactor feed. Stated symbolically, we've

$$C_o(t) = \begin{cases} 0 & t < 0 \\ (C_o) constant & t \geq 0 \end{cases}$$

The concentration of tracer in the feed to the reactor is kept at this level until the concentration in the effluent is indistinguishable from that in the feed.

$$C_O = \frac{\dot{m}}{v} \text{ and } \bar{t} = \frac{V}{v}$$

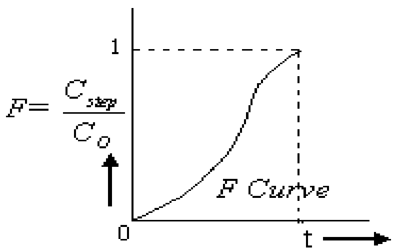


Shaded are under the Curve,

$$= \bar{t} \times C_O = \int_{O}^{C_O} t \times d\ C_{step}$$

$$\Rightarrow \bar{t} = \frac{1}{C_O} \int_{0}^{C_O} t.dC_{step}$$

- The dimensionless form of the C_{step} Curve is called *F Curve*. i.e. $\left(\frac{C_{step}}{C_O}\right) v/s\ t$

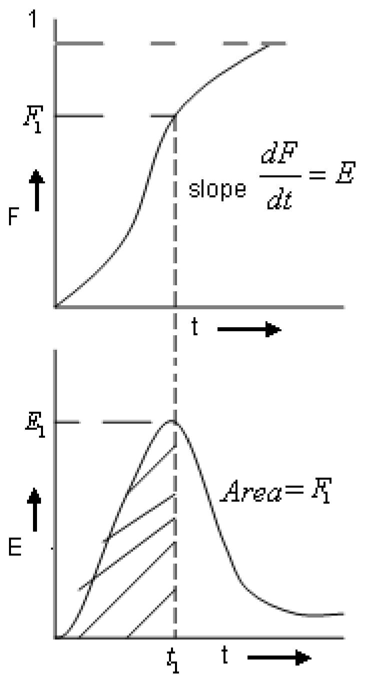


- **RELATION BETWEEN E & F CURVE**

Steady flow of white fluid through vessel at t = 0 we switch to red fluid and measure the rising concentration of red fluid in the effluent at t > 0.

$$F = \int_{o}^{t} .E\ dt \qquad Or \qquad \boxed{\frac{dF}{dt} = E}$$

- **GRAPHICAL REPRESENTATION**



a. MEAN RESIDENCE TIME ($\bar{t}$)

The first moment of the RTD function is the *mean residence time*.

$$\bar{t} = \frac{\int_{o}^{\infty} tE.dt}{\int_{o}^{\infty} E.dt} = \int_{o}^{\infty} tE.dt$$

Consider, we have reactor completely filled with white molecules.

At time *t = 0* we start to inject red molecules to replace white molecules to replace white molecules

In time *dt* the volume molecule leaving the fraction of those molecules that have been in the reactor for time *t* is *1 – f.* The volume of white molecules leaving the reactor at *dt* is $dV = v.(1-F)dt$

$$V = \int_{O}^{\infty} v.(1-F)\,dt$$

$$\frac{V}{v} = \int_{O}^{\infty} (1-F)dt$$

$$\tau = \int_{o}^{\infty} (1-F)dt$$

$$\int x \text{ dt} = \text{xy-}\int_{0}^{y} dt \text{ integration by part}$$

$$\Rightarrow \tau = \left[(1-F)t\right]_{o}^{\infty} + \int_{o}^{t} dF$$

$$At\ t = 0,\ F = 0$$
$$as\ t \to \infty\ \ F = 1$$
$$first\ term\ =\ 0$$

$$\tau = \int_{o}^{t} F.dt$$

$$Now,\ \frac{dF}{dt} = E$$
$$dF = Edt$$

$$\boxed{\tau = \int_{o}^{\infty} t.E.dt}$$

$\therefore$ $\boxed{\bar{t} = \tau}$ $\to$ *at* no dispersion

The second moment of RTD is called *variance* or square of standard deviation.

$$\sigma^2 = \int_0^{\infty} \left(t - \bar{t}\right)^2 E.dt$$

The magnitude of variance indicates the spread of the residence time distribution. The greater the value of variance indicates greater residence time distribution spread.

If σ^2 **>>** 1 $\Rightarrow$ RTD >> 1

And when RTD >> 1 $\Rightarrow$ Non ideality is more

$\sigma^2 = 0$ $\Rightarrow$ Reactor is ideal.

1.2 SOLVED PROBLEMS

1. At 500 K, reaction is 10 times faster than that of at 400 K, find the activation energy (E), by Collision theory?

Solution: from Collision theory, $k = T^{1/2}.e^{-E/RT}$

Or

$$\ln\left(\frac{k_2}{k_1}\right) = \ln\left(\frac{T_2}{T_1}\right) + \frac{E}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln\left(\frac{k_2}{k_1}\right) = \ln\left(\frac{500}{400}\right) + \frac{E}{1.98624}\left[\frac{1}{400} - \frac{1}{500}\right]$$

$$\boxed{E = 8707\frac{cal}{mol}}$$

2. For the series reaction $A \xrightarrow{k_1} R \xrightarrow{k_2} S$ and $k_1 = k_2$, find C_{Rmax} and t_{max}.

Solution:

We know that

$$-r_A = -\frac{dC_A}{dt} = k_1 C_A$$

$$\Rightarrow C_A = C_{Ao} e^{-k_1.t}$$

$$r_R = \frac{dC_R}{dt} = k_1 C_A - k_2 C_R$$

$$\Rightarrow \frac{dC_R}{dt} + k_1 C_R = k_1 C_{A0}.e^{-k_1 t} \qquad \because k_2 = k_1$$

$$C_R.e^{\int k_1 dt} = \int k_1 C_{A0.} e^{-k_1 t} e^{k_1 t} dt + C$$

$$C_R.\ e^{k_1 t} = \int k_1 C_{A0}.dt + C$$

$$C_R.e^{k_1 t} = k_1 C_{A0} t + \not{C}^0$$

$$C_R.e^{k_1 t} = k_1 C_{A0} t$$

$$C_R = k_1 C_{A0} t.\ e^{-k_1 t}$$

for $C_{R\max}$, *put* $\frac{dC_R}{dt} = 0,$

$$\frac{dC_R}{dt} = k_1 C_{A0}\left[t.e^{-K_1 t}\left((1-k_1\right) + e^{-k_1 t}\right] = 0$$

$$-k_1 t\ e^{-k_1 t} + e^{-k_1 t = 0}$$

$$k_1 t = 1$$

$$\boxed{t_{\max} = 1/k_1}$$

putting this value in C_R *expression, we get*

$$C_{R\max} = k_1 C_{A0} . e^{-k_1 . \frac{1}{k_1}} . \frac{1}{k_1}$$

$$\boxed{C_{R\max} = \frac{C_{A0}}{e}}$$

3. What is the overall conversion for two CSTR connected in series for the given reaction, if v_o = 5 litre / min, C_{Ao}= 10 mol / litre. And the volume of vessels are 15 litre each.

$$A \xrightarrow{k=4\min^{-1}} B$$

Solution: For CSTR, we know that

$$\tau = \frac{C_{Ao}.X_A}{(-r_A)} \quad \Rightarrow \tau = \frac{C_{Ao}.X_A}{k.C_A} \quad \Rightarrow \tau = \frac{C_{Ao} - C_A}{k.C_A}$$

$$\Rightarrow k\tau = \frac{C_A}{(1 - C_A)}$$

$$\Rightarrow C_A = \frac{C_{Ao}}{(1 + k\tau)}$$

$$\tau_1 = \tau_2 = \tau = \frac{V_1}{v_o} = \frac{15}{5} = 3 \text{ min}$$

$$for\ CSTR - 1,$$

$$\Rightarrow C_{A1} = \frac{C_{Ao}}{(1 + k\tau_1)}$$

$$for\ CSTR - 2,$$

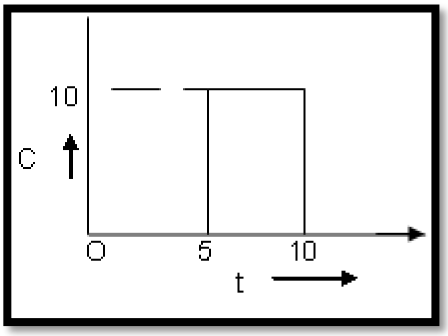
$$\Rightarrow C_{A2} = \frac{C_{A1}}{(1 + k\tau_2)} \qquad \Rightarrow C_{A2} = \frac{C_{Ao}}{(1 + k\tau)^2}$$

$$\Rightarrow C_{A2} = \frac{10}{(1 + 4 \times 3)^2} \qquad \Rightarrow C_{A2} = 0.05917 \frac{mol}{litre}$$

$$\therefore \Rightarrow X_{A2} = 1 - \frac{C_{A2}}{C_{Ao}} = 1 - \frac{0.05917}{10}$$

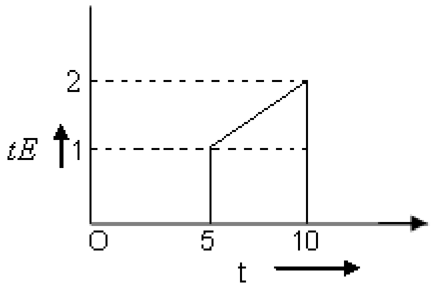
$$\Rightarrow \boxed{X_{A2} = 0.9941}$$

4. An isothermal pulse test is conducted on reactor, what is mean residence time of fluid?



Solution:

t	C	E	t.E
0	0	0	0
5	10	0.2	1
7.5	10	0.2	1.5
10	10	0.2	2



$\bar{t} = Area\ under\ curve$

$\bar{t} = (10-5)\times 1 + 0.5\times 5\times 1$

$\boxed{\bar{t} = 7.5\ \text{min}}$

5. An isothermal aqueous phase reversible reaction $P \rightleftarrows R$ is to be carried out in a mixed flow reactor. The reaction rate in (kmol / m^3 h) is given by r = 0.5 C_P – 0.125C_R. A stream containing only P enters the reactor. The residence time required (in hours) for 40% conversion of P would be?

Solution: We know the design equation of CSTR is given by

$$\tau = C_{PO}\frac{X_P}{-r_P} = C_{PO}\frac{X_P}{0.5C_P - 0.125C_R}$$

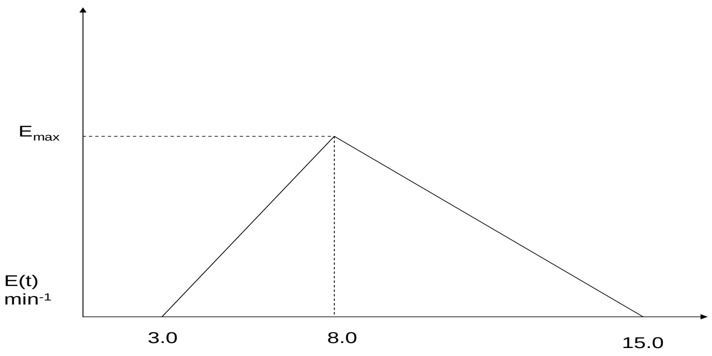
$$\tau = C_{PO}\frac{X_P}{0.5C_{PO}(1-X_P) - 0.125(C_{PO}X_P)} = \frac{X_P}{0.5(1-X_P) - 0.125X_P}$$

$At\ X_P = 0.4,$

$$\tau = \frac{0.4}{0.5\times 0.6 - 0.125\times 0.4}$$

$\boxed{\tau = 1.6\ hours}$

6. The residence time distribution E(t) (as shown below) of a reactor is zero until 3 minutes and then increases linearly to a maximum value E_{max} at 8 minutes after which it decreases linearly back to zero at 15 minutes. what is the value of E_{max}?



Solution:

$\because \int_0^\infty E(t)dt = 1$

$\therefore$ *Area under the curve* $= 1$

$$\text{Area of Left Triangle} = \frac{1}{2}\times(8-3)\times E_{max} = \frac{5}{2}E_{max}$$

$$\text{Area of Right Triangle} = \frac{1}{2}\times(15-8)\times E_{max} = \frac{7}{2}E_{max}$$

$$\therefore \text{Total Area} = \frac{5}{2}E_{max} + \frac{7}{2}E_{max} = 1$$

or $\frac{12}{2}E_{max} = 1$

or $\boxed{E_{max} = \frac{1}{6}}$

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CHAPTER 2
MASS TRANSFER OPERATIONS

2.1 Concise Notes

2.1.1 DIFFUSION

The flux is defined as anything is passing through per unit area per unit time. Molar flux of a given species is a vector quantity denoting the amount of the particular species, in either mass or molar units, that passes per given increment of time through a unit area normal to the vector. The flux of species defined with reference to fixed spatial coordinates, N_A is

$$N_A = C_A \nu_A$$

This could be written in terms of diffusion velocity of A, (i.e., $\nu_A - \nu$) and average velocity of mixture, ν, as

$$N_A = C_A(\nu_A - \nu) + C_A \nu$$

2.1.1.1 FICK'S LAW

An empirical relation for the diffusional molar flux, first postulated by Fick and, accordingly, often referred to as Fick's first law, defines the diffusion of component A in an isothermal, isobaric system. For diffusion in only the Z direction, the Fick's rate equation is

$$J_A = -D_{AB}\frac{dC_A}{dZ}$$

where D_{AB} is diffusivity or diffusion coefficient for component A diffusing through component B, and dC_A / dZ is the concentration gradient in the Z-direction.

2.1.1.2 STEADY STATE DIFFUSION OF A THROUGH A STAGNANT GAS B

Here $N_B = 0$, and N_A = constant, Thus

$$N_A = \frac{D_{AB}}{RT(z_2 - z_1)}\frac{(p_{A1} - p_{A2})}{p_{B,\ln}}$$

2.1.1.3 EQUIMOLAR DIFFUSION

Here $N_A = -N_B$ = constant, Thus

$$N_A = \frac{D_{AB}}{RT(z_2 - z_1)}(p_{A1} - p_{A2})$$

2.1.2 CONVECTIVE MASS TRANSFER

Molar flux N is given by,

$$N_A = k\Delta C$$

Where *k* is the mass transfer coefficient.

- **Mass Transfer theories**
  - **i. Film Theory**

$$k = D/\delta$$

  - **ii. Penetration theory**

$$k = 2\sqrt{D\delta/\pi V}$$

  - **iii. Surface renewal theory**

$$k = \sqrt{D/\tau}$$

2.1.3 DISTILLATION

Distillation is a method of separating the components of a solution which depends upon the distribution of the substances between a gas and a liquid phase, applied to cases where all components are present in both phases.

2.1.3.1 VAPOR LIQUID EQUILIBRIUM

The vapor liquid equilibria is the basic measure for the separation of components through distillation, it is the condition where the component composition in liquid phase is directly related to that in vapor phase and the chemical potential of the components in both phase is same.

2.1.3.2 RELATIVE VOLATILITY

This is the ratio of A and B in one phase to that in the other and is a measure of the separability.

$$\alpha = \frac{y^*/(1-y^*)}{x/(1-x)} = \frac{y^*(1-x)}{x(1-y^*)}$$

- If $\alpha = 1 \Rightarrow$ No separation is possible.

2.1.3.3 RAOULT'S LAW

For an ideal solution, the equilibrium partial pressure $\bar{p}^*$ of a constituent at a fixed temperature equals the product of its vapor pressure p, when pure at this temperature and its mole fraction in liquid. This is Raoult's Law.

$$\bar{p}_A^* = p_A x \qquad \bar{p}_B^* = p_B(1-x)$$

2.1.3.4 CONSTANT BOILING MIXTURES

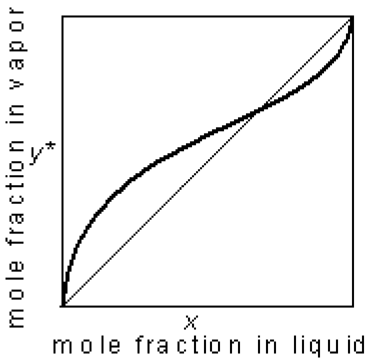
An azeotrope or constant boiling mixture are those in which composition of the liquid does not change as it is converted into vapor, vapor has the same composition as the liquid and the boiling point remains constant as vapor is generated.

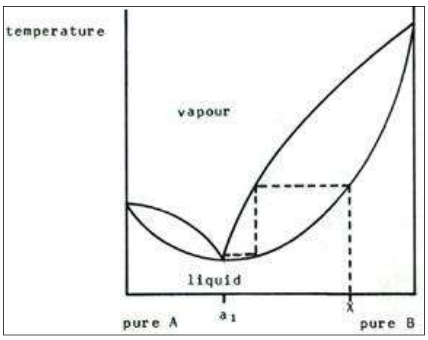
As the dew point and the bubble point are identical at the azeotropic composition, the azeotropes are called constant boiling mixtures.

i. MINIMUM BOILING MIXTURES

A minimum boiling azeotrope will boil at a temperature lower than the boiling points of the pure components.

A well-known example of a positive azeotrope is 95.63% ethanol and 4.37% water (by weight). In general, a positive azeotrope boils at a lower temperature than any other ratio of its constituents. Minimum boiling mixtures are also called **positive** azeotropes or **pressure maximum** azeotropes.

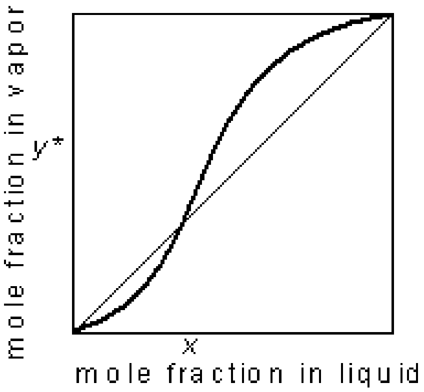


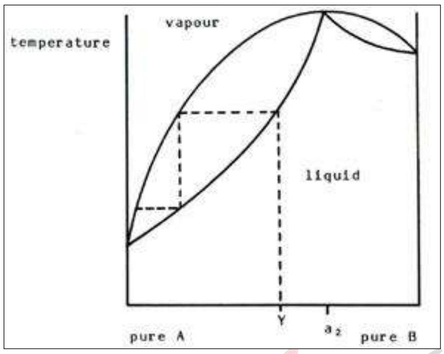


ii. MAXIMUM BOILING MIXTURES

A maximum boiling azeotrope will boil at higher temperature than the boiling points of pure components.

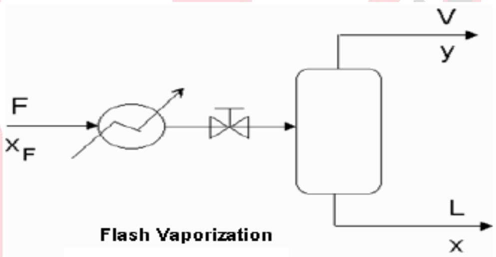
An example of a negative azeotrope is hydrochloric acid at a concentration of 20.2% and 79.8% water (by weight). In general, a negative azeotrope boils at a higher temperature than any other ratio of its constituents. Maximum boiling mixtures are also called **negative** azeotropes or **pressure minimum** azeotropes.





2.1.3.5 FLASH VAPORIZATION

Flash vaporization or also known as equilibrium distillation.



The product, D mol/time, richer in the more volatile substance, is in this case entirely a vapor. The material and enthalpy balances are

$$F = D + W$$

$$Fz_F = Dy_D + Wx_W$$

$$FH_F + Q = DH_D + WH_W$$

Solved simultaneously, these yield

$$-\frac{W}{D} = \frac{y_D - z_F}{x_W - z_F} = \frac{H_D - (H_F + Q/F)}{H_W - (H_F + Q/F)}$$

2.1.3.6 DIFFERENTIAL DISTILLATION

In the case of a differential distillation, the vapor at any time is in equilibrium with the liquid from which it rises but changes continuously in the composition.

Rayleigh's Equation:

$$\ln\frac{F}{W} = \int_{x_W}^{x_F} \frac{dx}{y^* - x}$$

For Constant α :

$$\ln\frac{Fx_F}{Wx_W} = \alpha \ln\frac{F(1-x_F)}{W(1-x_W)}$$

2.1.3.7 REFLUX RATIO

The reflux ratio is given by the ratio of the amount of distillate taken out in product to the amount of distillate from condenser recycle to column.

$$R = \frac{L}{D}$$

At the minimum reflux ratio (R_{min}), column requires infinite no. of trays. Optimum reflux ratio is about 1.2 – 1.5 times of R_{min}.

And can be calculated by

For saturated vapor feed:

$$R_m = \frac{1}{\alpha - 1}\left[\frac{\alpha x_D}{x_F} - \frac{1-x_D}{1-x_F}\right] - 1$$

For saturated liquid feed:

$$R_m = \frac{1}{\alpha - 1}\left[\frac{x_D}{x_F} - \frac{\alpha(1-x_D)}{(1-x_F)}\right]$$

2.1.4 GAS ABSORPTION

Absorption (also called gas absorption, gas scrubbing, or gas washing), refers to the transfer of one or more species from the gas phase to a liquid solvent. The species transferred to the liquid phase are referred to as *solutes or absorbate*.

2.1.4.1 SINGLE STAGE ABSORPTION

Most absorption or stripping operations are carried out in counter current flow processes, in which the gas flow is introduced in the bottom of the column and the liquid solvent is introduced in the top of the column. The mathematical analysis for both the packed and plated columns is very similar.

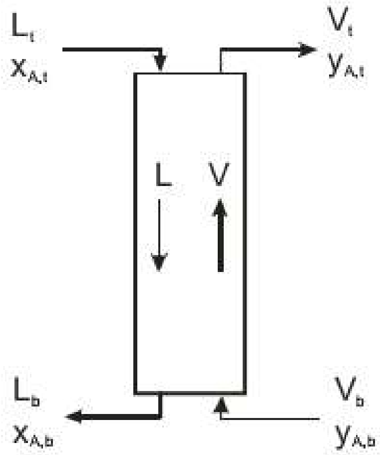


Figure 5.2-1 Countercurrent absorption process.

The overall material balance for a countercurrent absorption process is

$$L_b + V_t = L_t + V_b$$

where V = vapor flow rate
L = liquid flow rate
t, b = top and bottom of tower, respectively

The component material balance for species A is

$$L_b x_{A,b} + V_t y_{A,t} = L_t x_{A,t} + V_b y_{A,b}$$

where y_A = mole fraction of A in the vapor phase
x_A = mole fraction of A in the liquid phase

For some problems, the use of *solute-free* basis can simplify the expressions. The solute-free concentrations are defined as:

$$\bar{X}_A = \frac{x_A}{1-x_A} = \frac{\text{mole fraction of A in the liquid}}{\text{mole fraction of non-A components in the liquid}}$$

$$\bar{Y}_A = \frac{y_A}{1-y_A} = \frac{\text{mole fraction of A in the vapor}}{\text{mole fraction of non-A components in the vapor}}$$

If the carrier gas is completely insoluble in the solvent and the solvent is completely nonvolatile, the carrier gas and solvent rates remain constant throughout the absorber. Using

$\bar{L}$ to denote the flow rate of the nonvolatile and $\bar{V}$ to denote the carrier gas flow rate, the material balance for solute A becomes

$$\bar{L}\ \bar{X}_{A,b} + \bar{V}\ \bar{Y}_{A,t} = \bar{L}\ \bar{X}_{A,t} + \bar{V}\ \bar{Y}_{A,b}$$

or

$$\bar{Y}_{A,t} = \frac{\bar{L}}{\bar{V}}\bar{X}_{A,t} + \left(\bar{Y}_{A,b} - \frac{\bar{L}\bar{X}_{A,b}}{\bar{V}}\right)$$

The material balance for solute A can be applied to any part of the column. For example, the material balance for the top part of the column is

$$\bar{Y}_{A,J} = \frac{\bar{L}}{\bar{V}}\bar{X}_{A,I} + \left(\bar{Y}_A - \frac{\bar{L}\bar{X}_A}{\bar{V}}\right)$$

In this equation, $\bar{X}_A$ and $\bar{Y}_A$ are the mole ratios of A in the liquid and vapor phase, respectively, at any location in the column including at the two terminals. Above equation is called the operation line and is a straight line with slope $\frac{\bar{L}}{\bar{V}}$ when plotted on $\bar{X}_A$-$\bar{Y}_A$ coordinates.

2.1.4.2 CALCULATION FOR IDEAL NUMBER OF PLATES BY ABSORPTION FACTOR METHOD

In the case of L_{min} (Minimum Solvent requirement) y_b will be in equilibrium with x_b. and y_b can be calculated by equilibrium relation ($y^* = m\, x_b$) with the help of x_b.

And N is given by

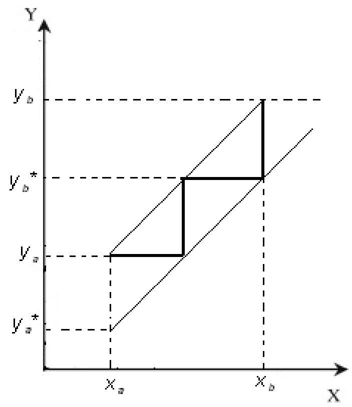
$$N = \frac{\ln\left[\frac{(y_b - y_b{}^*)}{(y_a - y_a{}^*)}\right]}{\ln A}$$

This is equation cannot be used when A = 1.

- **ABSORPTION FACTOR**

It can be defined as the ration of slope of operating line to the slope of equilibrium curve.

$$A = \frac{\text{slope of operating line}}{\text{slope of equilibrium curve}} = \frac{L/V}{m} = \frac{L}{mV}$$

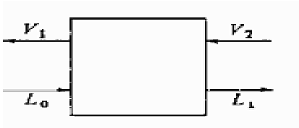


2.1.5 LIQUID EXTRACTION

It is a process of transferring a solute from one liquid phase to another immiscible or partially miscible liquid in contact with the first.

2.1.5.1 SINGLE-STAGE LIQUID EXTRACTION

In an extraction process we have two entering streams (L kg and V kg) which are not in equilibrium, as shown in figure given below. The solvent, as stream V_2, enters and the stream L_0 enters from the other side. The two entering streams are mixed and equilibrated and then exit as streams L_1 and V_1, which are in equilibrium with each other. To find the final product compositions in the two phases, it is required to know the mixture total mass and composition (point M). This can be obtained by material balances. After the point M is identified, the product composition can be found by the equilibrium tie line.



Material balances:

$$\text{Overall: } L_0 + V_2 = L_1 + V_1 = M$$
$$\text{(A): } L_0 x_{A0} + V_2 y_{A2} = L_1 x_{A1} + V_1 y_{A1} = M x_{AM}$$
$$\text{(C): } L_0 x_{C0} + V_2 y_{C2} = L_1 x_{C1} + V_1 y_{C1} = M x_{CM}$$

Since $x_A + x_B + x_C = 1$, an equation for B is not needed. because L0and V2are known, values of M, x_{AM}, and x_{CM}, can be found from all above equations. $L_1$and V_1 are obtained by drawing a tie line through point M.

By doing material balances, we have

$$\text{Overall: } V + L = M$$
$$\text{(A): } Lx_A + Vy_A == Mx_{AM}$$
$$\text{(C): } Lx_C + Vy_C == Mx_{CM}$$

From the combination of these equations, we get

$$\frac{L}{V} = \frac{x_{AM} - y_A}{x_A - x_{AM}}$$
$$\frac{L}{V} = \frac{x_{CM} - y_C}{x_C - x_{CM}}$$

On solving, we get

$$\frac{x_C - x_{CM}}{x_A - x_{AM}} = \frac{x_{CM} - y_C}{x_{AM} - y_A}$$

The left side is the slope of line LM and the right side is the slope of line MV. Because the two slopes are the same and the two lines have a common point M, the three points L, M, and V must be on a straight line. The lever-arm rule is

$$\frac{L}{V} = \frac{\overline{VM}}{\overline{LM}} \quad \& \quad \frac{L}{M} = \frac{\overline{VM}}{\overline{LV}}$$

2.1.5.2 MINIMUM SOLVENT RATE

If a solvent rate V_{N+1} is selected at too low a value, a limiting case will be reached with an operating line through Δ and a tie line being the same. Then an infinite number of stages will be needed to reach the desired separation. The minimum amount of solvent is reached. For actual operation a greater amount of solvent must be used.

The procedure to obtain this minimum solvent rate is as follows and shown in the right figure. Firstly line L_NV_{N+1} is extended, then all tie lines between L_0 and L_N are drawn to intersect the extended line L_NV_{N+1}. The intersection farthest from V_{N+1}(if Δ is in the L_N side, which is the case in the figure) or nearest V_{N+1}(if Δ is on the V_{N+1}side) is the Δ_{min} point for minimum solvent.

The actual position of Δ must be farther from V_{N+1} (if on the L_N side) or nearer to V_{N+1}(if on the V_{N+1}side) for a finite number of stages. The larger the amount of solvent, the fewer the number of stages.

This is proved as below.

If Δ is in the L_N side,

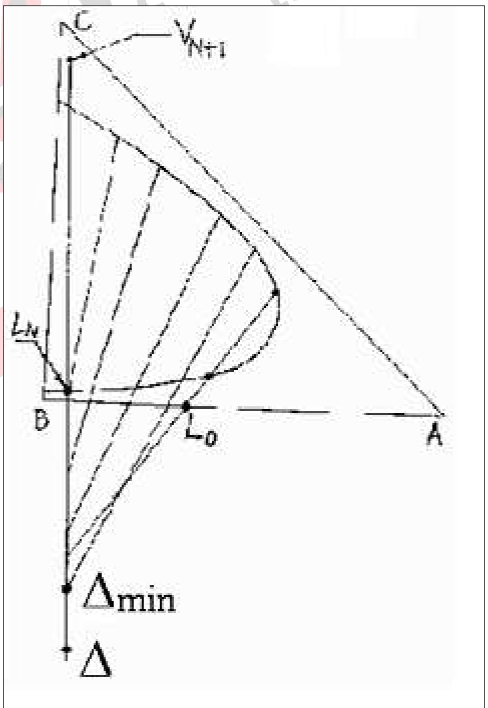
$$L_N - V_{N+1} = \Delta$$
$$L_N = V_{N+1} + \Delta$$

According to lever arm's rule:

$$(V_{N+1})\overline{V_{N+1}L_N} = \Delta\overline{\Delta L_N}$$
$$=(L_N - V_{N+1})\overline{\Delta L_N}$$
$$= L_N\overline{\Delta L_N} - V_{N+1}\overline{\Delta L_N}$$

$$V_{N+1} = \frac{L_N\overline{\Delta L_N}}{\overline{V_{N+1}L_N} + \overline{\Delta L_N}}$$

$$V_{N+1} = \frac{L_N}{\frac{\overline{V_{N+1}L_N}}{\overline{\Delta L_N}}+1}$$



Minimum solvent for countercurrent extraction.

Since L_N and $\overline{V_{N+1}L_N}$ are constant, V_{N+1}will achieve its maximum when the length of $\overline{\Delta L_N}$ is at its maximum.

If Δ is in the V_{N+1}side,

$$(V_{N+1})\overline{V_{N+1}\Delta} = L_N\overline{L_N\Delta}$$

$$V_{N+1} = \frac{L_N\overline{L_N\Delta}}{\overline{V_{N+1}\Delta}} = L_N\frac{\overline{L_N V_{N+1}} + \overline{V_{N+1}\Delta}}{\overline{V_{N+1}\Delta}}$$

$$= L_N\left(1 + \frac{\overline{L_N V_{N+1}}}{\overline{V_{N+1}\Delta}}\right)$$

Since L_N and $\overline{V_{N+1}L_N}$ are constant, V_{N+1} will achieve its maximum when the length of $\overline{V_{N+1}\Delta}$ is at its minimum.

2.1.6 DRYING

The separation operation of drying converts a solid, semi-solid or liquid feedstock into a solid product by evaporation of the liquid into a vapor phase via application of heat.

2.1.6.1 MOISTURE CONTENT

The liquid content present in the wet solid which to be dried by evaporation is called moisture content. We can classified the moisture content in the process as follows:

- **FREE MOISTURE CONTENT (X)**

It can be defined as the moisture content in excess of the equilibrium moisture content (hence free to be removed) at given air humidity and temperature.

- **UNBOUND MOISTURE CONTENT**

It can be defined as the moisture in solid which exerts vapor pressure equal to that of pure liquid at the same temperature.

- **BOUND MOISTURE CONTENT**

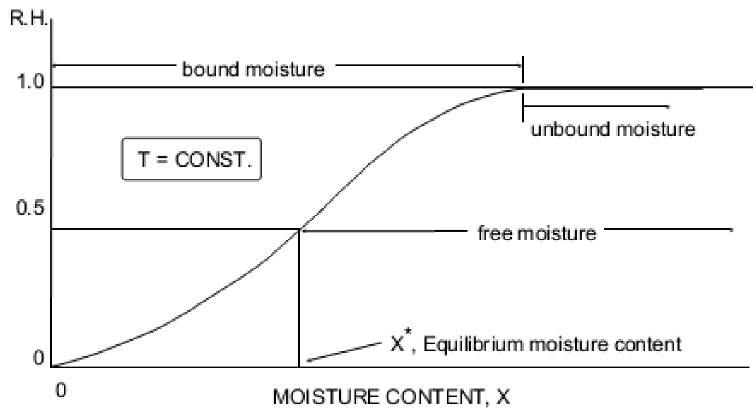
It can be defined as the moisture physically and/or chemically bound to solid matrix so as to exert a vapor pressure lower than that of pure liquid at the same temperature.

- **EQUILIBRIUM MOISTURE CONTENT (X*)**

It can be defined as the moisture content of moist solid in equilibrium with the gas-vapor mixture (zero for non-hygroscopic solids) at a given temperature and pressure.

- **CRITICAL MOISTURE CONTENT (X_C)**

It can be defined as the moisture content of the material at which the constant rate period ends and the falling rate period starts. It is the function of constant drying rate, material properties and particle size.



Various types of moisture content

2.1.6.2 TIME REQUIRED FOR DRYING

i. For Constant Rate Period

$$t_C = \frac{L}{AR_C}(X_1 - X_2)$$

ii. For Falling Rate Period

$$t_F = \frac{L}{AR_C}\left[(X_C - X^*)\ln\left(\frac{X_C - X^*}{X_2 - X^*}\right)\right]$$

2.2 SOLVED PROBLEMS

1. A sphere of naphthalene having a radius of 2mm is suspended in a large volume of shell air at 318 K and 1 atm. The surface pressure of the naphthalene can be assumed to be at 318 K is 0.555 mm Hg. The D_{AB} of naphthalene in air at 318 K is $6.92 * 10^{-6}$ m^2/sec. Calculate the rate of evaporation of naphthalene from the surface.

Solution:

Steady state mass balance over a element of radius r and r + δr leads to

$$SN_A\big|_r - SN_A\big|_{r+\delta r} = 0 \text{ ---------------------------- (1)}$$

where S is the surface are (= $4\pi r^2$)

dividing (1) by Sδr, and taking the limit as δr approaches zero, gives:

$$\frac{d\left(r^2 N_A\right)}{dr} = 0$$

Integrating $r^2 N_A$ = constant (or) $4 \pi r^2 N_A$ = constant

We can assume that there is a film of naphthalene – vapor / air film around naphthalene through which molecular diffusion occurs.

Diffusion of naphthalene vapor across this film could be written as,

$$N_A = -CD_{AB}\frac{dy_A}{dr} + y_A\left(N_A + N_B\right)$$

$N_B = 0$ (since air is assumed to be stagnant in the film)

$$N_A = -CD_{AB}\frac{dy_A}{dr} + y_A N_A$$

$$N_A = -CD_{AB}\frac{d}{dr}\left(\frac{y_A}{1-y_A}\right)$$

$$N_A = CD_{AB}\frac{d\left[\ln\left(1-y_A\right)\right]}{dr}$$

W_A = Rate of evaporation = $4 \pi r^2 N_A |_R$ = constant.

$$W_A = \frac{4\pi r^2 CD_{AB}\, d\left(\ln\left(1-y_A\right)\right)}{dr}$$

$$W_A \int \frac{dr}{r^2} = 4\pi D_{AB} \int C\, d\, \ln\left(1-y_A\right)$$

Boundary condition:

At r = R $\quad y_A = \frac{0.555}{760} = 7.303 * 10^{-4}$

$\ln(1 - y_A) = -7.3 * 10^{-4}$

At $r = \infty \quad y_A = 0 \quad \ln(1-y_A) = 0$

Therefore $W_A \int_R^{\infty} \frac{dr}{r^2} = 4\pi D_{AB}\, C \int_{-7.3*10^{-4}}^{0} d\left[\ln\left(1-y_A\right)\right]$

$$W_A \left[\frac{-1}{r}\right]_R^{\infty} = 4\pi D_{AB}\, C \left[\ln\left(1-y_A\right)\right]_{-7.3*10^{-4}}^{0}$$

$$W_A \left[0+\frac{1}{R}\right] = 4\pi D_{AB}\, C \left[0+7.3*10^{-4}\right]$$

$W_A = 4\,\pi\, R\, D_{AB}\, C * 7.3 * 10^{-4}$

$$C = \frac{P}{Gas\ constant * T} = \frac{1.01325*10^5}{8314*318}$$

$= 0.0383$ kmol/m^3

Initial rate of evaporation:

Therefore $W_A = 4 * 3.142 * 2 * 10^{-3} * 6.92 * 10^{-6} * 0.0383 * 7.3 * 10^{-4}$

$= 4.863 * 10^{-12}$ kmol / sec

$W_A = 1.751 * 10^{-5}$ mol/hr.

be analyzed.

2. Air at 1 atm is blown past the bulb of a mercury thermometer. The bulb is covered with a wick. The wick is immersed in an organic liquid (molecular weight = 58). The reading of the thermometer is 7.6 °C. At this temperature, the vapor pressure of the liquid is 5 kPa. Find the air temperature, given that the ratio fo heat transfer coefficient to the mass transfer coefficient (psychometric ratio) is 2 kJ/kg. Assume that the air, which is blown, is free from the organic vapor.

Solution:

For simultaneous mass and heat transfer, heat flux q and mass flux N_A are related as $q = N_A\,\lambda$ -- (1)

where λ is the latent heat of vaporization. Mass flux is given by

$$N_A = k_Y\left(Y'_\omega - Y'\right) \text{ --------------------------------- (2)}$$

Where

k_Y = mass transfer coefficient

Y'_{ω} = mass ratio of vapor in surrounding air at saturation; and

Y' = mass ratio of vapor in surrounding air.

Convective heat flux is given by

$$q = h\left(T - T_{\omega}\right) \text{ ---------------------------------- (3)}$$

Where

h = heat transfer coefficient;

T_{ω} = wet bulb temperature of air; and

T = dry bulb temperature of air.

Substituting for N_A and q from equation (2) and equation (3) in equation (1),

$$h\left(T - T'_{\omega}\right) = k_y\left(T'_{\omega} - Y'\right)\lambda$$

$$T - T_{\omega} = \frac{\lambda\left(Y'_{\omega} - Y'\right)}{h/k_Y} \text{ ------------------------ (4)}$$

Given: $Y' = 0$; λ = 360 kJ/kg; h/k_Y = 2 kJ/kg.K; and T_{ω} = 7.6°C

$$Y'_{\omega} = \frac{\text{kg organic vapor at saturation}}{\text{kg dry air}}$$

$$= \frac{5}{101.3 - 5}\,\frac{58}{29} = 0.1038$$

Substituting these in equation (4)

$$T - 7.6 = \frac{(360)(0.1038 - 0)}{2} = 18.69$$

T = 18.69 + 7.6 = 26.29°C

Temperature of air = 26.29°C.

3. A mixture of water and ethyl alcohol containing 0.16 mole fraction alcohol is continuously distilled in a plate fractionating column to give a product containing 0.77 mole fraction alcohol and a waste of 0.02 mole fraction alcohol. It is proposed to withdraw 25 per cent of the alcohol in the entering stream as a side stream containing 0.50 mole fraction of alcohol. Determine the number of theoretical plates required and the plate from which the side

stream should be withdrawn if the feed is liquor at its boiling point and a reflux ratio of 2 is used.

Solution: Taking 100 kmol of feed to the column as a basis, 16 kmol of alcohol enter, and 25 percent, that is 4 kmol, are to be removed in the side stream. As the side-stream composition is to be 0.5, that stream contains 8 kmol.

An overall mass balance gives:

$$F = D + W + S$$

That is: $100 = D + W + 8$ or $92 = D + W$

A mass balance on the alcohol gives:

$$(100 \times 0.16) = 0.77\,D + 0.02\,W + 4$$

$$12 = 0.77\,D + 0.02\,W$$

from which: distillate, $D = 13.55$ kmol and bottoms, $W = 78.45$ kmol.

In the top section between the side-stream and the top of the column:

$$R = L_n/D = 2, \text{ and hence } L_n = (2 \times 13.55) = 27.10 \text{ kmol}$$

$$V_n = L_n + D \quad \text{and} \quad V_n = (27.10 + 13.55) = 40.65 \text{ kmol}$$

For the section between the feed and the side stream:

$$V_s = V_n = 40.65, \quad L_n = S + L_s$$

and:

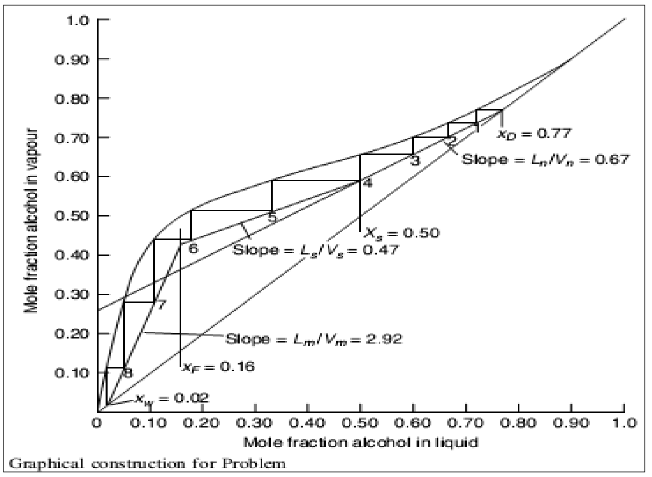
$$L_s = (27.10 - 8) = 19.10 \text{ kmol}$$

At the bottom of the column:

$$L_m = L_s + F = (19.10 + 100) = 119.10, \text{ if the feed is at its boiling-point.}$$

$$V_m = L_m - W = (119.10 - 78.45) = 40.65 \text{ kmol.}$$

The slope of the operating line is always L/V and thus the slope in each part of the column can now be calculated. The top operating line passes through the point (x_d, x_d) and has a slope of $(27.10/40.65) = 0.67$. This is shown in Figure 11b and it applies until $x_s = 0.50$ where the slope becomes $(19.10/40.65) = 0.47$. The operating line in the bottom of the column applies from $x_f = 0.16$ and passes through the point (x_w, x_w) with a slope of $(119.10/40.65) = 2.92$.



Graphical construction for Problem

The steps corresponding to the theoretical plates may be drawn in as shown and 8 plates are required with the side stream being withdrawn from the fourth plate from the top.

4. A vapor – air mixture at 302 K and 100 kPa having saturation temperature 291 K. Find out the Molal humidity, Relative humidity, Absolute humidity and percentage humidity.

Given Data: **Vapor pressure of air – vapor mixture at 291 K = 2.0624 kPa**

Vapor pressure of air – vapor mixture at 302 K = 4.004 kPa

Solution: We know that

Molal Humidity, $Y = \dfrac{\bar{p}_A}{p_t - \bar{p}_A} = \dfrac{2.0624}{100 - 2.0624} = 0.02106 \dfrac{\text{mol vapor}}{\text{mol dry air}}$

Absolute Humidity, $Y' = Y \times \dfrac{M_A}{M_B} = 0.02106 \times \dfrac{18}{29} = 0.01307 \dfrac{\text{kg vapor}}{\text{kg dry air}}$

Saturation Humidity, $Y_S = \dfrac{p_S}{p_t - p_S} = \dfrac{4.004}{100 - 4.004} = 0.04171 \dfrac{\text{mol vapor}}{\text{mol dry air}}$

Relative Humidity, $RH = \dfrac{\bar{p}_A}{p_S} = \dfrac{2.0624}{4.004} = 0.5151$

% Humidity, $Y_p = RH \times \dfrac{p_t - p_S}{p_t - \bar{p}_A} \times 100 = \dfrac{100 - 4.004}{100 - 2.0624} \times 100 = 50.5\%$

5. A batch of 120 kg wet solid has initial moisture content of 0.2 kg water / kg dry solid. The exposed area for drying is 0.05 m^2/kg dry solid. The rate of drying follows the curve given below. The time required (in hours) for drying this batch to a moisture content of 0.1 kg water/kg dry solid is

Solution: Time required for Falling rate Region is given by,

$$t = \frac{L_s}{a}\frac{(X_1 - X_2)}{(R_1 - R_2)}\ln\frac{R_1}{R_2}$$

$t = Time$ (h)

L_S = dry solid

A = Exposed content area (m^2)

X = Moisture content (kg water /kg dry solid)

Wet solid = 120 kg

Moisture content = 0.2 kg water / kg dry solid

Dry solid = 120 kg wet solid

$$\times\frac{1kg \text{ dry solid}}{(1+0.2)kg \text{ wet solid}}$$

$$\boxed{L_S = 100 \text{ kg dry solid}}$$

$$A = 0.05\frac{m^2}{kg \text{ dry solid}}\times 100 kg\ dry = 5m^2$$

$$X_1 = 0.2 \text{ kg water / kg dry solid /R}_1 = 10.2\ \frac{\text{Kg water}}{m^2 h}$$

$$X_2 = 0.1 \qquad \text{R}_2 = 0.6$$

$$t_0 = \frac{100}{5}\times\frac{(0.2-0.1)}{(1.2-06)}\ln\frac{1.2}{0.6}$$

$$\boxed{t_0 = 2.31\ hr}$$

CHAPTER 3
HEAT TRANSFER OPERATIONS

3.1 CONCISE NOTES

3.1.1 CONDUCTION

Conduction heat transfer is defined as heat transfer in solids and fluids without bulk motion.

3.1.1.1 FOURIER'S LAW

$$q_x = -k \cdot A \cdot \frac{\partial T}{\partial x}$$

3.1.1.2 CONDUCTION HEAT TRANSPORT EQUATION

$$q''' = -k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

Where α is called thermal diffusivity of material (m^2/sec):

$$\alpha = \frac{k}{\rho \cdot C}$$

- **HEAT TRANSFER THROUGH COMPOSITE WALL**

$$q = \frac{T_2 - T_1}{\sum \frac{x_i}{k_i . A}}$$

- **HEAT TRANSFER THROUGH CYLINDER**

$$q = \frac{T_2 - T_1}{\frac{1}{2\pi k L} \ln\left(\frac{R_2}{R_1}\right)}$$

3.1.1.3 CRITICAL RADIUS OF INSULATION

If the radius (for cylindrical or spherical surfaces) is greater than the critical radius, any addition of insulation on the tube surface decreases the heat loss. But if the radius is small the addition of insulation will increase the heat loss till it reaches the critical value.

i. For cylinder $r_C = k/h$

ii. For sphere $r_C = 2k/h$

3.1.1.4 LUMPED HEAT CAPACITY MODEL

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(hA/\rho CV\right)\tau\right]$$

3.1.2 CONVECTION

Convection is the mode of heat transfer between a solid surface and the adjacent fluid that is in motion, and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat transfer.

3.1.2.1 NEWTON'S LAW OF COOLING

$$q = hA\Delta T$$

3.1.2.2 NATURAL CONVECTION

In natural convection, the fluid motion occurs by natural means such as buoyancy. Si nce the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low.

3.1.2.3 FORCED CONVECTION

In forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or fan. Since the fluid velocity associated with force d convection is relatively high, the heat transfer coefficient encountered in forced convection is high.

3.1.3 RADIATION

Radiation differs from Conduction and Convection heat t transfer mechanisms, in the sense that it does not require the presence of a material medium to occur. Energy transfer by radiation occurs at the speed of light and suffers no attenuation in vacuum. Radiation can occur between two bodies separated by a medium colder than both bodies.

3.1.3.1 STEFAN-BOLTZMANN LAW

It states that, the total amount of radiant energy emitted by a blackbody is proportional to the fourth power of the absolute temperature such that,

$$E^* = \sigma \cdot T^4 \qquad W\ m^{-2}$$

where σ =5.67 x 10^{-8} W $m^{-2}K^{-4}$ is the Stefan-Boltzmann constant.

3.1.3.2 THE SHAPE FACTOR

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures. View factor (or shape factor) is a purely geometrical parameter that accounts for the effects of orientation on radiation between surfaces.

- $F_{i\rightarrow j}$ or F_{ij} = the fraction of the radiation leaving surface i that strikes surface j directly.

3.1.4 BOILING AND CONDENSATION

The heat transfer processes accompanied by phase change (liquid to vapor or vapor to liquid) are more complex than the process of heat transfer between fluids without phase change. A phase change vapor to liquid or liquid to vapor involves removal or addition of considerable amount of thermal energy at nearly constant temperature.

3.1.4.1 HEAT TRANSFER TO BOILING LIQUIDS

When a heated surface exceeds the saturation temperature of the surrounding coolant, boiling on the surface becomes possible. This is true whether the bulk fluid temperature is at or below the local saturation temperature. If the bulk fluid temperature is below the saturation temperature, boiling is referred to as "local" or "sub-cooled" boiling. If the bulk fluid temperature is equal to the saturation temperature, then "bulk" boiling is said to occur.

Bubbles formed on the heated surface depart the surface and are transported by the bulk fluid, such that a condition of two-phase flow is said to exist. Depending on the degree of sub-cooling and the length of the heated channel, the bubbles may or may not condense and collapse prior to exiting the channel. In sub-cooled boiling this process results in further heating of the fluid toward the saturation temperature.

3.1.4.2 HEAT TRANSFER TO CONDENSATION

Condensation is defined as the physical process by which a gas or vapor changes into a liquid. If the temperature of an object (e.g. grass, metal, glass) falls below what is known as the 'Dew Point' temperature for a given relative humidity of the surrounding air, water vapor from the atmosphere condenses into water droplets on its surface. This "dew point" varies according to the amount of water in the atmosphere (known as humidity). In humid conditions condensation occurs at higher temperatures. In cold conditions condensation occurs despite relatively low humidity.

3.1.5 HEAT EXCHANGERS

A heat exchanger is a device that is used to transfer thermal energy (enthalpy) between two or more fluids, between a solid surface and a fluid, or between solid particulates and a fluid, at different temperatures and in thermal contact.

- **Energy Balance**

The characteristics of fluids contribute to a fundamental property of heat exchangers—the heat-transfer rate ($\dot{Q}$). The heat transferred to the colder fluid must equal that transferred from the hotter fluid, according to the following equation:

$$\dot{Q} = \left[\dot{m} \times c_p \times \left(T_{out} - T_{in}\right)\right]_{cold} = -\left[\dot{m} \times c_p \times \left(T_{out} - T_{in}\right)\right]_{hot}$$

And the overall heat exchanger energy balance equation can be given as

$$\dot{Q} = UA\Delta T_{ln}$$

$$\dot{Q} = UA\Delta T_{ln} = \left[\dot{m} \times c_p \times \left(T_{out} - T_{in}\right)\right]_{cold} = -\left[\dot{m} \times c_p \times \left(T_{out} - T_{in}\right)\right]_{hot}$$

Where $$\Delta T_{ln} = \frac{\left(T_{in,hot} - T_{out,cold}\right) + \left(T_{out,hot} - T_{in,cold}\right)}{\ln\left(\dfrac{T_{in,hot} - T_{out,cold}}{T_{out,hot} - T_{in,cold}}\right)}$$

3.1.6 EVAPORATION

Evaporation is an operation used to concentrate a solution of a non volatile solute and a volatile solvent, which in many cases is water. A portion of solvent is vaporized to produce a concentrated solution, slurry or thick, viscous liquid.

3.1.6.1 CAPACITY

The capacity of an evaporator can be defined as the number of kilograms of water vaporized / evaporated per hour.

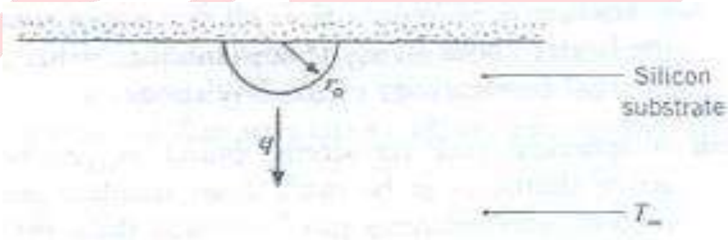
The higher the capacity of the evaporator, the more the designer can justify complex and expensive evaporation systems in order to provide high energy efficiency.

3.1.6.2 EVAPORATOR ECONOMY Economy of an evaporator is defined as the number of kilograms of water evaporated per kilogram of steam fed to the evaporator. It is alos called as steam economy.

In single effect evaporator the amount of water evaporated per kg of steam fed is always less than one and hence economy is less than one. The fact that the latent heat of evaporation of water decreases as the pressure increases tends to make the ratio of water vapor produced per kg of steam condensed less than unity.

3.2 SOLVED PROBLEMS

1. A transistor, which may be approximated as a hemispherical heat source of radius r_o= 0.1 mm, is embedded in a large silicon substrate (k = 125 W/m.K) and dissipates heat at a rate q. All boundaries of the silicon are maintained at an ambient temperature of T∞= 27 °C, except for a plane surface that is well insulated. Obtain a general expression for the substrate temperature distribution and evaluate the surface temperature of the heat source for q = 4 W.



Given data: A heat source embedded in a large silicon substrate, source and substrate boundary conditions. Require: substrate temperature distribution and surface temperature of heat source for q = 4 W.

Solution: From energy equation reduced to

$$\frac{1}{r^2}\frac{d}{dr}(kr^2\frac{dT}{dr}) = 0$$

At constant silicon thermal conductivity

$$\frac{d}{dr}(r^2\frac{dT}{dr}) = 0$$

By integration to the substrate radius

$$r^2\frac{dT}{dr} = C_1$$

$$T(r) = -\frac{C_1}{r} + C_2$$

Boundary conditions

$T(\infty) = T_\infty$, and $T(r_o) = T_s$

Then the constants

$C_2 = T_\infty$
$C_1 = r_o(T_\infty - T_s)$

The substrate temperature distribution

$$T(r) = (T_s - T_\infty) r_o / r + T_\infty$$

The heat rate is

$$q = -kA\frac{dT}{dr} = -k(2\pi\, r^2)\left(-(T_s - T_\infty) r_o / r^2\right) = 2\pi\, r_o (T_s - T_\infty)$$

The surface temperature of heat source for q = 4 W is

$$T_s = 4/125(2\pi\ X10^{-4}) + 27 = 78\ ^oC$$

2. A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is k = 0.15 W/m·K. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T_∞ = 30°C with a heat transfer coefficient of h = 12 W/m²·K, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

Solution:

The thermal conductivity of plastic is given to be k = 0.15 W/m·K. Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = \mathbf{V}I = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in above figure. The values of these two resistances are

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{conv} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot\text{K})(0.110\text{ m}^2)} = 0.76°\text{C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot\text{K})(5\text{ m})} = 0.18°\text{C/W}$$

And therefore

$$R_{total} = R_{plastic} + R_{conv} = 0.76 + 0.18 = 0.94°C/W$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total}$$
$$= 30°C + (80\ W)(0.94°C/W) = 105°C$$

Note that we did not involve the electrical wire directly in the thermal resis-tance network, since the wire involves heat generation.

The critical radius of insulation of the plastic cover can be determined from equation (22),

$$r_{cr} = \frac{k}{h} = \frac{0.15\ W/m\cdot K}{12\ W/m^2\cdot K} = 0.0125\ m = 12.5\ mm$$

Which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will enhance heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer will increase when the interface temperature T_1 is held constant, or T_1 will decrease when is held constant, which is the case here.

3. Air at 293 K flows over a thin plate with velocity of 2 m/s. the plate is 2m long and 1m wide. Find the boundary layer thickness at the trailing edge of the plate.Take kinematic viscosity of air = 15 x 10^{-6} m^2/s.

Solution: The critical length x_c at which the transition takes place is calculated as:

$$Re_{x=x_c} = 3\times10^5 = \frac{x_c u_\infty}{\upsilon}$$
$$\Rightarrow \quad x_c = \frac{3\times10^5 \times 15\times10^{-6}}{2} = 2.25\ m$$

As the actual length of the plate is 2 m which is less than critical length x_c, the flow is laminar over the entire plate.

The boundary layer thickness at any distance, x, is calculated from

$$\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$$

At trailing edge, x = L = 2 m and $\delta = \delta_L$

$$\frac{\delta_L}{L} = \frac{4.64}{\sqrt{\text{Re}_L}}$$

$$\text{Re}_L = \frac{u_\infty L}{\upsilon} = \frac{2\times 2}{15\times 10^{-6}} = 2.67\times 10^5$$

$$\delta_L = \frac{4.64\times 2}{\sqrt{2.67\times 10^5}} = 0.018\ m = \underline{18\ mm}$$

4. Two long planes A and B are maintained at 600 K and 300 K and their surface emissivities are 0.8 and 0.5 respectively. Two thin radiation shield C and D having emissivities 0.5 and 0.4 are introduced between the given planes. The given planes are in order A, C, D and B. Assuming all the planes to be infinitely long, find the rate of heat exchange per unit area and steady state temperature attained by the planes C and D.

Solution: In steady state, we can write

$$Q_{CD} = Q_{DB}$$

$$\frac{\sigma.\left(T_C^4 - T_D^4\right)}{\frac{1}{\varepsilon_C} + \frac{1}{\varepsilon_D} - 1} = \frac{\sigma.\left(T_D^4 - T_B^4\right)}{\frac{1}{\varepsilon_D} + \frac{1}{\varepsilon_B} - 1}$$

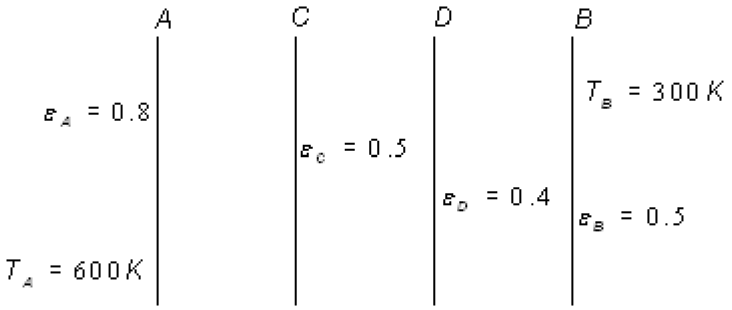
$$\varepsilon_C = 0.5,\ \varepsilon_D = 0.4,\ \varepsilon_B = 0.5$$

$$\frac{\left(T_C^4 - T_D^4\right)}{\frac{1}{0.5} + \frac{1}{0.4} - 1} = \frac{\left(T_D^4 - T_B^4\right)}{\frac{1}{0.4} + \frac{1}{0.5} - 1}$$

$$\left(T_C^4 - T_D^4\right) = \left(T_D^4 - T_B^4\right)$$

$$T_B = 300\text{ K}$$

$$T_D^4 = \frac{1}{2}\left(T_C^4 - T_B^4\right) = \frac{1}{2}\left(T_C^4 - 300^4\right)$$



Now,

$$Q_{AC} = Q_{CD}$$

$$\frac{\sigma.\left(T_A^4 - T_C^4\right)}{\frac{1}{\varepsilon_A} + \frac{1}{\varepsilon_C} - 1} = \frac{\sigma.\left(T_C^4 - T_D^4\right)}{\frac{1}{\varepsilon_C} + \frac{1}{\varepsilon_D} - 1}$$

$$\varepsilon_C = 0.5,\ \varepsilon_D = 0.4,\ \ \varepsilon_A = 0.8$$

$$T_A = 600\text{ K}$$

$$\frac{\left(600^4 - T_C^4\right)}{\frac{1}{0.8} + \frac{1}{0.5} - 1} = \frac{\left(T_C^4 - T_D^4\right)}{\frac{1}{0.5} + \frac{1}{0.4} - 1}$$

$$\frac{600^4 - T_C^4}{2.25} = \frac{T_C^4 - T_D^4}{3.5}$$

Putting value of T_D in terms of T_C, and solving, we get

T_C = 560.94 K

$$T_D^4 = \frac{1}{2}\left(T_C^4 - 300^4\right)$$

$$T_D^4 = \frac{1}{2}\left(560.94^4 - 300^4\right)$$

T_D = 461.73 K

$$\frac{Q}{A} = \frac{\sigma.\left(T_A^4 - T_C^4\right)}{\frac{1}{\varepsilon_A} + \frac{1}{\varepsilon_C} - 1} = \frac{5.67\times10^{-8}\times\left(600^4 - 560.94^4\right)}{\frac{1}{0.8} + \frac{1}{0.5} - 1}$$

$$\boxed{\frac{Q}{A} = 770.94\ W/m^2}$$

5. Water enters a counter flow, double pipe heat exchanger at 288 K, flowing at a rate of 1300 kg/h. it is heated by oil flowing at a rate of 550 kg/h from a inlet temperature of 367 K. for 1 m^2 area of heat transfer, determine the total heat transfer and outlet temperature of oil and water, and effectiveness.

Data: **Specific heat of oil = 2000 J / (kg.K)**

Specific heat of water = 4187 J / (kg.K)

Overall heat transfer coefficient = 1075 W / (m^2.K)

Solution:

Water : $m_c C_{p_c} = \frac{1300}{3600} \times 4187 = 1511.97\ W/K$

Oil : $m_h C_{p_h} = \frac{550}{3600} \times 2000 = 305.55\ W/K$

Heat capacity rate of hot fluid is smaller than that of water.

$$U = 1075\ W/(m^2.K), \qquad A = 0.1\, m^2$$

$$NTU = \frac{UA}{(m.C_p)_{small}} = \frac{UA}{m_h.C_{p_h}} = \frac{1075 \times 1}{305.55} = 3.52$$

$$C = \frac{m_h C_{p_h}}{m_c.C_{p_c}} = \frac{305.55}{1511.97} = 0.20$$

For counter current flow, we have

$$\varepsilon = \frac{1 - e^{-NTU(1-C)}}{1 - C.e^{-NTU(1-C)}} = \frac{1 - e^{-3.52(1-0.2)}}{1 - 0.2 \times e^{-3.52(1-0.2)}} = 0.9515$$

Effectiveness of exchanger = 0.9515

For $m_h C_{p_h}$ small, we have

$$\varepsilon = \frac{T_1 - T_2}{T_1 - t_1}$$

Where T_1 and T_2 are inlet and outlet temperatures of hot fluid (i.e. oil) and t_1 is the inlet temperature of cold fluid (i.e. water).

$$0.9515 = \frac{367 - T_2}{367 - 288}$$

$$T_2 = 291.83\ K$$

Outlet temperature of oil = 291.83 K

$$Q = m_c.C_{p_c}(t_2 - t_1) = m_h C_{p_h}(T_1 - T_2)$$

$$C = \frac{m_h C_{p_h}}{m_c.C_{p_c}} = \frac{t_2 - t_1}{T_1 - T_2}$$

$$0.20 = \frac{t_2 - 288}{367 - 291.83}$$

$$t_2 = 303.03K$$

Outlet temperature of water = 303.03 K

$Q = \text{Rate of heat transfer} = m_h C_{p_h}(T_1 - T_2)$

$$= \frac{550}{3600} \times 2000 \times (367 - 291.83)$$

$$= 22968.6 J/s$$

$$\boxed{Q = 22968.6\ W}$$

6. An evaporator operating at atmospheric pressure is fed at the rate of 10000 kg/h of weak liquor containing 4% caustic soda. Thick liquor leaving the evaporator contains 25% caustic soda. Find the capacity of the evaporator.

Solution:

Basis: 10000 kg/h of thick liquor leaving the evaporator.

Let m be the kg/h of thick liquor leaving the evaporator.

Material balance of caustic soda:

Caustic soda in feed = Caustic soda in thick liquor

0.04 x 10000 = 0.25 x m

m = 1600 kg/h

Overall material balance:

Kg/h of feed = kg/h water evaporated + kg/h of thick liquor

10000 = kg/h water evaporated + 1600

kg/h water evaporated = 8400 kg/h

Capacity of evaporator = 8400 kg/h

CHAPTER 4
FLUID MECHANICS

4.1 CONCISE NOTES

4.1.1 NEWTON'S LAW OF VISCOSITY

The *rate of shear stress* (shear stress per unit time, τ / time) is directly proportional to the *shear strain*.

$$\tau = -\mu \times \frac{du}{dy}$$

4.1.2 MASS DENSITY (ρ)

Mass Density is defined as the mass of substance per unit volume (kg / m^3).

4.1.3 SPECIFIC GRAVITY (s)

Specific Gravity is defined as the ratio of mass density of a substance to some standard mass density.

For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4 ^0C) at atmospheric pressure.

$$s = \frac{\rho}{\rho_w}$$

4.1.4 VISCOSITY

Viscosity is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress.

All fluids are viscous; "Newtonian Fluids" obey the linear relationship, given by Newton's law of viscosity. $\tau = \mu \frac{du}{dy}$.

Where τ is the shear stress; viscosity has Units of $N\, m^{-2}$; $kg\, m^{-1}\, s^{-2}$.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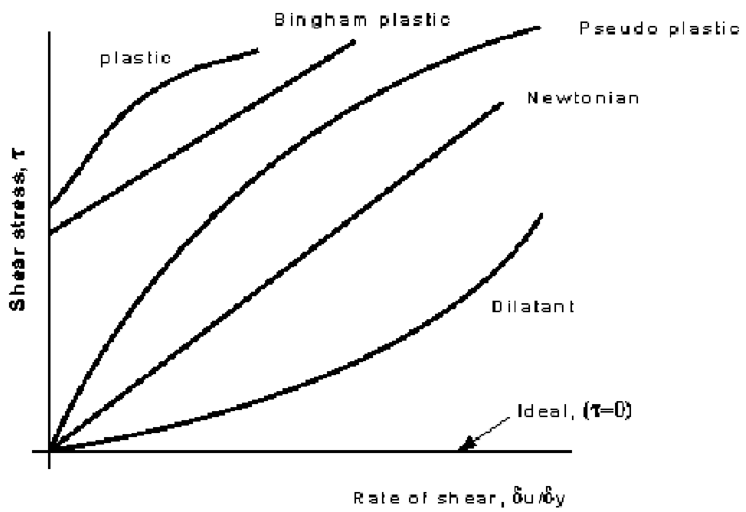
CS AIR 12 Rozy
CS AIR 12 Geminee
CS AIR 16 Moatoshi
ECE AIR 25 Rozy Marg
CS AIR 18 Akhilesh Singh
CS AIR 51 Yogesh Sharma

4.1.5 TYPES OF FLUID BEHAVIOR

When the measured values of shear stress or viscosity are plotted versus shear rate, various types of behavior may be observed depending upon the fluid properties, as shown in figure



Shear stress vs. Rate of shear strain δu/δy

i. NEWTONIAN FLUIDS

If the shear stress versus shear rate plot is a straight line through the origin (or a straight line with a slope of unity on a log-log plot), the fluid is said to be Newtonian:

$$\tau = \mu \frac{du}{dy}$$

Where μ is the coefficient of viscosity.

ii. NON NEWTONIAN FLUIDS

If the shear stress versus shear rate plot is not a straight line, the fluid is said to be Non Newtonian:

$$\tau = A + B\left(\frac{du}{dy}\right)^n$$

Where A, B and n are constants. For Newtonian fluids A = 0, B = μ and n = 1.

iii. PLASTIC FLUIDS

A fluid, in which the shear stress is more than the yield value is known as the plastic fluid. i.e. Shear stress must reach a certain minimum before flow commences.

iv. DILATANT FLUIDS

The fluids having no minimum shear stress and the Viscosity increases with rate of shear, and the value of n>1. e.g. *quicksand.*

v. BINGHAM PLASTIC FLUIDS

A minimum value of shear stress must be achieved before the fluid may start flow classification $n = 1$. An example is *sewage sludge.*

vi. PSEUDO PLASTIC FLUIDS

The fluids having no minimum shear stress and the viscosity decreases with rate of shear and the value of n < 1, e.g. *colloidal substances like clay, milk and cement*.

4.1.6 FLUID STATICS

Fluid statics deals with the fluids at rest.

4.1.6.1 PRESSURE

A fluid will exert a normal force on any boundary it is in contact with.

$$p = \frac{F}{A}$$

Units: Newton's per square metre, N m^{-2}.

4.1.6.2 HYDROSTATIC LAW

Consider a hypothetical differential cylindrical element of fluid of cross sectional area A and height $(z_2 - z_1)$ and pressure difference is $(P_2 - P_1)$

$$\boxed{\Delta P = \rho g \Delta z}$$

4.1.7 LAMINAR AND TURBULENT FLOW

A *Laminar flow* is one in which paths taken by the individual particles do not cross one another and move along well defined paths, this type of flow is also called *stream line flow* or *viscous flow.*

Example: *Flow through a capillary tube.*

A *Turbulent flow* is that flow in which fluid particles move in a disturbing (i.e. not in definite) way.

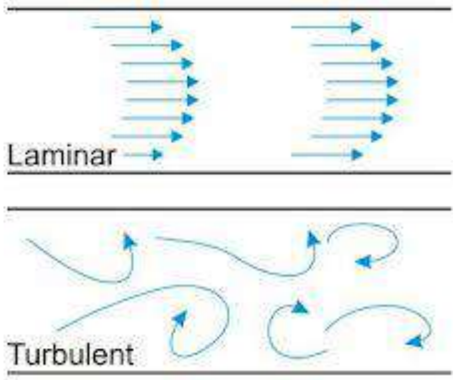
Example: *High velocity flow in a conduit of large size.*

Laminar and turbulent flows are characterized on the basis of Reynolds number.

For Reynolds number (Re) < 2000 ……..flow in pipes is *laminar.*

For Reynolds number (Re) > 4000 ……..flow in pipes is *turbulent.*

For 2000 < Re < 4000 ……..flow in pipes may be *laminar* or *laminar.*



4.1.8 CONTINUITY EQUATION

The *continuity equation* is based on the principle of conservation of mass. It states as follows:

"If no fluid is added or removed from the pipe in any length then the mass passes through different sections shall be same."

$$Q = A.u$$

4.1.9 BERNOULLI'S EQUATION

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of the pressure energy, kinetic energy and potential energy is constant along a stream line."

Mathematically, $\frac{p}{w} + \frac{v^2}{2g} + z = \text{constant}$

4.1.10 NAVIER STOKES EQUATIONS OF MOTION

$$\frac{DV}{Dt} = R - \frac{1}{\rho}\nabla p + \nu \nabla^2 V$$

4.1.11 HAGEN POISEUILLE EQUATION

$$\frac{p_1 - p_2}{w} = \frac{32\,\mu V_{av}\, l}{w\, d^2} = \frac{128\,\mu Q\, l}{\pi w\, d^4}$$

Evidently the head loss h_f over a length l of the pipe varies directly as the first power of the rate of discharge Q and inversely as the fourth power pipe diameter d.

Where, Velocity distribution is given by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$$u = V_{max} \times \left[1-\left(r/R^2\right)\right]$$

Equation is usually known as Hagen-Poisuille and is valid for a fully developed flow; a flow in which the velocity profile does not vary along the pipe axis.

$$\boxed{\frac{dp}{w} = \frac{32\,\mu V_{av}\, l}{w\, d^2} \times \frac{2V_{av}}{2V_{av}}} \qquad (w = \rho g)$$

Or

$$\boxed{\frac{dp}{w} = \frac{64}{(V_{av}\, d\, \rho)/\mu} \times \frac{l}{d} \times \frac{V_{av}^2}{2g}}$$

Comparing it with the Darcy – Weisback equation for head loss due to friction through a pipe line.

$$\boxed{h_f = 4f\,\frac{l}{d}\frac{V_{av}^2}{2g}}$$

we obtain: $$\boxed{f = \frac{16}{(V_{av} d\, \rho)/\mu} = \frac{16}{R_e}}$$

The above expression gives a relationship between friction coefficient f and the Reynolds number Re, for laminar flow through a circular pipe.

4.1.12 ORIFICE FLOW METER

An orifice meter is essentially a cylindrical tube that contains a plate with a thin hole in the middle of it. The thin hole essentially forces the fluid to flow faster through the hole in order to maintain flow rate. The point of maximum convergence usually occurs slightly downstream from the actual physical orifice this is the reason orifice meters are less accurate than Venturimeter, as we cannot use the exact location and diameter of the point of maximum convergence in calculations. Beyond the vena contracta point, the fluid expands again and velocity decreases as pressure increases.

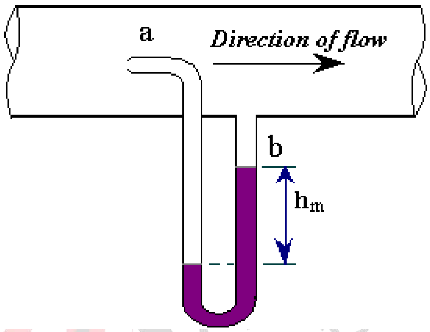
$$Q = C_0\left(\frac{\pi}{4}d^2\right)\sqrt{\frac{2gh(\rho_m-\rho)}{\rho\left(1-\left(\frac{d}{D}\right)^4\right)}} = C_0\left(\frac{\pi}{4}d^2\right)\sqrt{\frac{2(P_1-P_2)}{\rho\left(1-\left(\frac{d}{D}\right)^4\right)}}$$

4.1.13 VENTURI METER

The venturimeter has a converging conical inlet, a cylindrical throat and a diverging recovery cone. It has no projections into the fluid, no sharp corners and no sudden changes in contour. The following figure shows the venturi meter with uniform cylindrical section before converging entrance, a throat and divergent outlet.

The converging inlet section decreases the area of the fluid stream, causing the velocity to increase and the pressure to decrease. The low pressure is measured in the center of the cylindrical throat as the pressure will be at its lowest value, where neither the pressure nor the velocity will be changing. As the fluid enters the diverging section the pressure is largely recovered lowering the velocity of the fluid. The major disadvantages of this type of flow detection are the high initial costs for installation and difficulty in installation and inspection.

$$Q = C_V\left(\frac{\pi}{4}d^2\right)\sqrt{\frac{2gh(\rho_m-\rho)}{\rho\left(1-\left(\frac{d}{D}\right)^4\right)}} = C_V\left(\frac{\pi}{4}d^2\right)\sqrt{\frac{2(P_1-\ldots}{\rho\left(1-\left(\frac{d}{D}\ldots\right.\right.}}$$



4.1.14 PITOT TUBE

The *Pitot tube* is a device used to measure the local velocity along a streamline. The principle of the device is shown in figure given below. The opening of the impact tube *a* is perpendicular to the flow direction. The opening of the static tube *b* is parallel to the direction of flow. The two tubes are connected to the legs of a manometer or equivalent device for measuring small pressure differences. The static tube measures the static pressure p_0 since there is no velocity perpendicular to its opening. The impact opening includes a stagnation point *B* at which the stream line *AB* terminates.

The pressure p_s, is the stagnation pressure of the fluid measured by the impact tube *a*, then for incompressible fluids

$$u_0 = C_p\sqrt{\frac{2g(p_s-p_0)}{\rho}} \quad \text{........................(8.15)}$$

Where $(p_s - p_0)$ is the pressure difference measured by the manometer of the pitot tube, C_p is the coefficient of pitot tube and u_0 is local velocity of the point where the impact tube is located.

4.1.15 FLUIDIZATION

When a fluid is passed upward through a bed of particles, as illustrated in figure shown below, the pressure drop increases as the fluid velocity increases. The product of the pressure drop and the bed cross sectional area represents a net upward force on the bed, and when this force becomes equal to the weight of the bed (solids and fluid) the bed becomes suspended by the fluid. In this state the particles can move freely within the "*bed*" which thus behaves much like a boiling liquid. Under these conditions the bed is said to be ''*fluidized.*'' This freely flowing or bubbling behavior results in a high degree of mixing in the bed, which provides a great advantage for heat or mass transfer efficiency compared with a fixed bed.

Fluidized bed operations are found in refineries (i.e., fluid catalytic crackers), polymerization reactors, fluidized bed combustors, etc.

- If the fluid velocity within the bed is greater than the terminal velocity of the particles, however, the fluid will tend to entrain the particles and carry them out of the bed.
- If the superficial velocity above the bed (which is less than the interstitial velocity within the bed) is less than the terminal velocity of the particles, they will fall back and remain in the bed.

4.1.16 MINIMUM FLUIDIZATION VELOCITY

An equation for the minimum fluidization velocity can be obtain by setting the pressure drop across the bed equal to the weight of the bed per unit area of cross section, allowing for the buoyant force of the displaced fluid:

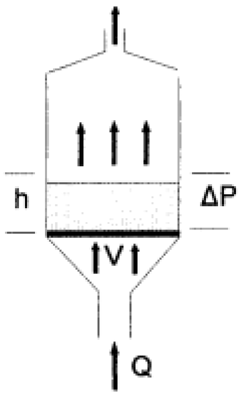
$$\Delta p = g(1-\varepsilon)(\rho_p - \rho)L$$

At incipient fluidization, ε is the minimum porosity ε_M, (if the particles themselves are porous, ε is the external void fraction of the bed.) Thus

$$\frac{\Delta p}{L} = g(1-\varepsilon_M)(\rho_p - \rho)$$

The expanded height may be obtained from ε and the value of L and ε at incipient fluidization, using the equation

$$L = L_M \frac{1-\varepsilon_M}{1-\varepsilon}$$



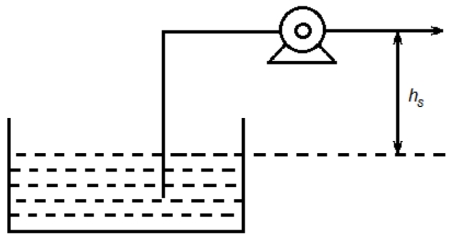
Fluidized bed.

4.1.17 NPSH (NET POSITIVE SUCTION HEAD)

To prevent cavitations, it is necessary that the pressure at the pump suction be sufficiently high that the minimum pressure anywhere in the pump will be above the vapor pressure.

This required minimum suction pressure (in excess of the vapor pressure) depends upon the pump design, impeller size and speed, and flow rate and is called the minimum required *net positive suction head (NPSH)*.

The NPSH is almost independent of impeller diameter at low flow rates and increases with flow rate as well as with impeller diameter at higher flow rates.



When Pump is above the water reservoir

Mathematically,

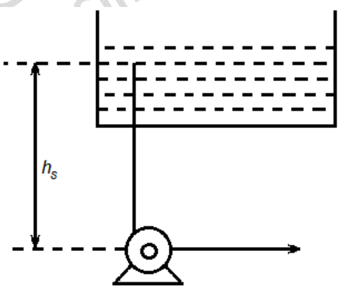
NPSH = Helping Head – Loosing Head

Where, *helping head* is the sum of the heads those support the fluid to flow at high velocity with minimum friction losses, and *losing head* is sum of the heads those are responsible for friction and other type of losses at the time of fluid flow.

- **CASE 1:**

The Pump is installed above the water reservoir,

$$NPSH = \left\{\frac{p_{atm}}{w}\right\} - \left\{\frac{p_v}{w} + h_f + h_s\right\}$$



When Pump is under the Water reservoir

Where, p_{atm} = atmospheric pressure

w = specific weight of fluid

p_v = vapor pressure of fluid

h_f = friction head loss

h_s = suction (static) head loss

- **CASE 2:**

The Pump is installed above the water reservoir,

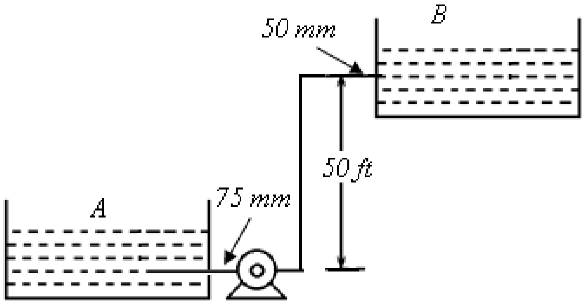
$$NPSH = \left\{\frac{p_{atm}}{w} + h_s\right\} - \left\{\frac{p_v}{w} + h_f\right\}$$

4.2 SOLVED PROBLEMS

1. In the equipment shown in the figure the pump draws a solution of specific gravity 1.84 from a storage tank A through a 75 mm I.D. tube. Pump efficiency is given as 60% , and velocity at suction point is 3 ft / s, the pump discharge through a 50 mm I.D. tube to a overhead tank B, the end of discharge pipe is 50 ft above the level of solution of feed tank. Friction losses in entire piping system are 29.9 J / kg. Calculate the power delivered to the fluid by the pump and what pressure must a pump delivered for that?

Solution: Given:

density = 1840 kg/m^3



$d_1 = 0.07$ m,

$d_2 = 0.05$ m,

$v_1 = 0.9144$ m / s,

$z_1 - z_1 = 50$ ft = 15.24 m,

$h_f = 29.9$ J / kg,

From continuity equation,

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1}{A_2} \times v_1$$

$$\Rightarrow v_2 = \left(\frac{d_1}{d_2}\right)^2 \times v_1 \Rightarrow v_2 = \left(\frac{0.075}{0.05}\right)^2 \times 0.9144 \quad \Rightarrow v_2 = 2.0574\ m/s$$

From Bernoulli's equation,

$$\eta W_p + \frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 + h_f$$

$$\Rightarrow W_p \times \eta = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) + h_f$$

$$\Rightarrow W_p \times 0.60 = 0 + \frac{2.0574^2 - 0.9144^2}{2} + 9.81 \times 15.24 + 29.9 \quad \Rightarrow \boxed{W_p = 301.8.38\ J/kg}$$

Pressure delivered to the fluid:

From Bernoulli's equation between inlet and outlet of pump,

$$\eta W_p + \frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 + h_f$$

$$\Rightarrow P_2 - P_1 = \rho \times \left[W_p \times \eta + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) + h_f \right]$$

$$\Rightarrow P_2 - P_1 = 1840 \times \left[301.838 \times 0.6 + \frac{2.0574^2 - 0.9144^2}{2} + 9.81 \times 0 + 0 \right]$$

$$\Rightarrow \boxed{\Delta P = 336.35\ kPa}$$

Power delivered to the pump:

$$Power = \dot{m} \times W_p$$

$$and\ \dot{m} = \rho \times A_1 \times v_1$$

$$\Rightarrow \quad \dot{m} = 1840 \times \frac{\pi}{4} \times (0.075)^2 \times 0.9144 \Rightarrow \quad \dot{m} = 7.433\ kg/s$$

Hence

$$Power = 7.433 \times 301.838 \quad \Rightarrow Power = 2243.574\ W$$

Thus, power delivered to the fluid by the pump:

$$P_{fluid} = Power \times \eta \Rightarrow P_{fluid} = 2243.574 \times 0.6$$

$$\Rightarrow \boxed{P_{fluid} = 1346.144\ W}$$

2. The velocity distribution for flow over a flat plate is given by $u = 2y - y^2$ where u is the velocity in m/s at a distance y *meters* above the plate. Determine the velocity gradient and shear stress at the boundary and *1.5 m* from it. Given Data: Dynamic viscosity of fluid is $0.9\ N.s/m^2$

Solution: $u = 2y - y^2$

$$\therefore \frac{du}{dy} = 2 - 2y$$

Velocity gradient, $\frac{du}{dy}$:

At the boundary: At $y = 0$, $\left(\frac{du}{dy}\right)_{y=0} = 2s^{-1}$

At 0.15 m from the oundary: At $y = 0.15$, $\left(\frac{du}{dy}\right)_{y=0.15} = 1.7s^{-1}$

Shear stress, τ:

At the boundary: $(\tau)_{y=0} = \mu.\left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 2 = 1.8\ N/m^2$

At 0.15 m from the oundary: $(\tau)_{y=0.15} = \mu.\left(\frac{du}{dy}\right)_{y=0.15} = 0.9 \times 1.7 = 1.53\ N/m^2$

3. A simple U – tube manometer is installed across an orifice meter. The manometer is filled with mercury (sp. Gravity = 13.6) and the liquid above the mercury is CCl_4 (sp. Gravity = 1.6). The manometer reads 200 mm. What is the pressure difference over the manometer in Newton per square meter?

Solution: Specific gravity of heavier liquid, S_{hl} = 13.6

Specific gravity of lighter liquid, S_{ll} = 1.6

Reading of the manometer, y = 200 mm

Pressure difference over the manometer: p

Differential head,

$$h = y\left[\frac{S_{hl}}{S_{ll}} - 1\right] \Rightarrow h = 200\left[\frac{13.6}{1.6} - 1\right]$$

$$h = 1500\ mm\ \text{of}\ CCl_4$$

Pressure difference over the manometer,

$$p = wh = (1.6 \times 9810) \times \left(\frac{1500}{1000}\right)$$

$$\boxed{p = 23544\ N/m^2}$$

4. In a fluid, the velocity field is given by

$$V = (3x + 2y)i + (2z + 3x^2)j + (2t - 3z)k$$

Determine:

(i) The velocity components *u, v, w* at any point in the fluid flow;
(ii) The speed at point (*1,1,1*)
(iii) The speed at time *t=2s* at point (*0,0,2*)

Also classify the velocity field as steady, or unsteady, uniform or non – uniform and one, two or three dimensional.

Solution: Given: Velocity field, $V = (3x + 2y)i + (2z + 3x^2)j + (2t - 3z)k$

(i) Velocity Components:

The velocity components are:

$$u = (3x + 2y),\ v = (2z + 3x^2),\ w = (2t - 3z)$$

(ii) Speed at point (1, 1, 1), $V_{(1,1,1)}$:

Substituting *x = 1, y = 1, z = 1* in the expression for *u, v* and *w,* we've:

$u = (3+2) = 5,\ v = (2+3) = 5,\ w = (2t-3)$

$V^2 = u^2 + v^2 + w^2$

$V^2 = 5^2 + 5^2 + (2t-3)^2$

$V^2 = 25 + 25 + 4t^2 - 12t + 9$

$V^2 = 4t^2 - 12t + 59$

$\Rightarrow \boxed{V_{(1,1,1)} = \sqrt{4t^2 - 12t + 59}}$

(iii) Speed at *t = 2s* at point (0, 0, 2), *V(1,1,1)*:

Substituting *t = 2, x = 0, y = 0, z = 2* in the expression for *u, v* and *w,* we've:

$u = 0,\ v = (2\times 2 + 0) = 4,\ w = (2\times 2 - 3\times 2) = -2$

$V^2 = u^2 + v^2 + w^2$

$V^2 = 0 + 4^2 + (-2)^2$

$V^2 = 16 + 4 = 20$

$\Rightarrow \boxed{V_{(0,0,2)} = \sqrt{20} = 4.472\ units}$

Velocity field, type:

(i) Since *V* at given (x, y, z) depends on t it is ***unsteady flow.***
(ii) Since *V* at given *t* velocity changes in the X direction it is ***non uniform flow.***
(iii) Since *V* depends on (x, y, z) it is ***three dimensional flow.***

5. The following function represents the possible irrotational flow or not?

$$\psi = A(x^2 - y^2)$$

Solution: For an irrotational fluid flow phenomenon ψ should satisfy Laplace equation.

$$\Rightarrow \frac{\partial \psi}{\partial x} = 2Ax; \qquad \frac{\partial \psi}{\partial y} = -2Ay$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = 2A; \qquad \frac{\partial^2 \psi}{\partial y^2} = -2A$$

From Laplace equation, we get :

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 2A - 2A = 0$$

Hence, $\psi = A(x^2 - y^2)$ **represents a possible irrotational flow.**

6. Water at the rate of *30 litres / s* is flowing through a *0.2 m*. I.D. pipe. A venturimeter of throat diameter *0.1 m* is fitted in the pipeline. A differential manometer in the pipeline has an indicator liquid M and the manometer reading is *1.16 m*. what is the relative density of the manometer liquid M? Venturi coefficient = *0.96*; Density of water = *998 kg / m³*.

Solution: Given: $Q = 30\ litres/s = 0.03\ m^3/s;\quad D_1 = 0.2\,m; D_2 = 0.1\,m;$

$$C_d = 0.96;\ \rho_w = 998\ \text{kg}/m^3;\ \ y = 1.16\ m$$

Assume venturimeter to be horizontal. The flow rate is given by,

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

Here, $A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.03141\,m^2$

$$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.1^2 = 0.007854\ m^2$$

Substituting the various values in flow rate expression, we get:

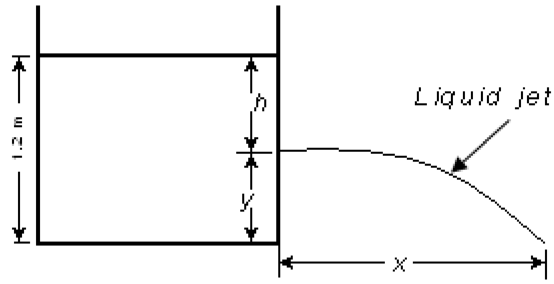
$$0.03 = 0.96 \times \frac{0.03141 \times 0.007854}{\sqrt{0.03141^2 - 0.007854^2}} \times \sqrt{2 \times 9.81} \times \sqrt{h}$$

$$\Rightarrow h = 0.756\ m$$

$$Also,\quad h = y\left(\frac{S_{hl}}{S_{ll}} - 1\right)$$

$$0.756 = 1.16\left(\frac{S_{hl}}{0.998} - 1\right)$$

$$\Rightarrow \quad \boxed{S_{hl} = 1.648}$$



7. It is required to place an orifice in the side of a tank at such an elevation that the jet will attain a maximum horizontal distance from the tank at the level of its base. What is the proper distance from the orifice to the free surface when the depth of liquid in the tank is maintained at *1.2 m*?

Solution: Depth of liquid in the tank = *1.2 m*

$$x = \sqrt{2gh} \times t$$

And, $y = -\frac{1}{2} g t^2$

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Eliminating *t*, we get:

$$y = -\frac{1}{2}g\left(\frac{x}{\sqrt{2gh}}\right)^2 \Rightarrow y = -\frac{x^2}{4h}$$

$$Also,\ 1.2 = h + y \quad \Rightarrow \quad y = 1.2 - h$$

$$\therefore \quad (1.2 - h) = -\frac{x^2}{4h}$$

$$\Rightarrow x^2 = -4h(1.2 - h) = -4.8h + 4h^2$$

For horizontal distance x **to be maximum** $\frac{dx}{dh} = 0$

$$\therefore \quad 2x\frac{dx}{dh} = -4.8 + 8h = 0 \Rightarrow \boxed{h = 0.6}$$

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CHAPTER 5
PROCESS DYNAMICS AND CONTROL

5.1 Concise Notes

5.1.1 LAPLACE TRANSFORMS

- **Laplace transforms of various function**

$$L\{f(t)\} = f(s) = \int_0^\infty f(t).e^{-st}.dt$$

Unit step : $1/s$

Unit ramp : $1/s^2$

Sin(ωt) : $\omega/(s^2+\omega^2)$

- **Initial value theorem**

$$\lim_{t\to 0}[f(t)] = \lim_{s\to\infty}[s.f(s)]$$

- **Final value theorem**

$$\lim_{t\to\infty}[f(t)] = \lim_{s\to 0}[s.f(s)]$$

5.1.2 First Order Systems

The transfer function is given as

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s+1}$$

- **Mercury Thermometer**

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s+1} \qquad where \qquad \tau = \frac{mc}{hA}$$

- **Liquid level Systems**

$$\frac{H(s)}{Q(s)} = \frac{R}{\tau s+1} \qquad where \quad \tau = AR$$

- **Mixing Process**

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s-1} \qquad where \quad \tau = \frac{V}{q}$$

5.1.3 TIME RESPONSE OF STEP INPUT

- **For First order systems**

For the step input of

$$X(t) = A$$
$$\Rightarrow X(s) = A/s$$

The transfer function for Ist order systems is given by

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{\tau s+1}$$
$$\Rightarrow Y(s) = \frac{X(s)}{\tau s+1} = \frac{A}{s(\tau s+1)}$$

Hence time response is

$$Y(t) = A(1-e^{-t/\tau})$$

5.1.4 ROUTH TEST

This method is used to determine the roots of characteristic equation and find out the stability of the system.

- **STEPS**

It has following steps:

- Find out open loop transfer function $G(s)$.
- Write your own characteristic equation; i.e. $1+G(s)=0$. And find out no. of roots.
- If any coefficient is negative then your system is definitely un-stable.
- If all coefficients are positive then your system may be stable or un-stable.
- Implement Routh Array.
- If all the elements of the first column are non-zero and positive then your system is stable.

- If any coefficient in the first column is negative then it indicates that one root of characteristic has positive real part and the system is said to be unstable.
- The no. of roots with positive real part is equal to no. of sign changes in the first column.
- If any pair of root is on the imaginary axis equi-distant from origin and all other roots are on L.H.S. then all the elements of n^{th} row will vanish, then the location of pair of imaginary roots on the imaginary axis can be found by using equation

$$\boxed{Cs^2 + D = 0}$$

Where, C,D are the elements of the $(n-1)^{th}$ row.

- **LIMITATIONS**
  - This Routh test used only for systems having characteristic equation.
  - It is not used for systems having exponential terms in their characteristic equation like transportation lag.
  - It gives information about the stability of the system but does not give any information about the degree of stability.

It does not give actual location of roots on the plane.

5.1.5 ROOT LOCUS

It is graphical method to find out the stability of control system.

- **STEPS**
  - Find out open loop transfer function $G(s)$.
  - Arrange open loop transfer function in the following manner:

$$\boxed{G(s) = K.\frac{N}{D} = K.\frac{(s-z_1)(s-z_2)..........(s-z_m)}{(s-p_1)(s-p_2)..........(s-p_n)}}$$

Where, z_i =open loop zeros and p_i = open loop poles

- Determine the no. of zeros and no. of poles.
- Locate these zeros and poles on the complex plane.
- Check whether $n \geq m$, if yes then proceed with following steps:
- The no. of root loci or branches = n.
- The root loci began at open loop poles and end at open loop zeros.
- If two open loop poles are adjacent to each other, the two root loci start from poles which intersect and leave the real axis.


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- If the sum of no. of poles and zeros is odd in no. then real axis become the part of root locus diagram for given system.
- The no. of asymptote = (n-m).

These asymptote are straight line start from centre of gravity (γ).

$$\gamma = \frac{\sum_{j=1}^{n} p_j - \sum_{i=1}^{m} z_i}{n-m}$$

- The angle made by asymptote with the real axis

$$\phi = \frac{\pi(2k+1)}{n-m}$$ where k=0,1,2,…..

- The break-away point is the point at which two real root loci immerging from adjacent poles on the real axis then intersect and leave the real axis. It is determined by following equation

$$\sum_{j=1}^{n} \frac{1}{s-p_j} = \sum_{i=1}^{m} \frac{1}{s-z_i}$$

- The location of pair of roots on the imaginary axis equi-distant from origin is find by Routh test.

$$Cs^2 + D = 0$$

5.1.6 BODE PLOT (DIAGRAM)

It is a way to represent the frequency response characteristics of a system. As we can see from following equations

$$AR = |G(j\omega)| \text{........................}(7.5)$$
$$\phi = \arg G(j\omega) \text{........................}(7.6)$$

The amplitude ratio and phase shift of the ultimate response of a system are functions of the frequency ω.

How the logarithm of the amplitude ratio varies with frequency

How the phase shift varies with the frequency.

- **First Order Systems**

Open loop transfer function,

$$G(s)=\frac{Y(s)}{X(s)}=\frac{1}{\tau s+1}$$

$$if\;\;X(t)=ASin\omega t$$

$$\Rightarrow X(s)=\frac{A\omega}{s^2+\omega^2}$$

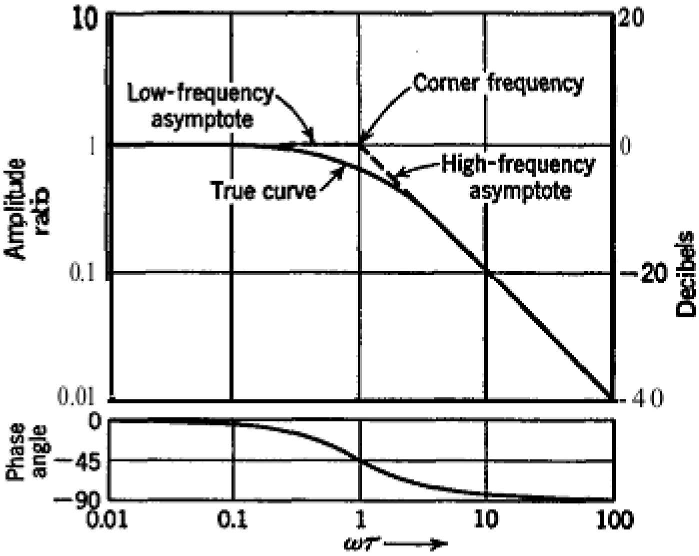
hence,

$$\Rightarrow \boxed{Y(t)=\frac{A\omega}{\sqrt{1+\tau^2\omega^2}}Sin(\omega t+\phi)}$$

Therefore

$$\boxed{AR=\frac{1}{\sqrt{1+\tau^2\omega^2}}}$$

$$\boxed{\phi=tan^{-1}(\tau\omega)}$$



- *Low frequency asymptote*

$$As\quad \omega\rightarrow 0\;\Rightarrow\; AR=1$$

- *High frequency asymptote*

$$As\quad \omega\rightarrow\infty\;\Rightarrow\; AR\approx\frac{1}{\omega\tau}$$

$$\Rightarrow \log AR=\log\left(\frac{1}{\omega\tau}\right)$$

$$\Rightarrow \boxed{\log AR=-\log(\omega\tau)}$$

5.1.7 BODE STABILITY CRITERIA

- The control system is unstable if an *A.R.* of an open loop transfer function is exceeding unity at the cross over frequency for which the phase angle is -180^0.
- it is applied to the system for which *A.R.* and phase lag curve decreases continuously with us.
  - If *A.R.* at *Φ = - 180^0 is < 1* ⇨ control system is stable.
  - If *A.R.* at *Φ = - 180^0 is > 1* ⇨ control system is un-stable.

- **CROSSOVER FREQUENCY (ω_{co})**

  It is the frequency at which the phase angle is -180^0 on the Bode plot.

- **RULES TO PLOT BODE DIAGRAMS**
  - Overall A.R. for the control system *A.R.* for the control system is obtained by multiplying the individual *A.R.* of different component.

$$\boxed{AR=AR_1\times AR_2\times.........}$$

For graphical addition individual *A.R.* which is above 1 on the diagram is taken as positive and below 1 is taken as negative.

$$\log AR = \log AR_1 + \log AR_2 + ...$$

- Overall phase angle for the control system is obtained by summing individual phase angle.

$$\phi = \phi_1 + \phi_2 + \phi_3 + ...$$

- Overall *A.R*. curve shift vertically upwards by a constant value if any constant is present in the overall transfer function of the system.
- The presence of constant in the overall transfer function has no effect on the overall phase angle.

5.2 SOLVED PROBLEMS

1. Find the Solution of the given function

$$\frac{d^4x}{dt^4} + \frac{d^3x}{dt^3} = Cost \quad where \;\; x(0) = x'(0) = x'''(0) = 0, \;\; x''(0) = 1$$

Solution: Taking Laplace transform of the given function, we get

$$\Rightarrow \left[s^4x(s) - s^3x(0) - s^2x'(0) - sx''(0) - x'''(0)\right] + \left[s^3x(s) - s^2x(0) - sx'(0) - x''(0)\right] = \frac{s}{s^2+1}$$

$$\Rightarrow \left[s^4x(s) - s\right] + \left[s^3x(s) - 1\right] = \frac{s}{s^2+1}$$

$$\Rightarrow x(s)s^3\left[s+1\right] + \left[s+1\right] = \frac{s}{s^2+1}$$

$$\Rightarrow (s+1)\left(x(s)s^3 - 1\right) = \frac{s}{s^2+1}$$

$$\Rightarrow \boxed{x(s) = \frac{s}{s^3(s^2+1)(s+1)} + \frac{1}{s^3}}$$

2. A thermometer follows first order dynamics with $\tau = 0.2\,\text{min}$ is placed at temperature bath keeping at 100 ^{0}C and is allowed to reach steady state. It is suddenly transfer to another bath keeping at 150 ^{0}C at time t=0, and is left for 0.2 min, it is immediately return to original bath at 100 ^{0}C. Calculate the thermometer reading at time t=0.1 min.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Solution:

$$Y(t) = A(1-e^{-t/\tau})$$
$$at \quad t = 0.1\min$$
$$\Rightarrow Y(t) = (150-1000)(1-e^{-0.1/0.2})$$
$$\Rightarrow y - y_s = 50(1-e^{-0.5})$$
$$\Rightarrow y = 19.667+100 \qquad as\ y_s = 100$$
$$\Rightarrow \boxed{y = 119.67\ ^0C}$$

3. Find out the transfer function H(s) / Q(s), for the liquid level system when

(i) Liquid level is at 1m.

(ii) Liquid level is at 3m.

Data: h_s= 1m , q_o= 10 m^3 / min

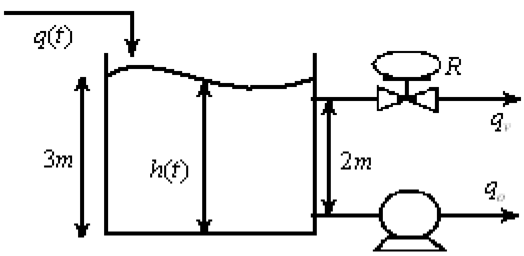
A= 1 m^2 , R= 0.5

Solution:

(i) Taking mass balance on tank when h= 1m,

$$q - q_o = A.\frac{dh}{dt} \qquad unsteady\ state$$
$$\Rightarrow q_s - q_o = A.\frac{dh_s}{dt} \qquad steady\ state$$
$$\Rightarrow q - q_s = A.\frac{d(h-h_s)}{dt}$$
$$\Rightarrow Q = A.\frac{dH}{dt}$$
$$taking\ Laplace\ transforms,\ we\ get$$
$$\Rightarrow Q(s) = A.sH(s)$$
$$\Rightarrow \boxed{\frac{H(s)}{Q(s)} = \frac{1}{s}}$$

(ii) Taking mass balance on tank when h= 3m,



$$q-\left(q_o+q_v\right)=A.\frac{dh}{dt} \quad \textit{unsteady state}$$

$$\Rightarrow q_s-\left(q_{os}+q_{vs}\right)=A.\frac{dh_s}{dt} \quad \textit{steady state}$$

$$\Rightarrow \left(q-q_s\right)-\left(q_v-q_{vs}\right)=A.\frac{d(h-h_s)}{dt}$$

$$\Rightarrow Q-Q_v=A.\frac{dH}{dt}$$

$$\Rightarrow Q-\frac{H}{R}=A.\frac{dH}{dt} \qquad as\ Q_v=\frac{H}{R}$$

taking Laplace transforms, we get

$$\Rightarrow Q(s)-\frac{H(s)}{R}=A.sH(s)$$

$$\Rightarrow \frac{H(s)}{Q(s)}=\frac{R}{ARs+1}$$

$$\Rightarrow \boxed{\frac{H(s)}{Q(s)}=\frac{0.5}{0.5s+1}}$$

4. if X(t) = 3 is a step change in input, for the transfer function $G(s)=\dfrac{10}{2s^2+0.3s+0.5}$ **find out overshoot, decay ratio, frequency and y(t)$_{max}$.**

Solution: We've

$$G(s)=\frac{10}{2s^2+0.3s+0.5} \Rightarrow G(s)=\frac{10}{0.5(4s^2+0.6s+1)} \Rightarrow G(s)=\frac{20}{4s^2+0.6s+1}$$

Comparing with

$$G(s)=\frac{K_p}{\tau^2 s^2+2\zeta\tau s+1}$$

we get,

$$\tau=2 \quad and \quad \zeta=0.15$$

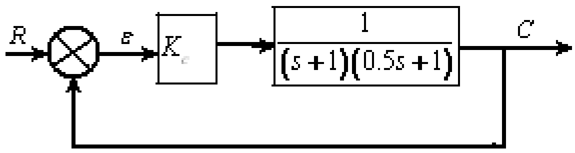
$$*Overshoot=\exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) \Rightarrow Overshoot=\exp\left(\frac{-\pi\times 0.15}{\sqrt{1-0.15^2}}\right) \Rightarrow \boxed{Overshoot=0.6209}$$

$$*\ Decay\ ratio=(Overshoot)^2 \Rightarrow Decay\ ratio=(0.6209)^2 \Rightarrow \boxed{Decay\ ratio=0.3855}$$

$$*\ frequency,\quad 2\pi f=\frac{\sqrt{1-\zeta^2}}{\tau} \Rightarrow frequency,\ f=0.0787$$

$$*\ y(t)_{\max}=A(1+Overshoot) \Rightarrow y(t)_{\max}=60(1+0.6209) \Rightarrow \boxed{y(t)_{\max}=97.254}$$

5. P Controller is used for 2 first order system. τ for systems are 1 and 0.5. the value of steady state gain of controller is 5, the set point of control system is given by step change of magnitude 0.5 then find out off set.



Solution: transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{K_c}{(s+1)(0.5s+1)+K_c} \Rightarrow \frac{C(s)}{R(s)} = \frac{5}{0.5s^2+1.5s+6}$$

$$\because R(t) = 0.5 \quad \Rightarrow R(\infty) = 0.5$$

$$\therefore R(s) = 0.5/s$$

$$\Rightarrow \frac{C(s)}{0.5/s} = \frac{5}{0.5s^2+1.5s+6} \Rightarrow sC(s) = \frac{2.5}{0.5s^2+1.5s+6}$$

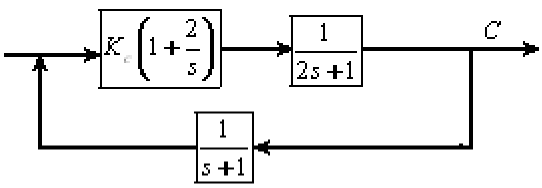
applying final value theorem, we've

$$\lim_{t\to\infty}\{C(t)\} = \lim_{s\to o}\{sC(s)\}$$

$$\Rightarrow C(\infty) = \frac{2.5}{0.5\times 0+1.5\times 0+6} = \frac{2.5}{6} \Rightarrow C(\infty) = 0.41667$$

$$\therefore \; off \;\; set = R(\infty) - C(\infty) \quad \Rightarrow off \;\; set = 0.5 - 0.41667 \Rightarrow \boxed{off \;\; set = 0.0833}$$

6. Check the stability of the following system and find out the ultimate value of K_c above which system is unstable.



Solution: the open loop transfer function of the system is given by

$$G(s) = K_c\left(1+\frac{2}{s}\right)\left(\frac{1}{2s+1}\right)\left(\frac{1}{s+1}\right)$$

Its characteristic equation is given by: 1+G(s) = 0

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$$\Rightarrow 1+K_c\left(1+\frac{2}{s}\right)\left(\frac{1}{2s+1}\right)\left(\frac{1}{s+1}\right)=0$$

$$\Rightarrow 2s^3+3s^2+s+K_cs+2K_c=0$$

$$\Rightarrow \boxed{2s^3+3s^2+(K_c+1)s+2K_c=0}$$

r_1	2	$K_c + 1$
r_2	3	$2K_c$
B	$\frac{3-K_c}{3}$	

for system stability,

$$b>0 \Rightarrow b=\frac{3-K_c}{3}>0 \Rightarrow \boxed{K_c<3}$$

hence, the ultimate value

$$\boxed{K_c=3}$$

The roots lie on imaginary axis at ultimate value of K_c,

$$\Rightarrow Cs^2+D=0$$

$$\Rightarrow 3s^2+2K_c=0$$

$$\Rightarrow 3s^2+2\times 3=0$$

$$\Rightarrow s\boxed{=\pm\sqrt{2}j}$$

7. Draw the root locus diagram for the given transfer function:

$$G(s)=\frac{K_c}{(s+1)(s+2)(s+3)}$$

Solution: We've

$$G(s)=K_c.\frac{1}{(s+1)(s+2)(s+3)}$$

no. of zeros, $m=0$ *and no. of poles,* $n=3$

$$p_1=-1,\ p_2=-2 \ \text{and}\ p_3=-3$$

$the\ no.\ of\ loci\ = 3$

$no.\ of\ asymptotes\ = n - m = 3 - 0 = 3$

$\square Centre\ of\ gavity = \frac{-1-2-3}{3} = -2$

$\square Asymptotes\ angle,$

$$\Rightarrow \phi_1 = \frac{\pi(2\times0+1)}{3} = \frac{\pi}{3} \quad \Rightarrow \phi_2 = \frac{\pi(2\times1+1)}{3} = \pi \quad \Rightarrow \phi_1 = \frac{\pi(2\times2+1)}{3} = \frac{5\pi}{3}$$

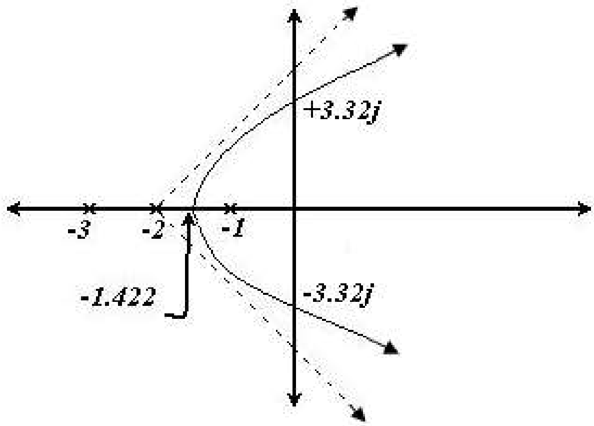
$*\ Break-away\ point,$

$$\sum_{j=1}^{n} \frac{1}{s-p_j} = \sum_{i=1}^{m} \frac{1}{s-z_i}$$

$$\Rightarrow \sum_{j=1}^{n} \frac{1}{s-p_j} = 0$$

$$\Rightarrow \frac{1}{s+1} + \frac{1}{s+2} + \frac{1}{s+3} = 0$$

$$\Rightarrow \boxed{s = -1.422, -2.577}$$



$\because Break-away\ point\ is\ between\ -1\ and\ -2,$

$\Rightarrow s = -1.422$

$characteristic\ equation \Rightarrow 1 + G(s) = 0$

$$\Rightarrow 1 + \frac{K_c}{(s+1)(s+2)(s+3)} = 0$$

$$\Rightarrow \boxed{s^3 + 6s^2 + 11s + 6 + K_c = 0}$$

$from\ Routh\ array,$

r_1	1	11
r_2	6	$6 + K_c$
b	$-K_c + 60$	

We get, the ultimate value of $K_c = 60$, for the range of unstability.

$$\Rightarrow Cs^2 + D = 0$$

$$\Rightarrow 6s^2 + (6 + K_c) = 0$$

$$\Rightarrow 6s^2 + (6 + 60) = 0$$

$$\Rightarrow \boxed{s = \pm 3.31j}$$

8. A PID controller output p(t), in time domain, is given by,

$$P(t) = 30 + 5e(t) + 1.25\int_0^t e(t)dt + 15\frac{de(t)}{dt}$$

Where, e(t) is the error at time t. The transfer function of the process to be controlled is $G_p(s) = \frac{10}{(200s+1)}$ **. The measurement of the controlled variable is instantaneous and accurate. The transfer function of the controller is**

Solution: We've

$$P(t) = 30 + 5\,e\,(t)\; + 1.25\int_0^t e(t)dt\; t\; 15\;\frac{de(t)}{dt}$$

$$\therefore\; \overline{P}\;(t) = p\;(t)\; - \;p\;(s)$$

$$= 5\;e\;(t) + 1.25\int_0^t e(t)dt + 15\frac{de\;(t)}{dt}$$

$$G_c\; = \frac{\overline{P}\;(s)}{e(s)} = 5 + \frac{1.25}{s} + 15s$$

$$= 5 + \frac{5}{4s} + 15s = \left[1 + \frac{1}{4s} + 3s\right] = \left[\frac{4s + 1 + 12s^2}{4s}\right]$$

$$Gc\;\; = \;5\left(\frac{12\;s^2 + 4s + 1}{4\;s}\right)$$

CHAPTER 6
MECHANICAL OPERATION

6.1 Concise Notes
6.1.1 Size Analysis

- **Sphericity**

$$\phi_s = \frac{\text{Surface to volume ratio of sphere having same volume as particle}}{\text{Surface to volume ratio of particle}} = \frac{6/D_p}{S_p/V_p}$$

- **Equivalent Diameter**

i. VOLUME – SURFACE MEAN DIAMETER

The volume – surface (Sauter) mean diameter defined as

$$\bar{D}_s = \frac{1}{\sum_{i=1}^{n}\left(x_i/\bar{D}_{pi}\right)}$$

ii. ARITHMETIC MEAN DIAMETER

$$\bar{D}_N = \frac{\sum_{i=1}^{n}\left(N_i\bar{D}_{pi}\right)}{N_T}$$

Where N_T = total number of particles in entire sample

iii. MASS MEAN DIAMETER

$$\bar{D}_w = \sum_{i=1}^{n}\left(x_i\bar{D}_{pi}\right)$$

iv. VOLUME MEAN DIAMETER

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{n}\left(x_i/\bar{D}_{pi}^3\right)}\right]^{1/3}$$

6.1.2 SIZE REDUCTION

Size Reduction may refer to the operation in which we reduce the size of coarse particles into fine or very fine particles.

6.1.2.1 LAWS OF CRUSHING

i. RITTINGER'S LAW

It states that the work required in crushing is proportional to the new surface created. It is written as:

$$\frac{P}{\dot{m}} = K_r \left(\frac{1}{\bar{D}_{sb}} - \frac{1}{\bar{D}_{sa}} \right)$$

Where, K_r = Rittinger's Constant

$\bar{D}_{sb}$, $\bar{D}_{sa}$ = Volume surface mean diameters of product and feed, respectively

$\frac{P}{\dot{m}}$ = power required per unit feed rate

This law assumes constant crushing efficiency which, for a given machine and feed material, is independent of the sizes of feed and product, this law also assumes equal sphericities of feed and product and constant mechanical efficiency.

ii. KICK'S LAW

It states that work required for crushing a given mass of material is constant for the same reduction ratio, i.e., the ratio of the initial particle size to the final particle size. This is written as:

$$\frac{P}{\dot{m}} = K_k \ln \frac{\bar{D}_{sa}}{\bar{D}_{sb}}$$

Where, K_k = Kick's law Constant

This law is based on stress analysis of plastic deformation within the elastic limit.

iii. BOND'S LAW

It states that the work required to form particles of size D_p from a very large feed is proportional to the square root of the surface to volume ratio of the product. This law is written as

$$\frac{P}{\dot{m}} = \frac{K_b}{\sqrt{D_p}}$$

Where K_b = bond's constant and depends upon the type of machine and on the material to be crushed.

- **WORK INDEX (W_i)**

The work index, W_i, is defined as the gross energy requirement in kWh per ton of feed needed to reduce a very large feed to such a size that 80 % of the product passes a 100 μm screen. This definition of W_i has been used to relate K_b and W_i, for D_p in mm, P in kW and $\dot{m}$ in tons per hour, the relation between K_b and W_i is given by

$$K_b = \sqrt{100\times10^{-3}}W_i = 0.3162W_i$$

If 80 % product of the feed passes a mesh size of D_{pa} mm and 80 % of the product passes a mesh size of D_{pb} mm, then the above equations can be reduced to

$$\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right)$$

6.1.2.2 CRITICAL SPEED OF BALL MILL

The critical speed (rps) is given by:

$$N_C = \frac{1}{2\pi}\sqrt{\frac{g}{(R-r)}}$$

Where *R* is the internal radius of shell in meters and r is the radius of balls used in mill in meters.

Ball mills are normally operated at around 75% of critical speed, so a mill with diameter 5 meters will turn at around ~14 rpm.

6.1.3 POWER REQUIRED FOR MIXING

$$N_{Po} = f\left(N_{\mathrm{Re}}, N_{Fr}, \frac{D_v}{D_i}, \frac{W_i}{D_v}, ...\right)$$

Where

N_{Po} = Power number = $\dfrac{P}{\rho N^3 D_i^5}$

N_{Re} = Reynolds number = $\dfrac{\rho N D_i^2}{\mu}$

N_{Fr} = Froude number = $\dfrac{N^2 D_i}{g}$

6.1.4 FILTRATION

Filtration is the separation of solid particles or liquid ones (droplets) from liquids and gases with the help of a filter medium also called a septum, which is essentially permeable to only the fluid phase of the mixture being separated. In earlier times, this process was carried out with felts, and the word "filter" has a common derivation with "felt". Often however, purification of a liquid or gas is called filtration even when no semi-permeable medium is involved (as in electro-kinetic filtration).

The liquid more or less thoroughly separated from the solids is called the filtrate, effluent, permeate or, more rarely, clean water. As in other separation processes, the separation of phases is never complete: Liquid adheres to the separated solids (cake with residual moisture) and the filtrate often contains some solids (solids content in the filtrate or turbidity).

- **CALCULATION FOR PRESSURE DROP FOR CAKE FILTRATION**

Overall pressure drop is given as

$$\Delta p = \Delta p_C + \Delta p_M$$

Where

$\Delta p_C = \dfrac{\mu c V u \alpha}{A}$ = pressure drop across cake

$\Delta p_M = \mu u R_m . A$ = pressure drop across medium

- **FOR CONSTANT PRESSURE FILTRATION**

The filtration design equation is given as

$$\frac{t}{V} = \frac{K_p}{2} V + B$$

Where

$$K_p = \frac{\mu \alpha c}{A^2 \Delta p_C}, \text{ s.m}^{-6}$$

$$B = \frac{\mu R_m}{A \Delta p_M}, \text{ s.m}^{-3}$$

6.1.5 CYCLONE SEPARATORS

Cyclonic separation is a method of removing particulates from an air, gas or liquid stream, without the use of filters, through vortex separation. Rotational effects and gravity are used to separate mixtures of solids and fluids. The method can also be used to separate fine droplets of liquid from a gaseous stream.

Separation factor, $S = \frac{v^2}{rg}$

6.2 SOLVED PROBLEMS

1.For a cyclone of diameter 0.2 m with a tangential velocity of 15 m/s at the wall, the separation factor is

Solution: Separation factor, $S = \frac{V_0^2}{gR}$

Where, V_0 tangential velocity =15m/s

R= radius = 0.1m

S= $\frac{(15)^2}{9.81 \times 0.1} = 229.59 \approx 230$

2. What is the terminal velocity in m/s, calculated from Stokes law, for a particle of diameter 0.1 x 10^{-3} m, density 2l800 kg/m^3 settling in water of density 1000 kg/m^3 and viscosity 10^{-3} kg/ms) ? (Assume g = 10 m/s^2)

Solution:

$$u = \frac{g D_P^2 (\rho_1 - \rho)}{18 \mu}$$

$$=\frac{10\times(0.1\times10^{-3})^2(2800-1000}{18\times10^{-3}}$$

$$=10^{-2}m/s$$

3. A bed fluidized by water is used for cleaning sand contaminated with salt. The particles of sand and salt have the same shape and size but different densities (ρ_{sand} = 2500 kg/m3 and ρ_{salt} = 2000 kg/m3). If the initial volume fraction of the salt in the mixture is 0.3 and if the initial value of the minimum fluidization velocity (U_{mf}) is 0.9 m/s, find the final value of the U_{mf} (in m/s) when the sand is washed free of the salt. Assume that the bed characteristics (bed porosity and solid surface area per unit volume) do not change during the operation and that the pressure drop per unit length is directly proportional to the fluid velocity

Solution:

For fluidized bed $\Delta\rho \propto VF$

$$\Delta\rho=(1-\varepsilon)\ (\rho_p)gL = Weight \text{ of bed - Buoyancy force}$$

for first condition,

$$\rho_{sand} = 2500\ \text{kg/m}^3, \rho_{salt} = 2000kg/m^3, and\ \text{U}_{mf} = 0.9m/s$$

$$Weight \text{ of bed} = \text{L}(1\text{-}\varepsilon)\times0.3\times\rho_{salt}\times g+(1-\varepsilon)\times L\times0.7\times\rho_{sand}\times g$$

$$Buoyancy = \text{L}(1\text{-}\varepsilon)\times\text{L}\times\rho_{wat}\times g$$

$$\Delta p = KU_{mf} = L(1-\varepsilon)\times0.3\times\rho_{salt}\times g+(1-\varepsilon)\times L\times0.7\times\rho_{sand}\times g-L(1-\varepsilon)\times L\times\rho_{water}\times g$$

For second condition,

For second condition,

When sand is washed free of salt, then

$$\text{KU}_{m_2} = (1-\varepsilon)\ \text{L}\times\rho_{sand}\times g-(1-\varepsilon)\rho_{water}\times g\times L$$

$$\frac{KU_{m_1}}{KU_{m_2}} = \frac{\text{L}(1\text{-}\varepsilon)\times0.3\times\rho_{salt}\times g+(1-\varepsilon)\times L\times0.7\times\rho_{sand}\times g-L(1-\varepsilon)\times\rho_{water}\times g}{L(1-\varepsilon)\times\rho_{sand}\times g-(1-\varepsilon)\rho_{water}}$$

$$\frac{0.3\times\rho_{salt}+0.7\times\rho_{sand}-\rho_{water}}{\rho_{sand}{}^{-\rho w}} = \frac{0.3\times2000+0.7\times2500-1000}{2500-100} = 0.9$$

$$\because \text{U}_m = 0.9$$

$$\therefore \text{U}_{m2} = 1.0$$

4.A filtration is conducted at constant pressure to recover solids from dilute slurry. To reduce the time of filtration, the solids concentration in the feed slurry is increased by evaporating half the solvent. If the resistance of the filter medium is negligible, the filtration time will be reduced by a factor of

Solution: For constant filtration, $\frac{t}{V} = \frac{K_p}{2} V + B$

Since filter medium resistance in negligible, $B = 0$

$\therefore \quad t \propto V^2$

$\Rightarrow \quad t_1 = KV^2$

$\Rightarrow \quad t_2 = \left(\frac{V}{2}\right)^2 = \frac{KV^2}{4}$

$\therefore \quad \frac{t_1}{t_2} = 4 \text{ Or } t_2 = \frac{t_1}{4}$

5. A continuous grinder obeying the Bond crushing law grinds a solid at the rate of 1000 kg/hr from the initial diameter of 10 mm to the final diameter of 1 mm. If the market now demands particles of size 0.5 mm, the output rate of the grinder (in kg/hr) for the same power input would be reduced to

Solution: According to Bond's Law, E= 100 E, $\left(\frac{1}{\sqrt{X_P}} - \frac{1}{\sqrt{X_F}}\right)$

Where E= work done,

E_i = Bond work index,

X_P = Product size,

X_F = Feed size

for initial conditions,

$$E_1 = 100E_i\left(\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{10}}\right) \quad ....(i)$$

and $$E_2 = 100E_i\left(\frac{1}{\sqrt{0.5}} - \frac{1}{\sqrt{10}}\right) \quad ....(ii)$$

Dividing equations (i) by equation (ii), we get

$$\frac{E_1}{E_2} = \frac{1-0.316}{1.414-0.316} = 0.623$$

$\therefore$ New output rate of grinder =623 kg/hr

CHAPTER 7
PROCESS CALCULATION

7.1 CONCISE NOTES

7.1.1 BASIC TERMS

- **Atomic Weight :** The atomic weight of an element is the mass of the atom of this element based on the scale that assigns carbon a mass exactly twelve.

- **Molecular Weight :** The molecular weight of a compound is the sum of the atomic weights of atoms that constitute a molecule of the compound.

- **Gram Atom :** It is used to specify the amounts of chemical elements. It is defined as the mass in grams of an element which is numerically equal to its atomic weight.

$$\text{Gram atoms of an element} = \frac{\text{Weight in grams}}{\text{Atomic Weight}}$$

- **Gram Mole :** It is use to specify the amounts of chemical compounds. It is defined as the mass in gram of a substance that is equal numerically to its molecular weight.

$$\text{Gram moles of an element} = \frac{\text{Weight in grams}}{\text{Molecular Weight}}$$

- **Equivalent Weight :** It is defined as the ratio of atomic weight or molecular weight to its valance. The valence of an element or a compound depends on the number of hydroxyl ions (OH^+) donated or the hydrogen ions (H^+) accepted for each atomic weight or molecular weight.

$$\text{Equivalent Weight} = \frac{\text{Molecular Weight}}{\text{Valence}}$$

7.1.2 COMPOSITION OF MIXTURES

The concentration of a solution containing a solid or a liquid solute can be expressed in terms o normality, molarity and molality.

- **Normality:** It is defined as the number of gram equivalents of solute dissolved in one litre of solution. It is designated by the symbol 'N'.

$$\text{Normality, N} = \frac{\text{gram equivalents of solute}}{\text{Volume of solution in litre}}$$

- **Molarity:** It is defined as the number of gram moles of solute dissolved in one litre of solution. It is designated by the symbol 'M'.

$$\text{Molarity, M} = \frac{\text{gram moles of solute}}{\text{Volume of solution in litre}}$$

- **Molality :** It is defined as the number of gram moles of solute dissolved in one kilogram of solvent. It is designated by the symbol 'm'.

$$\text{Molality, m} = \frac{\text{gram moles of solute}}{\text{Mass of solvent in kg}}$$

- **Concentration:** It is defined as the amount of solute in gram dissolved in one litre of solution. It is designated by the symbol 'C'.

$$\text{Concentration, C } (g/l) = \frac{\text{Mass of solute in gram}}{\text{Volume of Solution in litre}}$$

- **Weight Percent :** It is the weight of any component expressed as a percentage of the total weight of the system.

$$\text{Weight \% of A} = \frac{\text{Weight of A}}{\text{Total weight of system}} \times 100$$

$$\Rightarrow \boxed{\text{Weight \% of A} = \frac{W_A}{W} \times 100}$$

Where, W_A = weight of the component A

$W = W_A + W_B$ = weight of system…… for a binary system of A and B.

- **Volume Percent :** It is the pure component volume of any component expressed as a percentage of the total volume of the system.

$$\text{Volume \% of A} = \frac{\text{Pure component vollume of A}}{\text{Total volume of system}} \times 100$$

$$\Rightarrow \boxed{\text{Volume \% of A} = \frac{V_A}{V} \times 100}$$

Where, V_A = weight of the component A

V = Total volume of the system = $V_A + V_B$ …… for a binary system of A and B.

- **Mole Percent :** It is the moles of any component expressed as a percentage of the total moles of the system.

  For a binary mixture of A and B :

$$\text{Mole \% of A} = \frac{\text{Moles of A}}{\text{Total moles of system}} \times 100$$

$$\Rightarrow \boxed{\text{Mole \% of A} = \frac{n_A}{n} \times 100}$$

$$\Rightarrow \boxed{\text{Mole \% of A} = \frac{\frac{W_A}{M_A}}{\frac{W_A}{M_A} + \frac{W_B}{M_B}} \times 100}$$

Where, M_A and M_B are the molecular weights components A and B respectively.

The sum of the weight % of all the components present in a given system is equal to unity, or

$$\boxed{\sum_{i=1}^{n} x_i = 100\%}$$

7.1.3 MATERIAL BALANCE WITHOUT CHEMICAL REACTION

All material balance calculations are based on the law of conservation of mass, which states that matter can neither be created nor destroyed during s process (i.e. mass is conserved).

According to the law of conservation of mass, we have for any process

Input = Output + Accumulation

For steady state operations / process where in the accumulation of the material is constant or nil, equation becomes,

Input = Output

7.1.4 MATERIAL BALANCE WITH RECYCLE AND PURGE STREAMS

- **RECYCLING**

Recycling is returning back a portion of stream leaving the process unit to the entrance of a process unit for further processing. This operation is carried out under steady state.

$$\text{overall conversion} = \frac{\text{A in fresh feed} - \text{A in net product}}{\text{A in fresh feed}}$$

$$\text{single pass conversion} = \frac{\text{A in mixed feed} - \text{A in gross product}}{\text{A in mixed feed}}$$

- **Recycle Ratio :** It is the ratio of the quantity of recycle stream to the quantity of fresh feed stream.

$$\text{Recycle Ratio} = \frac{R}{F}$$

- **Combined Feed Ratio :** It is the ratio quantity of mixed feed stream to the quantity of fresh feed stream.

$$\text{Combined Feed Ratio} = \frac{M}{F}$$

- **PURGING :** Purge stream is the fraction of recycled stream which is continuously taken off in order to avoid an accumulation of inerts in the recycle loop.

  At steady state,

$$\text{Inerts in purge} = \text{Inerts in fresh feed}$$

$$\text{Purge Ratio} = \frac{P}{R}$$

7.1.5 MATERIAL BALANCE WITH CHEMICAL REACTION

$$\begin{Bmatrix}\text{Accumulation}\\ \text{within the system}\end{Bmatrix} = \begin{Bmatrix}\text{Input through the}\\ \text{system boundaries}\end{Bmatrix} - \begin{Bmatrix}\text{Output through the}\\ \text{system boundaries}\end{Bmatrix} + \begin{Bmatrix}\text{Generation}\\ \text{within the}\\ \text{system}\end{Bmatrix} - \begin{Bmatrix}\text{Consumption}\\ \text{within the system}\end{Bmatrix}$$

Limiting reactant is defined as the reactant that would disappear first if a reaction goes to completion. It is the one which decides the extent to which a reaction can proceed.

Excess Reactant is defined as the reactant which is excess of the theoretical or stoichiometric requirement as determined by the desired reaction.

- **PERCENT EXCESS**

The excess reactant involved in the reaction is generally specified in terms of percent excess. It is the amount in excess of the stoichiometric requirement expressed as a percentage of the stoichiometric requirement.

For any reaction $A+B \rightarrow C$

Where B is the excess reactant, then

$$\%\text{ excess of B} = \frac{\text{moles of B supplied} - \text{moles of B required theoretically}}{\text{moles of B required theoretically}} \times 100$$

- **CONVERSION**

Consider a chemical reaction:

$$A+B \rightarrow C$$

The conversion or fractional conversion of reactant A is the ratio of the amount of A reacted to the amount of A charged. The percentage conversion of A is the amount of A reacted expressed as a percentage of the amount of A charged.

$$\%\text{ conversion of A} = \frac{\text{moles of A reacted}}{\text{moles of A charged}} \times 100$$

- **YIELD AND SELECTIVITY**

The term yield and selectivity are used in case of multiple reactions to give the information regarding the degree to which the desired reaction predominates over the side reaction(s) involved.

Consider the series reactions: $A \rightarrow C \rightarrow D$

Where C is the desired product, D is the undesired product and A is the limiting reactant. Then the yield of C is given as:

$$\text{Yield of C} = \frac{\text{moles of A reacted to produce C}}{\text{moles of A totally reacted}} \times 100$$

Yield of the desired product is the quantity of the limiting reactant reacted to produce is expressed as the percentage of the quantity of the limiting reactant reacted totally.

Consider the parallel reactions:

$A \rightarrow C$

$A \rightarrow D$

Where C is the desired product, D is the undesired product. Then the selectivity is given as:

$$\text{Selectivity of C related to D} = \frac{\text{moles of C formed}}{\text{moles of D formed}}$$

Selectivity is the ratio of the moles of the desired product produced to the moles of the undesired product produced.

7.2 SOLVED PROBLEMS

1. A solution of caustic soda contains 20 % NaOH by weight. Taking density of the solution as 1.196 kg / l. find the normality, molarity and molality of the

Solution: Basis: 100 kg of solution.

The solution contains 20 kg NaOH and 80 kg water (solvent).

Density of solution = 1.196 kg / l

$$\text{Volume of Solution} = \frac{\text{weight of solution}}{\text{Density of solution}} = \frac{100}{1.196} = 83.62\ l$$

$$\text{Moles of NaOH in solution} = \frac{20}{40} = 0.5\ kmol = 500\ mol$$

$$\therefore \quad \text{Molarity (M)} = \frac{\text{moles of NaOH}}{\text{Volume of solution in litre}} = \frac{500}{83.52}$$

$$\Rightarrow \quad \boxed{\text{Molarity (M)} = 5.98}$$

For NaOH as valence = 1,

$$\text{Equivalent weight} = \text{Molecular weight}$$

$$\therefore \quad \text{Normality (N)} = \text{Molarity (M)} = 5.98$$

$$\Rightarrow \quad \boxed{\text{Normality (N)} = 5.98}$$

$$\text{Molality} = \frac{\text{moles of NaOH}}{\text{Weight of solvent in kg}} = \frac{500}{80}$$

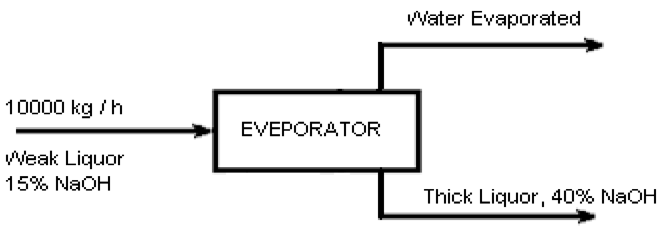
$$\Rightarrow \quad \boxed{\text{Molality (M)} = 6.25}$$

2. A single effect evaporator is fed with 10000 kg / h of weak liquor containing 15 % caustic by weight and is concentrated to get thick liquor containing 40 % by weight caustic. Calculate:

(a) kg / h of water evaporated and

(b) kg / h of thick liquor

Solution: Basis: 10000 kg / h of weak liquor.



Let x be the kg / h thick liquor obtained and y be the kg / h water evaporated.

Overall Material balance:

Total mass Input = Total mass output

kg / h weak liquor = kg / h water evaporated + kg / h thick liquor

$$10000 = x + y$$

Material Balance of NaOH:

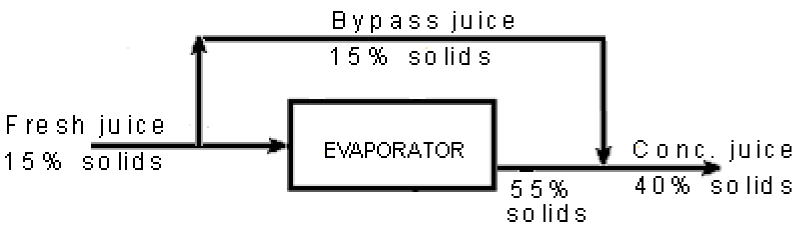
NaOH in the liquid stream = NaOH in output stream

NaOH in the weak liquor = NaOH in thick liquor

$$0.15 \times 10000 = 0.40x$$
$$\Rightarrow x = 3750 \text{ kg / hr}$$
Hence,
$$\Rightarrow y = 6250 \text{ kg / hr}$$

Water evaporated = 6250 kg / h and Thick liquor obtained = 3750 kg / h

3. Fresh juice contains 15% solids and 85 % water by weight and is to be concentrated to contain 40 % solids by weight. In a single evaporation system, it is found that volatile constituents of juice escape with water leaving the concentrated juice with a flat taste. In order to overcome this problem, part of the fresh juice bypasses the evaporator. The operation is shown schematically in figure given below:

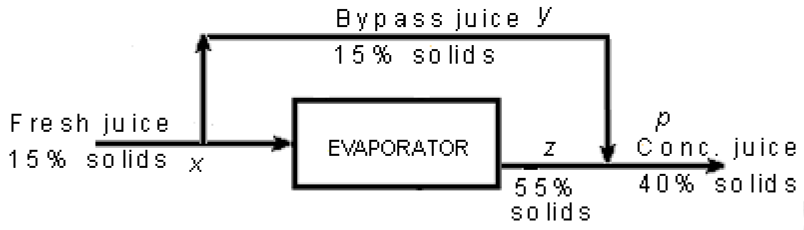


Calculate:

(a) The fraction of juice that bypasses the evaporator.
(b) The concentrated juice produced (containing 40 % solids) per 100 kg of fresh juice fed to the process.

Solution: Basis: 100 kg of fresh juice fed to the process.

Let x and y be the kg of juice fed to the evaporator and p be the kg of the concentrated juice obtained.



Material balance over the separation of bypass stream

$$x + y = 100 \ldots\ldots\ldots\ldots(i)$$

Material balance of solids over Evaporator:

$$0.15x = 0.55z \ldots\ldots\ldots\ldots(ii)$$

Overall material balance after Evaporator:

$$z + y = p \ldots\ldots\ldots\ldots(iii)$$

Material balance of solids after Evaporator:

$$0.55z + 0.15\,y = 0.40p \ldots\ldots\ldots\ldots(iv)$$

From equation (i), (ii), (iii) and (iv), we get

x = 85.94 kg, y = 14.06 kg, z = 23.44 kg, p = 37.5 kg

Fraction of juice that bypass evaporator in %,

$$= \frac{y}{100} \times 100 = \frac{14.06}{100} \times 100 = \mathbf{14.06\ \%}$$

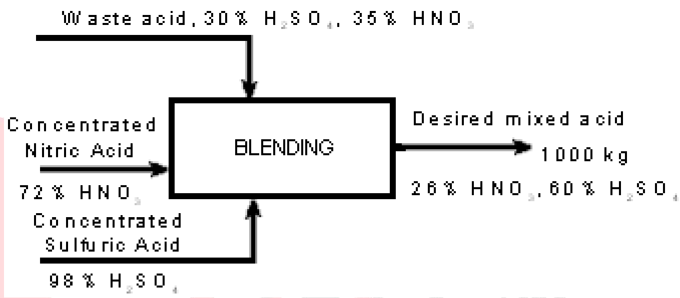
Quantity of concentrated juice obtained finally

p = 37.5 kg

4. The waste acid from a nitrating process contains 30 % H_2SO_4, 35 % HNO_3 and 35 % H_2O w/w. the acid is to be concentrated to contain 39 % H_2SO_4 and 42% HNO_3 by addition of concentrated sulfuric acid containing 98% H_2SO_4 and concentrated Nitric acid containing 72 % HNO_3. Calculate the quantities of three acids to be mixed to get 1000 kg of desired mixed acid.

Solution: Basis: 1000 kg of desired mixed acid.

Let x, y and z be the kg of waste acid, conc. Sulfuric acid and conc. Nitric acid required to make 1000 kg desired acid.



Overall Material Balance:

$x + y + z = 1000$…………..(i)

Material Balance of H_2SO_4:

$0.3x + 0.98y = 0.39 \times 1000$

⇨ $0.3x + 0.98y = 390$ ……………..(ii)

Material Balance of HNO_3:

$0.35x + 0.72z = 0.42 \times 1000$ ⇨ $0.35x + 0.72y = 420$……………..(iii)

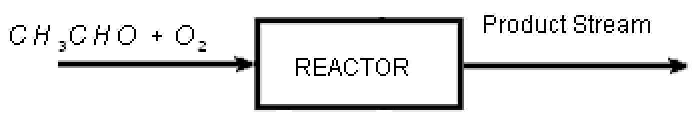
From equations (i), (ii) and (ii), we get

$x = 90.1$ kg, $y = 370.4$ kg, $z = 539.5$ kg

Thus,	**Amount of waste acid required**	**= 90.1 kg**
	Amount of conc. Sulfuric acid required	**= 370.4 kg**
	Amount of conc. Nitric acid required	**= 539.5 kg**

1. In manufacture of acetic acid by oxidation of acetaldehyde, 100 kmol of acetaldehyde is fed to a reactor per hour. The product leaving the reactor contains 14.81% acetaldehyde, 59.26 % acetic acid and rest oxygen (on mole basis). Find the % conversion of acetaldehyde.

Solution: Basis: 100 kmol / h acetaldehyde charged to reactor.



Reaction : $CH_3CHO + \frac{1}{2}O_2 \longrightarrow CH_3COOH$

The product contains 14.81% acetaldehyde and 59.26 % acetic acid by mole.

Let x be the kmol of product stream obtained per hour.

Acetic acid formed = 0.5926x kmol / h

Acetaldehyde un-reacted = 0.1481x kmol / h

From the rection, we have

1 kmol $CH_3CHO \equiv$ 1 kmol CH_3COOH

i.e., to produce 1 kmol CH_3COOH , 1 kmol CH_3CHO must reacts

thus, Acetaldehyde reacted to produce acetic acid

= $\frac{1}{1} \times 0.5926x = 0.5926x$ kmol / h

Material balance of CH_3CHO:

CH_3CHO fed to th reactor = CH_3CHO reacted 6+ CH_3CHO un reacted

CH_3CHO fed to th reactor = CH_3CHO reacted 6+ CH_3CHO un reacted

100 = 0.5926x + 0.1481x

x = 135 kmol / h

∴ Aceteldehyde reacted = 0.5926 (135) = 80 kmol / h

%Conversion of aceteldehyde = $\frac{\text{aceteldehyde reacted}}{\text{aceteldehyde charged}} \times 100 = \frac{80}{100} \times 100$

= 80 %

CHAPTER 8
CHEMICAL ENGINEERING THERMODYNAMICS

8.1 CONCISE NOTES

Thermodynamics is the branch of science that embodies the principles of energy transformation in macroscopic systems. The general restrictions which experience has shown to apply to all such transformations are known as the laws of thermodynamics. These laws are primitive; they cannot be derived from anything more basic.

- **SYSTEM AND SURROUNDING**

The words system and surroundings are similarly coupled. A system is taken to be any object, any quantity of matter, any region, and so on, selected for study and set apart (mentally) from everything else, which is called the Surroundings. The imaginary envelope which encloses the system and separates it from its surroundings is called the boundary of the system.

i. CLOSED SYSTEM (OR NON FLOW SYSTEM)

The system which can exchange energy with surroundings but which cannot transfer matter across the boundaries are known as *closed system.*

ii. OPEN SYSTEM (OR FLOW SYSTEM)

The system that can exchange both energy and matter with their environment.

iii.ISOLATED SYSTEM

An isolated system exchanges neither matter nor energy with its surroundings.

- ***Intensive properties*** are those that are independent of the mass of a system, such as temperature, pressure, and density.

- ***Extensive properties*** are those whose values depend on the size or extent of the system. Total mass, total volume and total momentum are some examples of extensive properties.

8.1.1 ZEROTH LAW OF THERMODYNAMICS

The *zeroth law of thermodynamics states* that if two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other.

8.1.2 FIRST LAW OF THERMODYNAMICS

"In a closed system undergoing any thermodynamic cycle, cyclic integral of heat and cyclic interval of work are proportional to each other when expressed in their own units and are equal to each other when expressed in the consistent (same) units"

$$\underbrace{\oint \partial Q}_{kcal} \propto \underbrace{\oint \partial W}_{kJ}$$

$$\partial Q - \partial W = dU = \text{internal energy}$$

- **INTERNAL ENERGY**

The part of heat energy which does not converted into work, just stored into the system is called increase in *internal energy* of the system.This is called the *law of conservation of energy.*Conclusively, first law of thermodynamics says that "*Heat and Work are mutually inter-convertible.*"

8.1.3 SECOND LAW OF THERMODYNAMICS

The observations which generally restricted on processes beyond that imposed by the first law.

The second law is equally well expressed in two statements that describe these restrictions:

(i) Kelvin Planck Statement
(ii) Clausius Statement

8.1.3.1 KELVIN – PLANCK STATEMENT

This statement is simply related to the work. And it state that

"It is impossible to construct a device that operates in a thermodynamic cycle and produces no effect other than net amount of positive work having heat exchange with a single thermal reservoir."

8.1.3.2 CLAUSIUS STATEMENT

This statement is simply related to the heat. And it state that

"It is impossible to construct a device that operates in a thermodynamic cycle and produces no effect other than transfer of heat from a body at low temperature toa body at high temperature."

- **THERMAL RESERVOIR**

Thermal reservoir is a body from which we can take / give any amount of heat without affecting its temperature. (i.e. its temperature remains constant)

Thermal reservoir has infinite heat capacity.

8.1.4 CLAUSIUS INEQUALITIES

when $\oint \frac{\delta Q}{T} < 0 \Rightarrow$ System is Actual $\Rightarrow$ Irreversible System

when $\oint \frac{\delta Q}{T} = 0 \Rightarrow$ System is Ideal $\Rightarrow$ Reversible System

when $\oint \frac{\delta Q}{T} > 0 \Rightarrow$ Impossible

8.1.5 ENTROPY

"*Entropy is a thermodynamic property that increases when heat is supplied and decreases when heat is rejected and remains constant if there is no heat transfer*"

$$dS = \frac{\delta Q}{T} \Rightarrow \delta Q = T.dS$$

- Entropy is independent of temperature.
- Entropy is subjected to the disorder (or order) of the molecules in the system.

as dS ↑ ⇒ disorder of molecules ↑

as dS ↓ ⇒ disorder of molecules ↓

disorder of molecules → random motion of molecules.

8.1.6 MAXWELL'S EQUATION

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial v}{\partial s}\right)_P$$

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

$$\left(\frac{\partial s}{\partial P}\right)_T = \left(\frac{\partial v}{\partial T}\right)_P$$

8.1.7 VAN DER WAALS EQUATION OF STATE

The van der Waals equation of state was proposed in 1873, and it has two constants that are determined from the behavior of a substance at the critical point. It is given by

$$\left(P + \frac{a}{v^2}\right)(V - b) = RT$$

8.1.8 SOLUTION THERMODYNAMICS

- **FUGACITY**

The concept of *fugacity* is widely used in solution thermodynamics to represent the behavior of real gases. The name fugacity is derived from the Latin for 'fleetness' or the 'escaping tendency'.

$$dG = VdP = RTd(\ln f)$$

- **FUGACITY COEFFICIENT**

The ratio of fugacity to pressure is referred to as *fugacity coefficient* and s denoted by ϕ. It is dimensionless and depends on nature of the gas, the pressure and the temperature. Integrating the equation $dG = RTd(\ln f)$ between pressures *f* and f^0,

$$G - G^0 = RT \ln \frac{f}{f^0}$$

- **ACTIVITY**

It is defined as the ratio of the fugacity to the fugacity at standard state.

$$a = \frac{f}{f^{sat}}$$

or, $$\ln(a) = \frac{V}{RT}\left(P - P^0\right)$$

- **CHEMICAL POTENTIAL**

The *chemical potential* (μ) is used as an index of chemical equilibrium in the same manner as temperature and pressure are used as indices of thermal and mechanical equilibrium.

The chemical potential μ_i of a component *i* in a solution is same as its partial molar free energy in the solution, $\bar{G}_i$. That is, chemical potential of a component *i* in a solution can be defined as

$$\mu_i = \bar{G}_i = \left(\frac{\partial G^t}{\partial n_i}\right)_{T,P,n_j}$$

8.1.9 GIBBS – DUHEM EQUATIONS

In a mixture, the partial molar properties of the components are related to one another by one of the most useful equations in thermodynamics, the Gibbs – Duhem equations. It tells us how the partial molar properties changes with compositions at constant temperature and pressure. The Gibbs – Duhem equation is given as

$$\sum n_i d\bar{M}_i = 0$$

- **For a binary mixture**

Consider a binary mixture made up of components 1 and 2 whose mole fractions in the solution are x_1 and x_2 respectively. Equation (xxxvii) can be written as

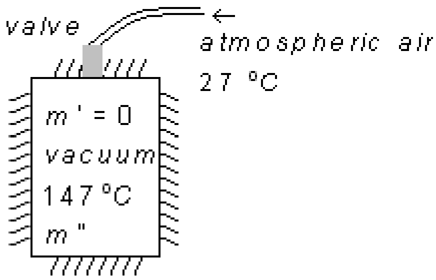
$$n_1 d\bar{M}_1 + n_2 d\bar{M}_2 = 0$$
$$\Rightarrow n_1 d\bar{M}_1 = -n_2 d\bar{M}_2$$
$$\Rightarrow x_1 d\bar{M}_1 = -x_2 d\bar{M}_2$$

Where $\bar{M}_1$ and $\bar{M}_2$ are the partial molar properties of components 1 and 2 respectively.

Dividing by dx_1 on both side of the equation, we get

$$x_1 \frac{\partial \bar{M}_1}{\partial x_1} = -x_2 \frac{\partial \bar{M}_2}{\partial x_1} \qquad (\because dx_1 = -dx_2)$$

$$\Rightarrow x_1 \frac{\partial \bar{M}_1}{\partial x_1} = x_2 \frac{\partial \bar{M}_2}{\partial x_2}$$



8.2 SOLVED PROBLEMS

1. When valve is open the atmospheric air at 27 °C and 1 atm pass into vacuum chamber and flow until its pressure become 1 atm. The new temperature of air would be?

Solution: The energy balance equation on chamber

$$m_1 h_1 + {}_1Q_2 + m'u' = {}_1W_{s2} + m_2 h_2 + m''u''$$

Where

m_1 is input air, m' is the initially present air,

m" is the finally present air, m_2 is the output air

$$\Rightarrow m_1 h_1 = m''u''$$
$$\Rightarrow h_1 = u'' \qquad (\because m_1 = m'')$$

We know that

$$h = u + Pv = C_v T + RT \qquad \text{and} \qquad u = C_v T$$

$$h = (C_v + R)T = C_p T \qquad \Rightarrow u'' = C_v T''$$
$$\Rightarrow h_1 = C_p T_1$$

Hence,

$$C_p T_1 = C_v T''$$
$$\Rightarrow T'' = \frac{C_p}{C_v}.T_1 \quad \Rightarrow T'' = \gamma.T_1 \quad \Rightarrow T'' = 1.4 \times 300$$
$$\Rightarrow \boxed{T'' = 420K}$$

2. A steel casing at temperature 725K and weighing 35 kg is quenched in 150 kg oil at 275 K. if there are no heat losses, determine the change in entropy. The specific heat (C_p) of steel is 0.88 kJ / kg.K and that of oil is 2.5 kJ / kg.K. and evaluate the loss in capacity for doing work when the steel casting in

Solution: Let T be the final temperature attained by the system. Then the heat balance gives

$$35(0.88)(725 - T) = 150(2.5)(T - 275)$$

Where T is the final temperature attained by the system. Solving this, we get, T = 309.15K. let ΔS_1 be the change in entropy of the casting and let ΔS_2 be that of oil. Then,

$$\Delta S_1 = 35 \times 0.88 \times \ln \frac{309.15}{725} = -26.25 \text{ kJ/K}$$

$$\Delta S_2 = 150 \times 2.5 \times \ln \frac{309.15}{275} = 43.90 \text{ kJ/K}$$

The total entropy change of the casting and oil together is

$$\Delta S = \Delta S_1 + \Delta S_2 = \underline{17.65 \text{ kJ/K}}$$

The loss in capacity for doing work is

$$T_0 \Delta S = 275 \times 17.65 = \underline{4853.75 \text{ kJ/K}}$$

3. The fugacity of component 1 in binary liquid mixture of components 1 and 2 at 298 K and 20 bar is given by $\hat{f}_1 = 50x_1 - 80x_1^2 + 40x_1^3$ Where *where* $\hat{f}_1$ is in bar and x_i is the mole fraction of component 1. Determine:

(a) The fugacity f_1 of pure component 1.
(b) The fugacity coefficient ϕ_1
(c) The Henry's law constant K_1
(d) The activity coefficient γ_1

Solution: (a) When the mole fraction approaches unity, the fugacity of component in the solution becomes equal to the fugacity of the pure component. That is,

$f_1 = \hat{f}_1$ when $x = 1$. Therefore, $f_1 = 50 - 80 + 40 = \underline{10\ bar}$

(b) $\phi_1 = f_1 / P = 10/20 = \underline{0.5}$

(c) By equation, Henry's Law constant is

$$K_1 = \lim_{x_1 \to 0} \frac{\hat{f}_1}{x_1} = \lim_{x_1 \to 0} \left(50 - 80x_1 + 40x_1^2\right) = \underline{50\ bar}$$

(d) The activity coefficient is given as

$$\gamma_1 = \frac{\hat{f}_1}{x_1 f_1} = \frac{\left(50 - 80x_1 + 40x_1^2\right)}{10x_1} = \underline{5 - 8x_1 + 4x_1^2}$$

4. The standard heat of formation and standard free energy of formation of ammonia at 298 K are – 46100 J / mol and – 16500 J / mol respectively. Calculate the equilibrium constant for the reaction

$$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$$

At 500 K assuming that the standard heat of reaction is constant in the temperature range 298 to 500 K.

Solution: The standard free energy of reaction is estimated from equation

$$\Delta G^0 = \sum_{products} |\nu_i| \Delta G^0_{i,f} - \sum_{reac\tan ts} |\nu_i| \Delta G^0_{i,f}$$

The second summation yields zero as the free energy of formation of the elements are zero.

$$\Delta G^0 = \sum_{products} |\nu_i| \Delta G^0_{i,f} = 2 \times -16500 = -33000\ J/mol$$

Using equation,

$$\ln K = -\frac{\Delta G^0}{RT} = \frac{33000}{8314 \times 298} = 133195$$

Therefore,

$$K \text{ at } 298 \text{ K } = 6.0895 \times 10^5$$

The standard heat of reaction at 298 K = 2 x – 46100 = – 92200 J / mol. This is assumed constant within the temperature range involved. Now using equation to evaluate the equilibrium constant.

$$\ln \frac{K}{K_1} = -\frac{\Delta H^0}{R}\left(\frac{1}{T} - \frac{1}{T_1}\right)$$

$$\Rightarrow \ln \frac{K}{6.0895 \times 10^5} = -\frac{92200}{8314}\left(\frac{1}{500} - \frac{1}{298}\right) = -15.0344$$

Therefore, the equilibrium constant at 500 K, K = 0.18

5. Ammonia synthesis reaction is represented by

$$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$$

The reactant stream consists of 1 mol N_2, 3 mol H_2 and 2 mol argon. The temperature and pressure of the reaction are 675 K and 20 bar. The equilibrium constant for the reaction is 2 x 10^{-4}. Determine how the conversion of nitrogen is affected by the presence of argon.

Solution: The total no. of moles of the initial mixture, n_0 = 1 + 3 + 2 = 6.

$$\nu = \sum \nu_i = -1 - 3 + 2 = -2$$

$$y_i = \frac{n_{i0} + \nu_i \varepsilon}{n_0 + \nu\varepsilon} = \frac{n_{i0} + \nu_i \varepsilon}{6 - 2\varepsilon}$$

Component	ν_i	n_{io}, mol	y_i
N_2	–1	1	$(1-\varepsilon)/(6-2\varepsilon)$
H_2	–3	3	$3(1-\varepsilon)/(6-2\varepsilon)$
NH_3	+2	0	$2\varepsilon/(6-2\varepsilon)$
Ar		2	
n_o		6	

$$K_y = \frac{K}{K_\phi} P^{-\nu} = KP^2$$

$$\frac{\left[2\varepsilon/(6-2\varepsilon)\right]^2}{\left[(1-\varepsilon)/(6-2\varepsilon)\right]3\left[(1-\varepsilon)/(6-2\varepsilon)\right]^2} = 2\times10^{-4}\times20^2$$

$$\Rightarrow \frac{\varepsilon(3-\varepsilon)}{(1-\varepsilon)^2} = 0.3674$$

On solving, we get ε=0.1022. thus, it is seen that the conversion of nitrogen decreases to 10.22%.

CHAPTER 9
PROCESS ECONOMICS

9.1 CONCISE NOTES

9.1.1 ESTIMATING EQUIPMENT COSTS BY SCALING

The capital cost of a project is related to capacity by the equation:

$$C_2 = C_1 \times \left(\frac{Q_2}{Q_1}\right)^n$$

Where, C_2 = capital cost of the project with capacity Q_2,

C_1 = capital cost of the project with capacity Q_1.

The value of the index n is traditionally taken as 0.6; the well-known **six-tenths rule**.

9.1.2 ESTIMATION OF TOTAL PRODUCT COST

Total product cost is the sum of the manufacturing cost and general expenses.

Total Product Cost = Manufacturing Cost + General Expenses

- **MANUFACTURING COSTS**

All expenses directly connected with the manufacturing operation or the physical equipment of a process plant itself are included in the manufacturing costs.

These expenses, as considered here, are divided into three classifications as follows:

A) Direct production costs (DPC),

B) Fixed charges (FC),

C) Plant-overhead costs (POC).

Manufacturing Costs = DPC + FC + POC

- **DIRECT PRODUCTION COSTS**

It includes expenses directly associated with the manufacturing operation. This type of cost involves expenditures for raw materials (including transportation, unloading, etc.,); direct operating labor; supervisory and clerical labor directly connected with the manufacturing operation; plant maintenance and repairs; operating supplies; power; utilities; royalties; and catalysts.

- **FIXED CHARGES**

Fixed charges are expenses which remain practically constant from year to year and do not vary widely with changes in production rate. Depreciation, property taxes, insurance, and rent require expenditures that can be classified as fixed charges.

- **PLANT-OVERHEAD COSTS**

Plant-overhead costs are for hospital and medical services; general plant maintenance and overhead; safety services; payroll overhead including pensions, vacation allowances, social security, and life insurance; packaging, restaurant and recreation facilities, salvage services, control laboratories, property protection, plant superintendence, warehouse and storage facilities, and special employee benefits.

- **GENERAL EXPENSES**

In addition to the manufacturing costs, other general expenses are involved in any company's operations. These general expenses may be classified as

a) Administrative expenses,

b) Distribution and marketing expenses,

c) Research and development expenses,

d) Financing expenses,

e) Gross-earnings expenses.

9.1.3 TYPES OF INTEREST

On the basis of accounting the interests can be divided as follows:

9.1.3.1 SIMPLE INTEREST

If P represents the principal, n the number of time units or interest periods and i the interest rate based on the length of one interest period, the amount of simple interest Z during n interest periods is

$$Z = P \times i \times n$$

The principal must be repaid eventually; therefore, the entire amount S of principal plus simple interest due after n interest periods is

$$S = P + Z = P(1 + i \times n)$$

9.1.3.2 ORDINARY AND EXACT SIMPLE INTEREST

If the interest rate is expressed on the regular yearly basis and *d* represents the number of days in an interest period, the following relationships apply:

$$Ordinary\ simple\ interest = P \times i \times \frac{d}{360}$$

$$Exact\ simple\ interest = P \times i \times \frac{d}{365}$$

9.1.3.3 COMPOUND INTEREST

The total amount of principal plus compounded interest due after *n* interest periods and designated as

$$S = P(1+i)^n$$

The term $(1+i)^n$ is commonly referred to as the discrete single-payment compound-amount factor.

9.1.3.4 NOMINAL & EFFECTIVE INTEREST RATES

S represents the total amount of principal plus interest due after *n* periods at the periodic interest rate *i.* Let *r* be the nominal interest rate under conditions where there are *m* conversions or interest periods per year.

Then the interest rate based on the length of one interest period is *r/m,* and the amount S after 1 year is

$$S_{after\ 1\ year} = P\left(1+\frac{r}{m}\right)^m$$

The effective interest rate as i_{eff}, the amount S after 1 year can be expressed in an alternate form as

$$S_{after\ 1\ year} = P(1+i_{eff})$$

Therefore, by combining above both the equation, we get expression for effective interest rate in terms of nominal interest rate as

$$Effective\ annual\ interest\ rate = i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$$

And *Nominal annual interest rate = r*

9.1.3.5 CONTINUOUS INTEREST

The effective annual interest rate $\boldsymbol{i_{eff}}$ which is the conventional interest rate that most executives comprehend, is expressed in terms of the nominal interest rate ***r*** compounded continuously as

$$i_{eff} = e^r - 1$$

Continuous interest compounding at a nominal annual interest rate of *r* the amount *S* and initial principal *P* will compound to in *n* years is as

$$S = Pe^{rn}$$

9.1.4 PRESENT WORTH AND DISCOUNT

The present worth (or present value) of a future amount is the present principal which must be deposited at a given interest rate to yield the desired amount at some future date.

For compound interest $\quad Present\ worth\ P = S\frac{1}{(1+i)^n}$

For continuous interest: $Present\ worth\ P = S\frac{1}{e^{rn}}$

9.1.5 RELATION BETWEEN AMOUNT OF ORDINARY ANNUITY AND THE PERIODIC PAYMENTS

Let *R* represent the uniform periodic payment made during *n* discrete periods in an ordinary annuity. The interest rate based on the payment period is *i*, and *S* is the amount of the annuity.

$$S = R\frac{(1+i)^n - 1}{i}$$

9.1.6 PRESENT WORTH OF AN ANNUITY

The present worth of un annuity is defined as the principal which would have to be invested at the present time at compound interest rate *i* to yield a total amount at the end of the annuity term equal to the amount of the annuity. Let *P* represent the present worth of an ordinary annuity.

For compound interest: $\quad S = R\frac{(1+i)^n-1}{i(1+i)^n}$

9.1.7 PERPETUITIES AND CAPITALIZED COSTS

If perpetuation is to occur, the amount *S* accumulated after *n* periods minus the cost for the replacement must equal the present worth *P*. Therefore, letting C_R represent the replacement

Cost then present worth given as

$$P = \frac{C_R}{(1+i)^n - 1}$$

The capitalized cost is defined as the original cost of the equipment plus the present value of the renewable perpetuity. Designating *K* as the capitalized cost and C_V as the original cost of the equipment then capitalized cost of the equipment is given as:

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

9.1.8 STRAIGHT-LINE METHOD

The annual depreciation cost may be expressed in equation form as follows:

$$d = \frac{V - V_s}{n}$$

Where, *d* = annual depreciation, *Rs*/year

V = original value of the property at start of the service-life period

V_s = salvage value of property at end of service life,

n = service life, years

The asset value (or book value) of the equipment at any time during the service life may be determined from the following equation:

$$V_a = V - a\,d$$

Where, V_a = asset or book value, dollars,

a = the number of years in actual use.

9.1.9 DECLINING-BALANCE (OR FIXED PERCENTAGE) METHOD

The depreciation cost for the first year of the property's life is V_f, where *f* represents the fixed-percentage factor.

At the end of the first year:

$$\text{Asset value} = V_a = V(1 - f)$$

At the end of n years (i.e., at the end of service life):

$$V_n = V(1 - f)^n = V_s$$

$$f = 1 - \left(\frac{V_s}{V}\right)^{1/n}$$

9.1.10 SUM-OF-THE-YEARS-DIGITS METHOD

Equation for determining annual depreciation is given by:

$$d_a = Depreciation\ for\ year\ a = \frac{(n-a+1)}{\sum_1^n a}(V - V_s)$$

$$d_a = \frac{2(n-a+1)}{n(n+1)}(V - V_s)$$

9.2 SOLVED PROBLEMS

1. A sale contract signed by a chemical manufacture is expected to generate a net cash flow of Rs 4.00,000 per year at the end of each year for a period of three years. The applicable discount rate is 15% The net present worth of the total cash flow is?

Solution: S = R $\left[\frac{(C+i)^n - 1}{i(1+i)^n}\right]$

S = 4,00,000 $\left[\frac{(1.15)^3 - 1}{0.15 \times (1.15^3}\right] = 9,13,290$

S = Rs. 913290

2. Fixed capital investment for a chemical plant is Rs 80 million with an estimated useful life of 10 years and a salvage value of Rs 10 million The rate of interest is 15 % tax is 25% of the annual taxable income in the first year of operation, the income from sales is Rs30 Million What is the rate of return on investment?

Solution: We know that

Rate of return = $\frac{yearly\ \text{profit}}{Total\ \text{initial investment}} \times 100$

Yearly profit = 3O-30X0.25=22.5

Rate of return = $\frac{22.5}{80} \times 100 = \underline{28\%}$

3. A compressor has an installed cost of Rs 24,000 and a 15 year estimated life. The salvage value of the compressor is zero at the end of 15 years. the compressor value after depreciation by the double declining method at the end of 10 years is?

Solution: We know that,

$$f = \frac{2}{n} = \frac{2}{15} = 0.133$$

$$V_n = V(1\text{-f})^n$$

$$= 24,000\ (1\text{-}0.133)^{10}$$

$$= \underline{5760}$$

4. A column costs of Rs 10 19 lakhs and has a useful like of 15 years. Using the double declining balance depreciation method, the book value of the unit at the end of 7 years is?

Solution:

Annual depreciation = $\frac{10,00,000}{15} = 66,666.67$

Depreciation rate = $\frac{66,666.67}{10,00,000} \times 100 = 6.67$

Double decline rate (f) =13.33%

$$V_n = V(1-f)^a$$

$$= 10,00,000(1-0.1333)^7$$

$$= \underline{3.67 \text{ lakhs}}$$

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