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HT-HEAT TRANSFER

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Thanks to all of You :

Oct 01, 14

HEAT TRANSFER

1.) Conduction :- 15 hrs
2.) Convection :- 20 hr
3.) Radiation 8 hrs

Heat Exchanger] 6 hr
Condensation } 4 hr
Boiling

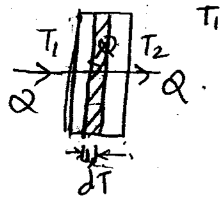
Conduction :-

Fourier's Law :-

$$\frac{Q}{A} \propto \frac{dT}{dx}$$

Heat flux Temp. gradient

Plane wall $T_1 > T_2$



A = Area normal to the direction of Heat transfer.

$$\frac{Q}{A} = -k \frac{dT}{dx}$$

k = constant of proportionality. (Thermal conductivity of material)

$$Q = -kA\,dT/dx$$

$$\frac{J}{s} = k\, m^2 \frac{K}{m}$$

$$k = \frac{J}{s\,m\,K} = W/mK$$

More H.T → Conduct Mat

Less H.T → Insulator Mat.

Assumption :-

1.) Isothermal Surfaces.
2.) Constant Thermal Conductivity in the given range of temp.
3.) Whatever heat enters, nothing is retained, entire amt of heat leaves the other side.
4.) Steady state.
5.) One-dimensional heat transfer (x-direction)

$$Q = -kA\,dT/dx$$

$$dx = -kA\,dT/Q$$

$$\int_1^2 dx = \int_1^2 -\frac{kA}{Q}\,dT$$

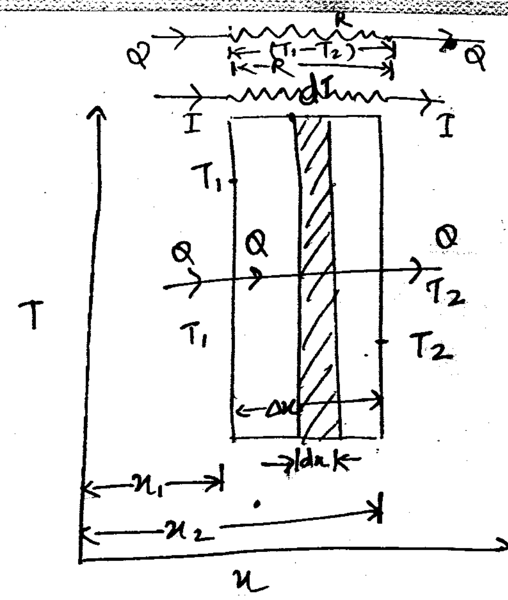
$$x\Big|_{x_1}^{x_2} = -\frac{kA}{Q}\,T\Big|_{T_1}^{T_2}$$

$$x_2 - x_1 = -\frac{kA}{Q}(T_2 - T_1)$$

$$= \frac{kA(T_1 - T_2)}{Q}$$

$$Q = \frac{kA(T_1 - T_2)}{(x_2 - x_1)}$$

$$= \frac{kA(T_1 - T_2)}{\Delta x}$$



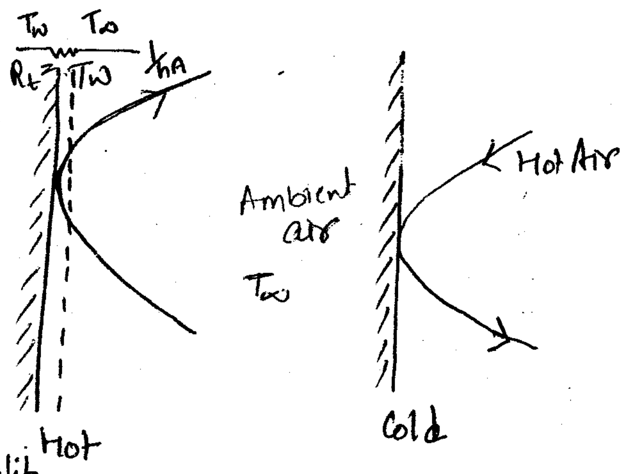
Ohm's law

$$I = V/R$$

$$Q = \frac{T_1 - T_2}{\Delta x / kA}$$

Thermal Resistance $R_t = \Delta x / kA = \frac{m}{\frac{W}{mK} \times m^2} = K/W$

Convection :-



Newton-Rikhman's Law

Heat flux $\propto$ Temp. diff.

$$\frac{Q}{A} \propto (T_w - T_\infty)$$

$$\frac{Q}{A} = h(T_w - T_\infty)$$

↳ Constant of proportionality

↳ Convective heat transfer co-efficient / film coeff.

$$Q = hA(T_w - T_\infty)$$

$$J/s = h\,m^2 K$$

$$h = J/s\,m^2 K = W/m^2 K$$

$$Q = \frac{T_w - T_\infty}{\frac{1}{hA}} \qquad I = \frac{V}{R}$$

$$R_t = \frac{1}{hA}$$

A very thin stagnant layer of air is formed. (film of air)

$$\frac{1}{hA} = \frac{1}{\frac{W}{m^2 K} \times m^2} = K/W$$

Example. shop - Tea cooling.

h is functn of velocity
if velocity ↑, h ↑ more
more velocity → more turbul

↑Q ← h ↑, A ↑

Example Tea → plate

Radiation :-

(Stefan Boltzman's Law)

$\frac{Q}{A} \propto$ Fourth power of the abs. temp.

$\frac{Q}{A} \propto T^4$, $\frac{Q}{A} = \sigma T^4$
↳ stefan Boltzman's Constant.

H.T is taking place bcz of the vibrational movement of source & its transfer of heat is don't affected by surface or medium. | ether medium

$$\frac{Q}{A} = \sigma (T_1^4 - T_2^4)$$

shape factor A_{12}, A_{21}

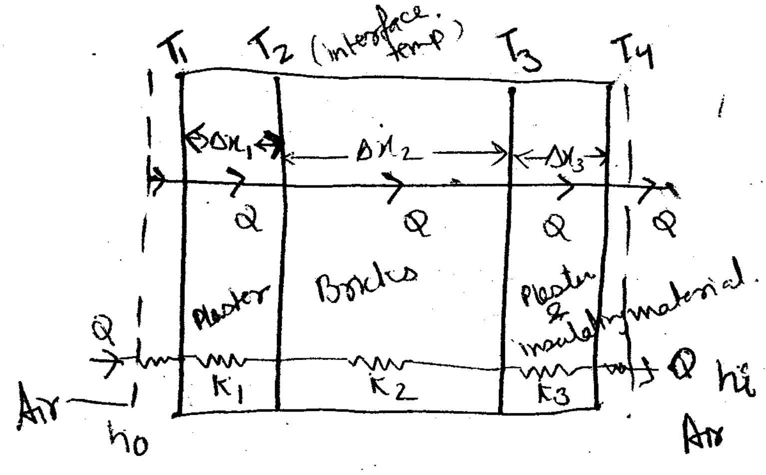
Oct 03, 14

Composite Wall :-

$$Q = \frac{T_0 - T_1}{1/h_o A_o}$$

$$Q = \frac{T_1 - T_2}{\frac{\Delta x_1}{K_1 A}}$$

$$Q = \frac{T_2 - T_3}{\Delta x_2 / K_2 A}$$



$$Q = \frac{T_3 - T_4}{\Delta x_3 / k_3 A} \quad , \quad Q = \frac{T_4 - T_i}{1/h_i A_i}$$

$$A_0 = A = A_i$$

$$\cancel{T_0} - T_1 = \frac{1}{h_0 A_0} Q$$

$$\cancel{T_1} - \cancel{T_2} = \frac{\Delta x_1}{k_1 A} Q$$

$$\cancel{T_2} - \cancel{T_3} = \frac{\Delta x_2}{k_2 A} Q$$

$$T_3 - \cancel{T_4} = \frac{\Delta x_3}{k_3 A} Q$$

$$\cancel{T_4} - T_i = \frac{1}{h_i A_i} Q$$

$$T_0 - T_i = \left(\frac{1}{h_0 A_0} + \frac{\Delta x_1}{k_1 A} + \frac{\Delta x_2}{k_2 A} + \frac{\Delta x_3}{k_3 A} + \frac{1}{h_i A_i} \right) Q$$

$$\boxed{Q = \frac{T_0 - T_i}{\left[1/h_0 A_0 + \frac{\Delta x_1}{k_1 A} + \frac{\Delta x_2}{k_2 A} + \frac{\Delta x_3}{k_3 A} + \frac{1}{h_i A_i} \right]}}$$

$$= \frac{T_0 - T_i}{\Sigma R_t}$$

$$Q = UA(T_0 - T_i)$$

$U \rightarrow$ overall heat transfer coefficient

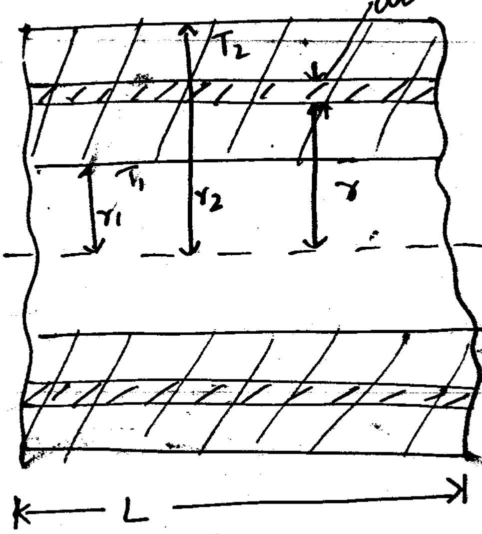
$$Q = \frac{(T_0 - T_i)}{1/UA}$$

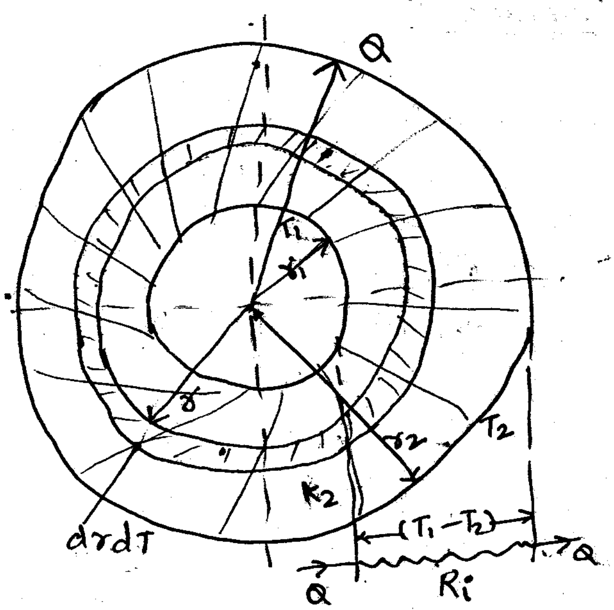
$$\frac{1}{U\cancel{A}} = \frac{1}{h_0 \cancel{A}} + \frac{\Delta x_1}{k_1 \cancel{A}} + \frac{\Delta x_2}{k_2 \cancel{A}} + \frac{\Delta x_3}{k_3 \cancel{A}} + \frac{1}{h_i \cancel{A}}$$

$$\boxed{\frac{1}{U} = \frac{1}{h_0} + \frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{\Delta x_3}{k_3} + \frac{1}{h_i}}$$

Hollow Cylinder or a Thick pipe :-

$T_1 > T_2$





$$Q = -kA \, dT/dr$$

$$= -k(2\pi r L)\, dT/dr$$

$$\int_{r_1}^{r_2} \frac{dr}{r} = -k \frac{2\pi L}{Q} \int_{T_1}^{T_2} dT$$

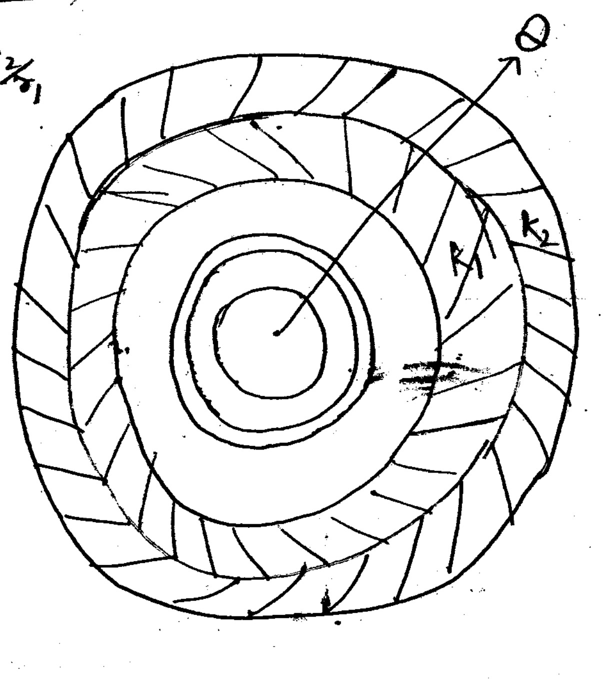
$$\ln r_2/r_1 = -k \frac{2\pi L}{Q} (T_2 - T_1)$$

$$Q = k \frac{2\pi L}{\ln \frac{r_2}{r_1}} (T_1 - T_2)$$

$$\boxed{Q = \frac{T_1 - T_2}{\frac{1}{k 2\pi L} \ln r_2/r_1}}$$

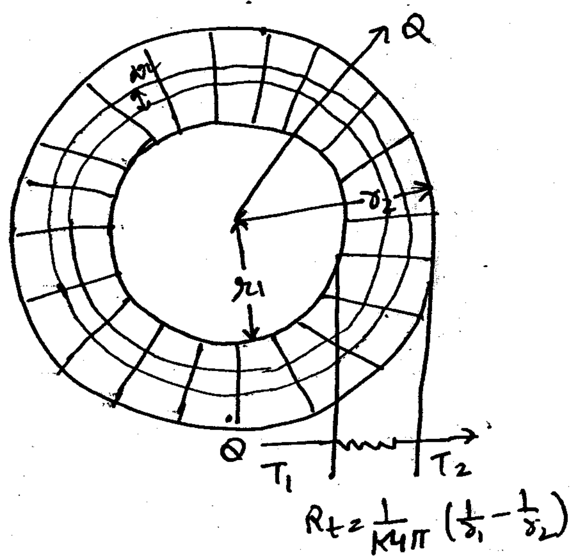
$$R_t = \frac{1}{k 2\pi L} \ln r_2/r_1$$

Which one shud be used. $K_2 = 0.5 K_1$



Good insulating material should be used inside as k_2 will act as good insulator So it should be used 1st as k_2 has

Sphere :-



$$Q = -kA\,dT/dr$$

$$= -k(4\pi r^2)\,dT/dr$$

$$\int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{k4\pi}{Q}\int_{T_1}^{T_2} dT$$

$$-\frac{1}{r}\Big|_{r_1}^{r_2} = -\frac{k4\pi}{Q}(T_2 - T_1)$$

$$\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{k4\pi}{Q}(T_1 - T_2)$$

$$Q = \frac{k4\pi(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$Q = \frac{(T_1 - T_2)}{\frac{1}{k4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$\boxed{Q = \frac{(T_1 - T_2)}{\frac{(r_2 - r_1)}{k4\pi r_1 r_2}}}$$

Mean Area :-

$$Q = -kA_m\left(\frac{dT}{dr}\right)$$

$$= \frac{-k(4\pi r_m^2)(T_2 - T_1)}{r_2 - r_1}$$

$$= \frac{(T_1 - T_2)}{\frac{1}{k4\pi r_m^2}(r_2 - r_1)}$$

$$\frac{\cancel{r_2 - r_1}}{\cancel{k4\pi} r_1 r_2} = \frac{\cancel{r_2 - r_1}}{\cancel{k4\pi} r_m^2}$$

$$r_m^2 = r_1 r_2$$

$$r_m = \sqrt{r_1 r_2} \quad \Big\}\text{ geometric mean.}$$

Mean radius of hollow sphere

$$Q = -kA_m\,dT/dr$$

this one? for cylinder

$$= -\frac{k(2\pi r_m L)(T_2 - T_1)}{r_2 - r_1}$$

$$= \frac{(T_1 - T_2)}{\frac{1}{(k 2\pi r_m L)} r_2 - r_1}$$

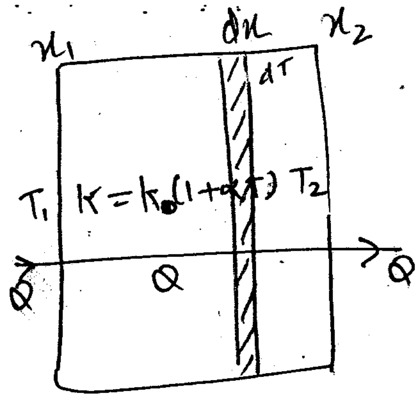
$$\frac{\cancel{T_1 - T_2}}{\frac{1}{\cancel{k 2\pi L}} \ln\frac{r_2}{r_1}} = \frac{\cancel{T_1 - T_2}}{\frac{1}{(\cancel{k 2\pi r_m L})}(r_2 - r_1)}$$

$$\ln r_2/r_1 = \frac{r_2 - r_1}{r_m}$$

$$\boxed{r_m = \frac{r_2 - r_1}{\ln r_2/r_1}}$$

Log mean radius (LMR)

Plane wall with variable thermal conductivity :-



$$K = k_0(1 + \alpha T)$$

$$Q = -kA\, dT/dx$$

$$= -k_0(1 + \alpha T) A\, dT/dx$$

$$dx = \frac{-k_0(1 + \alpha T) A}{Q} dT$$

$$\int_{x_1}^{x_2} dx = \int_{T_1}^{T_2} \frac{-k_0(1 + \alpha T) A\, dT}{Q}$$

$$= -\frac{k_0 A}{Q} \int_{T_1}^{T_2} (1 + \alpha T) dT$$

$$x_2 - x_1 = -\frac{k_0 A}{Q} \left| T + \alpha \frac{T^2}{2} \right|_{T_1}^{T_2}$$

$$x_2 - x_1 = \frac{-k_0 A}{Q}\left[\left(T_2 + \alpha\frac{T_2^2}{2}\right) - \left(T_1 + \alpha\frac{T_1^2}{2}\right)\right]$$

$$= -\frac{k_0 A}{Q}\left[(T_2 - T_1) + \frac{\alpha}{2}(T_2^2 - T_1^2)\right]$$

$$= \frac{k_0 A}{Q}\left[(T_2 - T_1)\left(1 + \alpha\frac{T_2 + T_1}{2}\right)\right]$$

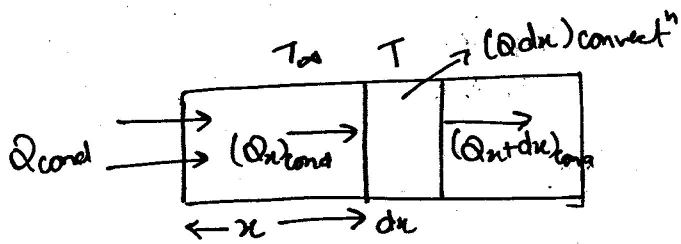
$$= \frac{K_0 A}{Q}\left[(T_1 - T_2)(1+\alpha T_m)\right]$$

$$Q = \frac{k_0 A\,(1+\alpha T_m)(T_1 - T_2)}{x_2 - x_1}$$

$$= \frac{K_m A\,(T_1 - T_2)}{x_2 - x_1}$$

$K_m = k_0(1+\alpha T_m)$
$\Delta x = x_2 - x_1$

$$\boxed{Q = \frac{T_1 - T_2}{\Delta x / K_m A}}$$



$$(Q_x)_{cond} = -kA\, dT/dx$$

$$(Q_{x+dx})_{cond} = ?$$

$$(Q_{x+dx})_{cond} = Q_x + \frac{d}{dx}(Q_x)\,dx$$

$$= -kA\,dT/dx + \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)dx$$

$$= -kA\,dT/dx + \left[-kA\frac{d^2T}{dx^2}\right]dx$$

$$(Q_x)_{cond} = (Q\,dx)_{convtn} + (Q_{x+dx})$$ convection conduction.

$$(Q\,dx)_{conv} = (Q_x)_{cond^n} - (Q_{x+dx})_{cond^n}$$

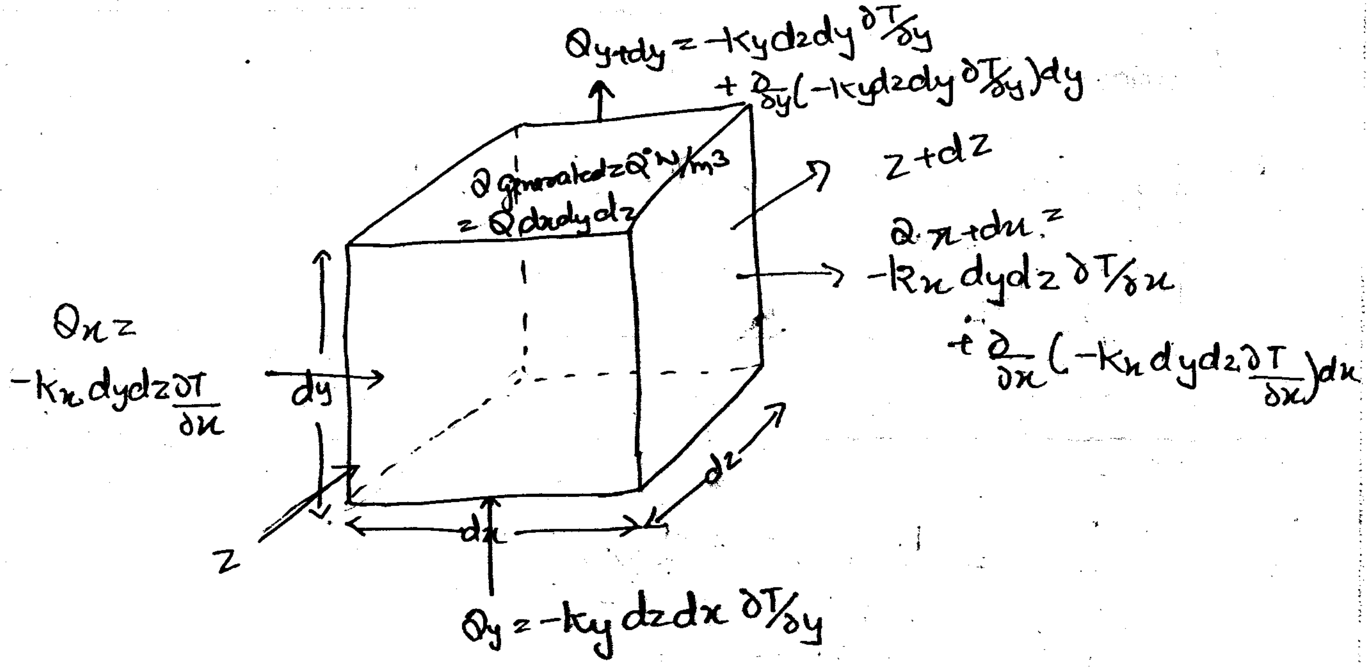
$$= \cancel{-kA\left(\frac{dT}{dx}\right)} - \left[\cancel{-kA\frac{dT}{dx}} + \left(-kA\frac{d^2T}{dx^2}\,dx\right)\right]$$

$$= kA\, \frac{d^2T}{dx^2}\,dx$$

$$hA\,dx\,(T - T_\infty) = kA\frac{d^2T}{dx^2}\,dx$$

$$kA\frac{d^2T}{dx^2} - hA(T - T_\infty) = 0$$

Unsteady state heat transfer in 3 dimensional wid variable thermal conductivity in 3 directions :-



$$Q_z = -k_z\,dx\,dy\,\partial T/\partial z$$

$$Q_{z+dz} = -k_z\,dx\,dy\,\partial T/\partial z + \frac{\partial}{\partial z}\left(-k_z\,dx\,dy\,\partial T/\partial z\right)dz$$

Change in I.E $= \rho\,dx\,dy\,dz\; c\,\frac{\partial T}{\partial z}$

$Q_x + Q_y + Q_z + Q_{generated} +$ change in I.E of element $= Q_{x+dx} + Q_{y+dy} + Q_{z+dz}$

$$\left(-k_x\,dy\,dz\,\frac{\partial T}{\partial x}\right) + \left(-k_y\,dx\,dz\,\frac{\partial T}{\partial y}\right) + \left(-k_z\,dy\,dz\,\frac{\partial T}{\partial z}\right) + \left(\dot{Q}\,dx\,dy\,dz\right)$$

$$= \left[\cancel{-k_x\,dy\,dz\,\partial T/\partial x} + \frac{\partial}{\partial x}\left(-k_x\,dy\,dz\,\frac{\partial T}{\partial x}\right)dx\right] +$$

$$\left[\cancel{-k_y\,dx\,dz\,\partial T/\partial y} + \frac{\partial}{\partial y}\left(-k_y\,dx\,dz\,\partial T/\partial y\right)dy\right] +$$

$$\left[\cancel{-k_z\,dy\,dx\,\partial T/\partial z} + \frac{\partial}{\partial z}\left(-k_z\,dy\,dx\,\partial T/\partial z\right)dz\right] +$$

$$\left(\rho\,dx\,dy\,dz\right) c\,\partial T/\partial z$$

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x\,\partial T/\partial x\right)\cancel{dx\,dy\,dz} + \partial/\partial y\left(k_y\,\partial T/\partial x\right)\cancel{dx\,dy\,dz} + \frac{\partial}{\partial z}\left(k_z\,\partial T/\partial z\right)\cancel{dx\,dy\,dz} +$$

$$\dot{Q}\,\cancel{dx\,dy\,dz} = \left(\rho\,\cancel{dx\,dy\,dz}\right) c\,\partial T/\partial z$$

$$\boxed{\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + Q^\circ = \rho c \frac{\partial T}{\partial \tau}}$$

Conduction Heat transport eqn

fourier's condition

Isotropic material, $k_x = k_y = k_z = k$ constant.

$$k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + Q^\circ = \rho c \, \partial T/\partial \tau$$

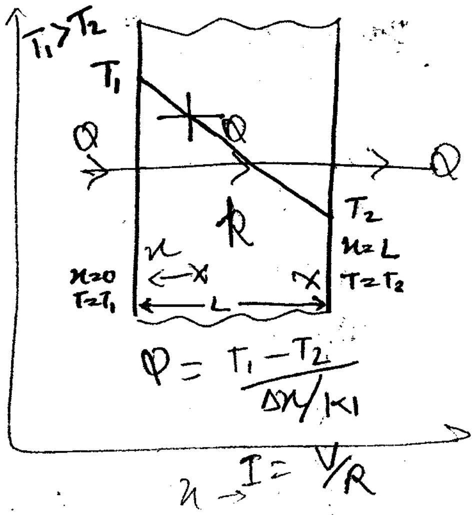
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{Q^\circ}{k} = \frac{\rho c}{k} \cdot \frac{\partial T}{\partial z} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

unsteady state

$$\frac{k}{\rho c} = \text{thermal diffusivity } (\alpha) \qquad \alpha = \frac{k}{\rho c}$$

$$\frac{\rho c}{k} = \frac{\text{heat capacity}}{\text{thermal cond}^n}$$

Oct 06, 14 No heat gnetn, Steady state



steady state, $\partial T/\partial \tau$

No heat generation, $Q = 0$

One dimensional $T = T(x)$

$$\frac{d^2 T}{dx^2} = 0.$$

The governing eqn is $\dfrac{d^2T}{dx^2} = 0$

integrating, $dT/dx = C_1$

integrating again, $T = C_1 x + C_2$

Boundary Conditions are :—

i) $x = 0$, $T = T_1$; $T_1 = C_2$

ii) $x = L$, $T = T_2$; $T_2 = C_1 L + C_2$

$$T_2 = C_1 L + T_1$$

$$C_1 = \frac{T_2 - T_1}{L} = -\frac{(T_1 - T_2)}{L}$$

$$\boxed{T = -\frac{T_1 - T_2}{L} x + T_1}$$ Temp. distribution eqn along d thickness of the wall.

$y = mx + c$; Comparing

$m =$ slope $= -\frac{(T_1 - T_2)}{L}$

also slope $= dT/dx$

$= -\frac{(T_1 - T_2)}{L}$

May also be written as

$T - T_1 = -(T_1 - T_2)\, x/L$

$T_1 - T = (T_1 - T_2)\, x/L$

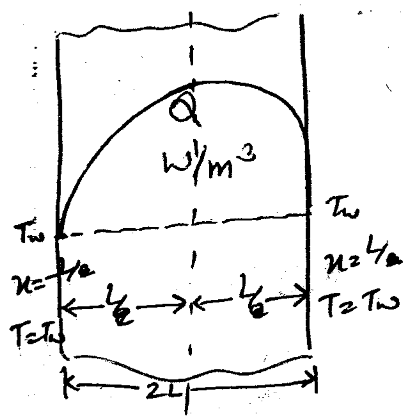
$$\boxed{\frac{T_1 - T}{T_1 - T_2} = x/L}$$

initial — Surr

Eqn is dimensionless form of temp. distribution

<u>1D steady state, with heat generation & const thermal conductivity</u>

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}} + Q/k = \frac{1}{\alpha}\cancel{\frac{\partial T}{\partial \tau}}$$



The governing eqn is $\frac{d^2T}{dx^2} + \frac{Q}{k} = 0$

Integrating, $\frac{dT}{dx} = -\frac{Q}{k}x + C_1$

Integrating again, $T = -\frac{Q}{k}\frac{x^2}{2} + C_1 x + C_2$

<u>Boundary conditions</u> are :—

i) $x = L$, $T = T_w$; $\quad T_w = -\frac{Q}{k}\cdot\frac{L^2}{2} + C_1 L + C_2$

ii) $x = -L$, $T = T_w$; $\quad T_w = -\frac{Q}{k}\cdot\frac{L^2}{2} - C_1 L + C_2$

$2C_1 L = 0$

$$\boxed{C_1 = 0}$$

for max temp.

$dT/dx = 0$, $\frac{d^2T}{dx^2} = -ve$

$\frac{dT}{dx} = -\frac{Q}{k}x = 0$ $\quad \therefore x = 0$

$$\frac{d^2T}{dx^2} = -Q/k$$

at centre, $x=0$, $T = T_{max} = T_0$

$$\boxed{T_0 = C_2}$$

$$T = -\frac{Q}{k}\cdot\frac{x^2}{2} + T_0$$

$$\boxed{T_0 - T = \frac{Q}{k}\cdot x^2/2}$$

$x = \pm L$, $T = T_w$, $T_0 - T_w = \frac{Q}{k} L^2/2$

$$\boxed{\frac{T_0 - T}{T_0 - T_w} = \frac{x^2}{L^2}}$$ dimensionless no.

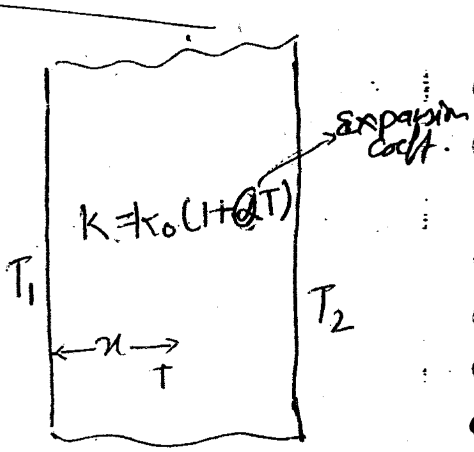
$Q_{generated} = \dot{Q} A (2L)$

Total heat $= 2\dot{Q}AL$

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + \dot{Q}} = \rho C \frac{\partial T}{\partial \tau}$$

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

$$\frac{d}{dx}\left[k_0(1+\alpha T)\frac{dT}{dx}\right] = 0$$



Integrating, $k_0(1+\alpha T)\frac{dT}{dx} = C_1$

Integrating again, $k_0\left(T + \alpha \frac{T^2}{2}\right) = C_1 x + C_2$

Boundary conditions are

i) $x = 0$, $T = T_1$.

ii) $x = L$, $T = T_2$

$$k_0\left(T_1 + \alpha \frac{T_1^2}{2}\right) = C_2$$

$$k_0\left(T_2 + \alpha \frac{T_2^2}{2}\right) = C_1 L + k_0\left(T_1 + \alpha\frac{T_1^2}{2}\right)$$

$$k_0\left((T_2 - T_1) + \frac{\alpha}{2}(T_2^2 - T_1^2)\right) = C_1 L$$

$$C_1 = \frac{k_0}{L}\left[(T_2 - T_1) + \frac{\alpha}{2}(T_2^2 - T_1^2)\right]$$

$$C_1 = \frac{k_0(T_2 - T_1)}{L}\left[1 + \alpha\frac{(T_1+T_2)}{2}\right]$$

$$\cancel{k_0}\left(T+\alpha\frac{T^2}{2}\right) = -\frac{\cancel{k_0}x}{L}(T_1-T_2)\left(1+\alpha\frac{T_1+T_2}{2}\right) + \cancel{k_0}\left(T_1+\alpha T_1^2/2\right)$$

$$\alpha\frac{T^2}{2} + T + \left[\frac{x}{L}(T_1-T_2)\left(1+\alpha\frac{T_1+T_2}{2}\right) - \left(T_1+\alpha T_1^2/2\right)\right] = 0$$

ax^2 bx c

$$\boxed{T = \frac{-1 \pm \sqrt{1 - 4\frac{\alpha}{2}\left[\frac{x}{L}(T_1-T_2)\left(1+\alpha\frac{T_1+T_2}{2}\right) - \left(T_1+\alpha\frac{T_1^2}{2}\right)\right]}}{\cancel{2}\alpha/\cancel{2}}}$$

$x = 0$, $T =$

$$T = \frac{-1 \pm \sqrt{1-2\alpha(-1)(T_1+\alpha T_1^2/2)}}{\alpha}$$

$$= \frac{-1 \pm \sqrt{1+2\alpha T_1+\alpha^2T_1^2}}{\alpha}$$

$$= \frac{-1 + (1+\alpha T_1)}{\alpha} = T_1$$

$x = L$

$$= \frac{-1 \pm \sqrt{1 - 2\alpha\left[(T_1-T_2)\left(1+\alpha\frac{(T_1+T_2)}{2}\right) - \left(T_1+\alpha T_1^2/2\right)\right]}}{\alpha}$$

$$= \frac{-1 \pm \sqrt{1 - 2\alpha\left(\left[(T_1-T_2)+\alpha\frac{(T_1^2-T_2^2)}{2}\right) - \left(T_1+\alpha T_1^2/2\right)\right]}}{\alpha}$$

we consider +ve bcz temp can't be -ve

$$= \frac{-1 \pm \sqrt{1 - \left[2\alpha(T_1-T_2) + \alpha^2(T_1^2-T_2^2) - 2\alpha T_1 \underline{+} \alpha^2T_1^2\right]}}{\alpha}$$

$$= -1 \pm \sqrt{1 - \cancel{2\alpha T_1} + 2\alpha T_2 - \cancel{\alpha^2T_1^2} + \alpha^2T_2^2 + \cancel{2\alpha T_1} + \cancel{\alpha^2T_1^2}}$$

$$= \frac{-1 \pm \sqrt{1+2\alpha T_2+\alpha^2T_2^2}}{\alpha}$$

$$\frac{-1 + (1+\alpha T_2)}{\alpha} = T_2$$

Cylindrical Co-ordinates

$$r, \theta, z$$

$$T = T(x, y, z, \tau)$$

$$T = T(r, \theta, z, \tau)$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \underbrace{\frac{\partial^2 T}{\partial z^2} + \frac{Q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

$$\frac{\partial T}{\partial x} = f(r, \theta)$$

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial T}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial T}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$\frac{\partial T}{\partial r} = \cos\theta\frac{\partial T}{\partial x} + \sin\theta\frac{\partial T}{\partial y} \quad \times \cos\theta / \times\sin\theta$$

$$\frac{\partial T}{\partial \theta} = -r\sin\theta\frac{\partial T}{\partial x} + r\cos\theta\frac{\partial T}{\partial y} \quad \times \frac{\sin\theta}{r} / \frac{\cos\theta}{r}$$

we multiply these factor for simplyfy our problem

- Substracting,

$$\cos\theta\frac{\partial T}{\partial r} - \frac{\sin\theta}{r}\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x}$$

- Adding

$$\sin\theta\frac{\partial T}{\partial r} + \frac{\cos\theta}{r}\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial y}$$

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Substracting both eq^n s :—

$$\cos\theta \frac{\partial T}{\partial r} - \frac{\sin\theta}{r}\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x}$$

$$\frac{\partial \cdot}{\partial x} = \cos\theta \frac{\partial \cdot}{\partial r} - \frac{\sin\theta}{r}\frac{\partial \cdot}{\partial \theta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \cos\theta \frac{\partial}{\partial r}\left(\frac{\partial T}{\partial x}\right) - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial x}\right)$$

$$\frac{\partial^2 T}{\partial x^2} = \cos\theta \frac{\partial}{\partial r}\left(\cos\theta \frac{\partial T}{\partial r} - \frac{\sin\theta}{r}\frac{\partial T}{\partial \theta}\right) - \frac{\sin\theta}{r}\left(\cos\theta\frac{\partial T}{\partial r} - \frac{\sin\theta}{r}\frac{\partial T}{\partial \theta}\right)$$

$$= \cos\theta\left[\cos\theta \frac{\partial^2 T}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial T}{\partial \theta} - \frac{\sin\theta}{r}\frac{\partial^2 T}{\partial r \partial \theta}\right]$$

$$- \frac{\sin\theta}{r}\left[-\sin\theta\frac{\partial T}{\partial r} + \cos\theta \frac{\partial^2 T}{\partial \theta \partial r} - \frac{\cos\theta}{r}\frac{\partial T}{\partial \theta} - \frac{\sin\theta}{r}\frac{\partial^2 T}{\partial \theta^2}\right]$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \cos^2\theta \frac{\partial^2 T}{\partial r^2} + \cancel{\frac{\sin\theta\cos\theta}{r^2}}\frac{\partial T}{\partial \theta} - \cancel{\frac{\sin\theta\cos\theta}{r}\frac{\partial^2 T}{\partial r\partial \theta}} + \frac{\sin^2\theta}{r}\frac{\partial T}{\partial r} - \cancel{\frac{\sin\theta\cos\theta}{r}\frac{\partial^2 T}{\partial \theta\partial r}} + \cancel{\frac{\sin\theta\cos\theta}{r^2}}\frac{\partial T}{\partial \theta} + \frac{\sin^2\theta}{r^2}\frac{\partial^2 T}{\partial \theta^2}}$$

$$\frac{\partial T}{\partial y} = \sin\theta \frac{\partial T}{\partial r} + \frac{\cos\theta}{r}\frac{\partial T}{\partial \theta}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) = \sin\theta\frac{\partial}{\partial r}\left(\sin\theta\frac{\partial T}{\partial r} + \frac{\cos\theta}{r}\frac{\partial T}{\partial \theta}\right)$$
$$+ \frac{\cos\theta}{r}\left(\sin\theta\frac{\partial T}{\partial r} + \frac{\cos\theta}{r}\frac{\partial T}{\partial \theta}\right)$$

$$\frac{\partial^2 T}{\partial y^2} = \sin\theta\left(\sin\theta\frac{\partial^2 T}{\partial r^2} - \frac{\cos\theta}{r^2}\frac{\partial T}{\partial \theta} + \frac{\cos\theta}{r}\frac{\partial^2 T}{\partial r\partial \theta}\right)$$

$$+ \frac{\cos\theta}{r}\left(\cos\theta\frac{\partial T}{\partial r} + \sin\theta\frac{\partial^2 T}{\partial \theta\partial r} - \frac{\sin\theta}{r}\frac{\partial T}{\partial \theta} + \frac{\cos\theta}{r}\frac{\partial^2 T}{\partial \theta^2}\right)$$

$$\boxed{\frac{\partial^2 T}{\partial y^2} = \sin^2\theta \frac{\partial^2 T}{\partial r^2} - \frac{\sin\theta\cos\theta}{r^2}\frac{\partial T}{\partial \theta} + \cancel{\frac{\sin\theta\cos\theta}{r}\frac{\partial^2 T}{\partial r\partial\theta}} + \frac{\cos^2\theta}{r}\frac{\partial T}{\partial r} + \cancel{\frac{\sin\theta\cos\theta}{r}\frac{\partial^2 T}{\partial\theta\partial r}} - \cancel{\frac{\sin\theta\cos\theta}{r^2}}\frac{\partial T}{\partial\theta} + \frac{\cos^2\theta}{r^2}\frac{\partial^2 T}{\partial\theta^2}}$$

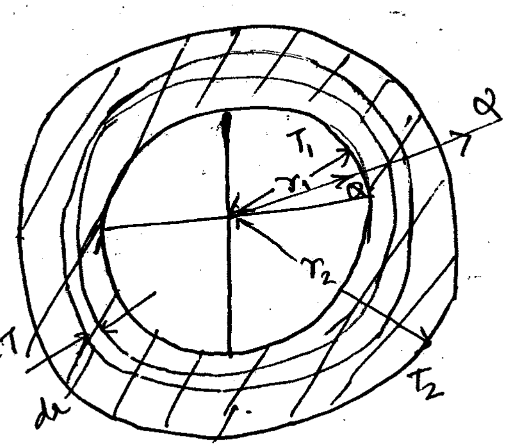
$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}}.$$

$$\boxed{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{Q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

3-Dim Conduction eqn of

$$r\frac{d^2 T}{\partial r^2} + \frac{dT}{dr} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



Integrating, $r\dfrac{dT}{dr} = C_1$

$$\frac{dT}{dr} = C_1/r$$

Integrating again, $T = C_1 \ln r + C_2$

Boundary conditions are:-

$r = r_1$, $T = T_1$; $\quad T_1 = C_1 \ln r_1 + \cancel{C_2}$

$r = r_2$, $T = T_2$; $\quad T_2 = C_1 \ln r_2 + \cancel{C_2}$

$$T_1 - T_2 = C_1(\ln r_1 - \ln r_2)$$

$$= C_1 \ln r_1/r_2$$

$$= -C_1 \ln r_2/r_1$$

$$T_1 = -\frac{T_1 - T_2}{\ln r_2/r_1}\ln r_1 + C_2$$

$$C_2 = T_1 + \frac{T_1 - T_2}{\ln r_2/r_1}\cdot \ln r_1$$

$$T = -\frac{T_1 - T_2}{\ln r_2/r_1} \ln r + T_1 + \frac{T_1 - T_2}{\ln r_2/r_1} \ln r_1$$

$$\boxed{T = T_1 - \frac{T_1 - T_2}{\ln r_2/r_1} \ln r/r_1}$$

Temp distribution along the thickness of pipe

$$\frac{T_1 - T_2}{\ln r_2/r_1} \cdot \ln r/r_1 = T_1 - T$$

$$\boxed{\frac{\ln r/r_1}{\ln r_2/r_1} = \frac{T_1 - T}{T_1 - T_2}}$$

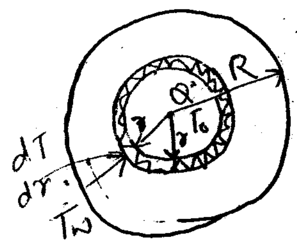
Dimensionless no.

$$Q = -kA \, dT/dr \Big|_{\substack{r = r_1 \\ \text{or} \\ r = r_2}}$$

$$= -k(2\pi r L) \Big| \left(-\frac{T_1 - T_2}{\ln r_2/r_1}\right) \cdot \frac{r_1}{r} \cdot \frac{1}{r_1} \Big|_{\substack{r = r_1 \\ \text{or} \\ r = r_2}}$$

$$= \frac{T_1 - T_2}{\frac{1}{k 2\pi L} \ln r_2/r_1}$$

Solid pipe w/o heat generation :-



$$Q_{gen} = \dot{Q} \pi R^2 L$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\dot{Q}}{k} = 0$$

$\times r$ check

$$r \frac{\partial^2 T}{\partial r} + \frac{\partial T}{\partial r} + r\frac{\dot{Q}}{k} = 0$$

$$\frac{d}{dr}\left(r \frac{dT}{dr}\right) = -r\frac{\dot{Q}}{k}$$

$$r\frac{dT}{dr} = -\frac{\dot{Q}}{k}\frac{r^2}{2} + C_1$$

$$\frac{dT}{dr} = \frac{\dot{Q}}{k}\frac{r}{2} + \frac{C_1}{r}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}}}_{\text{1-Dim.}} + \frac{\dot{Q}}{k} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}^{\;0}$$ for steady state

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\dot{Q}}{k} = 0$$

$$r\frac{d^2 T}{dr^2} + \frac{dT}{dr} + r\frac{\dot{Q}}{k} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{Q}}{k} r$$

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Integrating

$$r\,dT/dr = -\frac{\dot{Q}}{k}\frac{r^2}{2} + C_1$$

$$dT/dr = -\frac{\dot{Q}}{k}\frac{r}{2} + C_1/r$$

integrating again

$$T = -\frac{\dot{Q}}{k}\frac{r^2}{4} + C_1 \ln r + C_2$$

Boundary Conditions :-

$\Rightarrow$ $r = 0$, $\frac{dT}{dr} = 0$, $\frac{d^2T}{dr^2} = -ve$

C_1 must be zero.

$\Rightarrow$ $r = R$, $T = T_w$

$$T_w = -\frac{\dot{Q}}{k}\cdot\frac{R^2}{4} + C_2$$

$\Rightarrow$ $r = 0$, $T = T_0$; $T_0 = C_2$

$$T = -\frac{Q}{k}\cdot\frac{R^2}{4} + T_0$$

$$\boxed{T_0 - T = \frac{\dot{Q}}{k}\frac{R^2}{4}}$$

Temp distribution eqn along the radius of the cylinder.

~~$r = R$, $T = T_w$, $T_0 - T_w =$~~

$r=0, \quad T=T_0 \quad ; \quad T_0 = C_2$

$$T = -\frac{Q}{K}\cdot\frac{R^2}{4} + C_2$$

$$T = -\frac{Q^\circ}{K}\frac{r^2}{4} + T_0$$

$$T_0 - T_b = \frac{Q^\circ}{K}\, r^2/4$$

$$T_0 - T_w = \frac{Q^\circ}{K}\, R^2/4$$

$$\boxed{\frac{T_0 - T}{T_0 - T_w} = \frac{r^2}{R^2}}$$ Dimensionless no.

$$Q = -KA\, dT/dr \big|_{r=R}$$

$$Q = -KA\, dT/dr \big|_{r=R}$$

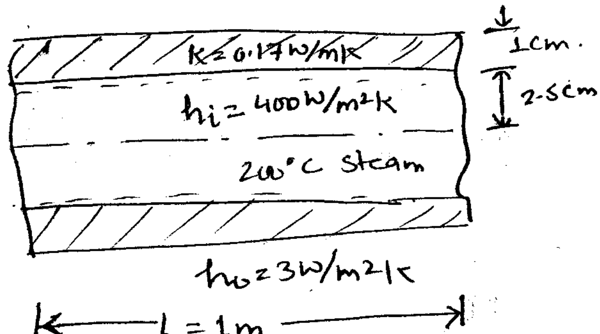
$$= -K(2\pi r L)$$

$$= -k(2\pi r L)\left(-\frac{Q^\circ}{K}\frac{r}{2}\right)\Big|_{r=R}$$

$$= Q^\circ \pi r^2 L \big|_{r=R}$$

$$= Q^\circ \pi R^2 L$$

Q.)



$$Q = \frac{T_i - T_o}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o}}$$

$$= \frac{200 - 20}{\frac{1}{400 \times 2\pi \times 0.025 \times 1} + \frac{1}{3 \times 2\pi \times 0.025 \times 1}}$$

84.19

1cm insulation

$$Q = \frac{T_i - T_o}{\frac{1}{h_i A_i} + \frac{1}{K 2\pi L}\ln\frac{r_2}{r_1} + \frac{1}{h_o A_o}}$$

$r_1 = 2.5$

$r_2 = 3.5 =$

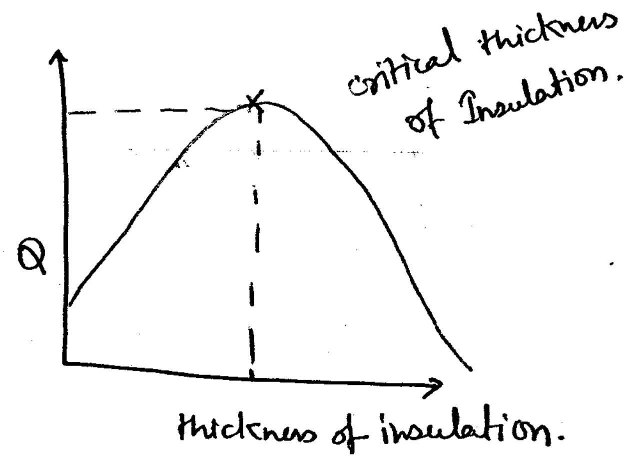
Q - ~~132.37~~ 97.47 W/m

2 cm Insulation

$r_1 = 2.5\,cm$

$r_2 = 4.5\,cm$.

$Q = 103$



$$Q = \frac{T_i - T_o}{\left[\frac{1}{k_i 2\pi r_1 L} + \frac{1}{k_{in} 2\pi L}\ln\frac{r_2}{r_1} + \frac{1}{h_o 2\pi r_2 L}\right]}$$

For Q to be maxm, the R_t i.e Denominator should be minm.

For R_t to be minm. $\frac{dR_t}{dr_2} = 0$ & $\frac{d^2R_t}{dr_2} = +ve$

$$\frac{dR_t}{dr_2} = 0 + \frac{1}{k_1 2\pi L}\,\frac{r_1}{r_2}\cdot\frac{1}{r_1} + \frac{1}{h_o \times 2\pi L} \times \left(-\frac{1}{r_2^2}\right)^2 = 0$$

$$\frac{1}{K_1 2\pi L}\,\frac{1}{r_i} = \frac{1}{h_o \times 2\pi L} \times \frac{1}{r_2^2} \qquad \boxed{r_2 = \frac{K_1}{h_o}}$$

r_c = critical radius of Insulation

$$\frac{d^2R_t}{dr_2^2} = \frac{1}{K_1 2\pi L}\left(-\frac{1}{r_2^2}\right) + \frac{1}{h_o 2\pi L}\cdot\frac{2}{r_2^3} = \frac{1}{2\pi L}\left[-\frac{1}{K_i}\cdot\frac{h_o^2}{K_i^2} + \frac{1}{h_o}\cdot\frac{2h_o^3}{K_i^3}\right]$$

$$= \left(-\frac{h_o^2}{k_i^3} + \frac{2h_o^2}{k_i^3}\right)\frac{1}{2\pi L} = \frac{1}{2\pi L}\,\frac{h_o^2}{K_1^3} + ve$$

$$\boxed{r_c = \frac{k_i}{h_o}} \Rightarrow \frac{W/mK}{W/m^2K} = m$$

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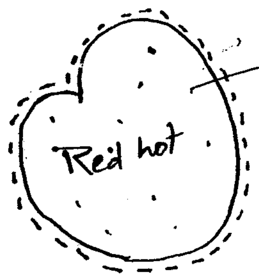
Unsteady State

Thermophysical Properties :-

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{Q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Unsteady state, with heat generation, constant thermal conductivity 3-dimensional conduction eqn



- High thermal conductivity
- Thermal Resistance offered for conduction is neglibly small

so here we talk abt convection

$Q_{convection}$ = Decrease in the Internal Energy of the material.

$$hA(T - T_\infty) = (\rho V)\, c \frac{dT}{d\tau}$$

Lumped heat Capacity Method :-

$$\frac{V}{A} = \text{characteristic length}, L$$

$$-\frac{hA}{\rho V c}\, d\tau = \frac{dT}{T - T_\infty}$$

Integrating, $\ln(T - T_\infty) = -\frac{hA}{\rho V c}\tau + C_1$

Initially, $\tau = 0$, $T = T_i$

$$\ln(T_i - T_\infty) = C_1$$

$$\ln(T - T_\infty) = -\frac{hA}{\rho V c}\tau + \ln(T_i - T_\infty)$$

$$\ln \frac{T-T_\infty}{T_i-T_\infty} = -\frac{h}{\rho L C}\tau$$

$$\boxed{\frac{T-T_\infty}{T_i-T_\infty} = e^{-\frac{h}{\rho L C}\tau}}$$

$$\frac{h}{\rho L C}\tau \times \frac{k}{k} \times \frac{L}{L}$$

$$\boxed{\frac{hL}{k}} \cdot \frac{k\tau}{\rho c\, L^2}$$

$$\alpha \tau / L^2$$

$$\frac{W}{m K} \frac{kg}{m^3} \cdot \frac{J}{kg K} \cdot m^2$$

$$\boxed{\frac{hL}{k} = \text{Biot's No} = Bi}$$

$$\boxed{\frac{\alpha \tau}{L^2} = \text{Fourier's No} = Fo}$$

$$\boxed{\frac{T-T_\infty}{T_i-T_\infty} = e^{-Bi\,Fo}} \quad ☆☆$$

$$\frac{T-T_\infty}{T_i-T_\infty} = \text{Exp}(-Bi\,Fo)$$

$$Bi = \frac{hL}{k} = \frac{L/kA}{\frac{1}{hA}}$$

$$= \frac{(R_t)\text{conduction}}{(R_t)\text{convection}}$$

$\frac{R_{cond}}{R_{conv}} < 0.1$

these Nu becomes Bi'.

Characteristic length, L

1) sheet $\quad \frac{V}{A} = \frac{\cancel{a \times b} \times t}{2(\cancel{a \times b})} = t/2$

2) Cube $\quad V/A = L^3/6L^2 = L/6$

3) Thin Cylinder, $\quad = \frac{\pi R^{\cancel{2}} \cancel{L}}{2\pi \cancel{R} \cancel{L}} = R/2$

4.) Sphere $\frac{4/3 \pi R^3}{4\pi R^2} = R/3$

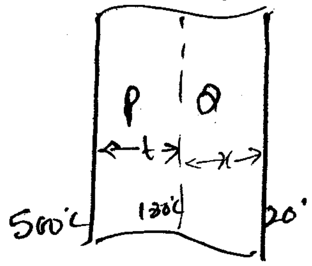
5.) Thick plate $\frac{V}{A} = \frac{a \times b \times t}{2(a\,t) + 2bt + 2ab}$

Q.) for A composite wall shown below

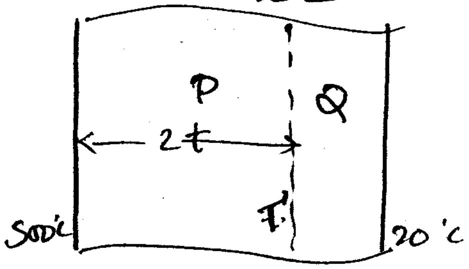
Case-I – The steady state interface temp is 180°C. if the thickness of layer P is doubled ~~in to~~ II.) then the rate of h.t. Assuming 1-D Heat conduction is reduced by ~~20~~

i.) 20% ii.) 40% iii.) 60% iv.) 70%

Case-I



Case-II



$Q = \cancel{\frac{T_1 - T_2}{\ln/2\pi L k}} \quad \frac{T_0 - T_1}{\Delta x / K_1 A} = \frac{\cancel{773} - \cancel{453}^{293}}{\frac{t}{K_1 A} + \frac{x}{K_2 A}} + \cancel{\frac{453 - 293}{x / K_2 A}}$

$= \cancel{\frac{500 - 180}{2 \times \pi \times t \times k}} = \cancel{\frac{320}{2\pi t k}}$

$Q' = \cancel{\frac{T_1 - T_2}{}} \quad \frac{\cancel{773} - 293}{2t/K_1 A + \frac{x}{K_2 A}} + \cancel{\frac{T' - 293}{x / K_2 A}}$

$\frac{Q}{Q'} = \frac{480}{\frac{t}{K_1 A} + \frac{x}{K_2 A}} \times \frac{\frac{2t}{K_1 A} + \frac{x}{K_2 A}}{480}$

$Q = \frac{500 - 20}{\frac{t}{K_1} + \frac{x}{K_2}}$

$Q = \frac{500 - 180}{t/K_1}, \quad Q = \frac{180 - 20}{x/K_2}$

$$Q' = \frac{500-20}{\frac{2t}{k_1} + \frac{x}{k_2}}$$

% Heat Reduction

$$\frac{Q-Q'}{Q} \times 100$$

$$\frac{t}{k_1} = \frac{320}{Q} \quad , \quad \frac{x}{k_2} = \frac{160}{Q}$$

$$Q' = \frac{500-20}{\frac{640}{Q} + \frac{160}{Q}}$$

$$= \frac{\overset{6}{\cancel{480}}\,Q}{\underset{10}{\cancel{800}}}$$

$$Q' = 0.6Q$$

So Reduction is 40%.

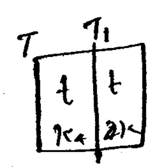
$$\Rightarrow \frac{Q - 0.6Q}{Q} \times 100$$

$$\Rightarrow 0.4 \times 100$$

$$\Rightarrow 40\%$$

Q→ Two plates of equal thickness t & crossectional Area are joined together as shown in fig. If the thermal conductivity of the plates are k & 2k and the effective thermal conductivity of composite wall is ?

$3k/2$, $4k/3$, $3k/4$, $2k/3$



$$\cancel{Q = T} \quad Q = \frac{T - T_0}{\frac{t}{k} + \frac{t}{2k}} = \frac{\Delta T}{2t/k_{eff}}$$

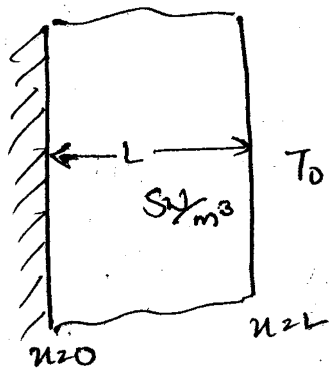
$$Q = \frac{T - T_1}{t/k} \quad , \quad Q = \frac{T_1 - T_0}{t/2k}$$

$$Q = \frac{\Delta T}{}$$

Q.) A slab of thickness L wid one side $x=0$ & insulated and the other side $x=L$ maintained at a constant temp & T_0 as shown. A Uniform distributed ~~k~~ internal heat source produces in the slab at the rate of $S\ W/m^3$. Assuming the heat conduction to be steady & 1-Dim along the x-direction.

1) T_{max} at $x=0$; $L/4$; $L/2$; L

2) Heat flux at $x=L$, 0 ; $\frac{SL}{4}$; $\frac{SL}{2}$; SL



$$\cancel{Q = \frac{KA(T_1-T_2)}{\Delta x}}$$

$$\frac{d^2T}{dx^2} + \frac{S}{K} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{S}{K}$$

Integrating, $\frac{dT}{dx} = -\frac{S}{K}x + C_1$

Wall insulated at $x=0$, $Q = -KA\frac{dT}{dx} = 0$

i.e $\frac{dT}{dx} = 0$

$$-\frac{S}{K}x + C_1 = \cancel{\frac{dT}{dx}}\ 0$$

$$\therefore C_1 = 0$$

$$\frac{dT}{dx} = -\frac{S}{K}x$$

for max^m temp,

$$\frac{dT}{dx} = 0 \;, \quad \frac{d^2T}{dx^2} = -ve$$

at $\boxed{x=0}$

so max temp occurs at $\boxed{x=0}$

2>

$$\frac{Q}{A}\bigg|_{at\,x=L} = \frac{-kA}{A}\frac{dT}{dx}\bigg|_{x=L}$$

$$= -k\left(-\frac{S}{k}L\right)$$

$$= SL$$

Q.> 3-dim heat conduction is governed by one of the following diff eqn

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla T + \psi k \,,\, \frac{1}{\alpha}$$

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla T + \psi/k$$

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla^2 T + \psi k$$

✓ $$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla^2 T + \psi/k$$

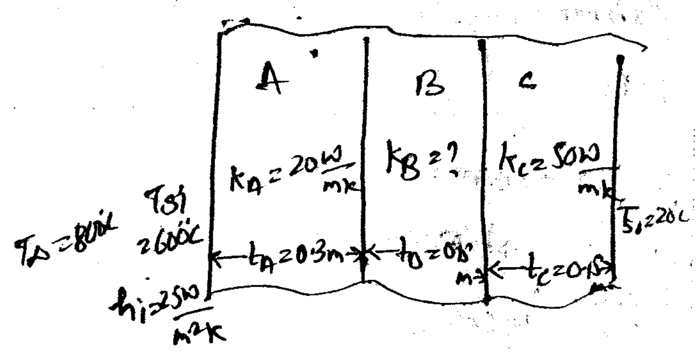
ψ = volumetric rate of heat generation

Q> The composite wall of oven is consist of 3 material A, B, & C. under steady state operating condition the outer surface $T_{so} = 20°C$ & inner surface temp as $T_{si} = 600°C$ and the oven air temp is ~~...~~ $T_\infty = 800°C$ for the following data.

$K_A = 20W/mK$, $K_C = 50W/mK$, $t_A = 0.3m$, $t_B = 0.15m$, $t_C = 0.15m$.

The inner wall heat transfer coeff is $25W/m^2K$. A thermal Conductivity of K_B of a material is calculated as

35 ; 1.53 ; 0.66 ; 0.03 .



$$Q = \frac{580}{\frac{0.3}{20\times A} + \frac{0.15}{K\times A} + \frac{0.15}{50\times B}}$$

$$Q = \frac{200}{\frac{0.3}{20A_1} + \frac{0.15}{K_B A_2}}$$

$$Q = \frac{780}{\frac{0.15}{K_B A_2} + \frac{0.15}{50\times A_3}}$$

$$Q = \frac{800-20}{\frac{1}{25} + \frac{0.3}{20} + \frac{0.15}{K_B} + \frac{0.15}{50}}$$

$$Q = \frac{800-600}{1/25}$$

$$200\times 25 = \frac{780}{\frac{1}{25} + \frac{0.3}{20} + \frac{0.15}{K_B} + \frac{0.15}{50}}$$

$$K_B = 1.53$$

Q7 A metallic ball density 2700 kg/m³ and specific heat 0.9 kJ/kg°C of dia 7.5cm is allowed to cool in air at 25°C. When the temp of the ball is 125°C. It is found to cool at the rate of 4°C/min. If the thermal gradient inside the ~~ball~~ wall are neglected. The H.T coeff is {2.034, 20.34, 81.36, 203.4} W/m²°C

$$hA(T-T_\infty) = -\rho V C \frac{dT}{dz}$$

Q> For a given ambient air temp (T_0) wrd increase in thickness of insulation of a hot cylinder by rate of heat loss from the surface would decrease

i> decrease

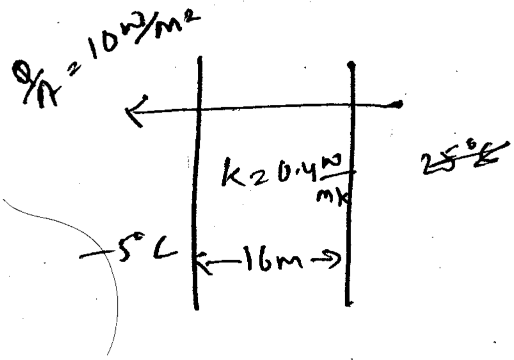
ii> first decrease then increase

iii> increase

iv> first increase then decrease.

Q> The heat flux from outside to inside across an insulating wall wrd thermal conductivity 0.04 W/mK and thickness 0.16m is $\frac{Q}{A} = 10 W/m^2$. The temp of inside wall is $-5°C$, the outside wall temp is ~~25°C~~.

25°C, 30°C, 35°C, 40°C



$$\frac{Q}{A} = -\frac{kA}{A}\frac{T_1 - T_2}{\Delta x}$$

$$10 = \frac{0.4}{16}(-5° - T_2)$$

$$\frac{160}{0.4}$$

Q> A metal ball of radius 0.1m at uniform temp of 90°C is left in air had 30°C the density & sp. heat of the metal are 3000 kg/m³ & 0.4 kJ/kgK resp., $h = 50$ W/m²K. The temp gradient inside the wall ... The tym taken in hrs for the ball to cool at 60°C is

555, 55.5, 0.55, 0.15 hr.

$$\boxed{\begin{gathered} hA(T - T_\infty) = -\rho V c \frac{dT}{d\tau} \\ \frac{50 \times 4\times\pi \times 0.1 \times 0.1\ (30-60)}{-3000 \times \frac{4}{3}\times\pi \times 0.1 \times 0.1 \times 0.1 \times 0.4\times10^3} \frac{dT}{dt} \end{gathered}}$$

37.5

$$\frac{T-T_\infty}{T_1-T_\infty} = e^{\frac{-h}{\delta LC}\tau}$$

$$\ln\frac{T-T_\infty}{T_1-T_\infty} = -\frac{h}{\delta LC}\tau$$

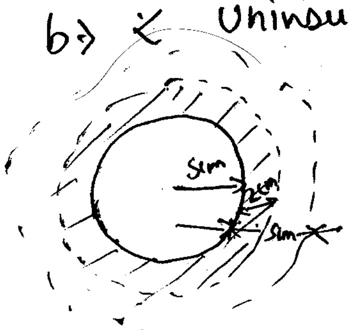
Q.) A 10cm dia steam pipe carrying steam at 180°C is covered with an insulation, $K_i = 0.6\ W/m^2K$. It looses heat to the surrounding at 30°C (T_∞). Assume a h.T coeff of $h = 0.8\ W/m^2K$ for h.t from surface to surrounding. Neglect wall resistance ocupy & film resistance of steam. If the insulation thickness is 2cm. The rate of heat loss from this insulated pipe will be a

a.) > uninsulated.

b.) < uninsulated

c.) = uninsulated

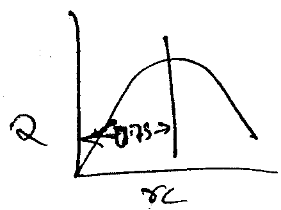
d.) < steam with 5cm insulation.



$r_c = \frac{K_i}{h_o}$

$\cancel{0.5 = \frac{0.6}{0.8}}$

$r_c = \frac{K_i}{h_o} = \frac{0.6}{0.8} = 0.75m$



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CONVECTION

$Q = hA\Delta T$

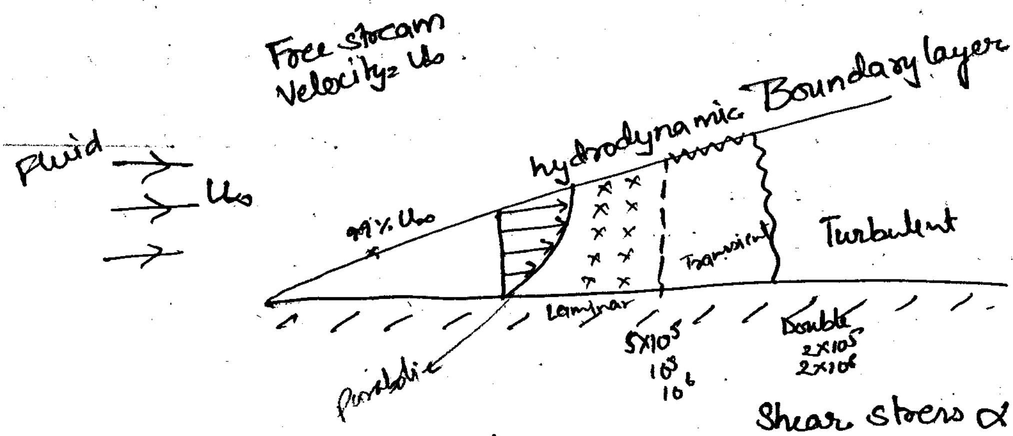
- ↳ Convective h.t coeff.
- ↳ Film coeff.

Free Convection or Natural Convection (ρ diff)

Forced Convection :-

fluid flow = hydro.

Agarbatti



Shear stress ∝ Velocity gradient.

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \qquad \frac{kg\,m}{s^2}\times\frac{s}{m^2}$$

$$\frac{kg\,m}{s\,m}$$

μ = dynamic viscosity.

$\frac{du}{dy}$ = change in velocity in y-direction.

⇒ Flow is laminar flow

Laminar flow

Laminas — flow in groups/moving in groups.

Reynold's No.

$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}}$$

$$= \cancel{\frac{\text{Inertia forces}}{\text{mass} \times \text{acceleration}}}$$

=

	Laminar	Turbulent
Reyno.	5×10^5	$-\ 10^6$
	10^5	$-\ 2\times10^5$
	10^6	$-\ 2\times10^6$

Inertia force

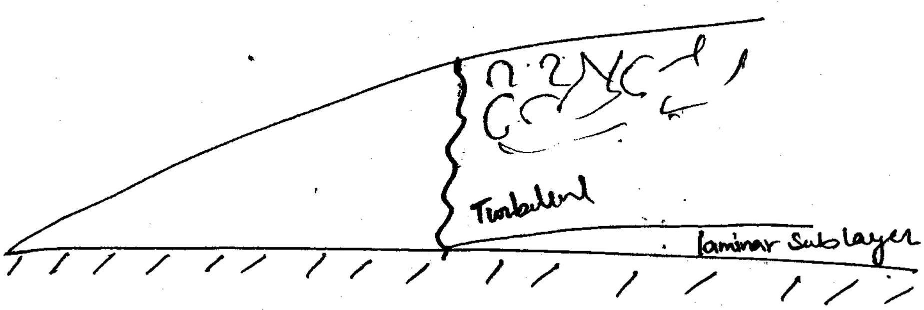
$$= \text{mass} \times \text{Accleration.}$$
$$= \rho L^3 \frac{u}{t}$$
$$= \rho \frac{L}{t} \cdot L^2 \cdot u$$
$$= \rho u^2 L^2$$

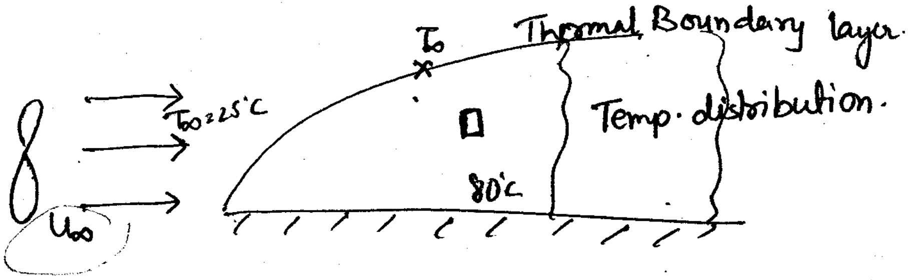
Viscous force

$$= \mu \cdot \frac{du}{dy} \cdot L$$
$$= \mu \cdot \frac{L}{t} \cdot \frac{L^2}{L}$$
$$= \mu U L$$

$$Re = \frac{\rho u^2 L^2}{\mu U L} = \frac{\rho U L}{\mu}$$

Train S_1 S_5 S_6





Conservation of Mass — Continuity eqn.

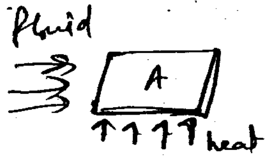
" " Momentum — Momentum eqn

" " Energy — Energy eqn

$$Q_{cond} = -kA\frac{dT}{dy}\Big|_{y=0}$$

$\downarrow$

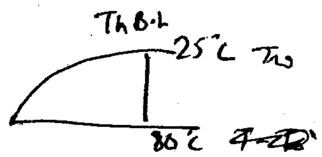
$$Q_{conv} = hA(T_w - T_\infty)$$



$$-kA\frac{dT}{dy}\Big|_{y=0} = hA(T_w - T_\infty)$$

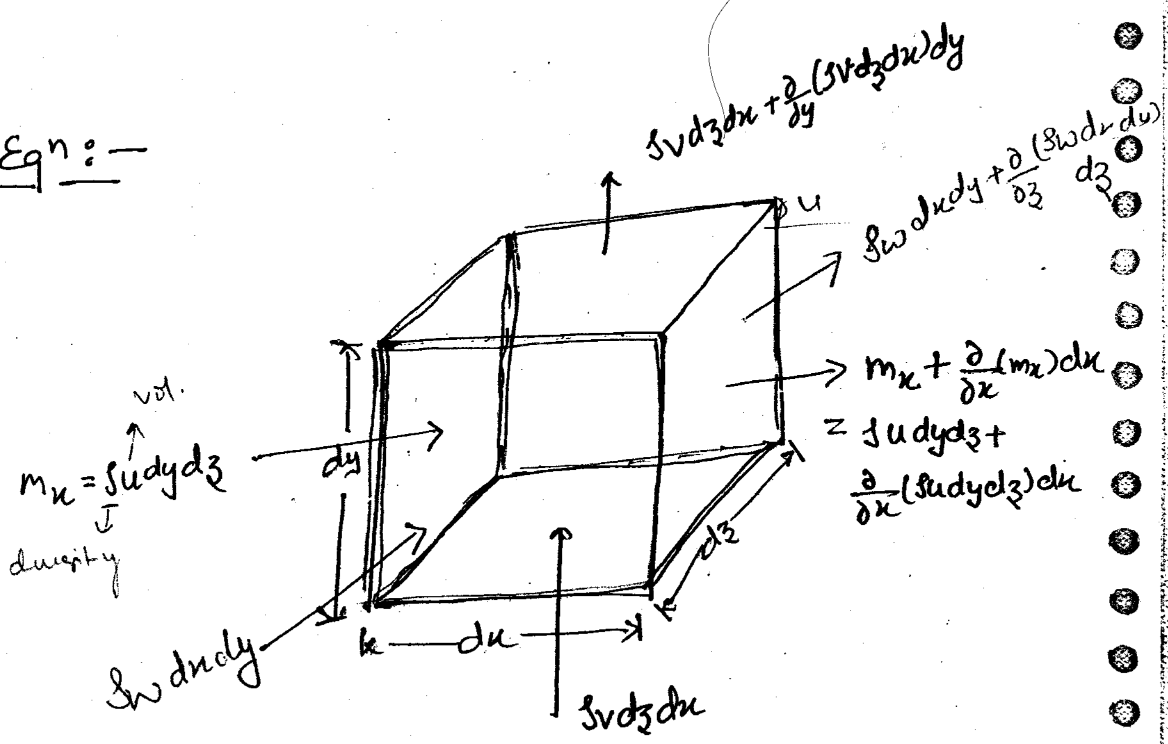
$$h = \frac{-k\frac{dT}{dy}\Big|_{y=0}}{(T_w - T_\infty)}$$

$$T = T(y)$$



$$\frac{dT}{dy}\Big|_{y=0} =$$

Continuity Eqn :-



$$\rho u\,dy\,dz + \rho v\,dz\,dx + \rho w\,dx\,dy$$
$$= \left[\rho u\,dy\,dz + \frac{\partial}{\partial x}(\rho u\,dy\,dz)dx\right] + \left[\rho v\,dz\,dx + \frac{\partial}{\partial y}(\rho v\,dz\,dx)dy\right]$$
$$+ \left[\rho w\,dx\,dy + \frac{\partial}{\partial z}(\rho w\,dx\,dy)dz\right]$$

$$\rho\,dx\,dy\,dz\frac{\partial u}{\partial x} + \rho\,dx\,dy\,dz\frac{\partial v}{\partial y} + \rho\,dx\,dy\,dz\frac{\partial w}{\partial z} = 0$$

$$\rho\,dx\,dy\,dz\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

3D Continuity -n

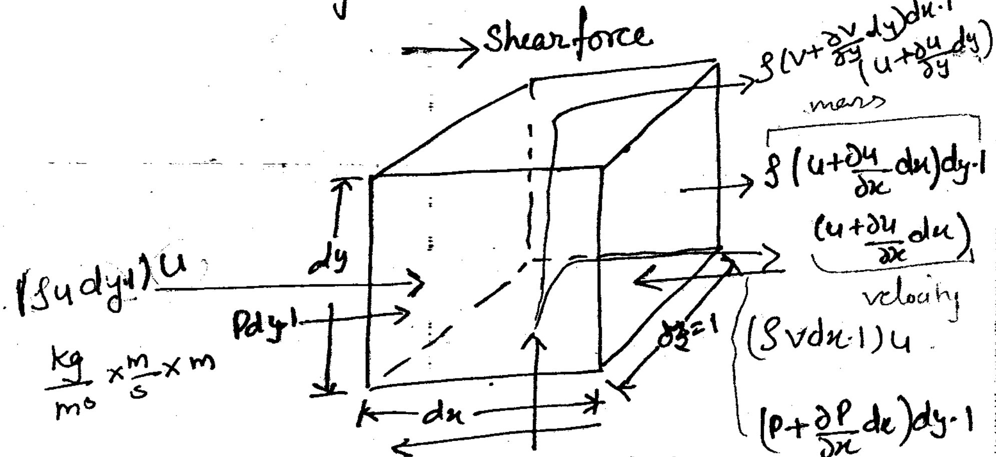
Oct 17, 14

Momentum Eqⁿ

as we have to study in x-direction only

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum Eqⁿ in x-direction only.
mV



$\mu \frac{du}{dy}\Big|_{y=0} dx = \tau_w dx\cdot 1$ Viscous shear force to oppose the flow

$$\Big[\rho\big(\cancel{u^2}+u\frac{\partial u}{\partial x}dx+u\frac{\partial u}{\partial x}dx\big)dy-\cancel{\rho u^2 dy}\Big]$$

$$+\Big[\rho\big(\cancel{uv}+v\frac{\partial u}{\partial y}dy+u\frac{\partial v}{\partial y}dy\big)dx-\cancel{\rho uv\,dx}\Big]$$

$$+\Big[\rho\big(uv+v\frac{\partial u}{\partial y}dy+u\frac{\partial v}{\partial y}dy\big)dx$$

$$=\Big[\mu\Big[\cancel{\frac{\partial u}{\partial y}}+\frac{\partial^2 u}{\partial y^2}dy\Big]dx-\cancel{\mu\frac{\partial u}{\partial y}dx}\Big]+\Big[\cancel{P}dy-\big(\cancel{P}+\frac{\partial P}{\partial x}dx\big)dy\Big]$$

$$\rho\,dx\,dy\Big[u\underbrace{\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\Big)}_{=0\ =\ \text{Continuity eq}^n}+\Big(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\Big)\Big]=\mu\frac{\partial^2 u}{\partial y^2}dx\,dy-\frac{\partial P}{\partial x}dx\,dy$$

Kinematic viscosity
momentum diffusivity.

Theory

$$\boxed{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho}\frac{\partial P}{\partial x}}$$

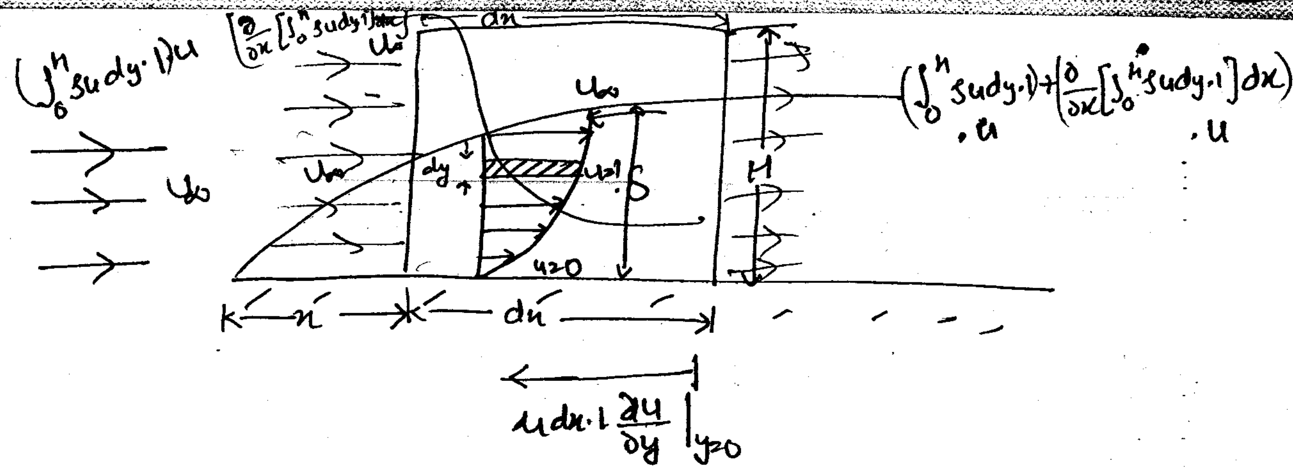
Pressure gradient is neglected

Numε

$$\boxed{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}}$$

Conservation of momentum eqⁿ

Momentum Integral Eqn



$$\left[\int_0^H \rho u^2 dy + \frac{\partial}{\partial x}\left[\int_0^H \rho u^2 dy\right]dx\right] - \left[\int_0^H \rho u^2 dy + u_\infty \frac{\partial}{\partial x}\left[\int_0^H \rho u\, dy\right]dx\right]$$

momentum leaving   momentum entering.

$$= -\mu dx \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\frac{\partial}{\partial x}\left[\int_0^H \rho u^2 dy\right]dx - \frac{\partial}{\partial x}\left[\int_0^H \rho u u_\infty dy\right]dx = -\mu dx \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\frac{\partial}{\partial x}\left[\int_0^H \rho(u^2 - u u_\infty)dy\right] = -\mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\frac{\partial}{\partial x}\left[\int_0^H \rho u(u - u_\infty)dy\right] = -\mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\frac{\partial}{\partial x}\left[\int_0^H \rho u(u_\infty - u)\right] = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

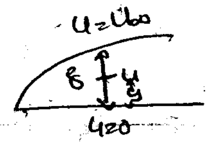
The Integrand becomes zero after, the hydrodynamic Boundary layer.

Hence $$\boxed{\frac{\partial}{\partial x}\left[\int_0^\delta \rho u(u_\infty - u)dy\right] = \mu \frac{\partial u}{\partial y}\Big|_{y=0}}$$ Momentum integral eqn.

L.H.S $$\frac{\partial}{\partial x}\left[\int_0^\delta \rho \frac{u}{u_\infty}\cdot u_\infty\; u_\infty\left(1 - \frac{u}{u_\infty}\right)dy\right] = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\rho u_\infty^2 \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right)dy\right]$$

$$\frac{u}{u_\infty} = f\left(\frac{y}{\delta}\right) = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3$$



Boundary Conditions

i) $y=0$, $u=0$; iii) $y=\delta$, velocity const $\frac{\partial u}{\partial y}=0$

ii) $y=\delta$, $u=U_\infty$; iv) $y=0$, $\frac{\partial^2 u}{\partial y^2}=0$

iv) $y=0$ $u=0$ $v=0$

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$

$0 = a$; from i) B.C.

$1 = a+b+c+d$; from (ii) B.C

maybe cubical parabola eqn ; $\frac{\partial u}{\partial y} = U_\infty\left(b\frac{1}{\delta} + c\cdot 2\left(\frac{y}{\delta}\right)\frac{1}{\delta} + d3\cdot\left(\frac{y}{\delta}\right)^2\frac{1}{\delta}\right)$

$= \frac{U_\infty}{\delta}\left[b + 2c\left(\frac{y}{\delta}\right) + 3d\left(\frac{y}{\delta}\right)^2\right] = 0$

$b + 2c\left(\frac{y}{\delta}\right) + 3d\left(\frac{y}{\delta}\right)^2 = 0$; as $\frac{U_\infty}{\delta} \neq 0$

$b + 2c + 3d = 0$; from (iii) B.C

$\frac{\partial^2 u}{\partial y^2} = \frac{U_\infty}{\delta}\left[2c\frac{1}{\delta} + 3d\cdot 2\left(\frac{y}{\delta}\right)\frac{1}{\delta}\right]$

$= \frac{U_\infty}{\delta^2}\left[2c + 6d\left(\frac{y}{\delta}\right)\right]$

$2c + \cancel{6d} = 0$

$\boxed{\therefore c = 0}$

$b+c+d = 1$

$b+d = 1$

$b+3d = 0$

$3d = -b$

$-2d = 1$

$d = -1/2$

$b = 3/2$

$$\boxed{\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3}$$

velocity — u; free stream velocity — u_∞

velocity distribution eqn within the hydrodynamic boundary layer.

Oct 27, 14

U20 4946

$$\int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\left[1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy$$

$$= \int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{9}{4}\left(\frac{y}{\delta}\right)^2 + \frac{3}{4}\left(\frac{y}{\delta}\right)^4 - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 + \frac{3}{4}\left(\frac{y}{\delta}\right)^4 - \frac{1}{4}\left(\frac{y}{\delta}\right)^6\right] dy$$

$$= \left| \frac{3}{2}\cdot\frac{1}{\delta}\cdot\frac{y^2}{2} - \frac{9}{4}\cdot\frac{1}{\delta^2}\cdot\frac{y^3}{3} + \frac{3}{4}\cdot\frac{1}{\delta^4}\cdot\frac{y^5}{5} - \frac{1}{2}\cdot\frac{1}{\delta^3}\frac{y^4}{4} + \frac{3}{4}\frac{1}{\delta^4}\cdot\frac{y^5}{5} - \frac{1}{4}\cdot\frac{1}{\delta^6}\cdot\frac{y^7}{7}\right|_0^\delta$$

$$= \cancel{\frac{3}{4}\delta} - \cancel{\frac{3}{4}\delta} + \frac{3}{20}\delta - \frac{1}{8}\delta + \frac{3}{20}\delta - \frac{1}{28}\delta$$

$$\left(\frac{3}{20} - \frac{1}{8} + \frac{3}{20} - \frac{1}{28}\right)\delta = \frac{42 - 35 + 42 - 10}{280}\delta$$

$$= \frac{39}{280}\delta$$

$$\frac{1}{u_\infty}\frac{\partial u}{\partial y}\bigg|_{y=0} = \left|\frac{3}{2}\cdot\frac{1}{\delta} - \frac{1}{2}\frac{1}{\delta^3}\cancel{3y^2}\right|_{y=0}$$

$$\frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{3}{2}\frac{u_\infty}{\delta}$$

$$\rho u_\infty^2 \frac{\partial}{\partial x}\left(\frac{39}{280}\delta\right) = \mu\frac{3}{2}\frac{u_\infty}{\delta}$$

$$\frac{13}{140}\frac{\partial\delta}{\partial x} = \frac{\mu}{\rho u_\infty \delta}$$

$$\delta\frac{\partial\delta}{\partial x} = \frac{140}{13}\frac{\mu}{\rho u_\infty}$$

Integrating $\quad \frac{\delta^2}{2} = \frac{140}{13}\frac{\mu}{\rho u_\infty}x + c \qquad \therefore c = 0$

$$\delta^2 = \frac{280}{13}\frac{\mu x}{\rho u_\infty}$$

$$\frac{\delta^2}{x^2} = \frac{280}{13}\frac{\mu x}{\rho u_\infty x^2}$$

$$\frac{\delta^2}{x^2} = \frac{280}{13}\frac{1}{\frac{\rho u_\infty x}{\mu}}$$

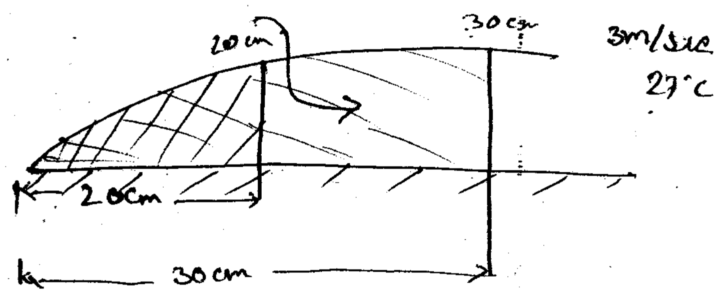
$$\frac{\delta}{x} = \sqrt{\frac{280}{13}}\frac{1}{\sqrt{Re_x}}$$

$$\boxed{\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}}$$

$\Rightarrow$ $$\boxed{\delta = \frac{4.64x}{\sqrt{Re_x}}}$$ δ = Approximate sol^n of Boundary layer thickness.

$\Rightarrow$ $$\boxed{\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}}$$ Exact Solution.

Q.) Air flow over a flat plate with a velocity of 3 m/sec at a temp of 27°C. Determine the mass flow introduced btween 20 cm & 30 cm from the ~~reading~~ leading edge of the plate.



Sol^n $\delta = 4.64 \frac{x}{\sqrt{Re_x}}$

$$Re_x = \frac{\rho U_\infty x}{\mu}$$

$$m_1 = \int_0^{\delta_1} \rho u dy \cdot 1$$

$$m_2 = \int_0^{\delta_2} \rho u dy \cdot 1$$

$$(m_2 - m_1) = ?$$

$$\int_0^{\delta} \rho u_\infty \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy$$

$$= \rho u_\infty \left[\frac{3}{2}\frac{y^2}{2}\frac{1}{\delta} - \frac{1}{2}\frac{1}{\delta^3}\frac{y^4}{4}\right]_0^{\delta}$$

$$= \rho u_\infty \left[\frac{3}{4}\delta - \frac{1}{8}\delta\right]$$

$$= \frac{5}{8}\rho u_\infty \delta$$

$$\frac{5}{8}\rho u_\infty(\delta_2 - \delta_1) = (m_2 - m_1)$$

$PV = RT$

$P = \rho RT$

$\frac{P}{RT} = \rho$

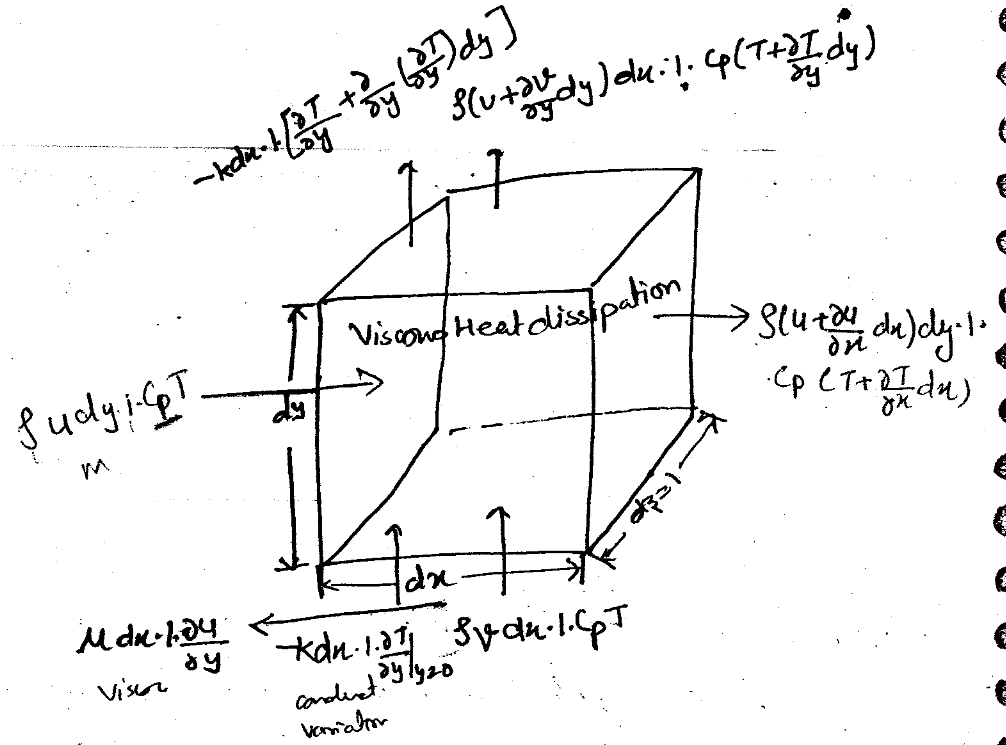
$$\frac{kN}{\frac{m^2 \cdot kJ}{kg\,K} \cdot K} = \frac{kg}{m^3}$$

$$\frac{kN}{m^2 \frac{kNm}{kg}}$$

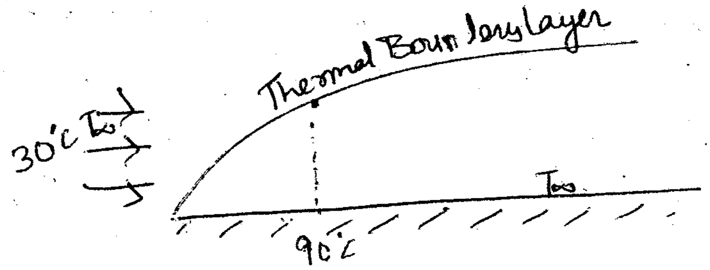
kg/m^3

$$R = \frac{R_u}{\text{Mol. mass}}$$

$$= \frac{8.314}{28.966} = 0.287$$



$Q = m\,C_p\,dT$



$$\rho u\cancel{C_pT}\,dy + \rho v\cancel{C_pT}\,dx + \left(-k\,dx\,\cancel{\frac{\partial T}{\partial y}}\right) + \mu\,dx\left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$= \rho\left[\left(\cancel{uT} + u\frac{\partial T}{\partial x}dx + T\frac{\partial u}{\partial x}dx\right)C_p\,dy\right] + \rho\left[\left(\cancel{vT} + v\frac{\partial T}{\partial y}dy + T\frac{\partial v}{\partial y}dy\right)C_p\,dx\right]$$

$$+\left[-k\,dx\left(\cancel{\frac{\partial T}{\partial y}} + \frac{\partial^2 T}{\partial y^2}dy\right)\right]$$

$$\mu\left(\frac{\partial u}{\partial y}\right)^2 dx\,dy + k\frac{\partial^2 T}{\partial y^2}dx\,dy = \rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)C_p\,dx\,dy$$

$$+\rho\left[T\left(\cancel{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}}\right)C_p\,dx\,dy\right]$$

; Continuity eqn.

$$\mu\left(\frac{\partial u}{\partial y}\right)^2 + k\frac{\partial^2 T}{\partial y^2} = \rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

$$\boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \cancel{\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2}}$$

viscous heat dissipation being very small may be neglected

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}}$$

Momentum eqn: u variation within BL in x-dirn

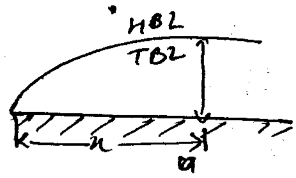
$$\boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}}$$

Energy Eqn

T variation with in B.L in x-dirn

$$\boxed{\nu = \alpha}$$

$$\frac{\mu}{\rho} = \frac{K}{\rho C_p}$$



$$\frac{\mu C_p}{k} = 1$$

Prandtl no → Pr.

$\boxed{Pr = 1}$ ⇒ HBL & TBL overlap each other.

Oct 28,14

h, convective heat transfer coefficient :-

$$h = f(K, u, L, \mu, C_p, \rho]$$

$$h = C(K^a, u^b L^c \mu^d C_p^e \rho^f)$$

$$J/_{sm^2K} = C\left(J/_{smK}\right)^a (m/s)^b (m)^c (kg/ms)^d (J/_{kgK})^e (kg/m^3)^f$$

$J \Rightarrow \quad 1 = a + e$

$s \Rightarrow \quad -1 = -a - b - d$

$m \Rightarrow \quad -2 = -a + b + c - d - 3f$

$K \Rightarrow \quad -1 = -a - e$

$kg \Rightarrow \quad 0 = d - e + f$

4 eqn & 6 variables

$$1 = a + e$$
$$1 = a + b + d$$
$$2 = a - b - c + d + 3f$$
$$0 = d - e + f$$

$a = 1 - e$ —— (i)

$d = -f + e$ —— (ii)

$b = f$ —— (iii)

$c = -1 + f$ —— (iv)

$$h = C(k)^{1-e}(u)^f(L)^{-1+f}(\mu)^{e-f}(C_p)^e(\rho)^f$$

$$h = C\left(\frac{k}{L}\right)^1\left(\frac{\mu C_p}{k}\right)^e\left(\frac{uL\rho}{\mu}\right)^f$$

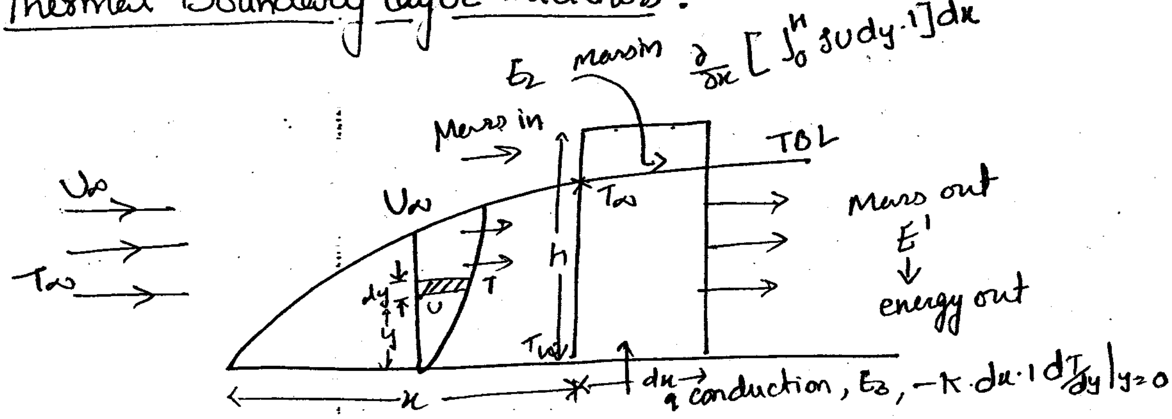
$$\frac{hL}{k} = C\left(\frac{UL\rho}{\mu}\right)^f\left(\frac{\mu C_p}{k}\right)^e$$

$$Nu \Rightarrow C\,Re^f\,Pr^e$$

$$Nu = f(Re, Pr)$$

<u>h.t.C</u> $\Rightarrow$ depends strongly on C_p & U so write other variables in the term of variable which are the power of C_p & U.

<u>Thermal Boundary layer Thickness</u> :—



$$E_2 = C_p T_\infty \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy.1\right]dx$$

$E_3, E_1, E_2 \rightarrow$ Energy enteering.

$$E' = \int_0^H \rho u\,dy.1.C_p.T + \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy.c_p.T\right]dx$$

Applying energy balance :—

$$\int_0^H \cancel{\rho u\,dy C_p T} + C_p T_\infty \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy\right]dx + \left[-k\,dx.1.\partial T/\partial y\right]$$
$$= \int_0^H \cancel{\rho u\,dy C_p T} + \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy\, C_p T\right]dx$$

$$\frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy.c_p T_\infty\right]dx - \frac{\partial}{\partial x}\left[\int_0^H \rho u\,dy.C_p T\right]dx$$
$$= k\,\frac{dT}{dy}\Big|_{y=0}$$

$$\frac{\partial}{\partial x}\left[\int_0^H \rho u c_p (T_\infty - T)dy\right] = k \, \partial T/\partial y\Big|_{y=0}.$$

$$\rho c_p \frac{\partial}{\partial x}\left[\int_0^H u(T_\infty - T)dy\right] = k \, \partial T/\partial y\Big|_{y=0}$$

$$\boxed{\frac{\partial}{\partial x}\left[\int_0^H u(T_\infty - T)dy\right] = \alpha \frac{\partial T}{\partial y}\Big|_{y=0}}$$

above T.B.L $\Rightarrow$ $T = T_\infty$ so Integral becomes zero after T.B.L

$$\boxed{\frac{\partial}{\partial x}\left[\int_0^{\delta t} \frac{u_\infty}{u_\infty} \times u \frac{(T_\infty - T)}{(T_\infty - T_w)} \times (T_\infty - T_w)\, dy\right] = \alpha \frac{\partial T}{\partial y}\Big|_{y=0}}$$

$$u_\infty (T_\infty - T_w)\frac{\partial}{\partial x}\left[\int_0^{\delta t} \frac{u}{u_\infty}\left(\frac{T_\infty - T}{T_\infty - T_w}\right)dy\right] = \alpha \frac{\partial T}{\partial y}\Big|_{y=0}$$

$$\frac{T - T_w}{T_\infty - T_w} = f\left(y/\delta_t\right) = a + b\left(\frac{y}{\delta_t}\right) + c\left(\frac{y}{\delta_t}\right)^2 + d\left(y/\delta_t\right)^3 \cdot$$

<u>B.C</u>

i) $y = 0$, $\delta_t = 0$, $T = T_w$ $\boxed{a = 0}$ — (i)

ii) $y = \delta_t$, $T = T_\infty = 1 = a + b + c + d = b + c + d$ — (ii)

iii) $y = \delta_t$ $\frac{\partial T}{\partial y} = 0$, $\frac{\partial T}{\partial y} = (T_\infty - T_w)\left[b \cdot \frac{1}{\delta_t} + c\frac{1}{\delta_t^2} 2y + \frac{d\,3y^2}{\delta_t^3}\right]$

$$0 \Rightarrow \frac{T_\infty - T_w}{\delta_t}\left[b + \frac{2cy}{\delta_t} + \frac{3dy^2}{\delta_t^2}\right]$$

$$b + 2c + 3d = 0 \text{ — (iii)}$$

$$\cancel{u\frac{\partial T}{\partial x}} + \cancel{v\frac{\partial T}{\partial y}} = \alpha\frac{\partial^2 T}{\partial y^2} \cdot \Rightarrow \text{so } \frac{\partial^2 T}{\partial y^2} = 0$$

iv) $y = 0$ $\frac{\partial^2 T}{\partial y^2} = 0$ $\frac{\partial^2 T}{\partial y^2} = \frac{T_\infty - T_w}{\delta_t}\left[\frac{2c}{\delta_t} + \frac{6dy}{\delta_t^2}\right]$

$$0 = \frac{T_\infty - T_w}{\delta t}\left[\frac{2c}{\delta t}\right]$$

$\boxed{c = 0}$ — (iv)

$$b + d = 1$$
$$b + 3d = 0$$
$$-2d = 1$$
$$\therefore d = -1/2 \quad , b = 3/2$$

$$\frac{T - T_w}{T_\infty - T_w} = 3/2 \left(\frac{y}{\delta_t}\right) - 1/2 \left(y/\delta_t\right)^3$$

$$\frac{T_\infty - T}{T_\infty - T_w} = 1 - \frac{T - T_w}{T_\infty - T_w} = 1 - 3/2\left(y/\delta_t\right) + 1/2\left(y/\delta_t\right)^3$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} \Rightarrow \left(\frac{3}{2\delta_t} - 1/2 \cdot \frac{1}{\delta_t^3} \cdot 3y^2\right)(T_\infty - T_w)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{3}{2}\frac{1}{\delta_t}(T_\infty - T_w)$$

Now from eqn

$$U_\infty \cancel{(T_\infty - T_w)} \frac{\partial}{\partial x}\left[\int_0^H \frac{u}{U_\infty}\left(1 - \frac{T - T_w}{T_\infty - T_w}\right) dy\right] = \alpha\, 3\frac{\cancel{(T_\infty - T_w)}}{2\delta_t}$$

$$\frac{\partial}{\partial x}\left[\underbrace{\int_0^H \frac{u}{U_\infty}\left(1 - \frac{T - T_w}{T_\infty - T_w}\right) dy}_{\text{solve this}}\right] = \frac{3\alpha}{2U_\infty \delta_t} \quad \text{——— (B)}$$

$$\int_0^{\delta_t}\left[\left(3/2\frac{y}{\delta} - 1/2\left(y/\delta\right)^3\right)\left(1 - 3/2\left(y/\delta_t\right) + 1/2\left(y/\delta_t\right)^3\right]dy$$

$$\int_0^{\delta_t} 3/2\frac{y}{\delta} - \frac{9}{4}\frac{y^2}{\delta\delta_t} + \frac{3}{4}\frac{y^4}{\delta\delta_t^3} - 1/2\frac{y^3}{\delta^3} + \frac{3}{4}\frac{y^4}{\delta_t\delta^3} - 1/4\frac{y^6}{\delta^3\delta_t^3}$$

$$\Rightarrow \left[\frac{3}{2\delta}\frac{y^2}{2} - \frac{9}{4}\frac{1}{\delta\delta_t}\frac{y^3}{3} + \frac{3}{4}\frac{1}{\delta\delta_t^3}\frac{y^5}{5} - 1/2\frac{1}{\delta^3}\frac{y^4}{4} + \frac{3}{4}\frac{1}{\delta^3\delta_t}\frac{y^5}{5} - \frac{1}{4}\frac{y}{\delta^3\delta_t^3}\frac{y^7}{7}\right]_0^{\delta_t}$$

$$\Rightarrow \left[\frac{3}{4}\frac{\delta_t}{\delta}\cancel{\cdot\delta_t} - \frac{3}{4}\frac{\delta_t}{\delta}\cancel{\cdot\delta_t} + \frac{3}{20}\frac{\delta_t}{\delta}\cdot\delta_t - \frac{1}{8}\left(\frac{\delta_t}{\delta}\right)^3\delta_t + \frac{3}{20}\left(\frac{\delta_t}{\delta}\right)^3\cdot\delta_t - \frac{1}{28}\left(\frac{\delta_t}{\delta}\right)^3\cdot\delta_t\right]$$

let $\frac{\delta t}{\delta} = \gamma$

$\delta t = \delta\gamma$

$$\left[\frac{3}{20}\gamma(\delta\gamma) + \left(-\frac{1}{8} + \frac{3}{20} - \frac{1}{28}\right)\gamma^3(\delta\gamma)\right] \Rightarrow$$

$$\Rightarrow \left(\frac{3}{20}\gamma^2 - \frac{3}{280}\gamma^4\right)\delta$$

for $\gamma \simeq 1$

$$\frac{\partial}{\partial x}\left[\frac{3}{20}\gamma^2\delta\right] = \frac{3\alpha}{2U_\infty\delta t} = \frac{3\alpha}{2U_\infty\gamma\delta}$$

$$\boxed{\frac{\partial}{\partial x}\left[\frac{1}{10}\gamma^2\delta\right] = \frac{\alpha}{U_\infty\gamma\delta}}$$

$$\gamma\delta\frac{\partial}{\partial x}(\gamma^2\delta) \Rightarrow \frac{10\alpha}{U_\infty}$$

$$\gamma\delta\left[\gamma^2\frac{\partial\delta}{\partial x} + \delta\, 2\gamma\cdot\frac{\partial\gamma}{\partial x}\right] \Rightarrow \frac{10\alpha}{U_\infty} \qquad \text{—— (C)}$$

$$\delta = \frac{4.64x}{\sqrt{Re_x}}$$

$$\delta^2 = \frac{280}{13}\frac{x^2}{\frac{\rho U_\infty x}{\mu}} \Rightarrow \frac{280}{13}\frac{\mu x}{\rho U_\infty}$$

$$\frac{2\delta\partial\delta}{\partial x} = \frac{280}{13}\frac{\mu}{\rho U_\infty}$$

$$\frac{\delta\partial\delta}{\partial x} = \frac{140}{13}\frac{\mu}{\rho U_\infty}$$

Now eq^n (C) becomes :-

$$\gamma^3\frac{140}{13}\frac{\mu}{\rho U_\infty} + 2\gamma^2\frac{280}{13}\frac{\mu x}{\rho U_\infty}\frac{\partial\gamma}{\partial x} = \frac{10\alpha}{U_\infty}$$

$$\frac{14}{13}\frac{\mu}{\rho}\left[\gamma^3 + 4\gamma^2 x\frac{\partial\gamma}{\partial x}\right] = \alpha$$

$$r^3 + 4r^2 x \frac{\partial r}{\partial x} = \frac{13}{14}\,\frac{\alpha}{\nu}$$

$$\boxed{r^3 + 4r^2 x \frac{\partial r}{\partial x} - \frac{13}{14\,Pr} = 0}$$

Cubical Eqn with differential.

let $r^3 - \frac{13}{14\,Pr} = y$

$$3r^2 \frac{\partial r}{\partial x} = \frac{\partial y}{\partial x}$$

$$r^2 \frac{\partial r}{\partial x} = \frac{1}{3}\,\frac{\partial y}{\partial x}$$

$$\Rightarrow \quad y + 4x\,\frac{1}{3}\,\frac{\partial y}{\partial x} = 0$$

$$3y\,\partial x + 4x\,\partial y = 0$$

$$\frac{3}{4}\,\frac{\partial x}{x} + \frac{\partial y}{y} = 0$$

| variable Separation.

| $\div$ by $4xy$

Integrating, $\frac{3}{4}\ln x + \ln y = 0$

$$\ln x^{3/4} + \ln y = \ln c$$

$$\ln y x^{3/4} = \ln c$$

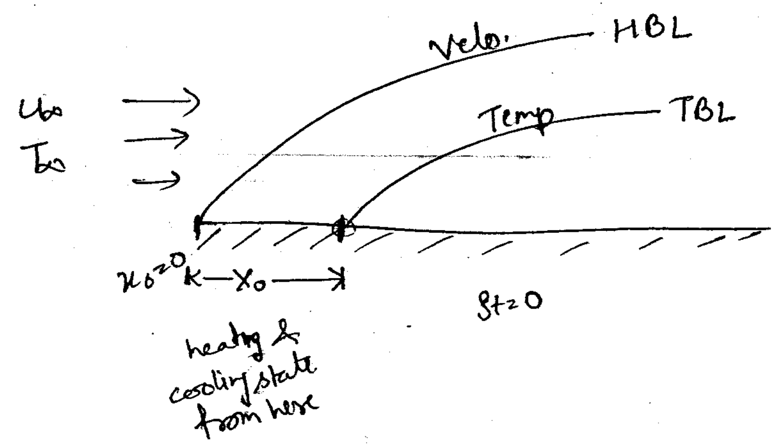
$$y \cdot x^{3/4} = c$$

$$\boxed{y = c \cdot x^{-3/4}}$$

c = constant

$$r^3 - \frac{13}{14\,Pr} = c \cdot x^{-3/4}$$

| to find c



$; \; r = \frac{\delta t}{\delta} = 0$

If given in question, du. consider it rest $X_0 = 0$ (start from here)

$x = x_0$, $\delta_t = 0$, $r = \frac{\delta t}{\delta} = 0$

ie $$0 - \frac{13}{14Pr} = C \cdot x_0^{-3/4}$$

hence, $$C = \frac{-13}{14Pr} \cdot x_0^{3/4}$$

$$\Rightarrow \quad r^3 - \frac{13}{14Pr} = \frac{-13}{14Pr} x_0^{3/4} \cdot x^{-3/4}$$

$$r^3 = \frac{13}{14Pr} \left[1 - \left(\frac{x_0}{x} \right)^{3/4} \right]$$

if $x_0 = 0$, $$r^3 = \frac{13}{14Pr} = \frac{1}{1.0769 Pr}$$

$$r = \left(\frac{1}{1.0769} \right)^{1/3} \frac{1}{Pr^{1/3}}$$

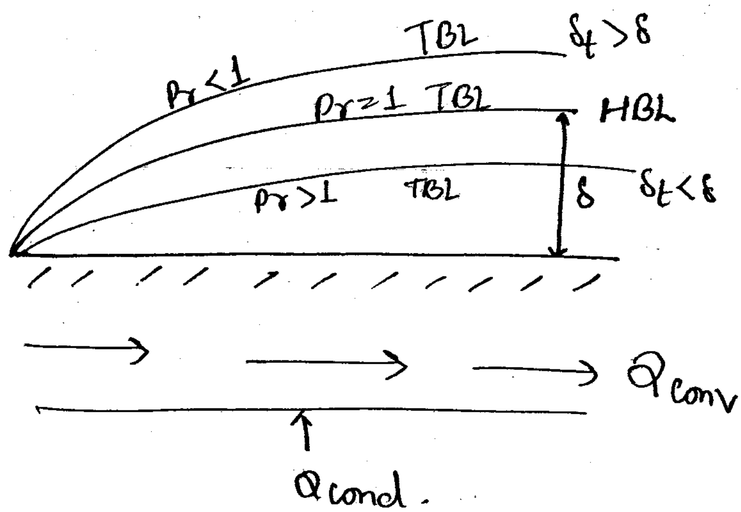
$$\boxed{r = \frac{1}{1.025 \; Pr^{1/3}}}$$

$$\boxed{r \simeq \frac{1}{Pr^{1/3}}}$$

i) $Pr > 1$, $r < 1$, means $\delta_t < \delta$.

ii) $Pr = 1$, $r = 1$, $\delta_t = \delta$

iii) $Pr < 1$, $r > 1$, $\delta_t \geq \delta$.



$$-KA \frac{\partial T}{\partial y}\Big|_{y=0} = hA(T_w - T_\infty)$$

$$h = \frac{-k \frac{\partial T}{\partial y}\Big|_{y=0}}{T_w - T_\infty}$$

$$\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

$$\frac{\partial T}{\partial y} = (T_\infty - T_w)\left[\frac{3}{2}\frac{1}{\delta_t} - \frac{1}{2}\frac{1}{\delta_t^3} 3\cancelto{0}{y^2}\right]$$

$$\boxed{\frac{\partial T}{\partial y} = \frac{3(T_\infty - T_w)}{2\delta_t}}$$

$$h = \frac{-k \frac{3}{2}\left(\frac{T_\infty - T_w}{\delta_t}\right)}{T_w - T_\infty}$$

☆☆ $\boxed{h = \frac{3}{2}\frac{k}{\delta_t}}$ $\delta_t = r\delta$.

$$h = \frac{3}{2} \cdot \frac{K}{\gamma \cdot \delta} = \frac{3K}{2 \cdot \frac{1}{1.025} Pr^{1/3} \cdot \delta}$$

$$h = \frac{\frac{3}{2} K}{\frac{1}{1.025} Pr^{1/3} \cdot \frac{4.64 x}{\sqrt{Re_x}}}$$

$$\boxed{h_x = 0.331 \frac{K}{x} Re_x^{1/2} \cdot Pr^{1/3}}$$

h_x = local convective h.t coeff

$$h = 0.331\, K/x \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

$$\boxed{\frac{h_x}{K} = 0.331\, Re_x^{1/2} \cdot Pr^{1/3}}$$

$$\boxed{Nu_x = 0.331\, Re_x^{1/2} \cdot Pr^{1/3}}$$

local Nusselt No.

*** very Imp

$$\boxed{Nu_x = 0.332\, Re_x^{1/2} \cdot Pr^{1/3}}$$ Exact

at a given distance x to find out local h & local K at a given distance x.

h_{avg} over the entire length of plate.

$$\bar{h} = \frac{1}{L} \int_0^L h_x \cdot dx.$$

$$\boxed{Nu_L = 0.664\, Re^{1/2} \cdot Pr^{1/3}}$$

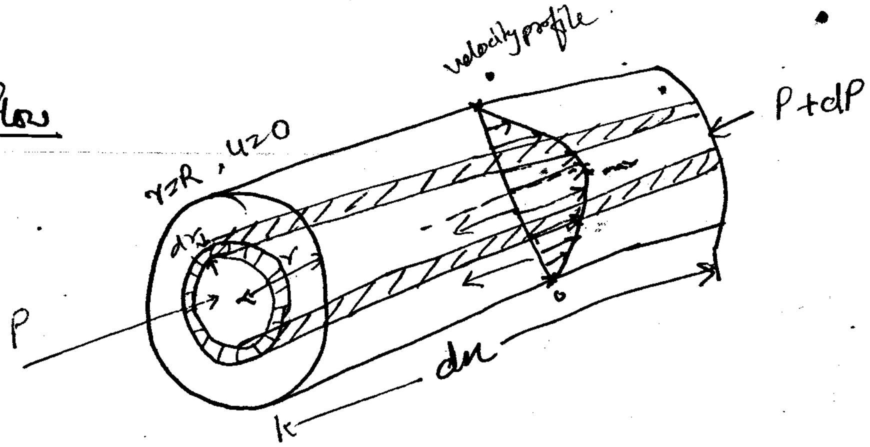
Avg. value of Nusselt No.
i.e Nu_L over the plate.

local $Nu_x = 0.332\, Re^{0.5} Pr^{0.33}$

$Nu_{avg} = 0.664\, Re^{0.5} Pr^{0.33}$

Oct 31, 14

Pipe flow & Tube flow



Pressure forces = Viscous shear force

$$\pi r^2 dP = \tau 2\pi r dx$$

$$\tau = \frac{r}{2} \frac{dP}{dx}$$

$$\mu \frac{du}{dr} = \frac{r}{2} \frac{dP}{dx}$$

$$du = \frac{r}{2\mu} \frac{dP}{dx}$$

dP = -ve
-ve = dP/dx Pressure gradient

Integrating, $u = \frac{r^2}{4\mu} \frac{dP}{dx} + c$

$u = 0$, $r = R$; $0 = \frac{R^2}{4\mu} \frac{dP}{dx} + c$

$$C = -\frac{R^2}{4\mu} \frac{dP}{dx}$$

R = Pipe dia
r = location dia.

$$u = \frac{r^2}{4\mu} \frac{dP}{dx} - \frac{R^2}{4\mu} \frac{dP}{dx}$$

$$\boxed{u = \frac{-R^2}{4\mu} \left(1 - \frac{r^2}{R^2}\right) \frac{dP}{dx}}$$

Velocity can't be -ve
here dP/dx is -ve
Pressure gradient

When $r = 0$, $u = U_{max} = U_0$

$$U_{max} = U_0 = \frac{-R}{4\mu} \frac{dP}{dx}$$

$$\boxed{\frac{u}{U_0} = 1 - \frac{r^2}{R^2}}$$

Hagen Poiseuille Eqn

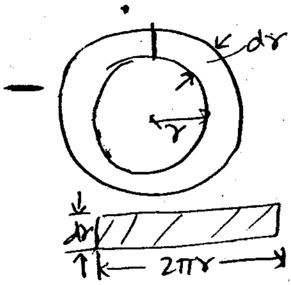
$$\rho u_{av} \pi R^2 = \int_0^R \rho u \, 2\pi r \, dr$$

$$= \rho 2\pi \int_0^R u r \, dr$$

$$= \rho 2\pi \int_0^R u_0 \left(1 - \frac{r^2}{R^2}\right) r \, dr$$

$$= \rho 2\pi u_0 \left\{ \frac{r^2}{2} - \frac{r^4}{4R^2} \Big|_0^R \right\}$$

$$= \rho 2\pi u_0 \left\{ \frac{R^2}{2} - \frac{R^2}{4} \right\}$$



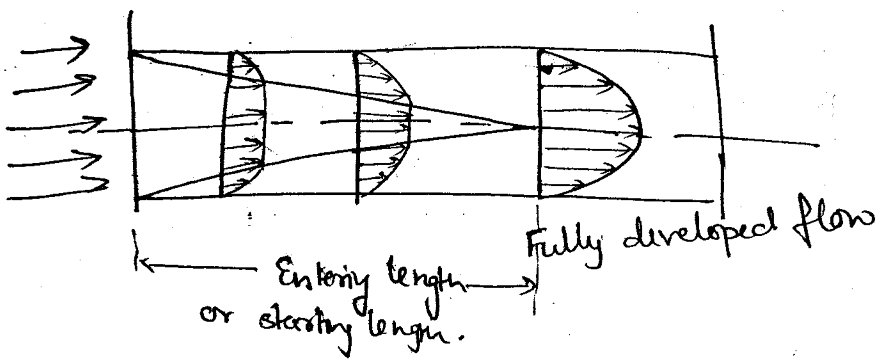
$\Rightarrow 2\pi r dr$

$$\cancel{\rho} u_{av} \cancel{\pi} \cancel{R^2} = \cancel{\rho} 2 \cancel{\pi} u_0 \cancel{R^2}/4$$

$$\boxed{u_{av} = u_0/2}$$

$$\boxed{U_{av} = \frac{U_{max}}{2}}$$

$u_0 = U_{max}$.



$$Re = \frac{u d \rho}{\mu}$$

$Re < 2100$ Laminar flow

$Re > 2300$ Turbulent flow

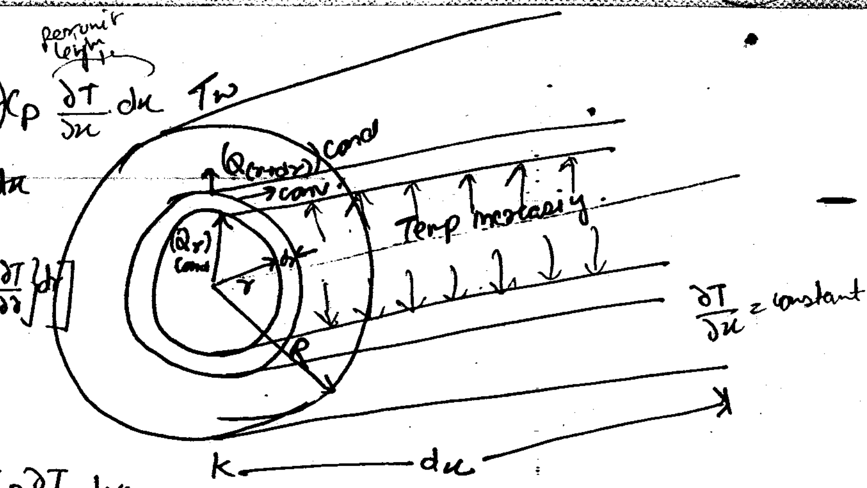
$$Re = \frac{U L \rho}{\mu}$$

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{ma}{\mu A \frac{u}{L}} = \frac{\rho L^3 u/t}{\mu L^2 u/L} = \frac{\rho \cancel{L^2} \frac{L}{t} \cancel{u}}{\mu \cancel{L^2} \cancel{u}/L} = \frac{\rho U L}{\mu}$$

$$-k(\cancel{2\pi r dx})\frac{\partial T}{\partial r} = \overbrace{(\rho u\, 2\pi r dr)}^{\text{mass}}c_p \overbrace{\frac{\partial T}{\partial x}}^{\text{per unit length}} dx$$

$$+\cancel{k(2\pi r dx}$$

$$+\left[\cancel{-k(2\pi r dx)\frac{\partial T}{\partial r}} + \frac{\partial}{\partial r}\left\{-k(2\pi r dx)\frac{\partial T}{\partial r}\right\}dr\right]$$



$$k\, 2\pi dx \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)dr$$

$$= \rho u 2\pi r dr c_p \frac{\partial T}{\partial x} dx$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{\rho c_p}{k}\cdot u \cdot r \frac{\partial T}{\partial x}$$

$-kA\, dT/dr$

$$\boxed{\frac{1}{ur}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial x}}$$ radial form / temp gradient.

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = ur\frac{1}{\alpha}\,\partial T/\partial x$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = u_0\left(1-\frac{r^2}{R^2}\right)r\frac{1}{\alpha}\frac{\partial T}{\partial x}$$

Integrating,
$$r\frac{\partial T}{\partial r} = u_0\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x} + C_1$$

$$\frac{\partial T}{\partial r} = u_0\left(\frac{r}{2} - \frac{r^3}{4R^2}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x} + C_1/r$$

Integrating again,
$$T = u_0\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x} + C_1 \ln r + C_2$$

Boundary condition

$r = 0$, $\frac{\partial T}{\partial r} = 0$, C_1 must be zero.

$r = R$, $T = T_w$

$$T_w = u_0\left(\frac{R^2}{4} - \frac{R^2}{16}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x} + C_2$$

$$C_2 = T_w - \frac{3}{16}R^2\frac{1}{\alpha}\frac{\partial T}{\partial x}$$

$$T = u_0\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x} + T_w - \frac{3}{16}u_0 R^2 \frac{1}{\alpha}\frac{\partial T}{\partial x}$$

$$T = T_w + u_0\left(\frac{r^2}{4} - \frac{r^4}{16R^2} - \frac{3}{16}R^2\right)\frac{1}{\alpha}\frac{\partial T}{\partial x}$$

$$\boxed{T = T_w + \frac{u_0 R^2}{4}\left(\frac{r^2}{R^2} - \frac{r^4}{4R^4} - \frac{3}{4}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x}}$$ temp distribution along radius

(mass x sp heat x temp) | Mixing cup temp.

Bulk Enthalpy

Bulk Enthalpy = Bulk mass x Sp. heat x Mean Bulk Temp.

Mean Bulk Temp:-

$$\text{Mean Bulk temp} = \frac{\text{Bulk Enthalpy}}{\text{Bulk mass} \times \text{Sp heat}}$$

$$T_b = \frac{\int_0^R (\rho u 2\pi r\,dr)\,C_p T}{\int_0^R (\rho u 2\pi r\,dr)\,C_p} = \frac{\int_0^R u T r\,dr}{\int_0^R u r\,dr}$$

$\Rightarrow$ $\int_0^R u r\,dr$

$$= \int_0^R u_0\left(1 - \frac{r^2}{R^2}\right) r\,dr$$

$$= u_0\left|\frac{r^2}{2} - \frac{r^4}{4R^2}\right|_0^R$$

$$= u_0\left(\frac{R^2}{2} - \frac{R^2}{4}\right) = u_0\frac{R^2}{4}$$

$\Rightarrow$ $$\int_0^R u T r\,dr = \int_0^R u_0\left(1 - \frac{r^2}{R^2}\right)\left[T_w + \frac{u_0 R^2}{4}\left(\frac{r^2}{R^2} - \frac{r^4}{4R^4} - \frac{3}{4}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x}\right] r\,dr$$

$$= u_0\int_0^R \left(r - \frac{r^3}{R^2}\right)\left[T_w + \frac{u_0 R^2}{4}\left(\frac{r^2}{R^2} - \frac{r^4}{4R^4} - \frac{3}{4}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x}\right] dr$$

$$= u_0\int_0^R \left[\left(r - \frac{r^3}{R^2}\right)T_w + \frac{u_0 R^2}{4}\left(\frac{r^3}{R^2} - \frac{r^5}{4R^4} - \frac{3}{4}r - \frac{r^5}{R^4} + \frac{r^7}{4R^6} + \frac{3r^3}{4R^2}\right)\frac{1}{\alpha}\frac{\partial T}{\partial x}\right] dr$$

$$= u_0 \left[\left(\frac{r^2}{2} - \frac{r^4}{4R^2} \right) T_w + \frac{u_0 R^2}{4} \left(\frac{r^4}{4R^2} - \frac{r^6}{24R^4} - \frac{3r^2}{8} - \frac{r^6}{6R^4} + \frac{r^8}{32R^6} + \frac{3r^4}{16R^2} \right) \frac{1}{\alpha}\frac{\partial T}{\partial x} \right]_0^R$$

$$= u_0 \left[\left(\frac{R^2}{2} - \frac{R^2}{4} \right) T_w + \frac{u_0 R^2}{4} R^2 \left(\frac{1}{4} - \frac{1}{24} - \frac{3}{8} - \frac{1}{6} + \frac{1}{32} + \frac{3}{16} \right) \frac{1}{\alpha}\frac{\partial T}{\partial x} \right]$$

$$= u_0 \left[\frac{R^2}{4} T_w + \frac{u_0 R^2 \cdot R^2}{4} \left(-\frac{11}{96} \right) \frac{1}{\alpha}\frac{\partial T}{\partial x} \right]$$

$$= \frac{u_0 R^2}{4} \left[T_w + \frac{u_0 R^2}{4} \left(-\frac{11}{96} \right) \frac{1}{\alpha}\frac{\partial T}{\partial x} \right]$$

$$= \frac{u_0 R^2}{4} \left[T_w - \frac{11}{96} u_0 R^2 \frac{1}{\alpha}\frac{\partial T}{\partial x} \right]$$

$$\Rightarrow \boxed{T_b = T_w - \frac{11}{96} u_0 R^2 \frac{1}{\alpha}\frac{\partial T}{\partial x}}$$

$$T_b = \frac{\frac{u_0 R^2}{4}[\;--\;]}{\frac{u_0 R^2}{4}} \Leftarrow (\;--\;)$$

Heat flux remains constant

$$\frac{Q}{A} = \text{constant}$$

$$= + \frac{k A\, \partial T/\partial r}{A} \Big|_{r=R}$$

$$Q_{cond} = -k A \frac{\partial T}{\partial r}\Big|_{r=R} = Q_{conv} = h A (T_w - T_b)$$

$$k \frac{u_0 R^2}{4} \left| \left(\frac{2r}{R^2} - \frac{4r^3}{4R^4} \right) \right|_{r=R} \frac{1}{\alpha}\frac{\partial T}{\partial x} = h (T_w - T_b)$$

$$k \frac{u_0 R^2}{4} \times \frac{1}{R} \cdot \frac{1}{\alpha}\frac{\partial T}{\partial x} = h \frac{11}{96_{24}} u_0 R^2 \frac{1}{\alpha}\frac{\partial T}{\partial x}$$

$$h = \frac{24}{11} k/R \times \frac{2}{2}$$

$$h = \frac{48k}{11D}$$

$$\frac{hD}{k} = \frac{48}{11} = 4.364$$

$\boxed{Nu = 4.364}$

T_w = const | Hausen's correlation

$$Nu = 3.66 + \frac{0.0668 \frac{d}{L} Re Pr}{1 + 0.04 \left(\frac{d}{L} Re Pr\right)^{2/3}}$$

flow is Laminar, Heat flux across the wall remains constant

Laminar flow, Tube wall temp constant.

$\boxed{Nu = 3.66}$

Turbulent flow in a tube

$\boxed{Nu = 0.023 Re^{0.8} Pr^{n}}$ Dittus Boelter Eqn

$n = 0.4$ for heating
$n = 0.3$ for cooling

Q.) for laminar flow in a tube for constant heat flux condition nusselt no is approximately equal to 4.36, 3.66, 5.78, 2

Q.) Laminar flow for constant wall temp. 4.36, 3.66, 5.78, 2

Q.) A fluid of thermal conductivity $k = 1.0$ W/mK flows in a fully developed flow with $Re = 1500$ through a pipe of dia 10cm. A H-T coeff (h) are (uniform heat flux & uniform wall temp) boundary conditions are respectively 36.57, 43.64 ; 43.64, 36.57 ; 43.64 for both case ; 36.57 for both the cases.

$Re = 1500 \Rightarrow$ Laminar flow.

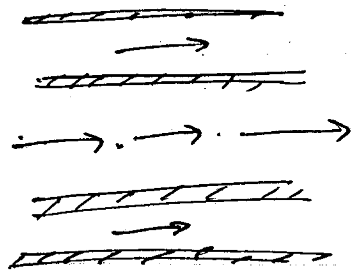
Q.) The nu no is related to Re no. in laminar and turbulent flow resp.

$Re^{-1/2}$, $Re^{0.8}$; $Re^{1/2}$, $Re^{0.8}$; $Re^{-1/2}$, $Re^{-0.8}$; $Re^{1/2}$, $Re^{-0.8}$

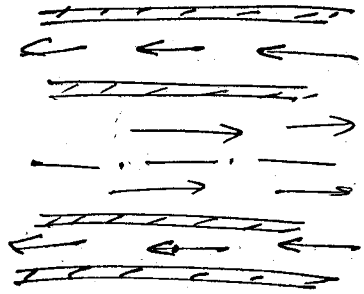
Nov 03, 14

Heat Exchanger

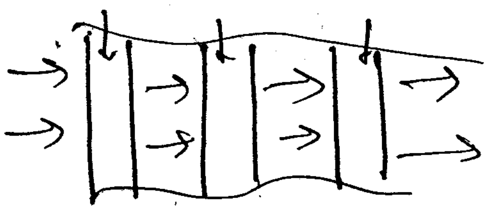
Hot fluid
Cold fluid



Parallel flow HE
&
Co-current HE.

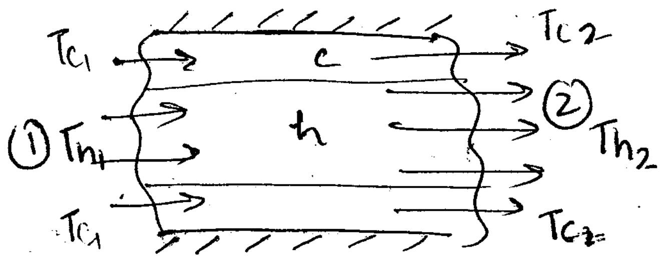


Counter flow HE or
Counter flow HE



Cross flow

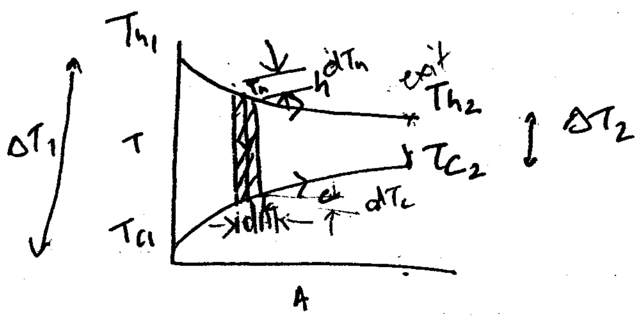
Parallel flow (Pipes & tubes both can be used)



$$Q = m_h C_h (T_{h_1} - T_{h_2}) = m_c C_c (T_{c_2} - T_{c_1})$$

$$Q = UA(\Delta T)_{mean}$$

Overall H.T coeff W/m²K.



$$\delta Q = U dA (T_h - T_c)$$

$$\delta Q = -m_h C_h dT_h$$

$$\delta Q = m_c c_c dT_c$$

$$dT_h = -\frac{\delta Q}{m_h C_h}$$

$$dT_c = \frac{\delta Q}{m_c c_c}$$

$$dT_h - dT_c = \delta Q \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right)$$

$$d(T_h - T_c) = -U dA (T_h - T_c) \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right)$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U dA \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right)$$

$$\int_1^2 \frac{d(T_h - T_c)}{(T_h - T_c)} = \int_1^2 -U dA \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right)$$

$$\ln \frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = -U \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right) A$$

$$\ln \frac{T_{h_1} - T_{c_1}}{T_{h_2} - T_{c_2}} = UA \left[\frac{T_{h_1} - T_{h_2}}{Q} + \frac{T_{c_2} - T_{h_1}}{Q} \right]$$

$$= UA \frac{(T_{h_1} - T_{c_1}) - (T_{h_2} - T_{c_2})}{Q}$$

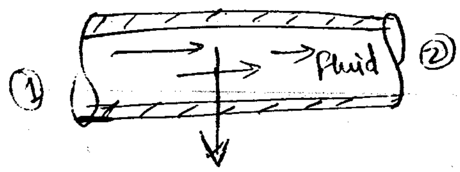
$$\ln \frac{\Delta T_1}{\Delta T_2} = UA \frac{(\Delta T_1 - \Delta T_2)}{Q}$$

$$Q = UA \frac{(\Delta T_1 - \Delta T_2)}{\ln \Delta T_1 / \Delta T_2}$$

$$\boxed{Q = UA (\Delta T)_{mean}}$$

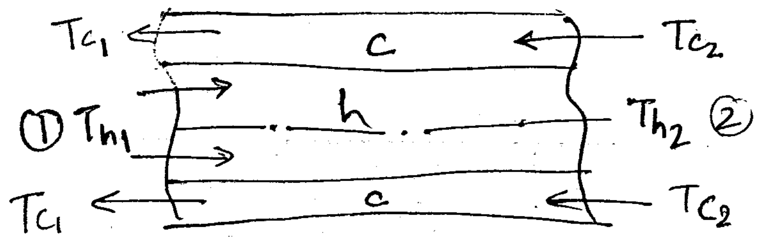
$$(\Delta T)_{mean} = \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1 / \Delta T_2}$$

logarithmic temp diff.
LMTD



Counter flow

(only tubes are tubes)

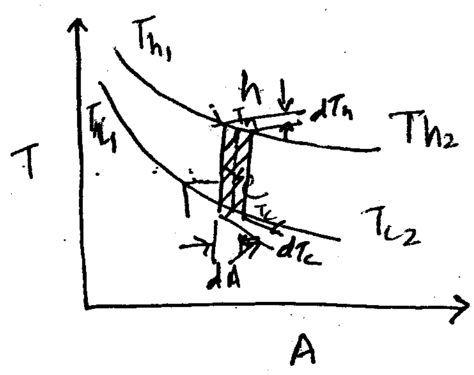


$$Q = m_h C_h (T_{h_1} - T_{h_2}) = m_c C_c (T_{c_1} - T_{c_2})$$

$T_{h_1}, T_{h_2}, T_{c_1}, T_{c_2}$ = const fixed

(LMTD)counter is more than (LMTD) parallel

for counter, check diff is same then that diff is LMTD,



$$\delta Q = -m_h C_h dT_h \,, \quad dT_h = -\frac{\delta Q}{m_h C_h}$$

$$\delta Q = -m_c C_c dT_c \,, \quad dT_c = -\frac{\delta Q}{m_c C_c}$$

$$dT_h - dT_c = -\delta Q \left(\frac{1}{m_h C_h} - \frac{1}{m_c C_c} \right).$$

Subtracting above eqns.

$$d(T_h - T_c) = -U dA (T_h - T_c) \left(\frac{1}{m_h C_h} - \frac{1}{m_c C_c} \right)$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = \int -U dA \left(\frac{1}{m_h C_h} - \frac{1}{m_c C_c} \right)$$

$$\int_1^2 d\frac{(T_h - T_c)}{(T_h - T_c)} = \int_1^2 -U dA \left(\frac{1}{m_h C_h} - \frac{1}{m_c C_c} \right)$$

$$\ln \frac{T_{h_2}-T_{c_2}}{T_{h_1}-T_{c_1}} = -UA\left(\frac{1}{m_h c_h} - \frac{1}{m_c c_c}\right)$$

$$\ln \frac{T_{h_1}-T_{c_1}}{T_{h_2}-T_{c_2}} = UA\left[\frac{T_{h_1}-T_{h_2}}{Q} - \frac{T_{c_1}-T_{c_2}}{Q}\right]$$

$$\ln \frac{T_{h_1}-T_{c_1}}{T_{h_2}-T_{c_2}} = UA\,\frac{(T_{h_1}-T_{c_1})-(T_{h_2}-T_{c_2})}{Q}$$

$$Q = UA\,\frac{\Delta T_1-\Delta T_2}{\ln \Delta T_1/\Delta T_2}$$

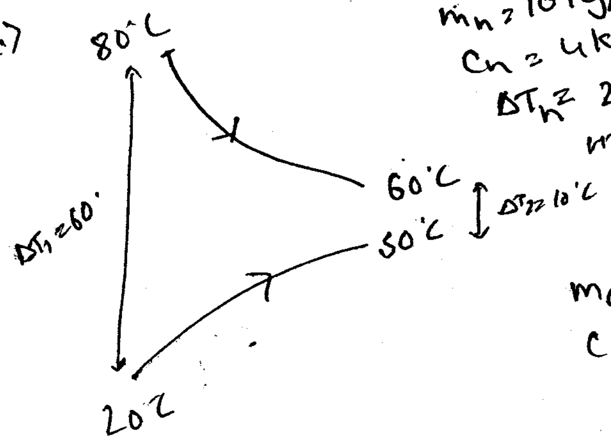
$$= UA\,\frac{\Delta T_2-\Delta T_1}{\ln \Delta T_2/\Delta T_1}$$

exp from LMTD is same in counter or parallel flow.

→ $Q = UA(LMTD)$

$U\uparrow$, $LMTD\uparrow$

Q.7

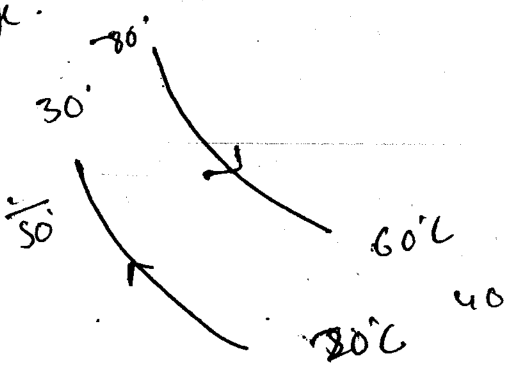


$m_h = 10$ kg/s
$c_h = 4$ kJ/kgK
$\Delta T_h = 20$ K
HT = 2500

$m_c = 15$ kg/s
$c_c = 16/9$ kJ/kgK
$\Delta T_c = 30°C$
$\delta Q = 800$

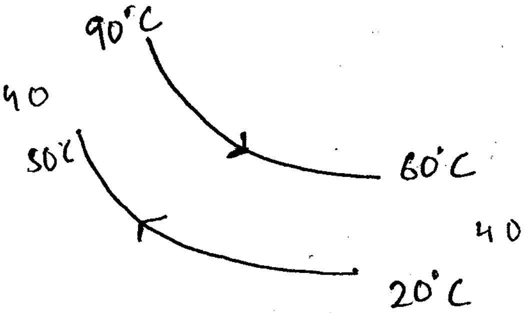
$$\frac{60-10}{\ln 60/10} = 27.9055°C$$

P.f flow is change.



$$= \frac{40-30}{\ln 40/30} = 34.76°C$$

Q>



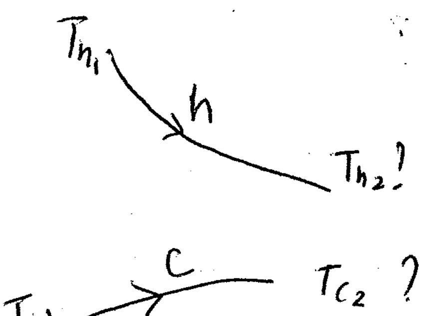
LMTD = ?

$Q = UA(\Delta T)_{mean}$

$$= UA\frac{(\Delta T_1 - \Delta T_2)}{\ln \Delta T_1/\Delta T_2}$$

40°C is mean temp diff.

<u>Effectiveness</u> of <u>the H.E Parallel flow</u>



$$= \frac{\text{Actual heat transfer}}{\text{Maximum heat transfer}}.$$

$$= \frac{m_h c_h (T_{h_1} - T_{h_2}) \text{ or } m_c c_c (T_{c_2} - T_{c_1})}{(mc)_{min}(T_{h_1} - T_{c_1})}$$

Cold fluid is the minm fluid.

$$\epsilon_c = \frac{m_c c_c (T_{c_2} - T_{c_1})}{\cancel{(mc)_{cold}} (T_{h_1} - T_{c_1})}$$

$$\epsilon_h = \frac{m_h c_h (T_{h_1} - T_{h_2})}{(mc)_{hot} (T_{h_1} - T_{c_1})}$$

$$\ln \frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = -UA\left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c}\right)$$

$$= \frac{-UA}{m_c c_c}\left(\frac{m_c c_c}{m_h c_h} + 1\right)$$

Let cold fluid be the minm fluid.

$$= \frac{-UA}{C_{min}}\left(\frac{C_{min}}{C_{max}} + 1\right)$$

Heat capacity = $C = mc$
mass x sp. heat.

U = overall ht. coeff $\frac{\frac{W}{m^2 K} \times m^2 \text{ Area}}{\cancel{kg/s} \times \cancel{J/kg K}}$

$\frac{UA}{C_{min}}$ = No of transfer unit NTU.

$\frac{C_{min}}{C_{max}} = C$

$= -NTU(1+C)$

NTU = N,

$$\boxed{\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = Exp\left[-N(1+C)\right]}$$

$$Q = m_h c_h (T_{h_1} - T_{h_2}) = m_c c_c (T_{c_2} - T_{c_1})$$

$$T_{h_1} - T_{h_2} = C(T_{c_2} - T_{c_1})$$

L.H.S

$$\frac{\left[T_{h_1} - C(T_{c_2} - T_{c_1})\right] - T_{c_2} + T_{c_1} - T_{c_1}}{T_{h_1} - T_{c_1}}$$

$$= \boxed{\frac{(T_{h_1} - T_{c_1}) - (T_{c_2} - T_{c_1}) - C(T_{c_2} - T_{c_1})}{T_{h_1} - T_{c_1}}}$$

$$1 - \epsilon - C\epsilon = \text{Exp}[-N(1+C)]$$

$$1 - \epsilon(1+C) = \text{Exp}[-N(1+C)]$$

$$1 - \text{Exp}[-N(1+C)] = \epsilon(1+C)$$

$$\boxed{\epsilon = \frac{1 - \text{Exp}[-N(1+C)]}{1+C}}$$

$\epsilon_c = \dfrac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}}$

$T_{c_2} = ?$

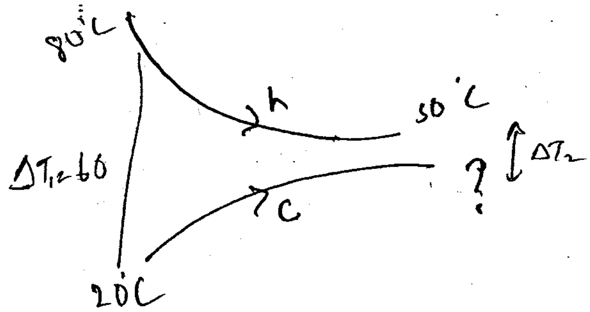
Q.) $$\epsilon = \frac{1 - \text{Exp}[-N(1-C)]}{1 - C\,\text{Exp}[-N(1-C)]}$$

Hot water (0.01 m³/min) tube side cocurrent shell & Tube H.E

80°C → 50°C

cold oil (0.05 m³/min), $\rho = 800$ kg/m³, $C = 2$ KJ/kgK enters at 20°C.

LMTD 32, 37, 45, 50°C



$$m_h C_h (T_{h_1} - T_{h_2}) = m_c C_c (T_{c_2} - T_{c_1})$$

$$\frac{0.01 \times 1000}{60} \times (4.1868 \times 10^3)(80 - 50)$$

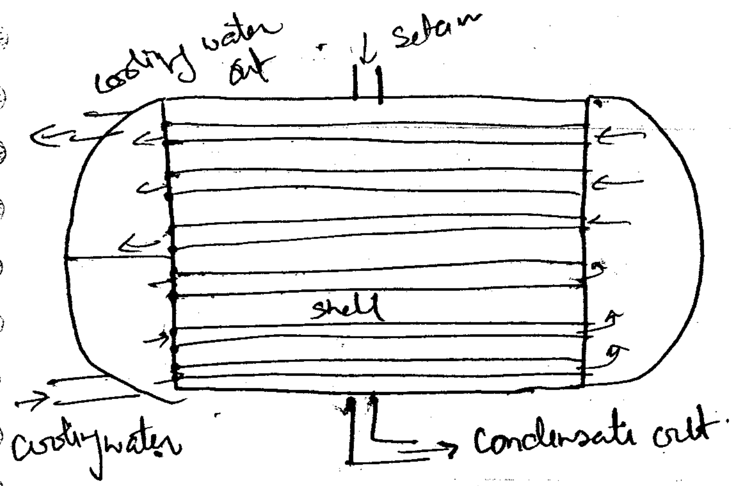
$$= \frac{0.05 \times 800}{60} \times (2 \times 10^3)(T_{c_2} - 20)$$

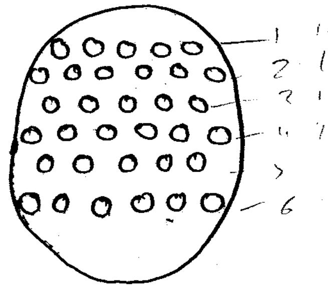
$$T_{c_2} = 35.675\,^\circ C$$

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1 / \Delta T_2}$$

$$= 32\,^\circ C$$

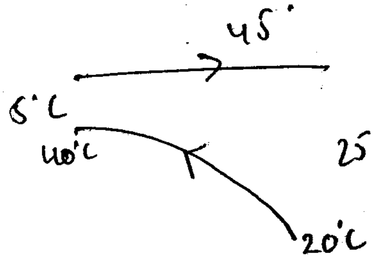
Shell = hot / baffles.
tube = cold.

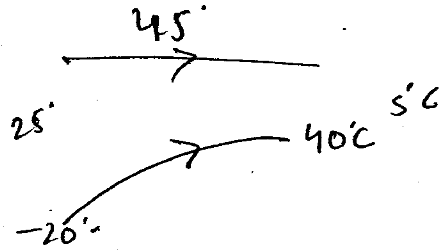




ek sides se jo water enter kra wo dusss ib padunch jati h to wo 1 pass hota h soits 2 pass

Two pass H.E
1-shell

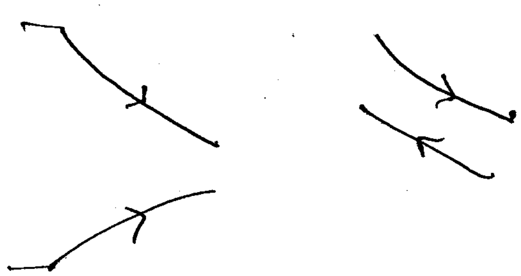




both parallel flow & counter flow is equally good or bad there z no effect.

$$\frac{25-5}{\ln 25/5}$$

$$\frac{25-5}{\ln 25/5}$$

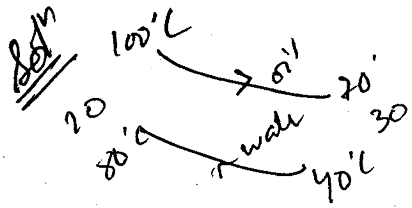


Q) A counter flow double pipe HE, is used to heat water flowing at 1kg/sec from 40°C to 80°C (water). Oil is used for heating the water & its temp changes from 100°C to 70°C. overall H.T coeff is $U = 300\ W/m^2 \cdot °C$. It is replaced by a 1-2 shell & Tube H.E wid counterflow configuration with water flowing in shell & oil in tubes. What is the excess area req. w.r.t double pipe HE. The correction factor for LMTD is 0.5!

H T coeff remains unchanged and the same inlet & outlet conditions are maintained. $C_{pw} = 4180 \ J/kg°C$, $C_{p_{oil}} = 2000 \ J/kg°C$

$\Rightarrow$ 0 , -20.15 , 22.6 , $9.69 \ m^2$

Soln



~~LMTD = 39.7005~~ ~~for Doublepipe~~ Chintiya

~~LMTD = 17.3802~~ ~~for 1-2 shell tube~~.

$LMTD = 24.86°C$

$$Q = UA\,(LMTD)$$

$$1 \times 4180 \times 40 = 300\,A\,24.66$$

$$A = 22.6 \, m^2$$

∵ the new LMTD is ↓ by 0.5 & the new Area ↑ by 2,
So change in area = Anew − A
$= 2A - A$
$= A$
$A = 22.6 m^2$

Now

Q.) A counter flow H.E, the pdt of mass flow rate & sp. heat is same of hot fluid & cold fluid. Determine the effectiveness.

$$\mathcal{E} = \frac{1 - Exp[-N(1-C)]}{1 - C\,Exp[-N(1-C)]}$$

$$C = \frac{C_{min}}{C_{max}} = \frac{m_c C_c}{m_h C_h} = 1$$

$$N = \frac{UA}{C_{min}} = \frac{UA}{m_c C_c}$$

$$= 0/0$$

} so cant be solved.

Let $\{-N(1-C)\} = x$

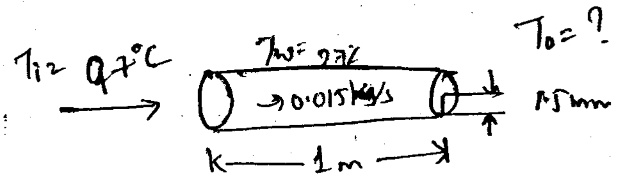
$$\epsilon = \frac{1-e^{x}}{1-Ce^{x}}$$

$$= \frac{1-[1+x+\cancel{\frac{x^2}{2}}+\cdots]}{1-C[1+x+\cdots]}$$

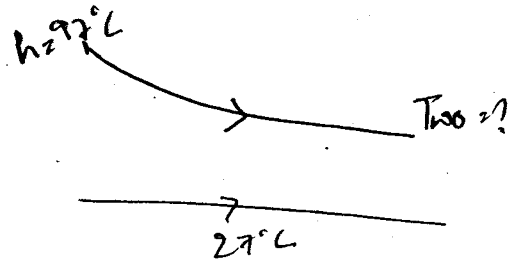
$$= \frac{-x}{1-C-Cx}$$

$$= \frac{N(1-C)}{1-C-CN(1-C)} = \frac{N}{1+CN} = \frac{N}{1+CN}$$

Q) Water enters a thin walled tube, length 1 m, dia 3mm at an inlet temp of 97°C & the mass flow rate of 0.015 kg/s. A tube wall is maintained at a constant temp of 27°C. Given thermo data. $\rho = 1000\ kg/m^3$, $\mu = 489 \times 10^{-6}\ NS/m^2$, $C_p = 4184\ J/kg\,C$. Inside H.T coeff $h_i = 12978\ W/m^2K$. The outlet temp of water in °C is 28°C, 37°C, 62°C, 96°C



$$Q = mC_{pw}(T_{wo} - T_{wi})$$
$$= UA(LMTD)$$



$$LMTD = \frac{(97-27)-(T_{wo}-27)}{\ln\frac{97-27}{T_{wo}-27}} = 37$$

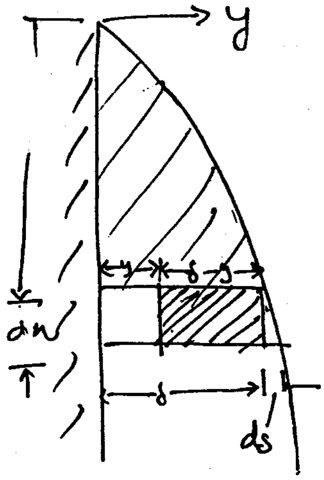
Nov 07, 14

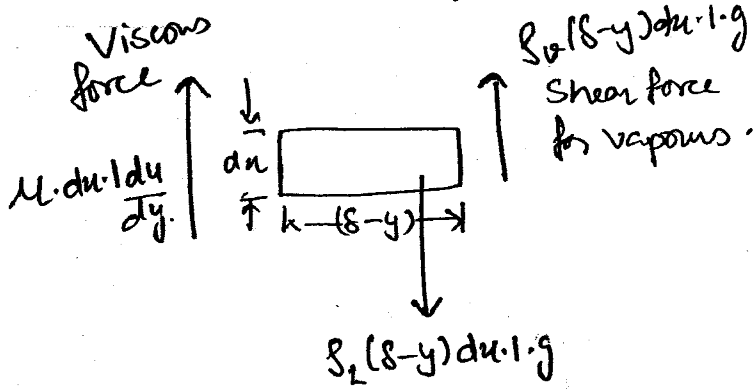
H.T wid change in phase

Filmwise condensation on a Flat Vertical plate for Laminar flow

Nusselt I —

$T_w < T_{sat}$.





gravity force = shear force + friction.

$$\mu \, dx \frac{du}{dy} + \rho_v (\delta - y) dx g = \rho_L (\delta - y) dx \cdot g$$

$$\mu \frac{du}{dy} = (\rho_L - \rho_v) g (\delta - y)$$

$$du = \frac{(\rho_L - \rho_v) g (\delta - y)}{\mu} dy.$$

Integrating

$$u = \frac{(\rho_L - \rho_v) g}{\mu} \left(\delta y - \frac{y^2}{2}\right) + C$$

$y = 0$, $u = 0$, $C = 0$

$$\boxed{u = \frac{(\rho_L - \rho_v) g}{\mu} \left(\delta y - \frac{y^2}{2}\right)}$$

$$m_x = \int_0^\delta \rho_L u\, dy$$

$$= \int_0^\delta \rho_L \frac{(\rho_L - \rho_v)}{\mu} g \left(\delta y - \frac{y^2}{2}\right) dy$$

$$= \frac{\rho_L(\rho_L - \rho_v) g}{\mu} \left[\frac{\delta y^2}{2} - \frac{y^3}{6}\right]_0^\delta$$

$$m_x = \frac{\rho_L(\rho_L - \rho_v) g}{\mu} \cdot \frac{\delta^3}{3}$$

x to $(x + dx)$, $\delta \to (\delta + d\delta)$

<u>Additional Mass</u>

$$= \frac{d}{dx}\left[\frac{\rho_L(\rho_L - \rho_v) g}{\mu} \cdot \frac{\delta^3}{3}\right] dx$$

$$= \frac{d}{d\delta}\left[\frac{\rho_L(\rho_L - \rho_v) g}{\mu} \cdot \frac{\delta^3}{3}\right] \frac{d\delta}{dx} \cdot dx$$

$$= \frac{\rho_L(\rho_L - \rho_v) g}{\mu} \frac{\cancel{3}\delta^2}{\cancel{3}} d\delta$$

Additional ht due to this additional mass :-

$$\frac{\rho_L(\rho_L - \rho_v) g}{\mu} \delta^2 d\delta\, h_{fg} = \frac{-k \cdot dx \cdot 1\,(T_w - T_g)}{\delta}$$

$m_x h_{fg}$

h_{fg} = latent heat of condensation

$$= \frac{K dx\,(T_g - T_w)}{\delta}$$

$$\delta^3 d\delta = \frac{K dx\,(T_g - T_w)\mu}{\rho_L(\rho_L - \rho_v) g\, h_{fg}}.$$

Integrating, $$\frac{\delta^4}{4} = \frac{Kx\mu(T_g - T_w)}{\rho_L(\rho_L - \rho_v) g h_{fg}} + C_1$$

$x = 0,\ \delta = 0,\ \therefore C_1 = 0$

*
$$\boxed{\delta = \left[\frac{4\mu K x (T_g - T_w)}{\rho_L(\rho_L - \rho_v) g h_{fg}}\right]^{1/4}}$$

$$\frac{K\,dx\,\cancel{(T_g - T_w)}}{\delta} = h \cdot dx \cdot 1\ \cancel{(T_g - T_w)}$$

$$\boxed{h_x = \frac{k}{\delta}}$$ x location

$$h_x = \frac{k}{\left[\frac{4\mu K x (T_g - T_w)}{\rho_L(\rho_L - \rho_v) g h_{fg}}\right]^{1/4}}$$

$$h_x = \left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} K^3}{4\mu x (T_g - T_w)}\right]^{1/4}$$

~~Local~~ heat transfer coeff. by convection during condensation. avg.

$$\bar{h} = \frac{1}{L}\int_0^L h_x\,dx$$

$$= \frac{1}{L}\left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{4\mu(T_g - T_w)}\right]^{1/4} \int_0^L \frac{1}{x^{1/4}}\,dx$$

Average h.t. coeff.

$$\bar{h} = \frac{1}{L}\left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{4\mu(T_g - T_w)}\right]^{1/4} \cdot \frac{4}{3} L^{3/4}$$

$$\boxed{\bar{h} = \frac{4}{3}\left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} K^3}{4\mu L(T_g - T_w)}\right]^{1/4}}$$

$$\boxed{\bar{h} = \frac{4}{3} h_e|_{x=L}}$$

$$\bar{h} = 0.943 \left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{\mu L (T_g - T_w)} \right]^{1/4}$$

for horizontal tube

$$\bar{h} = 0.725 \left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{\mu n d (T_g - T_w)} \right]^{1/4}$$ | n = no. of tubes.

⇒ 1.27 cm dia, 1 m length.

$$\bar{h}_{horizontal} = 0.725 \left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{\mu d (T_g - T_w)} \right]^{1/4}$$

$$\bar{h}_{vertical} = 0.943 \left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{\mu L (T_g - T_w)} \right]^{1/4}$$

$$\frac{\bar{h}_{horizontal}}{\bar{h}_{vertical}} = \frac{0.725}{0.943}\left(\frac{L}{d}\right)^{1/4} = \left(\frac{1m}{1.27 \times 10^{-2}}\right)^{1/4}$$

$$\frac{\bar{h}_{horizontal}}{\bar{h}_{vertical}} = 2.2902$$

⇒ horizontal arrangement is better arrangement than vertical.

2005 Q.) In film type condensation of liquid along a vertical tube the thickness of the condensate layer increases towards the bottom. This implies ~~the~~ ~~that~~ the local h.t coeff. ~~Heat along~~

a.) decreases from top to bottom ✓

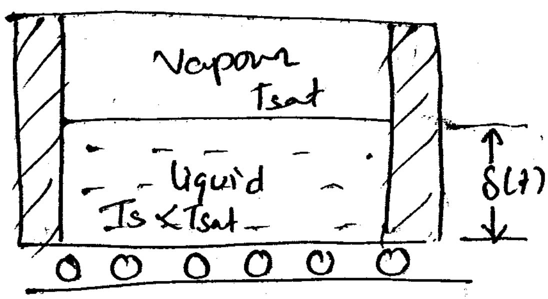
b.) remains constant from top to bottom

c.) first increases & then decreases from top to bottom.

d.) increases from top to bottom

Q.) Consider a liq. stored in a container exposed to its saturated
100? vapour at constant temp T_{sat}. The bottom surface of a container
is maintained at a temp T_s which $T_s < T_{sat}$. While the side walls
are insulated. The thermal conductivity K_L for the liq, λ latent
heat of vaporisation, & ρ_L are known. Assuming a
linear temp distribution in the liquid. Express for the growth
of the liq layer δ. A functn of time is given by.

$$\delta(t) = \left[\frac{4 K_L (T_{sat}-T_s) t}{\rho_L \lambda}\right]^{1/2}$$



$$= \left[\frac{K_L (T_{sat}-T_s) t}{2 \rho_L \lambda}\right]^{1/2}$$

$$= \left[\frac{2 K_L (T_{sat}-T_s) t}{\rho_L \lambda}\right]^{1/2}$$

$$= \left[\frac{K_L (T_{sat}-T_s) t}{\rho_L \lambda}\right]^{1/2}$$

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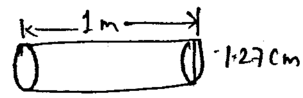
$$\frac{(\rho A \delta)\lambda}{t} = \frac{K A (T_{sat}-T)}{\delta}$$

$$\delta^2 = \frac{K (T_{sat}-T)}{\rho \lambda}$$

$$\delta = \left[\frac{K (T_{sat}-T) t}{\rho \lambda}\right]^{1/2}$$

Q) Steam condenses has atm press when the tube wall is maintained at a temp of 98°C. A tube dia 1.27 cm, $l = 1m$, properties at the film temp of 99°C or given $\rho_L = 960\ kg/cm^3$; $\mu_L = 2.82 \times 10^{-4}\ kg/ms$; $K_L = 0.68\ W/mK$ & $h_{fg} = 2255\ KJ/kg$. The rate of condensation when the tube is kept

1) vertical position
2) Horizontal position.



$$h_{vertical} = 0.943 \left[\frac{\rho_L(\rho_L - \rho_v) g h_{fg} k^3}{\mu L (T_g - T_w)}\right]^{1/4}$$

$$Q = \bar{h} A (T_g - T_w)\ J/s = m h_{fg}$$

$$\frac{kg}{s} \times \frac{J}{kg}$$

$$m = \cdots \frac{kg}{s} \times 3600 = kg/hr$$

$d = 1.27\ cm$

$l = 1\ m$

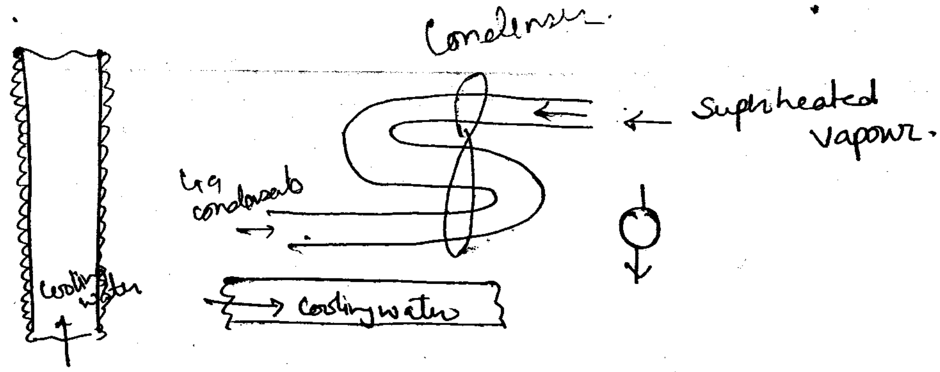
$T_f = 99°C = \frac{98 + 100}{2}$

$\rho_L = 960\ kg/m^3$

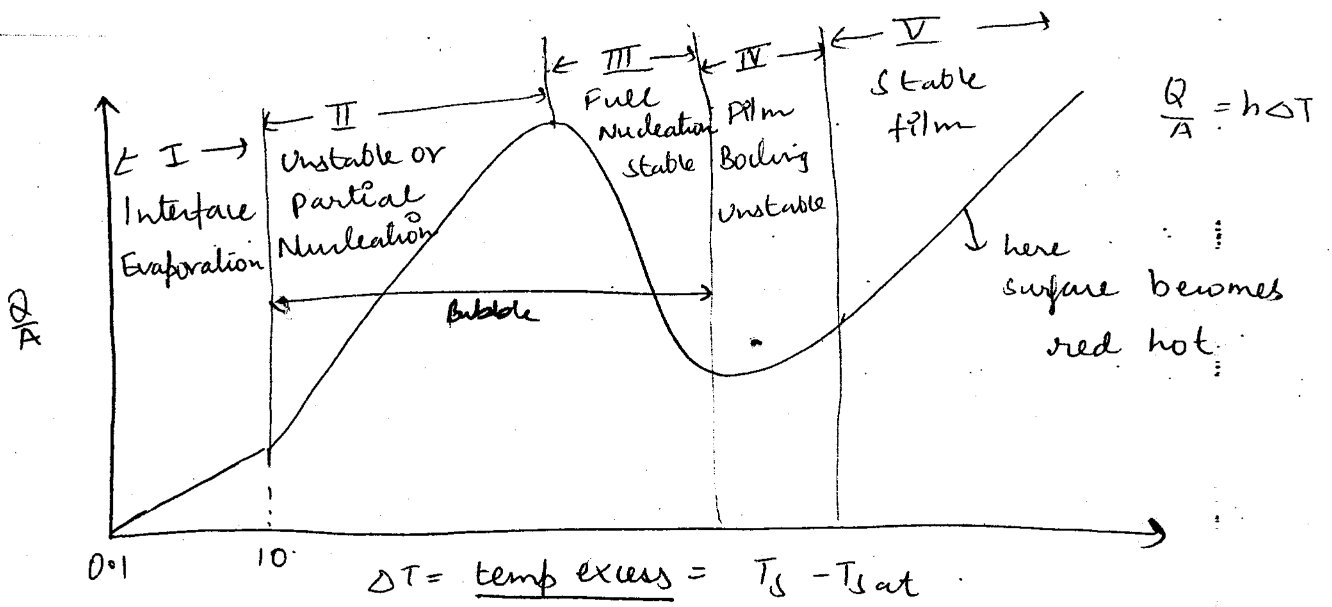
$\mu_L = 2.82 \times 10^{-4}\ kg/ms$

$k_L = 0.68\ W/mK$

$h_{fg} = 2255\ kJ/kg.$



Boiling (change in phase)



Boiling : Change in phase

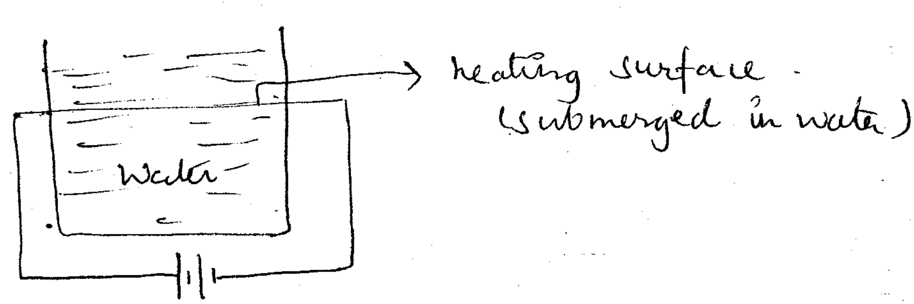
~~When~~ When a surface is exposed to a liquid

$T_s > T_{sat}$ of liquid

If surface is submerged in the liq

ex -

Platinum coil

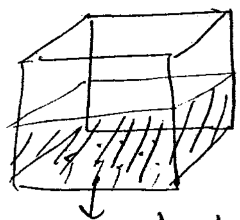


Water temp is T_{sat} corresponding to the Pressure.

Saturated Boiling or Bulk boiling → if temp. of water is T_{sat}

If temp of water < T_{sat} it is called as local boiling or subcooled Boiling.

→ When bubbles has been forming the process is called Nucleation or Nucleate Boiling.



a short film (resistance to HT).

Q- In which regimes HT coefficient are high.

Q: An electrically heated element is submerged in a pool of water at its saturation temperature. As the temp of element increases the max HT coefficient is observed.

(a) In the free convection regime.
(b) B/w a nucleate boiling and partial Nucleate boiling mixed with unstable film boiling regimes.
(c) In incipient Nucleate boiling regime.
(d) In the stable film ~~to~~ boiling regime without significant effects of radiation.

let 30 kg of air
29.5 kg is dry air
0.5 kg is of vapour → moisture water particles.

P_T = 760 mm of Hg.
P_v = 20 mm of Hg = 0.0267 bar $\xrightarrow{\text{corresponding}}$ T_{sat} = 22°C
P_d = 740 mm of Hg.

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Radiation

$$c = \lambda \times f$$

c – velocity of light; λ – wave length; f – frequency.

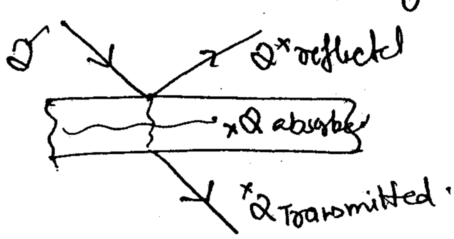
All rays are responsible for h.t, α, β, γ & radio waves are nt responsible for h.t. Sound waves of cell phones are also not responsible for h.t..

Wave length	frequency
longer λ	small
Smaller	~~Light~~ high

we want frequency shud be high..

0.1 to 100 μm
micron

Sun core (wave length) = 0.1 to 4 μm.



$$\text{Reflectivity, } \rho = \frac{Q_{reflected}}{Q}$$

$$\text{Absorptivity, } \alpha = \frac{Q_{absorbed}}{Q}$$

$$\text{Transmissivity, } \tau = \frac{Q_{Transmitted}}{Q}$$

$$\boxed{\rho + \alpha + \tau = 1}$$

Black Body :—

Idealbody ← $\alpha = 1, \; \rho = 0 = \tau$

↓

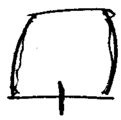
practically not possible

maxn value $\alpha = 0.985$ (Snow)

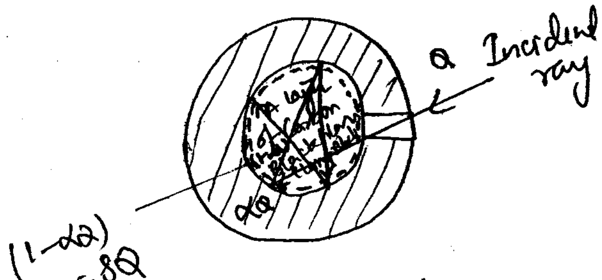
→ Dark Coloured Paint.

→ Lamp Black :—

$\alpha = 0.95$



→ Hohlraum



$\alpha + \rho = 1$

$1 - \alpha = \rho$

Black Body is a non-reflecting & non-transmitting surface.

Gray Body :—

All the incident radiation is not fully absorbed at a given temp. and is independant of the wave length.

Black Body depends on the nature of surface, structure, λ, T.

Gray Body is also an Ideal body.

Opaque Body :-

$\tau = 0, \quad \alpha + \rho = 1$

Transparent Body :- Diathermaneous

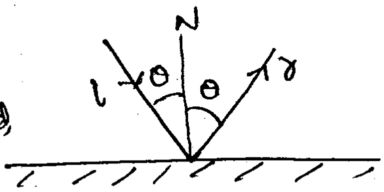
$\tau = 1, \quad \alpha = 0 = \rho$

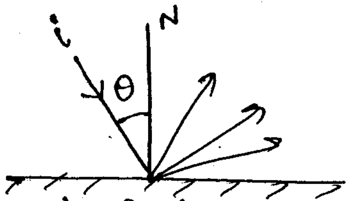
Specular Body :- (White body)

$\alpha = 0, \quad \tau = 0$

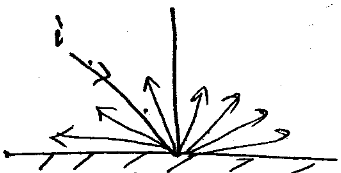
$\rho = 1.$

Highly Polished smooth





Absolute White Body



Actual surface.

Real white body.

E_b = Energy emitted by black body at a given temp for all the wave length per unit area in all the directions.

$(E_\lambda)_b$ = Monochromatic λ & $(\lambda + d\lambda)$

Emissive power of a body = E/E_b

Plancks Law :-

$$(E_\lambda)_b = 2\pi c^2 h \frac{\lambda^{-5}}{\mathrm{Exp}\left[\frac{ch}{\lambda kT}\right]-1}$$

c = velocity of light 2.998×10^{8} m/s

h = Plancks Constant. 6.6236×10^{-34} JS

k = Boltzman's Constant 13.802×10^{-24} J/K

Below 50°K, effect of radiation is neglected.

for shorter wavelength $\frac{ch}{\lambda kT} = \frac{C_2}{\lambda T}$

$$\frac{ch}{k} = \text{Constant} = C_2$$

$$2\pi c^2 h = \text{constant} = C_1$$

$\frac{C_2}{\lambda T}$ is quite high.

In denominator, 1 may be neglected.

$$E_\lambda = \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T}}$$

Wein's Law.

for longer wavelengths $\frac{C_2}{\lambda T}$ become quite small.

$$E_\lambda = \frac{C_1\lambda^{-5}}{\frac{C_2}{\lambda T}-1} = \frac{C_1\lambda^{5}}{e^{C_2/\lambda T}-1}$$

$$e^x = 1+x+\boxed{\cancel{\frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3}} + \dots}$$

$$E_\lambda = \frac{C_1 \lambda^{-5}}{\frac{C_2}{\lambda T}}$$

$$E_\lambda = \frac{C_1 T}{C_2 \lambda^4}$$

Jean & Rayleigh's law

Stefan Boltzman's Law :—

Planck's Law $\quad (E_\lambda)_b = \int_0^\infty 2\pi c^2 h \frac{\lambda^{-5}}{e^{\frac{C_2}{\lambda k T}} - 1} d\lambda$

Let $\frac{C_2}{\lambda T} = x \qquad = \int_0^\infty C_1 \frac{\lambda^{-5}}{e^{\frac{C_2}{\lambda T}} - 1} d\lambda$

$$\lambda = \frac{C_2}{xT}$$

$$d\lambda = -\frac{C_2}{x^2 T} dx$$

$\lambda = 0,\ x = \infty$
$\lambda = \infty,\ x = 0$

$$= \int_\infty^0 \frac{C_1}{\left(\frac{C_2}{xT}\right)^5} \cdot \frac{\left(-\frac{C_2}{x^2 T}\right) dx}{e^x - 1}$$

$$= \int_0^\infty \frac{C_1}{C_2^4} \cdot T^4 \frac{x^3}{e^x - 1} dx$$

$$= \frac{C_1}{C_2^4} T^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

$\lfloor 3 \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right]$

$\frac{\pi^4}{90}$.

$$\frac{C_1}{C_2^4} T^4 \frac{\pi^4}{15} = 5.67 \times 10^{-8} T^4$$

$5.67 \times 10^{-8} = \sigma$ = Stefan Boltzman Constant.

$$\boxed{(E_\lambda)_b = \sigma T^4}$$

for optimised heat transfer from a black body what should λ at a given temp.

$$\frac{d}{d\lambda}\left[(E_\lambda)_b\right] = 0$$

$$\& \quad \frac{d^2}{d\lambda^2}(E_\lambda)_b = -ve$$

$\{ (\lambda T)_{max} = \ldots$

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$$(E_\lambda)_b = 2\pi c^2 h \frac{\lambda^{-5}}{e^{\frac{ch}{\lambda kT}} - 1}$$

$$= c_1 \frac{\lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1}$$

To determine the maxm value of λ, so that $(E_\lambda)_b$ is maxm,

$$\frac{d}{d\lambda}\left[(E_\lambda)_b\right] = \frac{d}{d\lambda}\left[c_1 \frac{\lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1}\right]$$

$$= \frac{\left(e^{\frac{c_2}{\lambda T}} - 1\right)\left(-5c_1\lambda^{-6}\right) - c_1\lambda^{-5}\left(-\frac{c_2}{\lambda^2 T}\right)e^{\frac{c_2}{\lambda T}}}{\left(e^{\frac{c_2}{\lambda T}} - 1\right)^{2}} = 0$$

$$\left(e^{\frac{c_2}{\lambda T}} - 1\right)\left(-5c_1\lambda^{-6}\right) + c_1\lambda^{-5}\frac{c_2}{\lambda^2 T} \cdot e^{\frac{c_2}{\lambda T}} = 0$$

Divide by $e^{\frac{c_2}{\lambda T}} \cdot c_1\lambda^{-6}$

$$-5 + \frac{5}{e^{\frac{c_2}{\lambda T}}} + \frac{c_2}{\lambda T} = 0$$

$$\frac{1}{5}\cdot\frac{C_2}{\lambda T} + e^{-C_2/\lambda T} - 1 = 0$$

Let $\frac{C_2}{\lambda T} = x$, $\frac{x}{5} + e^{-x} - 1 = 0$

If $x = 5$, Error $= 0.0067379$

$x = 4.97$, " $= 0.000943$

⋮

$x = 4.965$, " $= -0.000022$.

$$\frac{C_2}{\lambda T} = 4.965$$

$$\lambda T = \frac{C_2}{4.965} = \frac{1.438\times10^{-2}}{4.965} = \cancel{1.438\times10^{-2}}$$

$C_2 = \frac{ch}{k}$

$$= \frac{2.998\times10^{8}\times6.6236\times10^{-34}}{13.807\times10^{-24}} = 0.2897\times10^{-2}\ mk$$

$= \cancel{1.438\times10^{-2}}$ $= 2.897\times10^{-6}\ mk$

$= 1.438\times10^{-2}$

$= 2897\ \mu mk$.

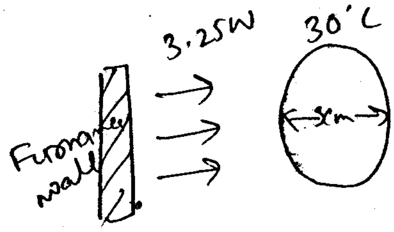
$\lambda_{maxm} = 0.5\ \mu m$

$(\lambda T)_{maxm} = 2897\ \mu mk$

$$T = \frac{2897}{0.5} = 5794\ k$$

$$Q = \sigma T^4 = 5.67\times10^{-8}\times(5794)^4$$
$= 6.3899\times10^{7}$
$= 63.9\times10^{6}\ W$
$= 63.9\ MW$

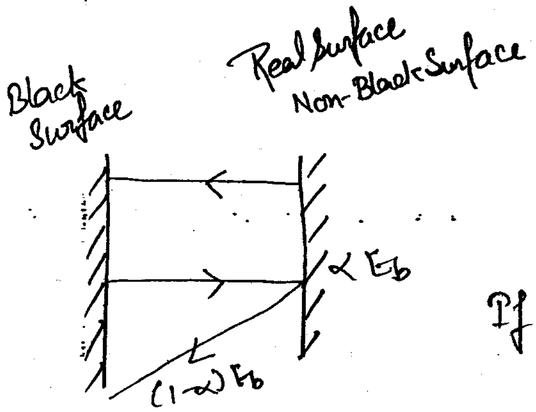
Q.> A thin metal plate of 5 cm dia is suspended in air at a temp of 25°C Radiation energy of 3.25 W falls from a furnance wall on one the face of the plate. The h.t coeff of the plate is estimated as 93 W/m² K. The plate attains a steady temp of 30°C. Find the reflectivity of the plate



$h = 93 W/m^2K$

Reflectivity of the plate = ?

Kirchoff's Law



Net Heat Exchange

$(E - \alpha E_b)$

If both the surfaces are at the same temp.

$E - \alpha E_b = 0$

$E = \alpha E_b$

$\frac{E}{\alpha} = E_b$

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \cdots = E_b$$

$$\underbrace{\frac{E_1}{E_b}}_{\alpha_1} = \underbrace{\frac{E_2}{E_b}}_{\alpha_2} = \underbrace{\frac{E_3}{E_b}}_{\alpha_3} = \frac{E_b}{E_b} = 1$$

$\boxed{\alpha = \epsilon}$ The same temp.

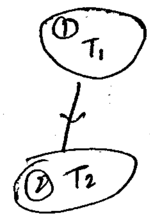
Kirchoff's Law

Orientation factor or (View factor) or (Shape factor) :-

Amt of energy emitted from surface (1) that reaches surface (2)

$$Q_{12} = A_1 F_{12}\, \sigma (T_1^4)$$

$$Q_{21} = A_2 F_{21}\, \sigma (T_2^4)$$



Net Heat Exchange $= A_1 F_{12}\, \sigma T_1^4 - A_2 F_{21}\, \sigma T_2^4$.

↓

Energy If the two bodies are at the same temp

Net Heat Exchange $= 0$, $T_1 = T_2 = T$ say.

$$\sigma T^4 (A_1 F_{12} - A_2 F_{21}) = 0$$

$$A_1 F_{12} - A_2 F_{21} = 0 \qquad \sigma, T \neq 0.$$

$$\boxed{A_1 F_{12} = A_2 F_{21}}$$ Reciprocity Theorem

Q.) The fraction of radiation leaving the surface Area A_2 that strikes itself is -

$\frac{1}{4}, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{3}{4}$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{22} = ?$$

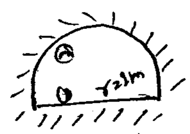
$$F_{21} + F_{22} = 1$$

$$F_{22} = 1 - A_1/A_2$$

$$= 1 - \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{3}{4}$$

Q.) Consider 2 black bodies with surfaces S_1 & S_2. Area $1m^2$, $4m^2$. They exchange heat only by radiation. 40% of the energy emitted by S_1, is received by S_2. The fraction of energy emitted by S_2 i.e. is recived by S_1 is 0.05, 0.1, 0.4, 0.6

Q.) A well insulated hemispherical surfaces radius = 1m is shown below



The self new factor. of radiation of the cone surface ② is

$\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

Soln

Free Convection

Flow is due to density diff.

$h.$

$\rho, C_p, \mu, k, L, \beta, \Delta T, g$

$$\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p$$

Buoyant force

$$= (\rho_1 - \rho_2) g$$

$$\rho_1 =$$

$$\rho_2 = \rho_1(1+\beta\Delta T)$$

$$\rho_1 - \rho_2 = \rho_1 - \rho_1(1+\beta\Delta T)$$

$$= -\rho_1 \beta \Delta T$$

$$\rho_2 \beta \Delta T$$

$$h = f[\rho, C_p, \mu, k, L, (\beta\Delta T g)]$$

$$h = C[\rho^a, C_p^b, \mu^c, k^d, L^e (\beta \Delta T g)^f]$$

$$\frac{J}{sm^2K} = C\left[\left(\frac{kg}{m^3}\right)^a \left(\frac{J}{kgK}\right)^b \left(\frac{kg}{ms}\right)^c \left(\frac{J}{smK}\right)^d (m)^e\right.$$

$$\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p = \frac{1}{T}$$

$J, \quad 1 = b+d$

$s, \quad -1 = -c-d-2f$

$m, \quad -2 = -3a-c-d+e+f$

$K, \quad -1 = -b-d$

$Kg, \quad 0 = a-b+c$

$$\boxed{d = 1-b}$$

$$0 = a - 2f$$

$$\boxed{a = 2f}$$

$$\boxed{c = b - a = b - 2f}$$

$$\boxed{e = 3f - 1}$$

$$h = C\left[\rho^{2f} \cdot C_p^{b} \mu^{b-2f} k^{1-b} L^{3f-1} (\beta \Delta T g)^{f}\right]$$

$$= C\left[\left(\frac{\rho^2 L^3 \beta \Delta T g}{\mu^2}\right)^{f} \left(\frac{C_p \mu}{k}\right)^{b} \frac{k}{L}\right]$$

$$\frac{hL}{k} = C\left[\left(\frac{\rho^2 L^3 \beta \Delta T g}{\mu^2}\right)^{f} \left(\frac{\mu C_p}{k}\right)^{b}\right]$$

$$Nu = C\left[Gr^{f} Pr^{b}\right] = C\left[Gr^{m} Pr^{n}\right]$$

$$\left(\frac{kg}{m^3}\right)^2 m^3 \frac{m}{s^2} \times \frac{1}{\left(\frac{kg}{ms}\right)^2}$$

$$Nu = f(Gr, Pr)$$

$$m = n$$

$$Nu = C\,(Gr \cdot Pr)^{n}$$

$Gr \cdot Pr =$ Rayleigh No. (Ra)

$$Nu = C\,Ra^{m}$$

		Plates	Tubes
Laminar flow	$Gr Pr < 10^{7}$	$Nu = 0.54\,Ra^{0.25}$	$Nu = 0.53\,Ra^{0.25}$
Turbulent	$Gr Pr > 10^{9}$	$Nu = 0.14\,Ra^{0.33}$	$Nu = 0.13\,Ra^{0.33}$

Condensation

It is defined as phys. process by which a gas or vap. changes into the liq. If the temp. of an object or surface falls below, what is known as dew pt-temp. for a given relative humidity of surrounding air.

Dew pt. varies accordinly to the amt of water present in atm, which is known as humidity. Thats y in humid condition, condensation occurs at higher temp.

Evaporation

It is an operation used to condensate a sol^n of a non-volatile solute & a volatile solvent (water generally). A portion of solvent is vaporised to produce a concentrated sol^n

Capacity

It can be defined as the no. of kg of water (solvent) vapourized or evaporated per hour.

Economy

Defined as a no. of kg of solvent evaporated per kg of steam fed to the evaporated. i.e also known as steam economy.