GATE MATHEMATICS

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Thanks to all of You :

horizontal rows = m
vertical columns = n

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MATRICES

A rectangular arrangement of m×n numbers arranged in the form of row & columns is known as matrix.

number - can be anything, any number, any variables.

→ The total no. of element in a matrix are mn.

$$\begin{bmatrix} 1 & 2 & 3 \\ \ln x & \sin x & 2 \\ x^2 & \ln x^3 & 5 \end{bmatrix}_{m \times n}$$

m×n = order; m: no. of rows, n: no. of columns

Square Matrix

The matrix in which the no. of rows & columns are equal. $\boxed{m=n}$, if $\boxed{m \neq n}$ is known as Rectangular or non-sq matrix.

Principle sub-Matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 1 & 4 \end{bmatrix}$$

✓ $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ ×

× $\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$ $\begin{bmatrix} 3 & 4 \\ 1 & 4 \end{bmatrix}$ ✓

The diagonal elements of the sub-matrix when matches with the diagonal elements of original matrix then the sub matrix is known as principle sub-matrix.

Diagonal Matrix (square matrix)

The matrix in which all the non-diagonal elements are zero is known as diagonal matrix. The diagonal elements may or may not be zero.

✓ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ✓

Properties:-

diag $[x \; y \; z]$ (representation)

i) $diag[x\ y\ z] + diag[a\ b\ c] = diag[x+a\ \ y+b\ \ z+c]$

ii) $diag[x\ y\ z] \times diag[a\ b\ c] = diag[xa\ \ yb\ \ zc]$

iii) $diag[x\ y\ z]^n = diag[x^n y^n z^n]$

ex $diag[1\ 2\ 3]^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 27 \end{bmatrix}$

iv) $|diag[x\ y\ z]| = xyz$

v) $diag[x\ y\ z]^{-1} = diag[1/x\ \ 1/y\ \ 1/z]$

vi) $diag[x\ y\ z]$

Eigen Values $= x, y, z$

vii) $k[diag[x\ y\ z]] = diag[kx\ \ ky\ \ kz]$

Scalar Matrix

The diagonal matrix in which all the diagonal elements are equal/same is known as the scalar matrix.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Identity Matrix

The diagonal matrix in which all the diagonal elements are unity/'1'

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

⇒ it is also scalar matrix.

⇒ eigen value $= (1, 1, 1)$

Properties

⇒ $|I| = 1$ ⇒ $I^{-1} = I$

⇒ $I^n = I$ ⇒ $I' = I$

Null Matrix :-

The matrix in which all the elements are zero. It may or may not be square.

Upper triangular Matrix:-
The matrix in which all the lower of diagonal elements are zero is known as upper triangular matrix.

$$A = \begin{bmatrix} 1 & 7 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 5 \end{bmatrix}$$

Lower triangular Matrix:-
The matrix in which all the upper of diagonal elements are zero is known as lower triangular matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 7 & 3 & 0 \\ 8 & 9 & 5 \end{bmatrix}$$

☆ Eigen Values of U.T & L.T Matrix are the diagonal elements.

Idempotent Matrix:- The matrix whose square is equal to the original matrix $\boxed{A^2 = A}$ ex→ $\boxed{I^2 = I}$

Involutary Matrix:- The matrix whose square is equal to identity $\boxed{A^2 = I}$

Nilpotent Matrix:-
The nilpotent matrix of order x is defined that matrix such that $\boxed{A^x = 0}$ but $\boxed{A^{x-1} \neq 0}$

Trace of Matrix:-
The sum of the diagonal elements is knew as the trace of matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 8 & 9 & 10 \end{bmatrix}$$

$$tr(A) = 1+5+10 = 16$$

$$tr[A+B] = tr A + tr B$$

$$tr[KA] = K tr A$$

☆ $$tr[AB] = tr[BA]$$

Transpose of Matrix:-

When the rows & columns are interchanged

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^T = A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

It is defined for square as well as non-square

$$\boxed{A_{m\times n} = A^T_{n\times m}}$$

Conjugate of Matrix:-

$$A = \begin{bmatrix} i & 2-i \\ 0 & 2+i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} -i & 2+i \\ 0 & 2-i \end{bmatrix}$$

Changing (i) to (-i) in all the elements of matrix.

$\Rightarrow$ $\boxed{\overline{(\bar{A})} = A}$

$\Rightarrow$ $\boxed{\bar{A} = A}$ A must be real

$\Rightarrow$ $\boxed{\bar{A} = -A}$ A must be imaginary

} These conditions are not satisfied for complex number 2+i

$\Rightarrow$ $\boxed{\overline{(kA)} = k[\bar{A}]}$

Transposed Conjugate of a matrix:-

The transpose of the conjugated matrix is known as the transpose conjugate of the matrix.

$$A = \begin{bmatrix} 2+i & 3 \\ 5 & 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2-i & 3 \\ 5 & 0 \end{bmatrix}$$

$$A^{\theta} = (\bar{A})' = \begin{bmatrix} 2-i & 5 \\ 3 & 0 \end{bmatrix}$$

Classification

Matrix
- Real (Transpose)
  - Symmetric
  - Skew-Symmetric
  - Orthogonal
- Complex (Transposed conjugate)
  - Hermition
  - Skew-hermition
  - Unitary

Symmetric:- The matrix whose transpose is equal to the original matrix. $\boxed{A^T = A}$, $A = A'$

for a symmetric matrix upper ~~&~~ lower of diagonal elements must be same.

$$\begin{bmatrix} 1 & 4 & 5 \\ 6 & 3 & 6 \\ 5 & 4 & 3 \end{bmatrix}$$

$\Rightarrow \boxed{\frac{A+A'}{2}}$ for any matrix A

Skew-Symmetric :- The matrix whose transpose is equal to the negative of symmetric matrix.

$\boxed{A' = -A}$, $\boxed{a_{11} = a_{22} = a_{33} = 0}$

For a skew-symmetric matrix, the digonal element must be zero. The upper of lower of diagonal must of opposite bt equal in magnitude.

$\Rightarrow \boxed{\frac{A-A'}{2}}$ for any matrix A

Orthogonal Matrix :- The matrix whose transpose is equal to its inverse.

$$\boxed{A^T = A^{-1}}$$

$$AA^{-1} = I$$

$$\boxed{AA^T = I}$$

$$|A|\,|A^T| = |I|$$

$$|A|\,|A^T| = 1$$

$$\boxed{|A^T| = |A|}$$

for orthogonal matrix $|A| = \pm 1$

$$|A^2| = 1$$

$$\boxed{|A| = \pm 1}$$

In conceptual questions the ± 1 is used but in numerical type 1 used

Hermition Matrix :-
The matrix whose transpose conjugate is equal to the original matrix

$$\boxed{A^\theta = A}$$

Skew-Hermition Matrix:-
The matrix whose transposed conjugate is equal to the negative of original matrix.

$$\boxed{A^\theta = -A}$$

Unitary matrix:-
The matrix whose transpose conjugate is equal to the inverse of matrix.

$$\boxed{A^\theta = A^{-1}}$$

Q.) Consider a matrix having order $X_{4\times3}$, $Y_{4\times3}$ & $P_{2\times3}$. Then

$$[P(X^TY)^{-1}P^T]^T$$

For multiplication
$A_{m\times n}$ $B_{p\times q}$
$\boxed{n = p}$
$AB_{m\times q}$

Ans $\Rightarrow$ 2×2

Q.) Real matrix $A_{3\times1}$, $B_{3\times3}$, $C_{3\times5}$, $D_{5\times3}$, $E_{5\times5}$, $F_{5\times1}$
B & E are symmetric.
Following statements are made w.r.t these matrices.
i.) Matrix product $[F^T, C^T BCF]$ is a scalar.
ii.) " " $[D^TFD]$ is always symmetric.
with the reference to above statement which of the following opth is correct.

1.) ✓ State I is true II is false
ii.) Statement I is false II is true
iii) both are true
iv) both are false.

Q.) For an orthogonal matrix, $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$. Find the value of $[AA^T]^{-1}$

$= I$ Ans. | for orthogonal $AA^T = I \Rightarrow [AA^T]^{-1} \Rightarrow [I]^{-1} = \boxed{I}$

Q.) For a matrix m, given as $\begin{bmatrix} 3/5, & 4/5 \\ x & 3/5 \end{bmatrix}$. The value of x if the given matrix is orthogonal is.

$$A^T = A^{-1}$$

$$|AA^T| = I$$

$$|A| = 1$$

$$\frac{9}{25} - \frac{4x}{5} = 1$$

$$x = -4/5$$

Determinants:-

Let a matrix defined as $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$|A| = a_{11}a_{22} - a_{12}a_{21}$$

Determinant is a no. which gives us the measure of central tendency of the matrix.

Minors :-

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 8 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

The minor of any element can be find out by deleting the row and the column in which the element lies and then solving the determinants for the rest of the element.

$$M_1 = \begin{vmatrix} 8 & 9 \\ 1 & 1 \end{vmatrix}$$
$$= 8 - 9 = -1$$

☆ for a matrix of order $n \times n$ the minor of any element has the order of $(n-1) \times (n-1)$

Co-factor:-

Co-factor is a minor with a proper sign:

$$A_{ij} = (-1)^{i+j}(M_{ij})$$

$$M_{3} = \begin{vmatrix} 7 & 9 \\ 1 & 1 \end{vmatrix}$$

$$= 7 - 9 = -2$$

$$A_{12} = (-1)^{1+2}(-2)$$

$$= 2$$

Adjoint of a Matrix:-

A transpose of a co-factor matrix is known as the adjoint of a matrix.

$$A = \begin{vmatrix} 2 & 4 & 5 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\text{cof }(A) = \begin{vmatrix} +(1) & -(2) & +(1) \\ -(-1) & +(-3) & -(-2) \\ +(-6) & -(-13) & +(-8) \end{vmatrix}^{T}$$

$$\text{adj of }(A) = \begin{vmatrix} 1 & 1 & -6 \\ -2 & -3 & +13 \\ 1 & 2 & -8 \end{vmatrix}$$

Inverse of a Matrix:-

The ratio of adjoint of a matrix to its determinant.

$$\boxed{A^{-1} = \frac{\text{adj } A}{|A|}}$$

→ A^{-1} will exists only for sq. matrix as $|A|$ is possible only for sq matrix.

$$\boxed{AA^{-1} = I}$$

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$2(1) + 4(-2) + 5(1)$$

* If determinant of A is zero, then inverse of the matrix is not define.

* If the determinant of A is zero, then the matrix is known as Singular matrix.

*For an $\underbrace{\text{invertible matrix}}_{\text{whose inverse is possible}}$, the determinant must be non-zero.

Properties of determinant :- (3×3)

i) If all the rows & the columns of a matrix are changed then the value of determinant remains the same.

$$\boxed{|A^T| = |A|}$$

ii) If two rows or two columns of a matrix are changed, the value of determinants become −ve.

$$x = \begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{vmatrix}$$

$$-x = \begin{vmatrix} 3 & 2 & 4 \\ 5 & 4 & 6 \\ 1 & 1 & 1 \end{vmatrix}$$ changed

iii) If any two rows or two columns of a determinant are same then the value of the determinant is zero.

iv) If any row or column of a ~~determinant~~ matrix is zero, then the value of the determinant is also zero.

v) If any row or column of a matrix is expressed as sum of two functions, then the determinant can also be expressed as the sum of two determinants.

$$\begin{bmatrix} 3+x & 1+a & 4 \\ 4+y & 2+b & 5 \\ 6+z & 3+c & 6 \end{bmatrix}$$ breaking one at a tym.

$$= \begin{vmatrix} 3 & 1+a & 4 \\ 4 & 2+b & 5 \\ 6 & 3-c & 6 \end{vmatrix} + \begin{vmatrix} x & 1+a & 4 \\ y & 2+b & 5 \\ z & 3+c & 6 \end{vmatrix}$$

vi) If we add or substract k times any row or column to any other row or column, then the value of the determinants remain the same.

$R_1 \longrightarrow R_1 \pm k R_3$ $\qquad |A| =$ same

$C_2 \longrightarrow C_2 \pm k C_1$

vi) If we multiply k to the matrix of order n, then the resulting determinant will be equals to the

$$|kA_{n\times n}| = k^n|A|$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 4 & 5 & 1 \end{vmatrix}$$

$1(-13) - 1(-10) + 1(2)$

$= -13 + 10 + 2$

$= -1$

If we multiply $2 \times A_{3\times3}$

$|A| = -1 \times 2^3$

Matrix = if we multiply any no., multiplication will be done to each & every element.

Determinant = if we multiply any no, we should mention the row & column.

$|A| = 3$

$A_{3\times3}$

$k = 3$

$|kA| = ?$

$3^3 \times 3 = 81$.

Q.) If Matrix A is given as $\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$. Find its inverse

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \frac{1}{6-0}\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} \Rightarrow \begin{vmatrix} 1/2 & 1/6 \\ 0 & 1/3 \end{vmatrix}$$

Q.) $A = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix} \Rightarrow \frac{1}{7-10}\begin{vmatrix} 7 & -2 \\ -5 & 1 \end{vmatrix}$$

$$A^{-1} \Rightarrow \begin{bmatrix} -7/3 & 2/3 \\ 5/3 & -1/3 \end{bmatrix}$$

$$\Rightarrow \quad (AB)^T = B^T A^T$$
$$\Rightarrow \quad (AB)^{-1} = B^{-1} A^{-1}$$

Q.) Let A, B, C, D be $n \times n$ matrix, each with non-zero determinant if $ABCD = I$, then value of $B^{-1} = ?$

$$ABCD = I$$

Pre-multiplying A^{-1} on the B.S.

$$\underbrace{A^{-1}A}BCD = \underbrace{A^{-1}I}$$
$$IBCD = A^{-1}$$
$$BCD = A^{-1}$$

Post-multiplying D^{-1} on the B.S

$$BC\underbrace{DD^{-1}} = A^{-1}D^{-1}$$
$$BC = A^{-1}D^{-1}$$
$$B\underline{CC^{-1}} = A^{-1}D^{-1}C^{-1}$$
$$B = A^{-1}D^{-1}C^{-1}$$
$$(B)^{-1} = (A^{-1}D^{-1}C^{-1})^{-1}$$
$$B^{-1} = (C^{-1})^{-1}(D^{-1})^{-1}(A^{-1})^{-1}$$
$$B^{-1} = CDA.$$

Q.) If A is given as

$$A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$$

$$a + b = ?$$

$$AA^{-1} = I$$

$$\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2a - 0.b \\ 0 & 3b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2a - 0.b = 0 \qquad b = 1/3$$
$$3b = 1$$

Q.) If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$ then top row R^{-1} is

$A_{11} = 5$

$A_{21} = -(3) = -3$

$A_{31} = 1$

$|A| = 1\times 5 + 2\times(-3) + 2\times(1)$

$= 1$

Ans. $= [5 \; -3 \; 1]$

Q.) for which value of x the given matrix is singular.

$$R = \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

$$8(0-12) + x(2\times 12 - 4\times 0) + 0 = 0$$

$$-96 + 24x = 0$$

$$96 = 24x$$

$$x = 4$$

Rank ÷ defined as no. of linearly independent ~~or~~ non-zero rows & columns.

06 July

For a matrix $(n\times n)$. $\boxed{r(A) \le n}$

Case I If the determinant is non-zero rank of A will be n

Case II if the determinant of A is zero. rank of A will be less than n

Check for all the minors of order $(n-1)$

Case II i.) If any of the minor $(n-1)$ order is non-zero. then the rank of matrix will be $(n-1)$

ii.) If all of the minor $(n-1)$ order are zero. then the rank of A will be less than $(n-1)$

* Now check for all the minors of order $(n-2)$ and so on till we get a non-zero minor determinant.

* The rank of matrix will be 1 at its minimum if it is not a null matrix, & rank is 0, only for the zero matrix.

* For a matrix having order $m \times n$, the rank will be equal to or less than minimum of mn.

$$\begin{bmatrix} 1 & 3 & 7 \\ 2 & 5 & 8 \end{bmatrix}_{2\times 3}$$

$$r(A) \leq 2$$

* The rank of A will be equal to its transpose. $\boxed{r(A) = r(A^T)}$ $|A| = |A^T|$

* For a matrix A & B, the rank of AB will be min^m of rank A & B

$$r(AB) \leq r(A \& B)$$

* The rank of matrix is same, wether we calculate the row rank or column rank.

<u>Echleon form</u> (Row to Row Ya Column to Column)

⇒ leading non-zero element in every row is behind leading non-zero element in previous row.

⇒ Elementary transformation don't alter the rank of a matrix.

Q) For a matrix A $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 6 & 3 & 4 & 7 \\ 4 & 2 & 1 & 3 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Any of 2×2 deter is non-zero so rank is 2

Q.> $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$r = (2)$

Q.> If $X = [x_1\, x_2\, x_3 \dots x_n]^t$ is a non-zero vector then, a matrix. $V = XX^t$ has the rank of

$A \leftarrow r(x) = 1$

$B \leftarrow r(x^t) = 1$

$r(AB) \leq r(A) \,\&\, r(B)$

$1 \,\&\, 1$

rank $= 1$

Orthogonality of Vectors :-

$[1\ 2\ 3]$ $\qquad$ $[3\ 4\ a]$

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$\theta = \pi/2$

$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\boxed{\vec{a} \cdot \vec{b} = 0}$

If 0 then orthogonal
if not 0, then non-orthogonal.

$3 + 8 + 3a = 0$

$a = -11/3$

$\boxed{a_1 b_1 + a_2 b_2 + a_3 b_3 = 0}$

Linear dependency or independency :-

— let $V_1 V_2 V_3$ represents the 3 vectors

$$V_1 \; [1 \; 3 \; 5]$$
$$V_2 \; [2 \; 6 \; 10]$$
$$V_3 \; [1 \; 2 \; 3]$$

$$\boxed{k_1 V_1 + k_2 V_2 + k_3 V_3 = 0}$$

Case I :- If atleast two of the scalars are non-zero such that this relation satisfied, vectors are said to be Linearly dependend.

$k_1V_1 + k_2V_2 + k_3V_3 + \dots k_nV_n = 0$

Supp $k_1 = k_2 = k_3 = \dots k_n = 0$

or $v_1 = [1, 2, 3]$
$v_2 = [2 \; 4 \; 6]$
$k_1 = (2)$, $k_2 (-1)$
then above eqn is satisfied

$k_1 = 2$
$k_2 = -1$
$k_3 = 0$

then the theorem satisfy.

Case II :-

$$V_1 = [1 \; 2 \; 5]$$
$$V_2 = [2 \; 4 \; 7]$$
$$V_3 = [9 \; 11 \; 12]$$

if no combination of non-zero Scalar is there to satisfy the relation or the relation satisfy only when the scalars are zero. Then the vectors are said to be linearly independent.

$\boxed{k_1 k_2 k_3 \dots k_n}$

Q:> Show that the vectors [1 2 3] & [2 -2 0] forms a linearly independent set.

$$k_1[1\ 2\ 3] + k_2[2\ -2\ 0] = 0$$

$$k_1 + 2k_2 = 0$$

$$2k_1 - 2k_2 = 0$$

$$3k_1 + 0k_2 = 0$$

$$k_1 = 0$$

$$k_2 = 0$$

Since k_1 & k_2 are zero ie they don't posses any value so they are independt of each.

k_1 & k_2 both are zero. So linearly independent.

Space & Spain :-
(order) (rank)

Q:> For the vectors [1,2,-1] [2 3 0] [-1 2 5]. Find the space & spain.

Space R^3

Spain R^3 → rank = 3

determinant is non-zero.

Q:> For the vector [1 2 3] [4 5 6] [7 8 9]. Find space & spain.

space → R^3

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$1(45-48) - 2(36-42) + 3(32-35)$$

$$-3 + 12 - 9$$

$$0$$

Spain → R^2

System of homogeneous Equation :-

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$AX = 0$$

$$r(A) = n$$

Solⁿ: Unique → Trival $\qquad x = y = z = 0$

if solⁿ exist, then it must be $x = y = z = 0$ otherwise it will be ...

$$r(A) < n$$

Infinite solⁿ

Homogeneous system is always consistent.

Non-Homogenous System of Equation :-

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \qquad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B$$

$$\left[\begin{array}{ccc:c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

$A \vdots B$ augmented sign represents the independency of A & B

Consistant $\begin{cases} r(A:B) = r(A) = n \rightarrow \text{Unique Sol}^n \\ r(A:B) = r(A) < n \rightarrow \text{Infinite Sol}^n \end{cases}$

Inconsistant. $[\, r(A:B) \neq r(A) \qquad \rightarrow$ No Solⁿ

$r(A:B)$ greater than $r(A)$

Q.) For what values of α & β the following simultaneous eq^n have infinite no. of sol^n.

$$x+y+z=5, \quad x+3y+3z=9, \quad x+2y+\alpha z=\beta$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - R_1 \\ R_2 \to R_2 - R_1 \end{array} \quad \left[\begin{array}{ccc:c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]$$

$$R_3 \to R_3 - \tfrac{1}{2} R_2$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & \alpha-2 & \beta-7 \end{array}\right]$$

$\alpha - 2 = 0$
$\alpha = 2$
$\beta = 7$

If unique
$\alpha \neq 2$
$\beta \neq 7$.

Q.) The following system of eq^n is given by
$x+y+z=3$
$x+2y+3z=4$
$x+4y+kz=6$
will not have a unique sol^n for $k=1$.

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array}\right|$$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & k-1 & 3 \end{array}\right| \quad R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k-7 & 0 \end{array}\right| \quad R_3 \to R_3 - 3R_2$$

$$\boxed{k=7}$$

Q.) For a system of eq^n $x+2y+z=6$
$2x+y+2z=6$
$x+y+z=5$

$$\left|\begin{array}{ccc:c} 1 & 2 & 1 & 6 \\ 2 & 1 & 2 & 6 \\ 1 & 1 & 1 & 5 \end{array}\right|$$

$R_3 \rightarrow R_3 - R_1$
$R_2 \rightarrow R_2 - 2R_1$

$$\left|\begin{array}{ccc:c} 1 & 2 & 1 & 6 \\ 0 & -3 & 0 & -6 \\ 0 & -1 & 0 & 1 \end{array}\right|$$

$R_3 \rightarrow R_3 + \frac{1}{3}R_2$

$$\left|\begin{array}{ccc:c} 1 & 2 & 1 & 6 \\ 0 & -3 & 0 & -6 \\ 0 & 0 & 0 & 3 \end{array}\right|$$

No sol^n

Q.) $A = (3 \times 4)$ real matrix and $AX=B$ is an inconsistant System of eq^n. The highest possible rank of A is

$A_{3\times4}$
$r(A) \leq \min(m \,\&\, n)$
$r(A) \leq 3$

$AX = B$
$\downarrow$
Inconsistent
$r(A) \neq r(A:B)$

$r(A) \leq 2$
$\therefore r(A) \leq 2.$

Q.) For what value of a will following system have a sol^n

$2x+3y=4$
$x+y+z=4$
$x+2y-z=a.$

$$\left|\begin{array}{ccc:c} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & -1 & a \end{array}\right|$$

$R_3 \to R_3 + R_2$

$$\left|\begin{array}{ccc:c} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 2 & 3 & 0 & a+4 \end{array}\right|$$

$R_3 \to R_3 - R_1$

$$\left|\begin{array}{ccc:c} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 0 & 0 & 0 & a \end{array}\right|$$

$\boxed{a = 0}$ Ans.

Q.) $x + y + z = 6$ No sol^n, find $\lambda = ?$ & $\mu = ?$

$x + 4y + 6z = 20$

$x + 4y + \lambda z = \mu$

for each possible case.

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right|$$

1) ~~No soln~~ Unique sol^n

$\lambda \neq 6$, $\mu \neq 20$

$R_3 \to R_3 - R_2$

2) Infinite sol^n

$\lambda = 6$, $\mu = 20$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & \lambda-6 & \mu-20 \end{array}\right|$$

3) No sol^n

$\lambda = 6$, $\mu \neq 20$

$\lambda = 6$

$\mu \neq 20$.

Q.) $2x+y-4z=\alpha$
$4x+3y-12z=5$
$x+2y-8z=7$

for <u>how many values</u> of α does this system of equation have <u>in</u>finitely many solⁿ.

$$\left[\begin{array}{ccc:c} 1 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 3/2 & -6 & 7-\alpha/2 \end{array}\right]$$

value of α will be diff. if there is interchange of row or column.

$R_3 \rightarrow R_3 - \frac{3}{2}R_2$

$$\left[\begin{array}{ccc:c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 0 & -12 & 7-\frac{\alpha}{2}-\cancel{4}\frac{3}{2}(5-\alpha) \end{array}\right]$$

* Since in the last eqⁿ row, α comes in linear equation. So the value of α is one, if α comes is square ~~form~~ form then value of α is two & so on.

Q.) for the set of eqⁿ $x_1+2x_2+x_3+4x_4=2$
$3x_1+6x_2+3x_3+12x_4=6$
, The following statement is true for the set of eqⁿ

i) only trivial solⁿ exist
ii) No solⁿ
iii) Unique solⁿ
iv) infinite solⁿ ✓

Eigen Value Problem:-

coeff matrix, variable matrix, any scalar

The problem of type $\boxed{AX = \lambda X}$ is eigen value problem

eigen vector, eigen value

$*$ $\boxed{X[A-\lambda I]=0}$ —— (i)

$\boxed{|A-\lambda I|=0}$ (Charateristic Eqn)

and the roots of the charateristic eqn are known as eigen values. Say $\lambda = \lambda_1, \lambda_2$ & λ_3 are eigen values. Then by putting λ in eqn (i). Then we can find out eigen vectors.

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 3 \\ 5 & 2-\lambda \end{vmatrix} = 0$$

Q) $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ Find the eigen value.

$$\Rightarrow \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 4-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(1-\lambda) - (4) = 0$$

$$\cancel{4} - 4\lambda - \lambda + \lambda^2 - \cancel{4} = 0$$

$$\cancel{\lambda^2} = 5\cancel{\lambda}$$

$$\boxed{\lambda = 5}, 0$$

$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\begin{vmatrix} 9 & 5 \\ 5 & 8 \end{vmatrix}$

$\left|\begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right|$

$\begin{vmatrix} 9-\lambda & 5 \\ 5 & 8-\lambda \end{vmatrix} = 0$

$(9-\lambda)(8-\lambda) - 25 = 0$

$72 - 17\lambda + \lambda^2 - 25 = 0$

$\boxed{\lambda^2 - 17\lambda + 47 = 0}$

$\lambda = \frac{17 \pm \sqrt{101}}{2}$

$\Rightarrow$ No. of egin value depends on (degree of eqn) order of matrix for each eigen value there is atleast 1 eigen vector.

Q.) $\begin{bmatrix}4 & 5\\ 2 & -5\end{bmatrix}$ find the eigen values.

$$\Rightarrow \begin{vmatrix}4-\lambda & 5\\ 2 & -5-\lambda\end{vmatrix}$$

$$\Rightarrow (4-\lambda)(-5-\lambda) - 10 = 0$$

$$\Rightarrow -20 - 4\lambda + 5\lambda + \lambda^2 - 10 = 0$$

$$\lambda^2 + \lambda - 30 = 0$$

$$\lambda^2 + 6\lambda - 5\lambda - 30 = 0$$

$$\lambda(\lambda+6) - 5(\lambda+6) = 0$$

$$(\lambda-5)(\lambda+6) = 0$$

$$\boxed{\lambda = 5, -6}$$

$$\therefore \cancel{\begin{bmatrix}4 & 5\\ 2 & -5\end{bmatrix}}$$

$$[A - \lambda I]X = 0 \Rightarrow \begin{bmatrix}4-5 & 5\\ 2 & -5-5\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

eqn is less
soln infinite.

$$\boxed{-x + 5y = 0}$$

$$2x - 10y = 0$$

$$\cancel{x = 5y}$$

$$\cancel{10y - 10y = 0}$$

$$\cancel{y = 0}$$

$x = k$ $\quad k : \frac{k}{5}$

$y = \frac{k}{5}$.

$1 : \frac{1}{5}$

$\boxed{5 : 1}$

$$10x + 5y = 0$$

$$2x + y = 0 \qquad \boxed{6:1}$$

$$\cancel{y = -2x}$$

Q.) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$ find the eigen vector.

i.) ~~[-1, -2, 0, 0]~~ $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$

ii.) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

iii.) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$

iv.) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$

$$\begin{vmatrix} 5-\lambda & 0 & 0 & 0 \\ 0 & 5-\lambda & 5 & 0 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 3 & 1-\lambda \end{vmatrix} = 0$$

we had taken this column & non-zero element is only one

$$(5-\lambda)\begin{vmatrix} 5-\lambda & 5 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 3 & 1-\lambda \end{vmatrix}$$

$$\boxed{\begin{array}{l} (5-\lambda)^2(2-\lambda)(1-\lambda) = 0 \\ \cancel{\lambda^2+25-10\lambda} \\ \cancel{(5-\lambda)(5-\lambda)(2-\lambda)(1-\lambda) = 0} \end{array}}$$

$$(5-\lambda)(5-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 3 & 1-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(5-\lambda)((2-\lambda)(1-\lambda)-3) = 0$$

$$(5-\lambda)(5-\lambda)(2-2\lambda-\lambda+\lambda^2-3) = 0$$

$$(5-\lambda)(5-\lambda)(\lambda^2-3\lambda-1) = 0$$

$$\lambda = 5, 5, \frac{3 \pm \sqrt{13}}{2}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 3 & -4 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

as we don't know the exact value of a, b & c, d = 0 so eigen vector

by multiplication

$5c = 0$
$-3c + d = 0$
$3c - 4d = 0$

$c = 0$
$d = 0$

$$\begin{bmatrix} a \\ b \\ 0 \\ 0 \end{bmatrix}$$

Q.) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ find the no. of linearly independent eign vectors.

$$\begin{vmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)^2 = 0$$

$$\lambda = 2, 2$$

$$\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\cancel{x_2 = 0} \quad \begin{bmatrix} K \\ 0 \end{bmatrix}$$

Q.) $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ The eign vectors of a matrix are given $\begin{bmatrix} 1 \\ a \end{bmatrix}$ $\begin{bmatrix} 1 \\ b \end{bmatrix}$. find the value of a+b

$$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) = 0$$

$$\lambda = 1, 2$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2a = 0$$

$$1a = 0$$

$$\boxed{a = 0}$$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-1 + 2b = 0$$

$$2b = 1$$

$$\boxed{b = 1/2}$$

$$\boxed{a + b = 1/2}$$

Q) $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. One of the eigen vector is

✓ i) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ — iii) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

ii) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ iv) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$
$$6 - 2\lambda - 3\lambda + \lambda^2 - 2 = 0$$
$$\lambda^2 - 5\lambda + 4 = 0$$
$$= \frac{5 \pm \sqrt{25-16}}{2}$$
$$= \frac{5 \pm 3}{2} = 4, 1$$

$$\begin{bmatrix} -2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-2x + 2y = 0$$
$$x + y = 0$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$x + 2y = 0$ $\quad x = k \quad 1 : -\frac{1}{2}$

$x + 2y = 0$ $\quad y = -\frac{k}{2} \quad 2 : -1$

Q) $\begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ one of the eigen value (-2). Then which one of the following is eigen vector

i) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ ii) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ iii) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ ✓ iv) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

$$\begin{vmatrix} 3-\lambda & -2 & 2 \\ 0 & -2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$(1-\lambda)\{(3-\lambda)(-2-\lambda)\} = 0$$

$$\lambda = 1, 3, -2$$

$$\begin{vmatrix} 2 & -2 & 2 \\ 0 & -3 & 1 \\ 0 & 0 & 0 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 - 2x_2 + 2x_3 = 0$$
$$-x_2 + x_3 = 0$$

$$\begin{bmatrix} 0 & -2 & 2 \\ 0 & -5 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_3 = 0.$$

$$\begin{bmatrix} 5 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = k_1$$
$$x_2 = k_2$$
$$x_3 = 0$$

Q.) The linear operation $L(x)$ is defined by the crosspdt $= B \times X$, where $B = [0\ 1\ 0]^t$ & $X = [x_1\ x_2\ x_3]^t$ are 3-dimensional vectors. The 3×3 unit matrix of these operations satisfies.

$$L(x) = M \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Then the eigen values of matrix M are.

Sol^n

$$L(x) = \underset{3\times3}{M}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1}$$

$$B \times X = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ x_1 & x_2 & x_3 \end{vmatrix}$$

$$= \hat{i}[x_3] - \hat{j}[0] + \hat{k}[-x_1]$$

$$L(x) = x_3\hat{i} - x_1\hat{k}$$

$$\begin{bmatrix} x_3 \\ 0 \\ -x_1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$M_{11}x_1 + M_{12}x_2 + M_{13}x_3 = x_3$$

$$M_{11} = 0, \quad M_{12} = 0, \quad M_{13} = 1$$

$$M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} -\lambda & 0 & 1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{vmatrix}$$

$$-\lambda(+\lambda^2) + 1(\mp\lambda) = 0$$

$$-\lambda^3 \mp \lambda = 0$$

$$-\lambda(\lambda^2+1) = 0$$

Q.> $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

i.> $[-1,1,1]$ ~~ii.>~~ $[1,2,1]$ iii.> $[1,-1,2]$ iv.> $[2,1,-1]$

$$\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & 2-\lambda & 2 \\ 0 & 0 & 3-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)((2-\lambda)(3-\lambda) = 0$$

$$\lambda = 1, 2, 3$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_2 = 0$

$x_2 + 2x_3 = 0$ $x_2 : x_3$ 111

$2x_3 = 0$

$-x_1 + \cancel{x_2 = 0}$ 3

$\cancel{x_3 = 0}$

x

Properties of Eigen Values:-

1.) If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of Matrix A, then $K\lambda_1, K\lambda_2, K\lambda_3$ are eigen values of matrix KA.

2.) If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of Matrix A, then $\lambda_1^k, \lambda_2^k, \lambda_3^k$ are the eigen values of matrix A^k.

$$\text{matrix } A = 2, 3$$
$$A^3 = 2^3, 3^3$$

3.) If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of Matrix A; $1/\lambda_1, 1/\lambda_2, 1/\lambda_3$ are the eigen values of matrix A^{-1}

$$A = 2, 3$$
$$A^{-1} = 1/2, 1/3$$

4.) If $\lambda_1, \lambda_2, \lambda_3$, are the eigen values of Matrix A, then

$\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}, \frac{|A|}{\lambda_3}$ are the eigen values of Matrix adj A.

$$A = 2, 3$$
$$|A| = 2$$
$$\text{adj } A = 2/2, 2/3.$$

5.) Eigen values of a real symmetric (or Hermition) matrix are alwaz real.

6.) Eigen values of a real skew-symmetric (or Skew Hermition) matrix are either zero or purely imaginary.

7.) Eigen value of an orthogonal matrix (or unitary matrix) is of unit modulus.

for orthogonal, the eigen value $|\lambda| = 1 \Rightarrow |A| = 1$

8.) Eigen value of a matrix is equal to ~~its transpose~~. eigen values of the transpose of a matrix.

$$EV[A] = EV[A]^t.$$

*9.) Sum of the eigen values is equal to the trace of matrix.

10.) The pdt. of eigen values is equal to the determinant of a matrix.

$$A = \lambda_1 \lambda_2$$

$$|A| = \lambda_1 \times \lambda_2$$

July 13, 14

Q.) for matrix $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ eigen vector is given $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$. Find eign value corresponding to eigen vector.

$$\begin{vmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(4-\lambda) - 4 = 0$$

$$\lambda^2 + 16 - 8\lambda - 4 = 0$$

$$\lambda^2 - 8\lambda + 12 = 0$$

$$(\lambda - 6)(\lambda - 2) = 0$$

$$\lambda = 6, 2$$

$$\boxed{|A - \lambda I| X = 0}$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} \begin{bmatrix} 101 \\ 101 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(4-\lambda)101 + 2(101) = 0$$

$$404 - 101\lambda + 202 = 0$$

$$606 = 101\lambda$$

$$\boxed{\lambda = 6}$$

Q.) let A be the 2x2 matrix wid elements $a_{11} = a_{12} = a_{21} = +1, a_{22} = -1$ then the eigen value of the matrix A^{20} is

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$(1-\lambda)(1-\lambda)-1=0$$

$$\cancel{\lambda^2+1-2\lambda-1=0}$$
$$\cancel{\lambda^2-2\lambda=0}$$
$$\cancel{\lambda(\lambda-2)=0}$$
$$\cancel{\lambda=0,2}$$

$$-1-\lambda+\cancel{\lambda}+\lambda^2-1=0$$

$$\lambda^2-2=0$$

$$\lambda=\pm\sqrt{2}$$

$$\Rightarrow (+\sqrt{2})^{20}, \quad (-\sqrt{2})^{20}$$

Q.) For the matrix $\begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$ one of the eigen value of the matrix is 3, the other two eigen values are

i) 2, -5 iii) 2, 5

ii) 3, -5 iv) 3, 5

$\cancel{2-\lambda(-1-\lambda)}$ $2+(-1)+0$

$\cancel{1(-2\lambda-2-\lambda-1\lambda)}$ $=1$

Q.) For the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. The eigen values are $-2, 6$. What is the 3rd eigen value

$$\lambda_1+\lambda_2+\lambda_3 = tr(A)$$
$$-2+6+\lambda_3=7$$
$$\lambda_3=7-6+2$$
$$\boxed{\lambda_3=3}$$

Q.) The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. What is the sum of eigen value

$$\lambda_1+\lambda_2+\lambda_3 = tr(A)$$
$$=7$$

Q.) If a square matrix A is real & symmetric, then the eigen values are.

Ans. always Real

Q) For the matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ one of the eigen value is 3. What is sum of other two eigen value.

$$3 + \lambda_2 + \lambda_3 = 1 + p$$

$$\lambda_2 + \lambda_3 = 1 + p - 3$$

$$= p - 2 .$$

Q7 The trace & determinant of a 2x2 matrix are -2, -35. Find eigen values.

$$\lambda_1 \lambda_2 = -35 \quad \Rightarrow \cancel{\lambda_1 = -35/\lambda_2}$$

$$\lambda_1 + \lambda_2 = -2$$

$$\lambda_1 = -(2 + \lambda_2)$$

$$-2\lambda_2 - \lambda^2 = -35$$

$$\lambda^2 + 2\lambda_2 - 35 = 0$$

$$(\lambda^2 + 7\lambda)(\lambda^2 - 5) = 0$$

$$\lambda = -7, 5$$

Q.) For the matrix $\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$. If the eigen values are 4, 8. Find the value of x, y.

$$2 + y = 12$$

$$y = 10$$

$$2y - 3x = 32$$

$$20 - 3x = 32$$

$$-3x = 12$$

$$x = -4$$

Cayley - Hamilton Theorem: —

⇒ Every matrix eqn satisfies its characteristic eqn

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 2\lambda + 3 = 0$$

$$\boxed{A^2 + 2A + 3I = 0}$$

This theorem is used to find the inverse of a matrix and the higher powers of a matrix

$$A^{-1}A^2 + 2A^{-1}A + 3A^{-1}I = A^{-1}0$$

$$A + 2I + 3A^{-1} = 0$$

$$\boxed{A^{-1} = \frac{-A-2I}{3}}$$

$$A^2 = -2A - 3I$$

$$A^4 = A^2 A^2$$

$$= (2A+3I)^2$$

$$= 4A^2 + 9I + 12A$$

$$= 4(-2A-3I) + 9I + 12A$$

$$= -8A - 12I + 9I + 12A$$

$$= 4A - 3I$$

Q.) Find the inverse of a matrix by $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ caley hamilton theorem.

$$\begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 3 \\ 4 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda)-12=0$$

$$2-\lambda-2\lambda+\lambda^2-12=0$$

$$\cancel{\lambda^2-3\lambda-10=0}$$

$$\cancel{\lambda^2+5\lambda-2\lambda-10=0}$$

$$\cancel{\lambda(\lambda-5)(\lambda+2)=}$$

$$\lambda^2-3\lambda-10=0$$

$$\boxed{A^2-3A-10I=0}$$

$$A^{-1}A^2-3A^{-1}A-10A^{-1}I=A^{-1}0$$

$$A-3I-10A^{-1}=0$$

$$A^{1}=\frac{A-3I}{10}$$

$$=\frac{\begin{vmatrix}1 & 3\\ 4 & 2\end{vmatrix}-\begin{vmatrix}3 & 0\\ 0 & 3\end{vmatrix}}{10}$$

$$A^{-1}=\frac{1}{10}\begin{vmatrix}-2 & 3\\ 4 & -1\end{vmatrix}$$

Q.) find A^5 for same.

$$A^5=A^2\cdot A^2\cdot A \qquad A^2=3A+10I$$

$$=(3A+10I)^2A$$

$$=(9A^2+100I+60A)A$$

$$=(9(3A+10I)+100I+60A)A$$

$$=(27A+90I+100I+60A)A$$

$$=(87A+190I)A^2$$

$$=87(3A+10I)+190A$$

$$=\cancel{75A+720I+190A}$$

$$=\cancel{265A+720I} \qquad 451A+870I$$

$= \cancel{265A + 720I}$ $1451A + 870I$

$= \begin{vmatrix} 265 & 795 \\ 1060 & 530 \end{vmatrix} - \begin{vmatrix} 720 & 0 \\ 0 & 720 \end{vmatrix}$

$= \begin{vmatrix} -455 & 795 \\ 1060 & 190 \end{vmatrix}$

Q.) $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. Express $2A^5 - 3A^4 + A^2 - 4I$ = polynomial into a linear polynomial.

$(3-\lambda)(2-\lambda) - (1 \cdot -1) = 0$

$6 - 3\lambda - 2\lambda + \lambda^2 + 1$

$\lambda^2 - 5\lambda + 7 = 0$

$A^2 - 5A + 7I = 0$

$A^2 = 5A - 7I$

$A^4 = (A^2)^2 = (5A - 7I)^2 = 25A^2 + 49I - 70A$

$A^5 = (25(5A - 7I) + 49I - 70A)A$

$= (125A - 175I + 49I - 70A)A$

$= (55A - 126I)A$

$= 55(5A - 7I) - 126A)$

$= 275A - 385I - 126A$

$A^5 = 149A - 385I$

$2(\cancel{149A - 385I}) - 3(55A - 126I) + (5A - 7I) - 4I = 0$

$298A - 770I - 165A + 378I + 5A - 7I - 4I = 0$

$= 138A - 403I$ = Ans find.

Q) $\begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ for the matrix find A^9 ? $A^9 = 511A + 510A$

$A^9 = A^2 \cdot A^2 \cdot A^2 A^2 \cdot A$

$\cancel{-3\lambda\lambda} + 2I = 0$

$(-3-\lambda)(0-\lambda) - (-2) = 0$

$3\lambda + \lambda^2 + 2 = 0$

$A^2 + 3A + 2I = 0$

$A^2 = -3A - 2I$

$A^4 = (9A^2 + 16I + 24A)$

$= 9(-3A - 4I) + 16I + 24A)$

$= -27A - 36I + 16I + 24A$

$= -3A - 20I$

$A^8 (A^4)^2 = (9A^2 + 400I + 120A)$

$= 9(-3A - 4I) + 400I + 120A)$

$= -27A - 36I + 400I + 120A$

$= (93A + 364I)A.$

$= 93(-3A - 4I) + 364A)$

$= -279A - 372I + 364A)$

$= 85A - 3$

Q) If $ABCD = I$ find B^{-1}

$(AB)^{-1} = B^{-1}A^{-1}$ inverse

$(AB)' = B'A'$ tr

Calculus

Limit:- It is an approximate value of a function, which tells us abt the continuity of a function and also helps to determine whether at a certain pt. the function is differentiable or not.

1.> $\lim_{x\to\infty} (1+1/x)^x = e \;\; | \; (1+a/x)^x = e^a \; | \; (1-a/x)^x = e^{-a}$

2.> $\lim_{x\to 0} (1+x)^{1/x} = e \;\; (1+ax)^{1/x} = e^a \; | \; (1-ax)^{1/x} = e^{-a}$

3> $\lim_{x\to 0} \frac{a^x-1}{x} = \ln a$

4.> $\lim_{x\to\infty} \frac{a^{1/x}-1}{x} = \ln a$

5> $\lim_{x\to 0} \frac{e^x-1}{x} = \ln e$

6.> $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

7> $\lim_{x\to 0} \frac{\sin x}{x} = 1$

8> $\lim_{x\to 0} \frac{\tan x}{x} = 1$

L-HOSPITAL Rule

By putting the limit, if we get $\overbrace{0/0, \frac{\infty}{\infty}}^{\text{indeterminant for } (0)^0, (\infty)^0, (1)^\infty}$. Then we differentiate the numerator as well as the denominator separately till we get a finite limit.

$$\lim_{x\to a} \frac{f(x)}{g(x)}$$

$$\lim_{x\to a} \frac{f'(x)}{g'(x)}$$

diff w.r.t to variable on which limit is applied & procedure is done till we find finite limit.

For the modulus funct^n, greatest integer funct^n, degree and for some power of bracket we cannot put the limit.

Q.) $\lim_{x\to 0} \frac{3^x-1}{\sqrt{x+1}-1}$

rationalise

$$\lim_{x\to 0} \frac{3^x-1}{\sqrt{x+1}-1} \times \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0} \frac{(3^x-1)(\sqrt{x+1}+1)}{x}$$

$$\lim_{x\to 0} \frac{(3^x-1)}{x} \cdot \lim_{x\to 0} \frac{(\sqrt{x+1}+1)}{x}$$

$\ln 3$.

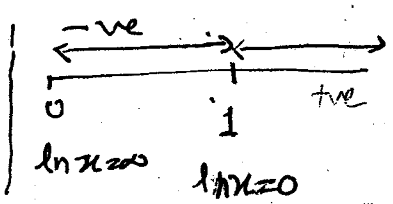
Q.) $\lim_{n\to\infty} \frac{4^{1/n}-1}{3^{1/n}-1}$

$$\lim_{n\to\infty} \frac{\frac{4^{1/n}-1}{n}}{\frac{3^{1/n}-1}{n}} = \frac{\ln 4}{\ln 3} = \ln_3 4$$

In mathematics we alwz work on base e so alwaz comm.

$$\boxed{\ln_a b = \frac{\ln_e b}{\ln_e a}}$$

$$\boxed{\ln_{c^d} a^b = \frac{b}{d} \ln_c a}$$



Domain is alwaz +ve

Q.) $\lim_{x\to 0} \frac{\sin \frac{2}{3}x}{x}$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$$\frac{\sin \frac{2}{3}x}{\frac{2}{3}x} \times \frac{2}{3}$$

$= \frac{2}{3}$

Q.) $\lim_{x\to 8} \frac{x^{1/3}-2}{x-8}$

$\lim_{x\to 8} \frac{\frac{1}{3}x^{1/3-1}}{1}$

$\frac{1}{3}\frac{1}{(8^{2/3})} = \frac{1}{12}$

Q.) $\lim_{x\to 0} \frac{e^x-(1+x+\frac{x^2}{\lfloor 2})}{x^3}$

$\lim_{x\to 0} \frac{(\cancel{1+x+\frac{x^2}{\lfloor 2}}+\frac{x^3}{\lfloor 3}+\dots)-(\cancel{1+x+\frac{x^2}{\lfloor 2}})}{x^3}$

$\lim_{x\to 0} (\frac{1}{\lfloor 3}+\frac{x}{\lfloor 4}+\dots)$

$\frac{1}{\lfloor 3} = \frac{1}{3\times 2} = \frac{1}{6}$

Q.) $\lim_{x\to 0} \frac{\sin^2 x}{x}$

$\lim_{x\to 0} \left(\frac{\sin x}{x}\right)\sin x$

$1\times 0 = 0$

Q.) $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

$\lim_{x\to 0} \frac{x\left[\frac{2\tan x}{1-\tan^2 x}\right]-2x\tan x}{(2\sin^2 x)^2}$

$\lim_{x\to 0} \frac{2x\tan x\left[\frac{1-1+\tan^2 x}{1-\tan^2 x}\right]}{4\sin^4 x}$

Formulas for trignometry :-

1.) $\sin 2x = \cancel{2\sin x \cos x} = \dfrac{2\tan x}{1+\tan^2 x}$

2.) $\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$ $\Rightarrow$ $\boxed{1+\cos 2\theta = 2\cos^2\theta}$

$= \dfrac{1-\tan^2 x}{1+\tan^2 x}$ $\Rightarrow$ $\boxed{1-\cos 2\theta = 2\sin^2\theta}$

3.) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ $\boxed{\sin n\theta = 2\sin \frac{n\theta}{2} \cos \frac{n\theta}{2}}$

4.) $\sin 3x = 3\sin x - 4\sin^3 x$ $\boxed{1+\cos n\theta = 2\cos^2 \frac{n\theta}{2}}$

5.) $\cos 3x = 4\cos^3 x - 3\cos x$

6.) $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

7.) $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

8.) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

9.) $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

10.) $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

11.) $\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$

12.) $\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$

13.) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

14.) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

15.) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

16.) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

17.) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$$\lim_{x\to 0} \frac{2x\tan x \cdot \tan^2 x}{4(1-\tan^2 x)\frac{\sin^4 x}{\cancel{x^4}}\cancel{x^4}}$$

$$\lim_{x\to 0} \frac{2x\left(\frac{\tan x}{x}\right)^{1}\times x \cdot \left(\frac{\tan^2 x}{x^2}\right)^{1}\cdot x^2}{4(1-\underbrace{\tan^2 x}_{0})\underbrace{\left(\frac{\sin^4 x}{x^4}\right)}_{1}\cdot x^4}$$

$$= \frac{2x^4}{4x^4} = \frac{1}{2}$$

Q.) $\lim_{x\to 0} \frac{x-\sin x}{x+\cos x} = \frac{0}{1} = 0$

Q.) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

$$\frac{\sin x}{2x}$$

$$\frac{\cos x}{2} = \frac{1}{2}$$

$$\frac{2\sin^2 x/2}{\frac{x^2}{4}\cdot 4} = \frac{2}{4} = \frac{1}{2}$$

Q.) $\lim_{x\to 0} \frac{1-\cos x}{x(2^x-1)}$

$$\frac{2\sin^2 x/2}{\cancel{x}\,\frac{(2^x-1)}{x}\,x\cdot\frac{4}{4}}\ 1$$

$$\frac{2\times 1}{\left(\frac{2^x-1}{x}\right)4}$$

$$\downarrow \ln 2$$

$$= \frac{2\times 1}{\ln 2\times 4} = \frac{1}{2\ln 2}$$

Q.) $\lim_{x\to 1} \dfrac{\sqrt{1-\cos 2(x-1)}}{x-1} = \cancel{0}/\cancel{1} = 0$

$\lim_{x\to 1} \dfrac{\sqrt{2\sin^2(x-1)}}{x-1}$

$\lim_{x\to 1} \sqrt{2}\,\dfrac{\sin(x-1)}{x-1}$

$\lim_{h\to 0} \sqrt{2}\,\dfrac{\sin h}{h}$

$\to \sqrt{2}$

i.) limit exist and it is $\sqrt{2}$

ii) limit exist and it is $-\sqrt{2}$

iii) limit doesn't exist bcz $(x\to 1)\to 0$

~~iv)~~ doesn't exist bcz LH limit not equal to RH limit

R.H.L.

$\lim_{x\to 1^+} \dfrac{\sqrt{1-\cos 2(x-1)}}{x-1}$

$x = 1+h$
$x \to 1^+$
$h \to 0$

$\lim_{h\to 0} \dfrac{\sqrt{1-\cos 2h}}{h}$

$\lim_{h\to 0} \dfrac{\sqrt{2\sin^2 h}}{h} = \sqrt{2}$

L.H.L

$\lim_{x\to 1^-} \dfrac{\sqrt{1-\cos 2(x-1)}}{x-1}$

$x = 1-h$
$x \to 1^-$
$h \to 0$

$\lim_{h\to 0} \dfrac{\sqrt{1-\cos 2(-h)}}{-h}$

$\lim_{h\to 0} \dfrac{\sqrt{1-\cos 2h}}{-h}$

$x = \cancel{1}-h$ ~~$x\to 1$~~ ~~$h\to$~~

$\sqrt{2\sin^2 h}$

$\sqrt{2}\,\dfrac{\sin h}{-h}$

$= -\sqrt{2} \Rightarrow$ RHL $\neq$ LHL.

Continuity :—

When LHL = RHL = value of the function at that point $f(a)$.
If these three are equal, then function is continous.

Q.) $\begin{cases} \frac{|x-2|}{2-x} & , \text{when } x \neq 2 \\ -1 & , x = 2 \end{cases}$

R.H.L

$\lim_{x\to 2^+} \frac{|x-2|}{2-x}$ $\quad x = 2+h$, $x \to 2^+$, $h \to 0$

$\lim_{h\to 0} \frac{h}{-h}$

$\lim_{h\to 0} -1 = -1$

L.H.L

$\lim_{x\to 2^-} \frac{|x-2|}{2-x}$ $\quad x = 2-h$, $x \to 2^-$, $h \to 0$

$\lim_{h\to 0} \frac{|h|}{h}$

$\lim_{h\to 0} 1 = 1$

$R.H.L \neq L.H.L = f(a)$

So function is not continous.

Q.) $f(x) = \begin{cases} \frac{x-|x|}{x} & , x \neq 0 \\ 2 & , x = 0 \end{cases}$

R.H.L

$\lim_{x\to 0^+} \frac{x-|x|}{x}$ $\quad x = 0+h$, $x \to 0^+$, $h \to 0$

$\frac{h-h}{h} = 0$

L·H·L

$$\lim_{x\to 0^-} \frac{x-|x|}{x} \qquad x\to 0-h,\ x\to 0^-,\ h\to 0$$

$$\lim_{h\to 0} \frac{-h-h}{-h} = +2h$$

$$= -2$$

$R\cdot H\cdot L \neq L\cdot H\cdot L \neq f(a)$ so function is discontinous.

Q.) $f(x)\begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1}, & x\neq 0 \\ 0, & x=0 \end{cases}$

R·H·L

$$\lim_{x\to 0^+} \frac{e^{1/x}-1}{e^{1/x}+1} \qquad x\to 0+h,\ x\to 0^+,\ h\to 0$$

$$\lim_{h\to 0} \frac{e^{1/h}-1}{e^{1/h}+1}$$

$$\lim_{h\to 0} \frac{e^{1/h}(1-1/e^{1/h})}{e^{1/h}(1+1/e^{1/h})}$$

$$\lim_{h\to 0} = \frac{1-1/e^{1/h}}{1+1/e^{1/h}}$$

$$= 1$$

$R\cdot H\cdot L \neq f(a)$ So function is discontinous.

Q.) $\begin{cases} f(x) = 2x+1, & x<2 \\ K, & x=2 \\ 3x-1, & x>2 \end{cases}$ If the function is continous then find the value of k.

Polynomial functions are alwaz continous & So simply put $x=2$ in 1st functⁿ

$2\times 2+1 = 5$

So $K = 5$,

Q.> $\begin{cases} \lambda(x^2-2x) & , x<0 \\ 4x+1 & , x>0 \end{cases}$ The function is continous find value of λ.

$4x+1$

$= 1$

$\lambda(x^2-2x) = 1$

$\lambda = 1$

L.H.L = 0

f(0) = 0

R.H.L = 1

for any value of λ function is not continous.

July 21, 14

Q.> $f(x)\begin{cases} ax+1 & , x\leq 3 \\ bx+3 & , x>3 \end{cases}$

Find relation b/w a & b, If f(x) is continuous.

Polynomial functions are continous function. So we directly put the limits.

$$3a+1 = 3b+3$$

$$\boxed{3a-3b=2}$$

Q.> $f(x)\begin{cases} \dfrac{1-\cos 4x}{x^2} & x<0 \\ a & x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x>0 \end{cases}$

$1-\cos n\theta = 2\sin^2\frac{n\theta}{2}$

$1+\cos n\theta = 2\cos^2\frac{n\theta}{2}$

$\sin n\theta = 2\sin\frac{n\theta}{2}\cos\frac{n\theta}{2}$

L.H.L = R.H.L = f(a)

L.H.L

$$\lim_{x\to 0}\frac{1-\cos 4x}{x^2} = \lim_{x\to 0}\frac{2\sin^2 2x}{4x^2}\times 4 = 8$$

R.H.L

$$\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}\times\frac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4} = \frac{\sqrt{x}\left(\sqrt{16+\sqrt{x}}+4\right)}{\cancel{16}+\sqrt{x}-\cancel{16}}$$

$$= 8.$$

★★★ We need 1 wid cos either +ve or -ve and sin alone.

Greatest Integer function:—

$$f(x) = [x]$$

$$\boxed{[x] \geq y} \longrightarrow \in I$$

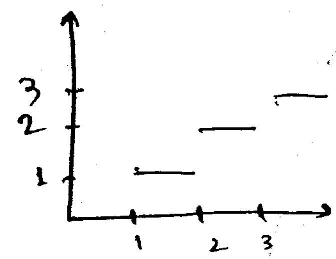
ex> $[5.7] = 5$, $[5.0001] = 5$, $[5.99999] = 5$.

(a+) (a)

$[-3.949] = -4$,

(a) (a+)

★ greatest integer function is discontinuous at all integral pts.



$a \in I$

$f(x) = [x]$

$f(a) = [a] = a$

R.H.L

$\lim\limits_{x \to a^+} [x]$ $\qquad$ $x = a+h$, $x \to a^+$, $h \to 0$

$\lim\limits_{x \to a^+} [x]$

$\lim\limits_{h \to 0} [a+h] = \lim\limits_{h \to 0} a = a$

L.H.L

$\lim\limits_{x \to a^-} [x]$

$\lim\limits_{h \to 0} [a-h]$

$= a - 1$

So the function is discontinuous at pt a.

$\cancel{a \notin I}$ $\qquad$ $N = a + f \longrightarrow 0.1$ to 0.9

$f(x) = [x]$

$f[N] = [a+f] = a$

R.H.L

$x = a+h$

$x \to a^+$

$h \to 0$

$$\lim_{x \to a^+} [x] \qquad x = [a + F + h] \quad x \to a^+ \quad h \to 0$$

$$\lim_{h \to 0} [a + P]$$

$$= a$$

L.H.L

$$\lim_{x \to a^-} [x]$$

$$\lim_{h \to 0} [x - h]$$

$$\lim_{h \to 0} [a + I - h]$$

$$\lim_{h \to 0} [a + q] \Rightarrow \lim_{h \to 0} a = a.$$

So the function is continuous at all non-integral pts.

Differentiability :-

1) $\frac{d x^n}{dx} = n x^{n-1}$

2) $\frac{d}{dx} (ax+b)^n = n(ax+b)^{n-1} \cdot a$

3) $\frac{d}{dx} \sin x = \cos x$

4) $\frac{d}{dx} \cos x = -\sin x$

5) $\frac{d \tan x}{dx} = \sec^2 x$

6) $\frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$

7) $\frac{d \sec x}{dx} = \sec x \tan x$

8) $\frac{d \operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cot x$

9) $\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$

$$\frac{d\cos^{-1}x}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$$

$$\frac{d\cot^{-1}x}{dx} = -\frac{1}{1+x^2}$$

$$\frac{d\sec^{-1}x}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d\,\mathrm{cosec}^{-1}x}{dx} = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{de^x}{dx} = e^x$$

$$\frac{da^x}{dx} = a^x \ln a$$

$$\frac{d\cdot\ln x}{dx} = \frac{1}{x}$$

$$\frac{d}{dx}|x| = \frac{x}{|x|} \text{ or } \frac{|x|}{x},\ x \neq 0$$

$$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$d(u\cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(u/v) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Power funct^n Angle
PFA
priorities while differentiating any function.

$$\frac{d\sqrt{\ }}{dx} = \frac{1}{2\sqrt{\ }}.$$

$$\frac{d}{dx}\,{}^{1/(\ )} = \frac{-1}{(\ )^2}$$

Q.) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -1x^{-1-1}\cdot 1 = -\frac{1}{x^2}$.

$$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{\frac{1}{2}-1}\cdot 1 = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Q.) $y = \cot(\sin^{-1}\sqrt{x})$

$$\frac{dy}{dx} \cot(\sin\sqrt{x}) = -\operatorname{cosec}^2(\sin\sqrt{x}) \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

Q.) $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$

$$\cancel{\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}}} = \cancel{\{(1+\sin 2x)(\cos 2x \cdot 2)\} - \{(1-\sin 2x)}$$

$$y = \sqrt{\frac{1-\cos(\pi/2 - 2x)}{1+\cos(\pi/2 - 2x)}}$$

$$y = \sqrt{\frac{2\sin^2(\pi/4 - x)}{2\cos^2(\pi/4 - x)}}$$

$$y = \tan(\pi/4 - x)$$

$$y' = \sec^2(\pi/4 - x)(-1)$$

Q.) $y = \sqrt{\tan^{-1} x^2}$

$$= \frac{1}{2\sqrt{\tan^{-1}x^2}} \cdot \frac{1}{1+(x^2)^2} \cdot 2x \cdot 1$$

$$= \frac{x}{(\sqrt{\tan^{-1}x^2})(1+x^4)}$$

Q.) $y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$

$$\tan^{-1}\sqrt{\frac{2\cos^2 x/2}{2\sin^2 x/2}}$$

$$y = \tan^{-1}.\cot x/2 = \tan^{-1}(\tan(\pi/2 - x/2))$$

$$= \cancel{\frac{1}{1+(\cot x/2)^2}(-\operatorname{cosec}^2 x/2)} \cdot \frac{1}{2}$$

$$= 0 - \frac{1}{2} = -\frac{1}{2}$$

Q.) $y = \tan^{-1}\sqrt{\dfrac{1+\sin x}{1-\sin x}}$

$= \tan^{-1}\sqrt{\dfrac{1+\cos(\pi/2 - x)}{1-\cos(\pi/2 - x)}}$

$= \tan^{-1}\sqrt{\dfrac{\cancel{2}\cos^2(\pi/4 - x/2)}{\cancel{2}\sin^2(\pi/4 - x/2)}}$

$= \tan^{-1}(\cot(\pi/4 - x/2))$

$= \tan^{-1}|\tan(\pi/2 - \pi/4 + x/2)$

$= \pi/4 + x/2$

$= 1/2$

Q.) $y = \tan^{-1}\left\{\dfrac{\cos x}{1+\sin x}\right\}$ $= \tan^{-1}\left\{\dfrac{\sin(\pi/2 - x)}{1+\cos(\pi/2 - x)}\right\}$

$= \tan^{-1}\sqrt{\dfrac{1+\cos x - 1}{1+\cos(\pi/2 - x)}}$ $= \tan^{-1}\left\{\dfrac{2\sin(\pi/4 - x/2)\cos(\pi/4 - x/2)}{2\cos^2(\pi/4 - x/2)}\right\}$

$= \tan^{-1}\sqrt{\dfrac{2\sin^2 x/2 - 1}{2\cos^2(\pi/4 - x/2)}}$ $= \tan^{-1}\{\tan(\pi/4 - x/2)\}$

$= -1/2$

Q.) $y = \tan^{-1}(\sqrt{1+x^2} + x)$

$y = \tan^{-1}(\sqrt{1+\tan^2\theta} + \tan\theta)$

$= \tan^{-1}(\sec\theta + \tan\theta)$

$= \tan^{-1}\left(\dfrac{1+\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}\left(\dfrac{1+\cos(\pi/2 - \theta)}{\sin(\pi/2 - \theta)}\right)$

$= \tan^{-1}\left(\dfrac{2\cos^2(\pi/4 - \theta/2)}{2\sin(\pi/4 - \theta/2)\cdot\cos(\pi/4 - \theta/2)}\right)$

$= \tan^{-1}|\tan(\pi/4 - \theta/2) = \pi/4 - \theta/2$

$\sqrt{a^2 - x^2}$, Put $x = a\sin\theta$
$\sqrt{a^2 + x^2}$, " $x = a\tan\theta$
$x^2 - a^2$, " $x = a\sec\theta$
$1 - x^2$, " $x = \sin\theta$
$1 + x^2$, " $x = \tan\theta$
$x^2 - 1$, " $x = \sec\theta$

$$= \frac{\pi}{4} - \frac{\tan^{-1}x}{2}$$

$$= -\frac{1}{2}\,\frac{1}{1+x^2}$$

Q.) $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}+1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta+1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1+\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\cancel{2}\cos^2\theta/2}{\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}}\right) = \tan^{-1}(\cot\theta/2)$$

$$= \tan^{-1}(\tan(\pi/2 - \theta/2))$$

$$= (\pi/2 - \theta/2) \Rightarrow \left(\pi/2 - \frac{\tan^{-1}x}{2}\right)$$

$$= -\frac{1}{2}\,\frac{1}{1+x^2}.$$

Q.) $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}+1}\right)$ Ans = $\frac{1}{2}\,\frac{1}{\sqrt{1-x^2}}$

Q.> $y = \tan^{-1}\left(\dfrac{\sqrt{1+a^2x^2}-1}{ax}\right)$

$ax = \tan\theta$

$\underline{\tan^{-1}ax}$

$= \tan^{-1}\left(\dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\dfrac{\sec\theta-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left\{\dfrac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right\}$

$= \tan^{-1}\{\tan\theta/2\}$

$= \theta/2 = \dfrac{\tan^{-1}ax}{2}$

$= \dfrac{1}{2}\cancel{\dfrac{1}{\sqrt{1-(ax)^2}}}\dfrac{1}{1+a^2x^2}\cdot a = \dfrac{a}{2}\dfrac{1}{1+(ax)^2}$

Q.> $y = \tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

$\tan(A\pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\tan^{-1}\left(\dfrac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y$

$= \tan^{-1}\left(\dfrac{a/b - \tan x}{1 + a/b(\tan x)}\right)$

$= \tan^{-1}\left(\dfrac{\tan\theta - \tan x}{1+\tan\theta\tan x}\right)$

$\dfrac{a}{b} = \tan\theta$

$= \tan^{-1}\{\tan(\theta \mp x)\}$

$= \cancel{\theta \pm x}\ \cancel{\tan}\ \theta - x$

$= \tan^{-1}\dfrac{a}{b} - x$

$= \cancel{\dfrac{1}{1+(a/b)^2}}\ 0 - 1 = -1$

Q.> $y = \cos^{-1}\left(\dfrac{2\cos x + 3\sin x}{\sqrt{13}}\right)$

$\dfrac{2}{\sqrt{13}} = \cos\alpha$

Also check the angle for both sin & cos.

$\sin\alpha = \sqrt{1-\cos^2\alpha}$

$= \sqrt{1-4/13} = \sqrt{\dfrac{9}{13}} = \dfrac{3}{\sqrt{13}}$

$= \cos^{-1}(\cos(\alpha - x))$

$= \alpha - x$

$= \cos^{-1}\dfrac{2}{\sqrt{13}} - x$

$y' = -1$

Q.) $y = \ln(\ln(\ln x))$ $\quad \ln x =$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{(\ln x)} \cdot \frac{1}{x}$$

Q.) $y = \ln_{10} x + \ln_x 10 + \ln_x x + \ln_{10} 10$ $\quad \boxed{\ln_b a = \frac{\ln_e a}{\ln_e b}}$

$$= \frac{\ln_e x}{\ln_e 10} + \frac{\ln 10}{\ln x} + 1 + 1$$ $\quad \boxed{\ln_a a = \frac{\ln_e a}{\ln_e a} = 1}$

$$= \cancel{\frac{\ln 10 \frac{1}{x} - \ln x \frac{1}{10}}{(\ln 10)^2}} +$$

$$y' = \frac{1}{\ln 10} \cdot \frac{1}{x} + \ln 10 \left[\frac{-1}{(\ln x)^2}\right] \cdot \frac{1}{x} + 0 + 0.$$

Q.) $ax + by^2 - \cos y = 0$. Find $\frac{dy}{dx}$. | function in which x, y are one same side $f(x,y) = 0$, Then function is known as Implicit function.

~~$ax+by^2 = \cos y$~~

$$a \cdot 1 + b \cdot 2y \cdot \frac{dy}{dx} + \sin y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}[2by + \sin y] = -a$$

$$\frac{dy}{dx} = \frac{-a}{2by + \sin y}.$$

$\frac{dy}{dx} = \frac{-f_x}{f_y}$ diff of function of x taking y as constant. } Partial derivative.

" " " " y " x " "

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{-(a)}{2by + \sin y}.$$

July 22, 14

Q.) $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$

$$y = \sqrt{\sin x + y}$$

$$y^2 - y - \sin x = 0$$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{\cos x}{2y-1}$$

Q.) $y = \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cfrac{\sin x}{\ddots \infty}}}}}$

$$y = \cfrac{\sin x}{1 + \cfrac{\cos x}{1+y}}$$

$$y = \cfrac{\sin x}{\cfrac{1+y+\cos x}{1+y}}$$

$$y + y^2 + y\cos x = \sin x(1+y)$$

$$y^2 + y + y\cos x - y\sin x - \sin x = 0$$

$$y^2 + (1+\cos x - \sin x)y - \sin x = 0$$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{y(\cos x - \sin x) - \cos x}{2y + (1+\cos x - \sin x)}$$

logarithmic differentiation :- when power of variable is variable.

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} \cdot 1 + \ln x \cdot 1$$

$$\frac{dy}{dx} = y(1+\ln x)$$

$$\frac{dy}{dx} = x^x(1+\ln x)$$

$$\left|\frac{dy}{dx} = (Ax)^{Bx} \Rightarrow Ax^{Bx}\left[\cancel{Bx \cdot \frac{dy}{dx} Ax + Ax \frac{dy}{dx} Bx}\right]\right.$$

$$y = (\sin x)^{\cos x}$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cancel{\tan x}\ \cos x \cdot \frac{1}{\sin x} \cos x + \ln \sin x\,(-\sin x)\right]$$

Q.) $y = x^{\ln x}$

$$= x^{\ln x}[\ln x \cdot x]$$

$$= x^{\ln x}\left[\ln x \cdot 1 + \cancel{x}\frac{1}{\cancel{x}}\right]$$

$$y' = x^{\ln x}[\ln x + 1]$$

Q.) $y = (5x)^{3\cos 2x}$ $\qquad 5x^{3\cos 2x}[3\cos 2x \cdot \ln 5x]$

$$= 5x^{3\cos 2x}\left[3\cos 2x / 5x + 5x\{3(-\sin 2x) \cdot 2\}\right]$$

$$= 5x^{3\cos 2x}[15\cos 2x + 30x \sin 2x]$$

$$= 5x^{3\cos 2x}\left[3\cos 2x \cdot \frac{1}{5x} \cdot 5 + \ln 5x\{3(-\sin 2x) \cdot 2\}\right]$$

Paramatric functions:-

$$y = \phi(t)$$
$$x = \psi(t)$$
$$x = at^2 \; , \; y = 2at$$

$$\frac{dx}{dt} = a \cdot 2t \quad , \quad \cancel{\frac{d^2x}{dt^2} = 2a}$$

$$\frac{dy}{dt} = 2a \qquad \cancel{\frac{d^2y}{dt^2} = 0}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= 2a \cdot \frac{1}{2at} = \frac{1}{t}$$

$$\frac{d^2y}{dx^2} = \left(\cancel{\frac{d^2y}{dt^2} \times \frac{dt^2}{dx^2}}\right) \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{t}\right)$$

$$= -\frac{1}{t^2} \cdot \frac{dt}{dx}$$

$$= -\frac{1}{t^2} \times \frac{1}{2at}$$

Q) $x = \sqrt{a^{\sin^{-1} t}} \; , \; y = \sqrt{a^{\cos^{-1} t}}$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{a^{\sin^{-1}t}}} \, a^{\sin^{-1}t} \, \cancel{\ln a} \, \frac{1}{\sqrt{1-t^2}} \cdot \cancel{1}$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{a^{\cos^{-1}t}}} \, a^{\cos^{-1}t} \, \cancel{\ln a} \, \frac{-1}{\sqrt{1-t^2}} \cdot \cancel{1}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\boxed{\cancel{\frac{1}{2\sqrt{a^{\cos^{-1}t}}} \cdot a^{\cos^{-1}t} \ln a \frac{-1}{\sqrt{1-t^2}} \cdot 1}}$$

$$-\frac{a^{\cos^{-1}t} \cancel{\ln a}}{\cancel{2\sqrt{a^{\cos^{-1}t}}} \cdot \cancel{\sqrt{1-t^2}}} \cdot \frac{\cancel{2\sqrt{a^{\sin^{-1}t}}} \cdot \cancel{\sqrt{1-t^2}}}{a^{\sin^{-1}t} \cancel{\ln a}}$$

$$-\frac{y^{\cancel{2}}}{\cancel{y}} \cdot \frac{x^{\cancel{x}}}{\cancel{x^2}}$$

$$\frac{dy}{dx} = -y/x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-y/x) = \frac{2y}{x^2}$$

Derivative of a function w.r.t to another function.

$z = \sin x$ $\qquad$ $\cos x = y.$

$$\frac{d \sin x}{d \cos x}$$

$$\frac{dz}{dy} = \frac{dz}{dx} \frac{dx}{dy}.$$

$$\frac{dz}{dx} = \cos x \quad \Rightarrow \quad \cos x \cdot \frac{1}{\sin x} = -\cot x.$$

$$\frac{dy}{dx} = -\sin x$$

Q.> $\underset{\overset{\shortparallel}{z}}{\dfrac{x^2}{1+x^2}}$ $\cdot$ $\underset{\overset{\shortparallel}{y}}{x^2}$

$$\frac{dz}{dy} = \frac{dz}{dx} \cdot \frac{dx}{dy}$$

$$\frac{dz}{dx} = \frac{(1+x^2)\,2x - x^2(2x)}{(1+x^2)^2}$$

$$= \frac{2x + \cancel{2x^3} - \cancel{2x^3}}{(1+x^2)^2} = \frac{2x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = 2x$$

$$\frac{dz}{dy} = \frac{dz}{dx} \cdot \frac{dx}{dy} = \frac{2x}{(1+x^2)^2} \cdot \frac{1}{2x}$$

$$\frac{dz}{dy} = \frac{1}{(1+x^2)^2}$$

if we see $\dfrac{\text{linear}}{\text{linear}}$ we make them same by any way

$$z = \frac{x^2+1-1}{1+x^2}$$

$$= \frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}$$

Q.> $\underset{\overset{\shortparallel}{z}}{x^x}$ w.r.t $\underset{\overset{\shortparallel}{y}}{x \ln x}$

$$\frac{dz}{dx} = x^x\,[x \cdot \ln x]$$

$$= x^x \left[\ln x \cdot 1 + x \frac{1}{\cancel{x}}\right]$$

$$= x^x\,[1 + \ln x]$$

$$\frac{dy}{dx} = \ln x\,[1] + \cancel{x}\frac{1}{\cancel{x}} = 1 + \ln x$$

$$\frac{dz}{dy} = x^x(1+\ln x) \times \frac{1}{(1+\ln x)}$$

$$= x^x$$

Q.) $x = 4z^2+5$, $y = 6z^2+7z+3$ find $\frac{d^2y}{dx^2}$

$$\frac{dx}{dz} = 8z$$

$\left(\frac{x-5}{4}\right)^{1/2}$

$$\frac{dy}{dz} = 12z+7 \qquad \frac{dy}{dz}\cdot\frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{12z+7}{8z}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left\{\frac{12z}{8z} + \frac{7}{8z}\right\} = \frac{d}{dx}\left\{\frac{3}{2} + \frac{7}{8z}\right\}$$

$$= \frac{3}{2} - \frac{7}{8z^2}\frac{dz}{dx}$$

$$= \frac{3}{2} - \frac{7}{8z^2}\cdot\frac{1}{8z}$$

$$= \frac{3}{2} - \frac{7}{64z^3}$$

Q.) $y = \tan x + \sec x$ check $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

$$\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$\frac{d}{dx}\left\{\frac{dy}{dx}\right\} \quad \sec^2 x(1+\tan x)$$

$$\frac{1+\sin x}{\cos^2 x}$$

$$= \cancel{1+\tan x} \quad \frac{1+\sin x}{\cos^2 x}$$

$$\sec^2 x + \frac{1+\sin x}{1-\sin^2 x}$$

$$\frac{\sin^2 x}{\cos} + \frac{\cos x + \sin}{\cos}$$

$$= \frac{1+\sin x}{1-\sin^2 x} = \frac{1+\sin x}{\cancel{(1+\sin x)}(1-\sin x)}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) \quad \frac{1}{(1-\sin x)} = \frac{-1}{(1-\sin x)^2}(-\cos x) = \frac{\cos x}{(1-\sin x)^2}$$

Q.) Successive differentiation

$y = A\sin nx + B\cos nx$. find $y'' + n^2 y$.

$y' = A\cos nx \cdot n + B(-\sin nx)(n)$

$= nA\cos nx - nB\sin nx$

$y'' = -nA\sin nx \cdot n - nB\cos nx \cdot n$

$= -n^2 A\sin nx - n^2 B\cos nx$

$= -n^2(A\sin nx + B\cos nx)$

$y'' = -n^2 y$

$y'' + n^2 y = -n^2 y + n^2 y = 0$

Q.) $e^y(x+1) = 1$. find the y'' as a function of y'.

$y'' = f(y')$

$e^y(x+1) = 1$

$e^y = \frac{1}{x+1}$

$y = \ln\left(\frac{1}{x+1}\right)$

$y' = \cancel{\log}\ \frac{1}{x+1}$

$y'' = \frac{-1}{(x+1)^2}$

$y'' = \cancel{} + y'^2$

$\cancel{y'' - y'^2 = 1}$

$y'' = y'^2$

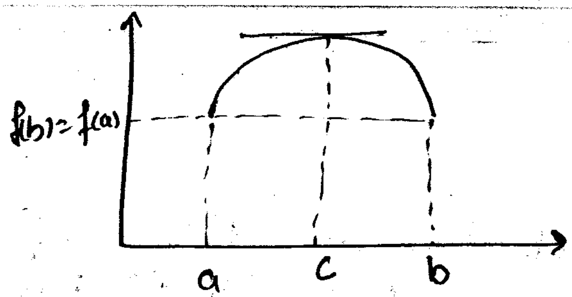
$e^y + (x+1)e^y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-e^y}{e^y(x+1)} = \frac{-1}{1+x}$

$\frac{d^2y}{dx^2} = \frac{1}{(1+x)^2} = \left(\frac{dy}{dx}\right)^2$

$y'' = (y')^2$

Rolle's Theorem:-



$f'(x)=0$
$f'(c)=0$

i.) $f(x)$ is continous on closed interval $[a,b]$ including a & b
ii.) $f(x)$ is derivable on open interval (a,b) excluding a & b
iii.) $f(a) = f(b)$

if these 3 conditions are satisfied then, there exist a value of $x=(c)$; where $c \in (a,b)$ Such $f'(c)=0$

Q.) $f(x) = x^3 - 4x$. Find the value of c, Rolle's theorem is satisfied:

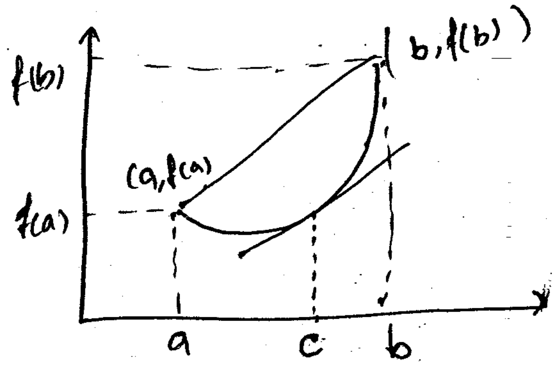
$f(0) = 0-0 = 0$, $f(2) = 8-8 = 0$.

$f'(x) = 3x^2 - 4$

$f'(\frac{2}{\sqrt{3}}) = 0$

$c = -2/\sqrt{3} \notin [0,2]$

Lagrange
Mean Value theorem:-



$$\boxed{f'(c) = \frac{f(b) - f(a)}{b-a}}$$

Rolle's theorem is a part of Mean Value theorem as $f(a) = f(b)$, $f'(c) = 0$ which is Rolle's theorem

Application

Increasing & Decreasing:-

If first derivative is greater than zero, then the functh is strictly increasing function. and if it is (≥ 0) then it is only increasing function.

$f'(x) > 0 \Rightarrow$ strictly increasing.

$f'(x) \geq 0 \Rightarrow$ — increasing.

$f'(x) \leq 0 \Rightarrow$ — decreasing

$f'(x) < 0 \Rightarrow$ strictly decreasing

Q) $f(x) = ax + b$, $a > 0$. check wether functh is strictly inc and decreas.

$f'(x) = a$.

$f'(x) > 0$

Strictly increasing.

Q) $f(x) = 10 - 6x - x^2$. Find the intervals for which the functh is increasing or decreasing.

$f'(x) = -6 - 2x$

$= -2(3+x)$

$f'(x) = 0$

$x = -3$

$-\infty$ ——— -3 ——— ∞

$(-\infty, -3)$ $(-3, \infty)$

for $(-\infty, -3)$
$\Rightarrow$ Increasing

for $(-3, -\infty)$
$\Rightarrow$ Decreasing.

July 23, 14

Q.) Which function is strictly decreasing in $(0, \pi/2)$

i.) $\cos x$ ii.) $\cos 2x$ iii.) $\cos 3x$ iv.) $\tan x$

?.

$f(x) = \cos x$

$f'(x) = -\sin x$.

$0 < x < \pi/2$ $f(x) = \cos 2x$ ——— Dec.

$0 < 2x < \pi$ $f'(x) = -\sin 2x \cdot 2$

$f(x) = \cos 3x$ ——— Doesn't decrease.

$0 < 3x < \frac{3\pi}{2}$ $f'(x) = -3\sin 3x$

Q.) $f(x) = \sin x + \cos x$ $[0, 2\pi]$. Break the interval into increasing interval and decreasing interval.

Sol^n

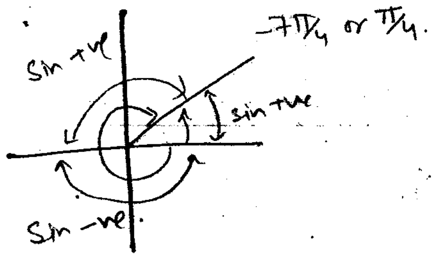
$$f'(x) = \cos x - \sin x$$
$$= \left(\frac{\cos x - \sin x}{\sqrt{2}}\right)\sqrt{2}$$
$$= (\sin \pi/4 \cos x - \cos \pi/4 \sin x)\sqrt{2}$$
$$= \sqrt{2}\sin(\pi/4 - x)$$

$$0 < x < 2\pi$$
$$0 > -x > -2\pi$$
$$\pi/4 > \pi/4 - x > -2\pi + \pi/4$$

$(\pi/4, \pi)$

$(\pi, 2\pi)$

$(0, \pi/4)$



Increasing.	Decreasing	Increasing
$\frac{\pi}{4} < \pi/4 - x < \pi$	$\pi \cancel{<} \pi/4 - x \cancel{<} 2\pi$	$0 < \frac{\pi}{4} - x < \pi/4$
$0 < -x < +3\pi/4$	$3\pi/4 \cancel{<} -x \cancel{<} +7\pi/4$	$-\pi/4 < -x < 0$
Clockwise $0 > x > -3\pi/4$,	$-3\pi/4 \cancel{>} x \cancel{>} -7\pi/4$	$+\pi/4 > x > 0$
Anti-clockwise $2\pi > x > 5\pi/4$.	$5\pi/4 > x > \pi/4$	

Q)

$I \rightarrow \left(0, \frac{5\pi}{4}\right) \quad \left(\frac{\pi}{4}, 0\right)$ we will change it into anti-clockwise dir.

$D \rightarrow \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

Q) $f(x) = \sin^4 x + \cos^4 x \qquad [0, \pi/2]$

$= \cancel{\sin^2 x + \cos^2 x}$

$\cancel{= 4\sin^3 x\,(\cos x) + 4\sin^3 x\,(-3}$

$f'(x) = \cancel{4\sin 3x\,(\cos x) + 4\cos^3 x\,(-\sin x)}$

$= \cancel{4\cos x\left[-\frac{\sin 3x + 3\sin x}{4}\right] \mp 4\sin x\,[}$

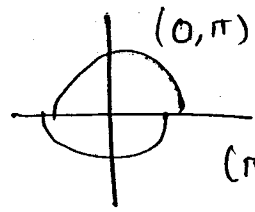
$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$

$= 4\sin x\cos x\,(\sin^2 x - \cos^2 x)$

$= -2\sin 2x\cos 2x$

$= -\sin 2(2x)$

$= -\sin 4x$



$0 < 4x < \pi$	$\pi < 4x < 2\pi$
$0 < x < \pi/4$	$\pi/4 < x < \pi/2$
Decreasing.	Increasing.

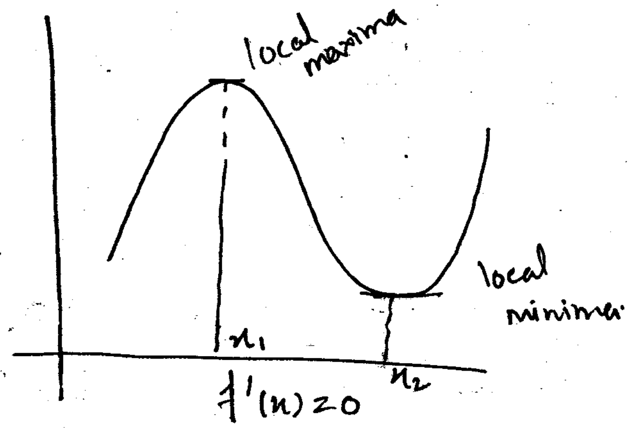
Maxima & Minima :—

i) find $f'(x)$

ii) put $f'(x) = 0$ & find values of x.

iii) Check the value of x for $f''(x)$

iv) If $f''(x) = -ve$ $\hookrightarrow$ Maxima.



$f''(x) = +ve$
$\hookrightarrow$ minima.

$f''(x) = 0$
we will proceed to $f'''(x)$

v) If the $f'''(x) = +ve$ or $-ve$
$\rightarrow$ point of inflexion occurs.

$f'''(x) = 0$
we will proceed to $f''''(x)$

vi) for all even derivative function is maxima or minima and
for " odd " " " point of inflexion.

Absolute maxima.

→ Absolute maxima & minima are the highest value & lowest value of the function in the given interval. Such that the end pts of the interval may or mayn't be the critical pts.

→ In case of Absolute question, u will not go for second derivative.

Q.) $2x^3 - 9x^2 + 12x - 5 = f(x)$. Find the local maxima & minima values.

$f'(x) = 6x^2 - 18x + 12$

$\Rightarrow 6x^2 - 18x + 12 = 0$

$6x^2 - 12x - 6x + 12 = 0$

$6x(x-2) - 6(x-2) = 0$

$(6x-6)(x-2) = 0$

$x = 1, 2$

$f''(x) = 12x - 18$

$f''(1) = -6$ So $x = 1 \Rightarrow$ Maxima.

$f''(2) = 6$ So $x = 2 \Rightarrow$ Minima.

$f(1) = 2 - 9 + 12 - 5 = 0$

$f(2) = 16 - 36 + 24 - 5 = -1$

Q> for same above question, $[0,3]$ find absolute maxima & minima?

Solⁿ

$~~x = 2, 1~~$

$f(1) = 0$

$f(2) = -1$

$f(0) = -5$ absolute minima.

$f(3) = 4$ absolute maxima.

0, 3 are not critical pts as it doesn't come under value we find.

Q> $f(x) = 12x^{4/3} - 6x^{1/3}$. Find the local maximum & minima values.

$f'(x) = 12 \cdot \frac{4}{3} x^{1/3} - 6 \cdot \frac{1}{3} x^{-2/3}$

$= 16x^{1/3} - \frac{2}{x^{2/3}}$

$\Rightarrow 16x^{1/3} - 2/x^{2/3} = 0$

$x = 1/8$.

$f''(x) = 16 \cdot 1/3\, x^{-2/3} - 2(-2/3) x^{-5/3}$

$f''(1/8) = 16 \cdot 1/3 (1/8)^{-2/3} - 2(-2/3)(1/8)^{-5/3}$

$= 64$ +ve

$x = 1/8$ = Minima.

$f(1/8) = -2.25$.

Q> for above $[-1, 1]$. find abs. maxᵐ, minᵐ

$f(-1) = 18$ ~~abs maxima~~

$f(1/8) = -2.25$ abs minima

$f(1) = 6$ ~~abs. maxima~~.

$(-1)^{1/3} = x = -1$

$-1 = x^3$

$x^3+1 = 0$

$\underbrace{(x+1)}_{\text{real root}} \underbrace{(x^2+1-x)}_{\text{Imaginary root}} = 0$

$x = -1$

$(-1)^{4/3} = x$

$(-1)^{4/3} = x^3$

$x^3 - 1 = 0$

$\underbrace{(x-1)}_{\text{real}} \underbrace{(x^2+x+1)}_{\text{Imaginary}} = 0$

$x = 1$.

2) It is given that $x=1$, the $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value in the interval $[0,2]$. Then find the value of a.

$f(x) = x^4 - 62x^2 + ax + 9$

$f'(x) = 4x^3 - 124x + a$

$f'(1) = 4 - 124 + a = 0$

$\boxed{a = 120}$

2) The minimum value of $y = x^2$ in the interval $[-1,5]$ is?

$f(x) = x^2$

$f'(x) = 2x$

$f'(x) \Rightarrow 2x = 0$

$x = 0$

$f(0) = 0$

$f(-1) = 1$

$f(5) = 25$

Q) $f(x) = x^2 \cdot e^{-x}$. Find the maximum value?

e^{x}/e^{x} = never be zero or -ve

$f'(x) = e^{-x} \cdot 2x - x^2 e^{-x}$

$x \cdot e^{-x}(2-x) = 0$

$x = 0, 2$

$f''(x) = e^{-x}[2-2x] + [2x - x^2](-e^{-x})$

$= e^{-x}[2 - 2x + 2x - x^2] \Rightarrow e^{-x}[x^2 - 4x + 2]$

$f''(0) = 2$ +ve

$f''(2) = \frac{1}{e^2}[4-8+2]$

$= -ve$

$x = 2 \Rightarrow$ Maxima.

$f(2) = 2^2 \cdot e^{-2} = 4/e^2$

Q) $f(x) = (x^2-4)^2$

~~i)~~ It has only one minimum

~~ii)~~ " " two "

iii) " " 3 "

iv) " " 3 maximum

for any eqn of degree n there are (n-1) roots & minima & maxima.

$f'(x) = 2(x^2-4)\cdot 2x$

$= 4x(x^2-4)$

$\Rightarrow 4x(x+2)(x-2) = 0$

$x = 2, -2, 0$

$f''(x) = 12x^2 - 16$

$f''(2) = 48$ mini

$f''(-2) = 48$ "

$f''(0) = -16$ maxima

Q) A cubic polynomial wid real coeff

i) can possibly have no extrimum & no zero crossings.

ii) May have upto 3 extrimum & upto two zero crossings.

~~iii)~~ Cann't have more than 2 extrimum and 3 zero crossings.

iv) will alwaz have an equal no. of extrimum & zero crossings.

In general for n^{th} degree polynomial, it can't have more than n zero crossing (Roots) and (n-1) extrimums.

Q) ~~$f(x)$~~ $= e^y = x^{1/x}$. Then y has a

~~i)~~ Maxima at $x=e$

ii) Minima at $x=e$

iii) Maxima at $x = 1/e$

iv) Minima at $x = 1/e$

$e^y = x^{1/x}$

$f(x) = y = \frac{1}{x}\ln x$

$f'(x) = \frac{1}{x^2}(1-\ln x)$

$\boxed{x=e}$

$f''(x) = \dfrac{x^2(-\frac{1}{x}) - (1-\ln x)2x}{x^4}$

$f''(e) = \dfrac{-e}{e^4} = -ve$

$x = e \Rightarrow$ Maxima

Q.) The no. of distint extrimum for the curve $3x^4-16x^3-24x^2+37$

$$f(x) = 3x^4-16x^3-24x^2+37$$

$$f'(x) = 12x^3-48x^2-48x=0$$

$$x^3-4x^2-4x=0$$

$$\cancel{x^3(x-4)=4}$$

$$\cancel{x=\pm 2, 4}$$

$$x(x^2-4x-4)=0$$

$$x=0, +2, -2$$

no. of extrim = 3

July 28, 14

Application of Maxima & Minima :-

Q.) Divide 14 into 2 parts such that there pdt is maximum.

14 — Given.

x , $y = (14-x)$

$$P = xy.$$

$$= x(14-x)$$

$$\frac{dP}{dx} = 14x-x^2$$

$$= 14-2x \qquad 2(x-7)=0 \Rightarrow x=7$$

$$\frac{d^2P}{dx^2} = -2$$

Q.) Prove that area of a rectangle of a given perimeter is maximum when it is a square.

Given things are alwe constant.

$$P = 2x+2y.$$

$$y = \frac{P-2x}{2}$$

$$A = xy \Rightarrow x\left(\frac{P-2x}{2}\right)$$

$$\frac{dA}{dx} = \frac{1}{2}(P-4x)$$

$$P = 4x \Rightarrow x = P/4, \; y = P/4$$

$$\frac{d^2A}{dx^2} = -4.$$

Q.) A wire of length 28 m is to be cut into two pieces. One of the pieces is converted into a square and the other into a circle, from where the wire be cut so that the combined area is minimum.

$$l = 28 \quad fm$$

$$x \quad\quad y.$$

$$L = x + y$$

$$y = 28 - x$$

Square $A_1 = x \times x$

Circle $a_2 = 2\pi r = y.$

$$r = \frac{28-x}{2\pi}$$

$$a_2 = \pi r^2 = \cancel{\pi} \times \frac{(28-x)^2}{4\pi \cdot \pi}$$

$$\frac{dA}{dx} = x^2 + \cancel{\pi}\left(\frac{28-x}{2\pi}\right)^2 \quad \frac{1}{4\pi}(28-x)^2$$

$$= 2x + \frac{\cancel{2}}{\cancel{4}} \frac{1}{2\pi}(28-x)(-1)$$

$$= 2x + \frac{1}{\pi}\left(14 - \frac{x}{2}\right)(-1)$$

$$= 2x + \frac{x}{2\pi_{6.28}} - \frac{14}{\pi}$$

$$= \frac{5x}{2} - 14$$

$$\frac{5x}{2} = 14$$

$$x = 28/5$$

$$\frac{x}{2}\pi = 56 - 2x$$

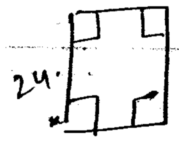
$$x\left(\frac{\pi}{2} + 2\right) = 56$$

$$x = \frac{112}{4+\pi}$$

$$\frac{d^2A}{dx} = \frac{5}{2}.$$

Q.) A square piece of tin of side 24cm is to be made into a box without top. By cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that a volume of a box is maximum.

Side = 24 cm =



side of box formed = 24−2x

$V = (24-2x)^{\cancel{3}\,2} \cdot x$ ⋮ height might be changed.

$\frac{dV}{dx} = 3(24-2x)^2(-2)$

$= -\cancel{6}\,(24-2x)^2 = 0$

$x = 12, 12.$

$2 \cdot x \cdot (24-2x) \cdot (-2) + (24-2x)^2 = 0$

$4x\,\cancel{(24-2x)} = (24-2x)^{\cancel{2}}$

$6x = 24$

$\boxed{x = 4}$

$\cancel{\frac{d^2V}{dx^2} = -12(24-2x)(-2)}$

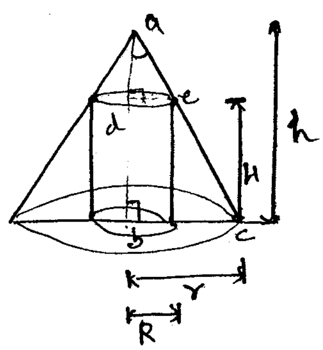
$\cancel{= 24(24-2x)}$

$\frac{d^2V}{dx^2} = \cancel{4} - 6$

Q.) ~~Show that~~ the height of right circular cylinder of maximum volume that can be inscribed in a given right circular cone is.

RC cone = given

RC Cylinder



$\frac{ab}{ad} = \frac{bc}{de}$

$\frac{h}{h-H} = \frac{r}{R}$

$R = \frac{r}{h}(h-H)$

$V = \pi R^2 H$

$V = \underbrace{\pi \frac{r^2}{h^2}}_{c} (h-H)^2 H.$

$V = c\,(h^2H + H^3 - 2hH^2)$

$V' = c\,(h^2 + 3H^2 - 4hH) = 0$

$H = \frac{+4h \pm \sqrt{16h^2 - 12h^2}}{6}$

$= \frac{4h \pm 2h}{6} = h, h/3$

Q> A box w/o top having square base is to have a given volume. The area of the box is

$$V = x^2 y$$

$$y = V/x^2$$

$$A = x^2 + (xy)4$$

$$A = x^2 + \left(x\frac{V}{x^2}\right)4$$

$$\frac{dA}{dx} = x^2 + 4\frac{V}{x}$$

$$A' = 2x + -4\frac{V}{x^2}$$

$$\frac{4V}{x^2} = 2x$$

$$2V = x^3$$

$$x = (2V)^{1/3}$$

Q> Show that the volume of a right circular cone inscribed in a given sphere of radius R is maximum.

PARTIAL DERIVATIVES :-

If z is a function of two or more independent variables then the partial derivative of z, w.r.t any 1 of the independent variable is the ordinary derivative of z wrt that variable keeping all the other variables as constant.

$$z = f(x, y, t) \qquad z = x^2 + y^2$$

$$\left.\frac{\partial z}{\partial x}\right|_{y,t} = 2x$$

$$\boxed{\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}}$$

Q.) If $u = \tan^{-1} \frac{x^2+y^2}{x+y}$ find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

$$\left.\frac{\partial u}{\partial x}\right|_y = \frac{1}{1+\left(\frac{x^2+y^2}{x+y}\right)} \cdot \left\{\frac{(x+y)\, 2x - (x^2+y^2)}{(x+y^2)\, 2}\right\}$$

$$\left.\frac{\partial u}{\partial y}\right|_x =$$

Q.) $u = x^y$. find $\frac{\partial^3 u}{\partial x^2 \partial y} = ?$ & Prove it $= \frac{\partial^3 u}{\partial x^2 dy} = \frac{\partial^3 u}{\partial x \partial y \partial x}$

$$\left.\frac{\partial u}{\partial y}\right|_x = x^y \ln x$$

$$\cancel{\left.\frac{\partial^2 u}{\partial x \partial y}\right|_y = y\, x^{y-1} + \ln x}$$

$$= (1 + \ln x)$$

$$\cancel{\left.\frac{\partial^3 u}{\partial x \partial x \partial y}\right|_y} = \left(1 + \frac{1}{x}\right)$$

$$\left.\frac{\partial^2 u}{\partial x \partial y}\right|_y = \ln x \; y \cdot x^{y-1} + x^y \frac{1}{x}$$

$$= x^{y-1}(1 + y\ln x)$$

$$= (1 + y\ln x)(y-1)x^{y-2} + x^{y-1}(y/x)$$

$$= x^{y-2}\,(y + y^2 \ln x - y - y\ln x + y]$$

$$\left.\frac{\partial u}{\partial x}\right|_y = y\, x^{(y-1)}$$

$$\left.\frac{\partial^2 u}{\partial y \partial x}\right|_x = y x^{y-1} \ln x + x^{y-1} \cdot 1$$

$$= x^{y-1}(y\ln x + 1)$$

$$\left.\frac{\partial^3 u}{\partial x (\partial y \partial x)}\right|_y = x^{y-1}\left(y \cdot \frac{1}{x}\right) + (y\ln x + 1)\,(y-1)x^{y-2}$$

$$= x^{y-2}\left[y + y^2 \ln x - y\ln x + y - 1\right]$$

Q.) i) $x = r\cos\theta$, $y = r\sin\theta$, $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$

$\cancel{\frac{\partial r}{\partial x}}$ $r = \frac{x}{\cos\theta}$ $\quad r = x/\cos\theta$

$= \frac{1}{r}\cos^{-1}x$

$= \frac{1}{r}\sin^{-1}x$

$\cancel{\frac{\partial r}{\partial x} = \frac{\cos\theta \cdot 1 + x(-\sin\theta)}{\cos^2\theta}}$

$\cancel{= \frac{\cos\theta - x\sin\theta}{\cos^2\theta}}$

$\cancel{\frac{\partial x}{\partial r} = \cos\theta}$

$\frac{\partial r}{\partial x} = \frac{1}{\cos\theta}$

$\left.\frac{\partial x}{\partial r}\right|_\theta = \cos\theta$

$r = \sqrt{x^2+y^2}$

$\left.\frac{\partial r}{\partial x}\right|_y = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}} = \frac{r\cos\theta}{r}$

$= \cos\theta$

Q.) ii) $\frac{1}{r}\frac{\partial x}{\partial \theta} = r\,\frac{\partial \theta}{\partial x}$

$\left.\frac{\partial x}{\partial \theta}\right|_r = -r\sin\theta$

$\theta = \tan^{-1} y/x$

$\left.\frac{\partial \theta}{\partial x}\right|_y = \frac{1}{1+(y/x)^2}\, y\left(-1/x^2\right)$

$= -\frac{1}{1+\tan^2\theta}\,\frac{r\sin\theta}{r^2\cos^2\theta}$

$= -\frac{1}{\sec^2\theta}\,\frac{1}{r}\,\frac{\sin\theta}{\cos^2\theta}$

$= -\frac{1}{r}\sin\theta$

$\frac{1}{r}\,\cancel{r}\,\frac{\partial x}{\partial \theta} = -\sin\theta$

$r\,\frac{\partial \theta}{\partial x} = -\sin\theta$

Homogeneous functions:—

$u = f(x,y) = x+y/\sqrt{x}+\sqrt{y}$

~~f(tx, ty) = tx+ty~~

then by replacing $x \to tx$, $y \to ty$,
if we regain the original function. Then the function is homogeneous and the degree of t is known as degree of homogenity.

$$f(tx,ty) = \frac{tx+ty}{\sqrt{tx}+\sqrt{ty}} = \frac{t}{\sqrt{t}}\,\frac{x+y}{\sqrt{x}+\sqrt{y}} = t^{1/2} f(x,y),$$

Euler's theorem of first Order:—

If u is a homogenous function, in x & y of degree of homogenity n then,

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu}$$

Q) $u = \sin^{-1} x/y + \tan^{-1} y/x$. find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{1-(x/y)^2}}\cdot 1 + \frac{1}{1+(y/x)^2}\cdot\left(-\frac{1}{2x^2}\right)$$

$$= \frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}} - \frac{1}{2\left(\frac{x^2+y^2}{x^2}\right)x^2}$$

$$= \frac{1}{y\sqrt{y^2-x^2}} - \frac{1}{2(x^2+y^2)}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{1-(x/y)^2}}$$

$f(x,y) = \sin^{-1} x/y + \tan^{-1} y/x$

$f(tx,ty) = \sin^{-1}\frac{tx}{ty} + \tan^{-1}\frac{ty}{tx}$

$= t^0 \sin^{-1} x/y + \tan^{-1} y/x$

$n = 0$.

$\therefore x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

Q)

Q) $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$,

$$u = f(x,y) = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$f(tx,ty) = \sin^{-1}\left(\frac{tx+ty}{\sqrt{tx}+\sqrt{ty}}\right)$$

$$u = \sin^{-1} t^{1/2}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$\sin u = t^{1/2}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$\Rightarrow x\frac{\partial \sin u}{\partial x} + y\frac{\partial \sin u}{\partial y} = \frac{1}{2}\sin u$$

$$x \cos u \frac{\partial u}{\partial x} + y \cos u \frac{\partial u}{\partial y} = \frac{1}{2} \sin u$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$$

Eulers theorem of Second Order:-

$$\boxed{x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u}$$

Composite functions:-

$$z = f(x, y) \qquad x = \phi(t), \quad y = \varphi(t)$$

if indenpent is one then, total derivatives used, if independent is more then one. then partial derivative is used.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\Big|_y \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y}\Big|_x \cdot \frac{dy}{dt}$$

$$z = f(x, y) \qquad x = \phi(t, s), \quad y = \varphi(t, s)$$

$$\frac{\partial z}{\partial t}\Big|_s = \frac{\partial z}{\partial x}\Big|_y \frac{\partial x}{\partial t}\Big|_s + \frac{\partial z}{\partial y}\Big|_x \frac{\partial y}{\partial t}\Big|_s$$

$$\frac{\partial z}{\partial s}\Big|_t = \frac{\partial z}{\partial x}\Big|_y \frac{\partial x}{\partial s}\Big|_t + \frac{\partial z}{\partial y}\Big|_x \frac{\partial y}{\partial s}\Big|_t$$

Q.) $u = \sin^{-1}(x-y)$, $x = 3t$, $y = 4t^3$. find $\frac{du}{dt}$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\Big|_y \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y}\Big|_x \cdot \frac{dy}{dt}$$

$$\frac{\partial u}{\partial x}\Big|_y = \frac{1}{\sqrt{1-[(x-y)]^2}} \cdot 1 \quad , \quad \frac{\partial u}{\partial y}\Big|_x = \frac{1}{\sqrt{1-[(x-y)^2]}} (-1)$$

$$\frac{dx}{dt} = 3 \quad , \quad \frac{dy}{dt} = 12t^2$$

$$\frac{du}{dt} = \frac{3}{\sqrt{1-(x-y)^2}} - \frac{12}{\sqrt{1-(x-y)^2}}$$

~~At t=1~~

$$\frac{du}{dt} = \frac{3}{\sqrt{1-(3t-4t^3)}} - \frac{12t^2}{\sqrt{1-(3t-4t^3)}}$$

At $t=0$

$$= 3$$

Q) Transform $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar co-ordinates.

$u = (r,\theta)\,(r;\theta) = f(x,y)$

$$\frac{\partial u}{\partial x}\Big|_y = \frac{\partial u}{\partial r}\Big|_\theta \frac{\partial r}{\partial x}\Big|_y + \frac{\partial u}{\partial \theta}\Big|_r \frac{\partial \theta}{\partial x}\Big|_y$$

$$\frac{\partial}{\partial x}\Big|_y = \cos\theta \frac{\partial}{\partial r}\Big|_\theta - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\Big|_r$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \left(\cos\theta\frac{\partial}{\partial r}\Big|_\theta - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\Big|\right)\left(\cos\theta\frac{\partial u}{\partial r} - \frac{\sin\theta}{r}\frac{\partial u}{\partial\theta}\right)$$

$$= \cos\theta\frac{\partial}{\partial r}\Big|_\theta\left(\cos\theta\frac{\partial u}{\partial r}\right) + \cos\theta\frac{\partial}{\partial r}\Big|_\theta\left(-\frac{\sin\theta}{r}\frac{\partial u}{\partial\theta}\right)$$

$$- \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left[\cos\theta\frac{\partial u}{\partial r}\right) - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\Big|_r\left(-\frac{\sin\theta}{r}\frac{\partial u}{\partial\theta}\right)$$

$$= \cos^2\theta\frac{\partial^2 u}{\partial r^2} - \sin\theta\cos\theta\left[\frac{1}{r}\frac{\partial^2 u}{\partial r\partial\theta} + \frac{\partial u}{\partial\theta}\left(-\frac{1}{r^2}\right)\right]$$

$$- \frac{\sin\theta}{r}\left[\cos\theta\frac{\partial^2 u}{\partial\theta\partial r} + \frac{\partial u}{\partial r}(-\sin\theta)\right] + \frac{\sin\theta}{r^2}\left[\sin\theta\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial u}{\partial\theta}(\cos\theta)\right]$$

$$\frac{\partial u}{\partial y}\Big|_x = \frac{\partial u}{\partial r}\Big|_\theta\frac{\partial r}{\partial y}\Big|_x + \frac{\partial u}{\partial\theta}\Big|_r\frac{\partial\theta}{\partial y}\Big|_x$$

$$=$$

July 30, 14

Jacobian

another way to find composite funct.
$z = f(u,v) \ne \phi(x,y)$

If u & v are ~~functions of two independent~~ variables x & y, then

$$\begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix}$$

is known as 'jacobian of u & v w.r.t x & y and is denoted by $\boxed{J\frac{(u,v)}{(x,y)}}$

Q.) Find $J\frac{(x,y)}{(r,\theta)}$

$x = r\cos\theta$
$y = r\sin\theta$

$$\begin{vmatrix} \partial x/\partial r & \partial x/\partial \theta \\ \partial y/\partial r & \partial y/\partial \theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & +r\cos\theta \end{vmatrix}$$

$$r\cos^2\theta - (-r\sin^2\theta) = r(\cos^2\theta + \sin^2\theta)$$
$$= \underline{\underline{r}}$$

$\theta = \frac{\sin\theta}{y}$

Q.) Find $J\frac{(r,\theta)}{(x,y)}$

$$\begin{vmatrix} \partial r/\partial x & \partial r/\partial y \\ \partial \theta/\partial x & \partial \theta/\partial y \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\frac{1}{r}\sin\theta & \frac{1}{r}\cos\theta \end{vmatrix}$$

$$= \frac{1}{r}\cos^2\theta + \frac{1}{r}\sin^2\theta = 1/r$$

Taylor Series :-

(for one variable) or expansion of any funct^n and any pt.

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{\lfloor 2}f''(a) + \ldots\ldots$$

→ about $x = a$

by default $\boxed{x=0}$

Mclaurian about $x = 0$

$\boxed{x=0}$

$$f(x) = f(0) + x f'(a) + \frac{x^2}{\lfloor 2} f''(0) + \ldots$$

Q) Write expansion for $\sin x$, $\cos x$, $\tan x$ about $x=0$.

$f(x) = \sin x \qquad f(0) = 0$

$f'(x) = \cos x \qquad f'(0) = 1$

$f''(x) = -\sin x \qquad f''(0) = 0$

$f'''(x) = -\cos x \qquad f'''(0) = -1$

$$0 + x \cdot 1 + \frac{x^2}{\underline{|2}} \cdot 0 + \frac{x^3}{\underline{|3}}(-1) + \dots$$

$$\boxed{\sin x = x - \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} + \dots}$$

for $\cos x$

$f(x) = \cos x \qquad f(0) = 1$

$f'(x) = -\sin x \qquad f'(0) = 0$

$f''(x) = -\cos x \qquad f''(x) = -1$

$f'''(x) = \sin x \qquad f'''(0) = 0$

$f''''(x) = \cos x \qquad f''''(0) = 1.$

$$f(0) + x f'(0) + \frac{x^2}{\underline{|2}} f''(0) + \frac{x^3}{\underline{|3}} f'''(0) + \dots$$

$$1 + x \cdot 0 + \frac{x^2}{\underline{|2}} \cdot (-1) + \frac{x^3}{\underline{|3}} \cancel{f'''(0)} (0) + \frac{x^4}{\underline{|4}} (1) + \dots$$

$$\boxed{\cos x = 1 - \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} + \dots}$$

for $\tan x$

$f(x) = \tan x \qquad f(0) = 0$

$f'(x) = \sec^2 x \qquad f'(0) = 1$

$f''(x) = 2\sec x \cdot \sec x \tan x \qquad f''(0) = 0$

$\Rightarrow 2\sec^2 x \tan x$

$f'''(x) = 2\tan x \{2\sec x \cdot \sec x \tan x)$

$+ 2\sec^2 x \sec^2 x$

$= 4\sec^2 x \tan^2 x + 2\sec^4 x \qquad f'''(0) = 2$

$f^{IV}(x) = 2\left[4\sec^4 x \tan x + 2\left(\sec^2 x\, 2\tan x \sec^2 x + \tan^2 x\, 2\sec^2 x \tan x\right)\right]$ $\quad f^{IV}(x) = 0$

$\tan x = 0 + x\cdot 1 + \frac{x^2}{\lfloor 2}\cdot 0 + \frac{x^3}{\lfloor 3}\cdot 2 + \frac{x^4}{\lfloor 4}\cdot 0 + \cdots$

$$\boxed{\tan x = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots}$$

Q.) $f(x) = \ln|1+x|$

$f(x) = \ln(1+x)$ $\quad f(0) = 0$

$f'(x) = \frac{1}{(1+x)}$ $\quad f'(0) = 1$

$f''(x) = -\frac{1}{(1+x)^2}$ $\quad f''(0) = -1$

$f'''(x) = \frac{2}{(1+x)^3}$ $\quad f'''(0) = 2$

$f^{IV}(x) = \frac{-6}{(1+x)^4}$ $\quad f^{IV}(0) = -6$

$0 + x\cdot 1 + \frac{x^2}{\lfloor 2}\cdot(-1) + \frac{x^3}{\lfloor 3}\cdot 2 + \frac{x^4}{\lfloor 4}(-6) + \cdots$

$$\boxed{\ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots}$$

Q) $f = e^{\sin x}$ $\quad f(0) = 1$

$f'(x) = e^{\sin x}\cdot\cos x$ $\quad f'(0) = 1$

$f''(x) = \cos x\, e^{\sin x}\cos x + e^{\sin x}(-\sin x)$ $\quad f''(0) = 1$

$= e^{\sin x}(\cos^2 x - \sin x)$

$f'''(x) = e^{\sin x}(-\cos x) + (-\sin x)e^{\sin x}.\cos x$ $\quad f'''(0) = -1 + 1 = 0$

$+ e^{\sin x}(-\sin 2x) + \cos^3 x\, e^{\sin x}$

$= 0 + x\cdot 1 + \frac{x^2}{\lfloor 2}(+1) + \frac{x^3}{\lfloor 3}2 + \frac{x^4}{\lfloor 4}(-6) + \cdots$

$1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3}\times 0 + \cdots$

$$\boxed{e^{\sin x} = 1 + x + \frac{x^2}{\lfloor 2} + \frac{\cancel{x^3}}{\lfloor 3}}$$

In the taylor series expansion of $e^{\sin x}$ at ~~(x-2)~~ about $(x=2)$ the coeff of $(x-2)^4$ is.

$$f(x) = f(2) + (x-2) f'(2) + \frac{(x-2)^2}{\lfloor 2} f''(2) + \frac{(x-2)^3}{\lfloor 3} f'''(2) + \frac{(x-2)^4}{\lfloor 4} f^{iv}(2) + \ldots$$

$$\text{Coeff} = \frac{1}{\lfloor 4} f''''(2)$$

$$= \frac{e^2}{\lfloor 4}$$

$$(x-a)^n \leftarrow \boxed{\text{Coeff} = \frac{1}{\lfloor n} f^n(a)}$$

Q.> Gate Question

Same quest. e^4 ||

$f(x) = e^{-x}$. linear approximation about $x=2$ will be.

$$f(x) = f(2) + (x-2) f'(2)$$

$$= e^{-2} + (x-2)(-e^{-2})$$

$$= e^{-2}[1 - x + 2]$$

$$= \frac{3-x}{e^2}$$

Q.> Which of the following functions would have only odd powers of x in its taylor series expansion about $x=0$.

a.> ~~cosec x^0~~ $\cos x^2$

b.> $\sin x^2$

~~c.>~~ $\sin x^3$

d.> $\cos x^3$

~~$x^3 \sin x$~~

Q.) In the taylor series expansion of $e^{(x+\sin x)}$ about the point $x = \pi$. A coefficient of $(x-\pi)^3$

$$(x-a)^n = \frac{1}{\lfloor n} f^n(a)$$

$$(x-\pi)^3 = \frac{1}{\lfloor 3} f'''(\pi)$$

$$f(x) = e^x + \sin x \quad f'(x) = e^x + \cos x \quad f''(x) = e^x - \sin x$$

check it

$$= \frac{e^\pi + 1}{\lfloor 3}$$

Taylor Theorem for function of two variables:-

$$f(x,y) = f(x,y) + [(x-a) f_x(a,b) + (y-b) f_y(a,b)]$$

$$+ \frac{1}{\lfloor 2} [(x-a)^2 f_{xx}(a,b) + (y-b)^2 f_{yy}(a,b) + 2(x-a)(y-b) f_{xy}(a,b)]$$

$$+ \frac{1}{\lfloor 3} [(x-a)^3 f_{xxx}(a,b) + (y-b)^3 f_{yyy}(a,b) + 3(x-a)^2(y-b) f_{xxy}(a,b) + 3(x-a)(y-b)^2 f_{xyy}(a,b)].$$

$f_x(x,y) = \sin y \cdot e^x$	$f_x(0,0) = 0$
$f_y(x,y) = e^x \cos y$	$f_y(0,0) = 1$
$f_{xx}(x,y) = \sin y\, e^x$	$f_{xx}(0,0) = 0$
$f_{yy}(x,y) = e^x(-\sin y)$	$f_{yy}(0,0) = 0$
$f_{xy}(x,y) = \cos y \cdot e^x$	$f_{xy}(0,0) = 1$
$f_{xxx}(x,y) = \sin y\, e^x$	$f_{xxx}(0,0) = 0$
$f_{yyy}(x,y) = e^x(-\cos y)$	$f_{yyy}(0,0) = -1$
$f_{xxy}(x,y) = \cos y\, e^x$	$f_{xxy}(0,0) = 1$
$f_{xyy}(x,y) = (-\sin y) e^x$	$f_{xyy}(0,0) = 0.$

Maxima & Minima :-

$$z = f(x,y)$$

$$\frac{\partial z}{\partial x} = 0 \qquad \frac{\partial z}{\partial y} = 0$$

$$x_1, x_2 \qquad y_1, y_2$$

$$(x_1, y_1) \quad (x_2, y_2)$$

$$\frac{\partial^2 z}{\partial x^2} = r, \quad \frac{\partial^2 z}{\partial y^2} = t, \quad \frac{\partial^2 z}{\partial x \partial y} = s$$

$\boxed{(rt - s^2) > 0}$ then only maxima & minima exist

$\boxed{\begin{array}{l} r > 0 \Rightarrow \text{Minima.} \\ r < 0 \Rightarrow \text{Maxima.} \end{array}}$

$\boxed{(rt - s^2) < 0 \Rightarrow \text{No critical pt.}}$

$\boxed{rt - s^2 = 0}$ may or may not be exist.

Q A box having a rectangular base open at top is to have a given volume. Find the dimensions of a box if the surface area is maximum.

$$v = xyh$$

$$h = \frac{V}{xy}$$

$$A = xy + 2xh + 2yh$$

$$A = xy + 2x \cdot \frac{V}{xy} + 2y\frac{V}{xy}$$

$$A = \left(xy + \frac{2V}{y} + \frac{2V}{x}\right)$$

$$\left.\frac{\partial A}{\partial x}\right|_y = y - \frac{2V}{x^2} \Rightarrow y - \frac{2V}{x^2} = 0 \Rightarrow y = \frac{2V}{x^2}$$

$$\left.\frac{\partial A}{\partial y}\right|_x = x - \frac{2V}{y^2} \Rightarrow x - \frac{2V}{y^2} = 0 \Rightarrow x = 2V/y^2$$

$$y = \frac{2V}{(2V/y^2)^2} \Rightarrow y = \frac{2V}{4V^2} \cdot y^4 \Rightarrow y\left(y^3 \cdot \frac{1}{2V} - 1\right) = 0$$

Not possible so rejected
$$\begin{cases} y \neq 0, & y = (2V)^{1/3} \\ x \neq 0, & x = (2V)^{1/3} \end{cases}$$

$$r = \frac{4V}{x^3} \Rightarrow \frac{4V}{(2V)'} = 2$$

$$t = \frac{4V}{y^3} \Rightarrow \frac{4V}{(2V)'} = 2$$

$$s = 1 \Rightarrow 1.$$

$$rt - s^2 \Rightarrow 2 \times 2 - 1 = 3 > 0$$

$$r > 0 \Rightarrow \text{Minima}.$$

$$h = \frac{V}{(2V)^{2/3}} = \frac{V^{1/3}}{2^{2/3}} = \left(\frac{V}{4}\right)^{1/3}$$

Q:> Find the maximum & minimum distance of the pt (3,4,12) from unit sphere.

Sol^n.

$$x^2 + y^2 + z^2 = 1$$

Aug 04, 14

Integral Calculus

1.) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

2.) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

Power formula → can be applied only when the diff of bracket in num is const

3.) $\int \sin x \, dx = -\cos x + C$

4.) $\int \cos x \, dx = \sin x + C$

5.) $\int \sec^2 x \, dx = \tan x + C$

6.) $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

7.) $\int \sec x \tan x \, dx = \sec x + C$

8.) $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + C$

9.) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

10.) $\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$

11.) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

12.) $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$

13.) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

14.) $\int \frac{-dx}{x\sqrt{x^2-1}} = \operatorname{cosec}^{-1} x + C$

15.) $\int \frac{1}{x} dx = \ln x + C$

16.) $\int \frac{1}{\text{linear}} dx = \frac{\ln|\text{linear}|}{\frac{d}{dx}(\text{linear})} + C$ | $\int \frac{1}{(ax+b)} = \frac{\ln(ax+b)}{a} + C$

17.) $\int a^x dx = \frac{a^x}{\ln a} + c$

18.) $\int e^x dx = e^x + c$

19.) $\int \tan x\, dx = \ln|\sec x| + c = -\ln|\cos x| + c$

20.) $\int \cot x\, dx = \ln|\sin x| + c$

21.) $\int \sec x\, dx = \ln|\sec x + \tan x| + c = \ln|\tan(\pi/4 + x/2)| + c$

22.) $\int \operatorname{cosec} x\, dx = \ln|\operatorname{cosec} x - \cot x| + c = \ln|\tan x/2| + c$

Q.) $\int (3x-4)^{1/3} dx$

$= \cancel{\frac{4}{3}\,\frac{(3x-4)}{4/3}\cdot\frac{3}{3}}$

$= \cancel{\frac{3(3x-4)}{3}}$

$= \frac{(3x-4)^{1/3+1}}{(1/3+1)\cdot 3} + C$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)\frac{d}{dx}(ax+b)}$

This formula is

The differential of bracket of the variable should be constant otherwise formula is not valid.

Q.) $\int \left(x + \frac{1}{x}\right)^3 dx$

$= \cancel{\frac{3\,(x + 1/3x)^{4}}{4}}$

$= \int \left(x^3 + \frac{1}{x^3} + 3x^2\frac{1}{x} + 3x\frac{1}{x^2}\right) dx$

$= \left[\frac{x^4}{4} + \frac{x^{-2}}{(-2)} + \frac{3x^2}{2} + 3\ln x + c\right]$

Q.) $\int \sec^2(7-4x)\,dx$

$= \frac{\tan(7-4x)}{-4} + C$

Q.) $\int \frac{x^4+1}{x^2+1}\,dx$

$$\begin{array}{r|l} & x^2-1 \\ x^2+1 & x^4+1 \\ & \underline{x^4+x^2} \\ & -x^2+1 \\ & \underline{-x^2-1} \\ & 2 \end{array}$$

$\int \frac{f(x)}{g(x)}\,dx$

If $\deg f(x) > \deg g(x)$

$g(x) \overline{)\, f(x)}$

$= \int \left\{ (x^2-1) + \frac{2}{x^2+1} \right\} dx$

$= \frac{x^3}{3} - x + 2\tan^{-1}x + C$

Q.) $\int \sin^2 x/2 \, dx$

$\int \frac{1-\cos x}{2}\,dx$

$= \frac{1}{2}x - \frac{\sin x}{2} + C$

if there are even & odd powers of sin & cos then that powers are converted into linear angles.

Q.) $\int \frac{dx}{1-\sin x}$

$= \int \frac{dx}{1-\cos(\pi/2 - x)}$

$= \int \frac{dx}{2\sin^2(\pi/4 - x/2)}$

$= \frac{1}{2}\int \operatorname{cosec}^2(\pi/4 - x/2)\,dx = \frac{-\cot(\pi/4 - x/2)}{-1/2}$

$$\boxed{\begin{aligned} \sin n\theta &= 2\sin\frac{n\theta}{2}\cos\frac{n\theta}{2} \\ 1+\cos n\theta &= 2\cos^2 n\theta/2 \\ 1-\cos n\theta &= 2\sin^2 n\theta/2 \end{aligned}}$$

Q.) $\int \frac{1}{\sin^2 x \cos^2 x} dx$

$\sin 2x = 2 \sin x \cos x$

$\frac{\sin^2 2x}{4} = \sin^2 x \cos^2 x$

$\int \frac{4}{\sin^2 2x} dx$

$\int \operatorname{cosec}^2 2x \, dx$

$-\frac{\cot 2x}{2} + C$

Q.) $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

$\int \frac{(\sin^4 x + \cos^4 x)\,(\sin^2 x)^2 - (\cos^2 x)^2}{1 - 2\sin^2 x \cos^2 x} dx$

$\int \frac{(\sin^4 x + \cos^4 x)\,(\sin^2 x + \cos^2 x)\,(\sin^2 x - \cos^2 x)}{1 - 2\sin^2 x \cos^2 x} dx$

$\sin^2 x + \cos^2 x = 1$

$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cos^2 x$

$\int \frac{(\cancel{1 - 2\sin^2 x \cos^2 x})\,(\sin^2 x - \cos^2 x)}{(\cancel{1 - 2\sin^2 x \cos^2 x})} dx$

$\cos 2x = \cos^2 x - \sin^2 x$

$\int -\cos 2x \, dx$

$= -\frac{\sin 2x}{2} + C$

Q.) $\int 2\sin 5x \sin 3x \, dx$

$\int (\cos 2x - \cos 8x) \, dx$

$= \frac{\sin 2x}{2} - \frac{\sin 8x}{8} + C$

Whenever sin A & cos B are in multiplication then we change this multiplication to add & sub.

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

Q.) $\int \frac{x}{\sqrt{x-a}}\,dx$

$\int \frac{\text{linear}}{\text{linear}}\,dx$.

Match Num Deno.

$\int \frac{x-a+a}{\sqrt{x-a}} = \int \frac{x-a}{\sqrt{x-a}}\,dx + \int \frac{a}{\sqrt{x+a}}\,dx$

$= \int \sqrt{x-a}\,dx + \int \frac{a}{\sqrt{x+a}}\,dx$

$= \left(\frac{(x-a)^{3/2}}{3/2}\right) + a\left(\frac{(x+a)^{+1/2}}{1/2}\right)$

Q.) $\int \frac{dx}{1+\cos x} = \int \frac{dx}{2\cos^2 x/2}$

$= \frac{1}{2}\int \cancel{\frac{dx}{2x}}$

$= \cancel{\cos^{2} x/2}\ \frac{1}{2}\int \sec^2 x/2\,dx = \frac{1}{2}\,\frac{\tan x/2}{1/2} + c$

Q.) $\int \sqrt{1-\sin x}\,dx$

$= \int (\sqrt{1-\cos \pi/2 - x})\,dx$

$= \int \sqrt{2\sin^2(\pi/4 - x/2)}\,dx$

$= \sqrt{2}\int \sin(\pi/4 - \cancel{x}/2) + \cancel{c}$

Q.) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\,dx$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \frac{1-\tan^2 x}{1+\tan^2 x}$

$\int \frac{2\cos^2 x - 1 - (2\cos^2\alpha - 1)}{\cos x - \cos\alpha}\,dx$

$\int \frac{2\cos^2 x - 2\cos^2\alpha}{\cos x - \cos\alpha}\,dx$

$$2\int \frac{(\cos x+\cos\alpha)\,\cancel{(\cos x-\cos\alpha)}}{\cancel{(\cos x-\cos\alpha)}}\,dx$$

$$2\int(\cos x+\cos\alpha)\,dx$$

$$2\{\sin x+\cos\alpha\cdot x+c\}$$

Q.) $\displaystyle\int \frac{(a^x+b^x)^2}{a^x b^x}\,dx$

$$\int\left\{\frac{(a^x)^2}{a^x b^x}+\frac{(b^x)^2}{a^x b^x}+\frac{2\cancel{a^x b^x}}{\cancel{a^x b^x}}\right\}dx$$

$$\int\left\{\frac{a^x}{b^x}+\frac{b^x}{a^x}+2\right\}dx$$

$$\int\left(\frac{a}{b}\right)^x dx+\int\left(\frac{b}{a}\right)^x dx+\int 2\,dx \qquad \int a^x\,dx=\frac{a^x}{\ln a}$$

$$\frac{a/b}{\ln(a/b)}+\frac{b/a}{\ln(b/a)}+2x+c$$

Q.) $\displaystyle\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}\,dx$

$\cancel{(\sin^6 x)}\cdot\cancel{\int \tan^2 x\cdot\operatorname{cosec}^2 x\,dx}+\cancel{\int \operatorname{cosec}^2 x}$

$\cancel{\int \tan^2 x\,\operatorname{cosec}^2 x\,dx+\int \cot^2 x\cdot\sec^2 x\,dx}$

$$\int\frac{(\sin^2 x)^3+(\cos^2 x)^3}{\sin^2 x\cos^2 x}\,dx$$

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$\int\frac{(\sin^2 x+\cos^2 x)(\sin^4 x+\cos^4 x-\sin^2 x\cos^2 x)}{\sin^2 x\cos^2 x}\,dx$$

$$\int \frac{(\sin^2 x+\cos^2 x)(1-2\sin^2 x\cos^2 x-\sin^2 x\cos^2 x)}{\sin^2 x\cos^2 x)}$$

$$\int \frac{1}{\sin^2 x\cos^2 x}\,dx - \int \frac{3\,\cancel{\sin^2 x\cos^2 x}}{\cancel{\sin^2 x\cos^2 x}}\,dx$$

$$\cancel{\int \operatorname{cosec}^2 x\sec^2 x\,dx - \int 3\,dx}$$

I from ques. 67)

$$= -4\frac{\cot 2x}{2} + 3x + C$$

Q.) $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$

$$= \int \tan^{-1}\sqrt{\frac{1-\cos(\pi/2-x)}{1+\cos(\pi/2-x)}}\,dx$$

$$= \int \tan^{-1}\sqrt{\frac{2\sin^2(\pi/4-x/2)}{2\cos^2(\pi/4-x/2)}}\,dx$$

$$= \int \tan^{-1}\tan(\pi/4-x/2)\,dx$$

$$= \pi/4\,x - \frac{x^2}{4} + C$$

Q.) $\int \frac{x^2}{\sqrt{1-x^6}}\,dx$

$x^3 = t$

$3x^2dx = dt$

$x^2dx = dt/3$

$$\int \frac{dt}{\sqrt{1-t^2}}$$

$$= \sin^{-1}t + C$$

$$= \sin^{-1}x^3 + C$$

Q.) $\int \frac{(x^4-x)^{1/4}}{x^5}\,dx$

$\int \frac{\left(x^{4\cdot 1/4}\left(1-\frac{x}{x^4}\right)^{1/4}\right)}{x^5}\,dx$

$\int \frac{\left(1-\frac{1}{x^3}\right)^{1/4}}{x^4}\,dx$

$\frac{1}{x^3}=t$

$\frac{dx}{x^4}$

$\frac{1}{x^3}=t$

$\frac{dx}{x^4}=-\frac{dt}{3}$

$-\int \frac{(1-t)^{1/4}}{3}\,dt$

$\cancel{-\left[\frac{t}{3}-\frac{t^2}{6}\right]dt}$

$\Rightarrow -\frac{1}{3}\int (1-t)^{1/4}\,dt$

$\Rightarrow -\frac{1}{3}\,\frac{(1-t)^{(1/4+1)}}{(1/4+1)(-1)}$

$\Rightarrow +\frac{1}{3}\,\frac{(1+1/x^3)^{5/4}}{5/4}$

$\Rightarrow \frac{4}{15}\left(1+\frac{1}{x^3}\right)^{5/4}+C$

Q.) $\int \frac{\sin 2x}{5+4\sin^2 x}\,dx$

$\sin^2 x = t$

$2\sin x\cdot\cos x\,dx = dt$

$\cos^2 x$

$-2\cos x\cdot\sin x$

Applicable for both sin & cos

$\int \frac{dt}{5+4t}$

$=\frac{\ln|5+4t|}{4}+C$

$=\frac{\ln|5+4\sin^2 x|}{4}+C$

Q.) $\int \frac{\tan x}{a+b\tan^2 x} = \int \frac{\sin x/\cos x}{a+b\frac{\sin^2 x}{\cos^2 x}}\,dx$

$= \int \frac{\sin x\cos x}{a\cos^2 x+b\sin^2 x}\,dx = \frac{1}{2}\int \frac{2\sin x\cos x}{a\cos^2 x+b\sin^2 x} = \frac{1}{2}\int \frac{\sin 2x}{a\cos^2 x+b\sin^2 x}$

$\sin^2 x = t$

$= \frac{1}{2}\int \frac{dt}{a(1-t)+bt}$

$= \frac{1}{2}\int \frac{dt}{t(b-a)+a}$

$= \frac{\ln|t(b-a)+a|}{(b-a)}$

$= \frac{\ln|\sin^2 x(b-a)+a|}{b-a}.$

Q.) $\int \frac{1}{e^x+e^{-x}}\,dx = \int \frac{dx}{e^x+1/e^x} = \int \frac{dx}{\frac{e^x.e^x+1}{e^x}}$

$\Rightarrow \int \frac{e^x\,dx}{(e^x)^2+1}$ $\qquad e^x = t$, $e^x dx = dt$

$\Rightarrow \int \frac{dt}{t^2+1}$

$\Rightarrow \tan^{-1} t + C$

$\tan^{-1} e^x + C$

Q.) $\int \frac{dx}{(1+x)^{1/2}+(1+x)^{1/3}}$

if there is any qustion in which basis are same and powers are in fraction change th substitution or change the basis

$1+x = t^6$

$dx = 6t^5\,dt$

$$\Rightarrow \int \frac{6^{3} t^5 dt}{t^3+t^2}$$

$$\Rightarrow \int \frac{6\, t^{\cancel{8}\,3} dt}{\cancel{t^2}(t+1)}$$

$$\Rightarrow 6 \int \frac{t^3 dt}{t+1}$$

$$\begin{array}{r|l} & t^2-t+1 \\ \hline t+1 & t^3 \\ & \cancel{t^3}+t^2 \\ & -\quad - \\ \hline & -t^2 \\ & \mp \cancel{t^2} \mp t \\ \hline & t \\ & \pm \cancel{t} \pm 1 \\ \hline & -1 \end{array}$$

$$\Rightarrow 6 \int \left(t^2-t+1 - \frac{1}{t+1} \right) dt$$

$$\Rightarrow 6 \left| \frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1| \right|$$

$$\Rightarrow 6 \left\{ \frac{(1+x)^{1/6 \cdot 3}}{3} - \frac{(1+x)^{1/6 \cdot 2}}{2} + (1+x)^{1/6} - \ln\left|(1+x)^{1/6}+1\right| \right\}$$

$$\Rightarrow 6 \left\{ \frac{(1+x)^{1/2}}{3} - \frac{(1+x)^{1/3}}{2} + (1+x)^{1/6} - \ln\left|(1+x)^{1/6}+1\right| \right\}$$

Q.) $\int \frac{\cos x}{\cos(x-a)} dx = \int \frac{\cos(x-a+a)}{\cos(x-a)} dx$

$$= \int \left[\frac{\cancel{\cos(x-a)}\cos a}{\cancel{\cos(x-a)}} - \frac{\sin(x-a)\sin a}{\cos(x-a)} \right] dx$$

$$= \int \cos a \, dx - \int \tan(x-a) \cdot \sin a \, dx$$

$$= x \cdot \cos a - \frac{\ln \sec|x-a|}{1} \cdot \sin a$$

Q.) $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$

$$\Rightarrow \frac{1}{\cos(b-a)} \int \frac{\cos[(x-a)-(x-b)]}{\sin(x-a)\cos(x-b)} dx$$

we can never introduce variable
→ introduce functions
if in denomin introduce
Sin Sin, cos cos → sin
Sin cos, cos sin → cos.

$$= \frac{\cos(x-a)\cos(x-b)}{\sin(x-a)\cos(x-b)} + \frac{\sin(x-a)\sin(x-b)}{\sin(x-a)\cos(x-b)}$$

$$= \frac{\cancel{\sin(x-a)\cos(x-b)}}{\cancel{\sin(x-a)\cos(x-b)}} - \frac{\cancel{\cos(x-a)\sin(x-b)}}{\sin(x-a)\cos(x-b)}$$

$$= \cancel{1} - \cancel{\cot}(x-a)\tan(x-b$$

$$= \int \cot(x-a)\,dx + \int \tan(x-b)\,dx$$

$$= \left\{ \frac{\ln|\sin(x-a)|}{1} + \frac{\ln|\sec(x-b)|}{1} + C \right\} \frac{1}{\cos(b-a)}$$

Q.) $\displaystyle\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}} = \int \frac{dx}{\sqrt{\sin^3 x(\sin x\cos\alpha + \cos x\sin\alpha)}}$

$$\cos\alpha + \frac{\cos x}{\sin x}\sin\alpha$$

$$= \int \frac{dx}{\sin^2 x\sqrt{\cos\alpha + \cot x\sin\alpha}} = \int \frac{\operatorname{cosec}^2 x\,dx}{\sqrt{\cos\alpha + \cot x\sin\alpha}} = t$$

$$-\operatorname{cosec}^2 x\,dx = \frac{dt}{\sin\alpha}$$

$$= -\frac{1}{\sin\alpha}\int \frac{dt}{\sqrt{t}} = \frac{t^{-1/2+1}}{-1/2+1}\cdot\frac{(-)1}{\sin\alpha}$$

$$= -2\,\frac{(\cos\alpha + \cot x\sin\alpha)^{1/4}}{\sin\alpha}$$

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Q.) $\displaystyle\int \frac{dx}{x-\sqrt{x}}$

$$x = t^2$$
$$dx = 2t\,dt$$

$$\int \frac{2t\,dt}{t^2-t} = 2\int \frac{t\,dt}{t(t-1)} = 2\int \frac{dt}{(t-1)}$$

$$= 2\ln|t-1| + C$$
$$= 2\ln|\sqrt{x}-1| + C$$

Q.) $\int \frac{(1-e^x)}{(1+e^x)} dx$

$\Rightarrow \int \frac{-(-1+e^x)+1-1}{(1+e^x)} dx$

$\Rightarrow$ $\cancel{\int -(e^x+1+1)}$ $\Rightarrow \int -\frac{(e^x+1)}{(1+e^x)} dx + 2\int \frac{dx}{(1+e^x)}$

$\Rightarrow$

Q.) $\int \frac{\sin 5x}{\sin 3x \sin 2x} dx$

$\Rightarrow \int \frac{\sin(3x+2x)}{\sin 3x \sin 2x} dx$

$\Rightarrow \int \frac{2\sin 3x \cos 2x + \cos 3x \sin 2x}{\sin 3x \sin 2x} dx$

$\Rightarrow \int \cot 2x \, dx + \int \cot 3x \, dx$

$\frac{\ln|\sin 2x|}{2} + \frac{\ln|\sin 3x|}{3} + C$

Q.) $\int \frac{\cos ax - \cos bx}{\sin ax + \sin bx} dx$

$= \int \frac{-2\sin x\frac{(a+b)}{2} \sin x\frac{(a-b)}{2}}{2\sin x\frac{(a+b)}{2} \cos x\frac{(a-b)}{2}} dx$

$= -\int \tan x\frac{(a-b)}{2} dx$

$= \ln\left|\sec x\frac{(a-b)}{2}\right| + C$

1.> $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c$

2.> $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right|+c$

3.> $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right|+c$

4.> $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$x = a\tan\theta$

5.> $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$ $x>a$

$x = a\sec\theta$

6.> $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c$ $a>x$

$x = a\sin\theta$

If coeff of x is -ve then ans will be in $\sin^{-1}$ otherwise ans will be in log. If ans is in log, it will be log of variable + same function.

Q.> $\int \frac{ax^3+bx}{x^4+c}\,dx$

$x^4+c=t$

$4x^3+0=dt$

$\frac{1}{4}\int\frac{(4ax^3+4bx)dx}{t}$

$\frac{a}{4}\int\frac{dt}{t} + b\int\frac{x}{x^4+c}dx$; $x^2=t$, $2x\,dx=dt$

$\frac{a}{4}\ln t + \frac{b}{2}\int\frac{dt}{t^2+(\sqrt{c})^2}$

$\frac{a}{4}\ln t - \frac{b}{2}\tan^{-1}\frac{x^2}{c}$

If degree of $f(x)$ is one less than degree of $g(x)$ then you will set the derivation $g(x)$ in $f(x)$

Q.> $\int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}$

$= \int \frac{\sec^2 x\,dx}{1+2\tan^2 x}$; $\tan x = t$

$\int\frac{dt}{1+2t^2} = \frac{1}{2}\int\frac{dt}{t^2+(1/\sqrt{2})^2}$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{1/\sqrt{2}}+c$

Q.) $\int \frac{dx}{3x^2+13x-10}$ Completing square method:

$= \int \frac{dx}{3\left(x^2+\frac{13x}{3}-\frac{10}{3}\right)}$

$= \int \frac{dx}{3\left(x^2+\frac{13}{3}x-\frac{10}{3}+\frac{169}{36}-\frac{169}{36}\right)}$

$= 3\int \frac{dx}{\left(x+\frac{13}{6}\right)^2-\left(17/6\right)^{2}}$

$\frac{1}{2-\frac{17}{6}} \ln\left|\frac{x+\frac{13}{6}+\frac{17}{6}}{x+13/6+17/6}\right| + c$

Q.) $\int \frac{x}{x^4-x^2+1}dx$ (By taking completing square method)

Put $x^2 = t$

$x = dt/2$

$= 1/2 \int \frac{dt}{t^2-t+1+1/4-1/4}$

$= 1/2 \int \frac{dt}{(t-1/2)^2+(\sqrt{3}/2)^2 t}$

<u>H.W</u>

(Q) $\int \frac{dx}{x+2\sqrt{x}+3}$

Q) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$

Integration by Parts :-

PLATE

$$\int \underset{\mathrm{I}}{u}\,\underset{\mathrm{II}}{v}\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx + c$$

{funct^n whose diff is possible} {whose integration is possible}

Q.) $\int x \sin x \, dx$

$\Rightarrow x\int \sin x\, dx - \int [1 \int v dx] dx$

$= -x\cos x + \int \cos x\, dx$

$= -x\cos x + \sin x + c$

Q) $\int \sec^3 x \, dx$

$= \int \underset{\mathrm{I}}{\sec x}\ \underset{\mathrm{II}}{\sec^2 x}\, dx$

$= \sec x \tan x - \int [\sec x \tan x\ \tan x] dx$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$

$= \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

$I = \sec x \tan x - I + \ln(\sec x + \tan x) + c$

$I = \frac{1}{2}[\sec x \tan x + \ln(\sec x + \tan)] + c$

Q.) $\int e^{ax} \cos bx\, dx$

$= e^{ax}\frac{\sin bx}{b} - \int [e^{ax} \cdot a \frac{\sin bx}{b}] dx$

$= e^{ax}\frac{\sin bx}{b} - a\int e^{ax}\frac{\sin bx}{b} dx$

$= \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$

$$\boxed{\int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]}$$

Q.) $\int e^{ax}\sin bx\,dx$

$$= -e^{ax}\frac{\cos bx}{b} + \int \left[e^{ax}\cdot a\,\frac{\cos bx}{b}\right]dx$$

$$= -e^{ax}\frac{\cos bx}{b} + \frac{a}{b}\left[e^{ax}\frac{\sin bx}{b} + e^{ax}\right.$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$

$$\boxed{\int [fx + f'(x)]e^{x}dx = e^{x}f(x)+c}$$

Q.) $\int e^{x}\left(\frac{\sin x\cos x - 1}{\sin^2 x}\right)$

$$= \int e^{x}\left(\frac{\sin x\cos x}{\sin^2 x} - \frac{1}{\sin^2 x}\right)$$

$$= \int e^{x}\int \cot x - (-\operatorname{cosec}^2 x)\,dx$$

$$= e^{x}(\operatorname{cosec}^2 x)$$

Q.) $\int \frac{2}{1+\cos 2x} + \frac{\sin 2x}{1+\cos 2x}$

$$= \frac{2}{2\cos^2 x} + \frac{2\sin x\cos x}{2\cos^2 x}$$

$$= \sec^2 x + \tan x$$

Q.) $\int \left[\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right]dx$

$\ln x = t$ $\qquad \int \left(\frac{1}{t} - \frac{1}{t^2}\right)e^{t}dt$

$x = e^{t}$

$dx = e\cdot dt$

$$= e^{t}\frac{1}{t} + c$$

$$= e^{\ln x}\frac{1}{\ln x} + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1} x/a + c$$

$$\int \sqrt{a^2+x^2}\,dx = x/2\sqrt{a^2+x^2} + a^2/2 \ln\left|x+\sqrt{a^2+x^2}\right| + c$$

$$\int \sqrt{x^2-a^2}\,dx = x/2\sqrt{x^2-a^2} - a^2/2 \ln\left|x+\sqrt{x^2-a^2}\right| + c$$

$\frac{\text{Variable}}{2}$ (same fun) $+ \frac{a^2}{2}\int \frac{dx}{\text{function}}$.

Aug 06, 14

$f(x)\cdot g(x)$

$$\int \frac{f(x)}{g(x)}\,dx$$

Q.) $\int (x+1)\sqrt{2x^2+3}\,dx$

$$\frac{1}{4}\int 4x\sqrt{\underbrace{2x^2+3}_{t}}\,dx + 1\int\sqrt{2x^2+3}\,dx$$

$$\frac{1}{4}\int\sqrt{t}\,dt + \sqrt{2}\int\sqrt{x^2+\left(\sqrt{3/2}\right)^2}\,dx$$

$$\frac{1}{4}\frac{t^{3/2}}{3/2} + \sqrt{2}\left[\frac{x}{2}\sqrt{x^2+(\sqrt{3/2})^2} + \cancel{3}/2 \ln\left|x+\sqrt{x^2+(\sqrt{3/2})^2}\right|\right] + c$$

Q.) $\int \frac{dx}{5+4\sin x}$

$$\int \frac{dx}{A\cos x + B\sin x + C}$$

$$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$$

$$\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$$

$$\Rightarrow \int \frac{dx}{5+4\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right)}$$

$$\Rightarrow \int \frac{\sec^2 x/2\,dx}{5+5\tan^2 x/2 + 8\tan x/2}$$

$\tan x/2 = t$

$\frac{1}{2}\sec^2 x/2\,dx = dt$

$$= 2\,\cancel{\sec^2}\int \frac{dt}{5t^2+8t+5}$$

$$= \frac{2}{5}\int \frac{dt}{t^2+\frac{8t}{5}+1}$$

Q.) $\int \frac{dx}{5+7\cos x+\sin x}$

$= 2\int \frac{dt}{-2t^2+2t+12} = -1\int \frac{dt}{t^2-t-6} = -1\int \frac{dt}{t^2-t-6+1/4-1/4}$

$= -1\int \frac{dt}{(t+\frac{1}{2})^2-(\frac{5}{2})^2} = \int \frac{dt}{(\frac{5}{2})^2-(t+\frac{1}{2})^2}$

$= \frac{1}{2\cdot\frac{5}{2}} \ln \left| \frac{\frac{5}{2}+t+\frac{1}{2}}{\frac{5}{2}-(t+\frac{1}{2})} \right|$

Q.) $\int \frac{\sin x+8\cos x}{2\sin x+3\cos x}$

$\int \frac{A\sin x+B\cos x}{C\sin x+D\cos x}\,dx$

$\boxed{\text{Num} = P\frac{d}{dx}(\text{Den})+Q(\text{Den})}$

$\sin x+8\cos x = P(2\cos x-3\sin x) + Q(2\sin x+3\cos x)$

$1 = -3P+2Q$
$8 = 2P+3Q$

$-2 = +6P \mp 4Q$
$24 = 6P+9Q$

$26 = 13Q$
$Q = \frac{26}{13} = 2$
$P = \frac{8-6}{2} = 1$

$\int \frac{1(2\cos x-3\sin x)\,dx}{2\sin x+3\cos x}$, with $t = 2\sin x+3\cos x$

$\int \frac{dt}{t} + 2\int 1\,dx$

$\ln t + 2x + C$

$\ln|2\sin x+3\cos x| + 2x + C$

Q.) $\int \frac{x^2+1}{x^4+1}\,dx$

$\Rightarrow \int \frac{x^2(1+1/x^2)\,dx}{x^2(x^2+1/x^2)}$

$x - 1/x = t$

$(1+1/x^2)\,dx = t$

$(x-\frac{1}{x})^2 = t^2$

$x^2+1/x^2-2 = t^2$

$$= \int \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}}$$

Q.) $\displaystyle\int \frac{1}{x^4+x^2+1}\,dx$

$$\Rightarrow \frac{1}{2}\int \frac{(x^2+1)-(x^2-1)}{x^4+x^2+1}\,dx$$

$$\Rightarrow \frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1} - \frac{1}{2}\int \frac{(x^2-1)}{x^4+x^2+1}\,dx$$

$$\frac{1}{2}\left\{\frac{1}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} - \frac{1}{2}\int \frac{dt}{t^2-1}\,dx\right\}$$

$$\frac{1}{2}\left\{\frac{1}{\sqrt{3}} \tan^{-1} \frac{(x-\frac{1}{x})}{\sqrt{3}} - \frac{1}{2} \ln\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right|\right\} + C$$

Q.) $\displaystyle\int \sqrt{\tan\theta}\,d\theta$

$\sqrt{\tan\theta} = t$

$\tan\theta = t^2$

$\sec^2\theta\,d\theta = 2t\,dt$

$d\theta = \dfrac{2t\,dt}{1+t^4}$

$$= \int \frac{t\,2t\,dt}{1+t^4} = \int \frac{2t^2\,dt}{t^4+1}$$

$$= \int \frac{t^2+t^2+1-1}{t^4+1}\,dt$$

$$= \int \frac{t^2+1}{t^4+1}\,dt + \int \frac{t^2-1}{t^4+1}\,dt$$

H.W

$$\int (\sqrt{\tan\theta}+\sqrt{\cot\theta})\,d\theta.$$

Partial Fractions

degree of Num should be less than the degree of denominator only then partial fraction is applicable.

$$\frac{(ax+\cancel{b})}{(cx+d)\,(px+q)} = \frac{A}{cx+d} + \frac{B}{px+q}$$

$$\frac{(ax+b)}{(cx^2+d)} = \frac{Ax+B}{cx^2+d}$$

$$\int\frac{(ax+b)}{(cx+d)(px+q)^2} = \frac{A}{cx+d} + \frac{B}{px+q} + \frac{C}{(px+q)^2}$$

Q.) $\int\frac{x+5}{(x-3)(x-2)}dx = \frac{A}{x-3} + \frac{B}{x-2}$

$x+5 = A(x-2) + B(x-3)$

for x^2 $0 = 0$

x $1 = A+B$

$5 = -2A-3B$

$-2 = -2A-2B$

$5 = -2A-3B$

$-7 = B$

$A = +8$

$$= \int\left\{\frac{8}{x-3} + \frac{(-7)}{x-2}\right\}dx$$

$$= 8\ln|x-3| - 7\ln|x-2| + C$$

for case of linear factors: put $x=0$ and find consts for each factor.

Q.) $\int\frac{2x+5}{(x+2)(3x-3)(x+1)}$

$$\int\frac{1}{(-9)\quad(x+2)(-1)} + \int\frac{9}{(3)(3x-3)(2)} + \int\frac{3}{(x+1)\,(1)(-6)}$$

Q.) $\int \frac{(x^2+1)}{(x^2+2)\cdot(2x^2+1)}$ | $x^2 = t$ (trick)

$\int \frac{-1}{(x^2+2)(-3)}dx + \int \frac{1/2}{(2x^2+1)\,3/2}dx$

$\frac{1}{3}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{1}{3}\cdot\frac{1}{\sqrt{1/2}\cdot 2}\tan^{-1}\sqrt{2}x + C$

Q.) $\int \frac{dx}{x(x^n+1)}$

$= \int \frac{x^{n-1}}{x^{n-1}\cdot x(x^n+1)}dx = \int \frac{x^{n-1}dx}{x^n(x^n+1)}$ | $x^n = t$

$\Rightarrow \frac{1}{n}\int \frac{dt}{t(t+1)}$

$= 1/n\left[\frac{1}{t} - \frac{1}{t+1}\right] = \frac{1}{n}\ln\frac{t}{t+1}$

$$\boxed{\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\ln\left|\frac{x^n}{x^n+1}\right| + c}$$

Q.) $\int \frac{x^3+x+1}{x^2 \cancel{+} 1}dx$

$x + \cancel{2/x}$

$x^2-1 \,\overline{)\, x^3+x+1}$

$\quad \cancel{x^3} \mp x$

$\quad 2x+1$

$\quad \cancel{2x \ldots 1/2}$

tab tk karo jab tk degree baabr ya kam n hojaye.

$\int x\,dx + \int \frac{2x+1}{x^2-1}dx$

$\frac{x^2}{2} +$

$$\int \frac{dx}{\text{linear}\sqrt{\text{linear}}} = \sqrt{\text{linear}} = t$$

$$\int \frac{dx}{\text{linear}\sqrt{\text{Quadratic}}} \qquad \text{Linear} = 1/t$$

$$\int \frac{dx}{\text{Quadratic}\sqrt{\text{linear}}} = \sqrt{\text{linear}} = t$$

$$\int \frac{dx}{\text{Quadratic}\sqrt{\text{Quadratic}}} = \underset{\text{variable}}{x = 1/t} \qquad dx = -1/t^2$$

Q.) $\int \frac{dx}{(x+1)\sqrt{x+2}}$

Q.) $\int \frac{dx}{(x^2-4)\sqrt{x+1}}$ $\quad \sqrt{\text{linear}} = t$

Q.) $\int \frac{dx}{(x+1)\sqrt{x^2+1}}$

Q.) $\int \frac{dx}{(x^2+1)\sqrt{x^2+1}}$ $\quad \left\{ \begin{matrix} x \to t \to z \to u \\ z \to x \end{matrix} \right\}$ $\quad$ 3 time Substitution

$x = 1/t$.

Definite Integral

$$\int_a^b f(x)\,dx$$

$\downarrow$ x

$$\frac{x^2}{2}\Big|_a^b$$

$$\left[\frac{b^2}{2} - \frac{a^2}{2}\right]$$

Here c is not added as we get the final result

Properties of definite Integral :—

1.) $\int_a^b f(x)\,dx = \int_a^b f(z)\,dz$; $x = z$, $dx = dz$

2.) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3.) $\int_a^b f(x)\,dx = \int_a^b f(b+a-x)\,dx$ | $\int_5^8 (x^2+2)\,dx = \int (13-x)^2+2\,dx$

*4.) $\boxed{\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx}$

5.) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ f(x) is even

$= 0$ f(x) is odd

$f(x) = \sin x$

$f(-x) = \sin(-x) = -\sin x$

$\boxed{f(-x) = -f(x)}$ odd function

6.) $\int_0^{2a} f(x)\,dx = 2\int_a^{2a} f(x)\,dx$ f(x) is even

$= 0$ f(x) is odd.

$f(x) = \cos x$

$f(-x) = \cos(-x) = \cos x$

$\boxed{f(x) = f(x)}$ even function

Q) $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$

$= \int_0^{\pi/2} \frac{\sin(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx$

$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$

$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\cos x + \sin x} dx$

$= x \Big|_0^{\pi/2}$

$2I = \pi/2$

$I = \pi/4$

Q) $\int_0^1 x(1-x)^n dx$

$= \int_0^1 (1-x)(1-1+x)^n dx$

$= \int_0^1 (1-x) x^n dx$

$= \int_0^1 \{x^n - x^{n+1}\} dx$

$= \left| \frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right|_0^1$ $\cancel{+c}$

$=$

Q) $I = \int_0^{\pi/2} \ln \tan x \, dx$

$I = \int_0^{\pi/2} \ln \cot x \, dx$

$2I = \int_0^{\pi/2} (\ln \tan x + \ln \cot x) dx$

$$2I = \int_0^{\pi/2} \ln \tan x \cdot \cot x \cdot dx.$$

Q.> $I = \int_0^{\pi/2} \ln \sin x\, dx = \frac{-\pi}{2} \ln 2$

Q> $\int_{-2}^{2} |x+1|\, dx$

$x+1 = 0$
$x = -1$

$\int_{-2}^{-1} -(x+1)\, dx + \int_{-1}^{2} (x+1)\, dx$

value of fun before breaking pt is −ve after breaking pt value to +ve.

Q.> $\int_{-1}^{2} |x^3 - x|\, dx$

$x^3 - x = 0$
$x(x^2-1) = 0$
$x(x-1)(x+1) = 0$
$x = 0, +1, -1$

$\int_{-1}^{0} (x^3 - x) + \int_{0}^{1} -(x^3 - x)$
$+ \int_{1}^{2} (x^3 - x)\, dx$

Q.) $\int_0^2 [x^2] dx$ not continuous at non integral pts.

$0 < x < 2$

$0 < \underset{x^2}{\underline{x^2}} < 4$

$\Rightarrow 0 < x^2 < 1$, $\overbrace{0 < x < 1}^{dx} \downarrow$

$\Rightarrow 1 < x^2 < 2$, $1 < x < \sqrt{2}$

$\Rightarrow 2 < x^2 < 3$, $\sqrt{2} < x < \sqrt{3}$

$\Rightarrow 3 < x^2 < 4$, $\sqrt{3} < x < 2$

$$\int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^2 3\,dx$$

Q.) $I = \int_0^{\pi/2} \ln \sin x\,dx = -\pi/2 \ln 2$

$\cancel{I \neq 2}\ I = \int_0^{\pi/2} \ln \cos x\,dx$

$I + I = \int_0^{\pi/2} (\ln \cos x + \ln \sin x)\,dx$

$2I = \int_0^{\pi/2} \ln \dfrac{2 \sin x \cos x}{2}$

$= \int_0^{\pi/2} \ln \dfrac{\sin 2x}{2}$

$= \int_0^{\pi/2} \ln \sin 2x - \int_0^{\pi/2} \ln 2$

$2x = t$
$2dx = dt$

Unit also changes.

$= \frac{1}{2} \int_0^{2\pi/2} \ln \sin t\,dt$

$= 2\int_0^{\pi/2} \ln \sin t\,dt$

$\int_0^{2a} f(x)\,dx = 2\int_0^a f\,dx$

$f(x) = \ln \sin x$

$f(-x)$
$= \ln \sin(-x)$
$= \ln(-\sin x)$
$= \ln \sin x.$

Aug 11, 14

1.) $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx \quad = 1/2$

2.) $\int_1^\infty \frac{1}{x^3}\,dx$

3.) $\int_0^1 xe^x\,dx$

4.) $\int_{-a}^{a} (\sin^6 x + \sin^7 x)\,dx$

5.) $\int_0^{\pi/4} \frac{1-\tan x}{1+\tan x}\,dx$

6.) $\phi(x) = \int_0^{x^2} \sqrt{t}\,dt$. Find $d\phi$

7.) $\int_{-8}^{8} (\sin^{293} x + x^{295})\,dx$

8.) $\int_0^\pi \ln(1+\cos x)\,dx$

9.) $\int_{-\pi/2}^{\pi/2} |\sin x|\,dx$

1.) $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\,dx$

$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{2(\sqrt{x}+\sqrt{a-x})}\,dx$

$2I = \frac{x}{2}\Big|_0^a$

$I = a/2$

2.) $I = \int_0^1 \underset{I}{x}\,\underset{II}{e^x}\,dx$

$xe^x - \int 1 \cdot e^x dx$

$\left[xe^x - e^x\right]_0^1$

$(1 \cdot e^1 - e^1) - (0 \cdot e^0 - e^0)$

$= 0 - (-1)$

$= 1$

3.) $I = \int_{-a}^{a} (\sin^6 x + \sin^7 x)\,dx$ $\qquad 2\int_0^a \sin^6 x\,dx$

5.) $\int_0^{\pi/4} \frac{1 - \tan x}{1 + \tan x}\,dx$

$= \int_0^{\pi/4} \frac{\tan \pi/4 - \tan x}{1 + \tan x \cdot \tan \pi/4}$ $\qquad \tan \pi/4 = 1$

$= \int_0^{\pi/4} \tan(\pi/4 - x)\,dx$

$I = \int_0^{\pi/4} \tan(\pi/4 - \pi/4 + x)\,dx$

$= \int_0^{\pi/4} \tan x\,dx$

$= \log \sec \pi/4\, x + c$

6 Q.) $\left.\frac{t^{3/2}}{3/2}\right|_0^{x^2}$

$\frac{2x^3}{3} - 0 = \phi(x)$

$\frac{d\phi}{dt} = 0$

$\frac{d\phi}{dx} = 2 \cdot x^2$

7.) $\int_{-8}^{8} (\sin^{293} x + x^{295})\, dx$

both odd functions

$\Rightarrow 0$.

$I = \int_0^{\pi} \ln |2\cos^2 x/2|\, dx$

$= \ln 2 + \ln \cos^2 x/2$

$= \int \ln 2 + 2\int_0^{\pi} \ln \cos x/2$

$x/2 = t$

$= \pi \cdot \ln 2 + 2 \cdot 2 \int_0^{\pi/2} \ln \cos t$

$= \pi \ln 2 + 2 \cdot 2\left(-\frac{\pi}{2} \ln 2\right) = -\pi \ln 2$.

8.) $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx$

$2\int_0^{\pi/2} (\sin x)\, dx$.

$= 2\{\cos x\, dx\left.\right|_0^{\pi/2}$

$f'(x) > 0$	$f'(x) < 0$
$-2(3+x) > 0$	$(3+x) > 0$
$(3+x) < 0$	$x > -3$
$x < -3$	$(-3, \infty)$
$(-\infty, -3)$	

Here we suppose in starting.

Q.) $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6((x-3)(x-2))$

$f'(x) = 0$

$x = 3, 2$

Check any value in between interval and check them in $f'(x)$

$-\infty \quad 2 \quad 3 \quad \infty$

$(-\infty, 2)$ I $\quad (2, 3)$ D $\quad (3, \infty)$ I

Solution of inequality :-

$(x-a)(x-b) > 0$

< 0

i) To make the coeff. of x as +ve one.

$(x+3)(2x+5) > 0$

$2(x+3)(x+5/2) > 0$

ii) Write the factors into general form.

$\boxed{2(x-(-3))(x-(-5/2)) > 0}$

iii) Now look at the sign of inequality.

Case I ≥ 0

then the value of x is greater than the greater value and lesser then the lesser value.

$x > -5/2$

$x < -3$

Case II : '<0'

then the value of x is in between the two value

$$\boxed{2(x-(-3))\,(x-(-5/2))<0}$$

$\Rightarrow \quad -3<x<-5/2 \quad \Rightarrow (-3,-5/2)$

Q.) $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x-3)(x-2)$

$f'(x) > 0$

$(x-3)(x-2) > 0$

$x > 3$

$x < 2$

$(-\infty, 2) \cup (3, \infty)$

$f'(x) < 0$

$(x-3)(x-2) < 0$

$2 < x < 3$

$(2,3)$

Q.) $f(x) = (x-1)^3 (x-2)^2$

$= \cancel{(3x^3 - 1 + 3x - 3x^2)}$

$\cancel{2x^2-4x}$
$+2x+2$
x^2-3x+2

$f'(x) = (x-1)^3 \cdot \{2(x-2)\} + (x-2)^2\{3(x-1)^2\}$

$= 2(x-1)^3(x-2) + 3(x-2)^2(x-1)^2$

$= (x-1)^2(x-2).\{2(x-1) + 3(x-2)\}$

$= (x-1)(x-2)\{2x^2+1+4x + 3x^2 - 9x + 6\}$

$= (x-1)(x-2)\{5x^2 - 5x + 7\}$

$= (x-1)^2(x-2)(5x-8)$

$f'(x) = 0$

$\therefore x = 1, 2, 8/5$

Interval for increasing

$\Rightarrow (-\infty, 1) \cup (1, 8/5) \cup (2, \infty)$

Decreasing Interval

$\Rightarrow (8/5, 2)$

$-\infty \quad 1 \quad 8/5 \quad 2 \quad \infty$

$(-\infty, 1) \quad (1, 8/5) \quad (8/5, 2) \quad (2, \infty)$

I $\quad$ I $\quad$ D $\quad$ I

Gamma functions :-

$$\boxed{\Gamma(n) = \int_0^\infty e^{-z} z^{n-1} dz}$$

$n = 1/2$

$$\boxed{\Gamma(1/2) = \sqrt{\pi}}$$

$$\Gamma(n+1) = n\Gamma(n)$$

Q.) $I = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\left(\frac{x^2}{8}\right)} dx$

by comparing.

$\frac{x^2}{8} = z \Rightarrow \frac{2x\,dx}{8} = dz$

$= \frac{1}{\sqrt{2}\sqrt{\pi}} \int_0^\infty e^{-z} dz \frac{4}{\sqrt{8z}}$

$= \frac{1}{\sqrt{2}\sqrt{\pi}} \sqrt{2} \int_0^\infty e^{-z} z^{-1/2} dz$

$n = 1/2$

$= \frac{1}{\sqrt{2\pi}} . \sqrt{2} . \Gamma 1/2$

$= \frac{1}{\sqrt{2\pi}} \sqrt{2} . \sqrt{\pi}$

$\frac{3}{2} - 1$

Q.) $\Gamma 3/2 = \frac{1}{2} \Gamma 1/2$.

$= \frac{1}{2} \sqrt{\pi}$

Q.) $\Gamma(-1/2) = \frac{\Gamma(-1/2+1)}{-1/2} = \frac{\sqrt{\pi}}{-1/2} = -2\sqrt{\pi}$

$$\Gamma\left(-\tfrac{3}{2}\right) = \frac{\Gamma\left(-\tfrac{3}{2}+1\right)}{-\tfrac{3}{2}}$$

$$= \frac{-2\sqrt{\pi} \times 2}{-3} = \frac{4}{3}\sqrt{\pi}$$

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

Q.) $\int_0^\infty y^{1/2} e^{-y^3} dy$

$z = +y^3$

$dz = 3y^2$

$$\int_0^\infty \cancel{y^{1/2}}\ e^{-z}\ z^{1/6}\ \frac{dz}{3z^{2/3}}$$

$n = \cancel{\frac{-4}{6}} =$

$\frac{7}{6} - 1$

$n - 1 = \frac{1}{6}$

$n = \frac{1}{6} + 1$

$= \frac{7}{6}$

Boundary functions :-

Those functions which remain bounded {define} even when the input is unbounded {not define}

$y = f(x)$

$$\boxed{\lim_{x \to \infty} y = \text{Defined}}$$

Q.> Which one of the following functions is bounded?

a.> e^x b.> x^2

c.> e^{-x^2} d.> None of these.

even when input is undefined then, o/p is defined.

Q.> A continuous time system is described by $y(t) = e^{-|x(t)|}$ output is bounded. Only

i.> only when input is bounded

ii.> only when input is non-negative

iii.> Even when input is unbounded

iv.> None of these.

$e^{-\text{undu.}}$

$y(t) =$ defin.

in/p = undu

o/p = defined

Q.> $\int_0^{3/2} |x \cos \pi x| \, dx$

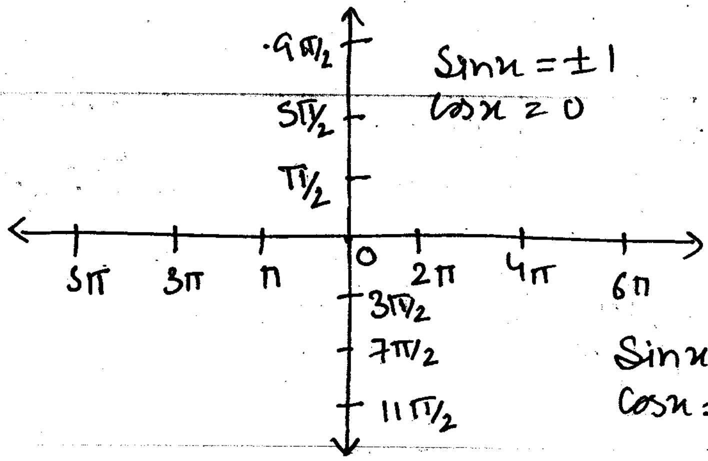
$\int_0^{1/2} (x \cos \pi x) + \int_{1/2}^{3/2} (-x \cos \pi x) \, dx$

$x \cos \pi x = 0$

$x = 0$, $\cos \pi x = 0$

$\pi x = \pi/2, 3\pi/2, 5\pi/2, \ldots$

$x = 1/2, 3/2, 5/2, \ldots$



$\sin x = \pm 1$
$\cos x = 0$

$\sin x = 0$
$\cos x = \pm 1.$

Q.) Which of the following function is unbounded.

a.) $\int_0^{\pi/4} \tan x \, dx$

$|\ln|\sec x||_0^{\pi/4}$ = defined

b.) $\int_0^{\infty} \frac{1}{x^2+1} dx$ $\quad \tan^{-1}x\Big|_0^{\infty} \Rightarrow$ Defined.

c.) $\int_0^{\infty} x e^{-x} dx$

$x \cdot \frac{e^{-x}}{-1} - \int 1 \cdot \frac{e^{-x}}{-1} dx$

$-xe^{-x} + \int e^{-x} dx$

$-xe^{-x} + e^{-x}$ $\quad [e^{-x}(x+1)$

$\lim_{x\to\infty} \frac{x+1}{e^x} - \lim_{x\to 0} \frac{x+1}{e^x}$ defined.

$\lim_{x\to\infty} \frac{1}{e^x}$

$= \frac{1}{\infty} = 0$
defined.

d.) $\int_0^1 \frac{1}{1-x} dx$

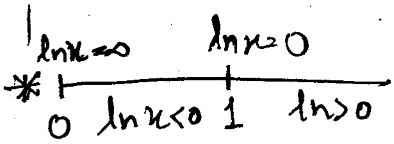
$-\ln|1-x|\Big|_0^1$

log 0 is not defined..

Q) $\int_{1/e}^{e} |\ln x| \, dx$ $\quad = \left\{ 2\left(1-\frac{1}{e}\right) \right\}$ Ans

$\int_{1/e}^{1} -\ln x \, dx + \int_1^e \ln x \, dx$
+ve $\qquad$ +ve

Also for modulus
$\ln x = 0$
$x = e^0 = 1$



Q.) $\int_{-1}^{1} \ln\left(\frac{2-x}{2+x}\right)dx$

∵ when the limit is $-a$ to a $(-1$ to $1)$

Check $funct^n$ (even or odd)

∵ the $funct^n$ is odd.

∴ ⇒ 0.

$f(x) = \ln\left|\frac{2-x}{2+x}\right|$

$f(-x) = \ln\left|\frac{2+x}{2-x}\right|$

$= \ln\left|\frac{2-x}{2+x}\right|^{-1}$

$= -\ln\left|\frac{2-x}{2+x}\right| = -f(x)$

$\int \underset{I}{\ln x} \cdot \underset{II}{1}\, dx$

$\ln x \cdot x - \int \frac{1}{x} \cdot x\, dx$

$\boxed{x\ln x - x = x(\ln x - 1)}$

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Laplace Transforms

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$$

Let $f(t)$ be a function define for all (+ve) values of t
Laplace transf involves the transformation of $\{t \to s\}$ and its use in the solution of the ODE

1.) $L\{1\} = 1/s$

2.) $L\{e^{at}\} = 1/s-a$

3.) $L\{\sin at\} = \frac{a}{s^2+a^2}$

4.) $L\{\cos at\} = \frac{s}{s^2+a^2}$

5.) $L\{t^n\} = \frac{\lfloor n}{s^{n+1}}$, $n \in I$

6.) $L\{t^n\} = \frac{\lfloor n+1}{s^{n+1}}$, $n \notin I$; Gamma funcⁿ.

7.) $L\{\sinh at\} = \frac{a}{s^2-a^2}$

8.) $L\{\cosh at\} = \frac{s}{s^2-a^2}$

First Shifting Property :-

$$\boxed{L\{f(t)\} = \bar{f}(s)}$$

$$\boxed{L\{e^{at} f(t)\} = \bar{f}(s-a)}$$

$$L\{e^{at} \sin at\} = \frac{a}{(s-b)^2+a^2}$$

Change of Scale Property :-

$$L\{f(t)\} = \bar{f}(s)$$

$$\boxed{L\{f(at)\} = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)}$$

L.T of derivatives :-

$$\boxed{L\{f^n(t)\} = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) \dots f^{n-1}(0)}$$

$$* \quad \boxed{L\{f''(t)\} = s^2 \bar{f}(s) - s f(0) - f'(0)}$$

*** Derivative = Multiplication by s

L.T of Integrals :-

$$L\{f(t)\} = \bar{f}(s)$$

$$\boxed{L\left\{ \int_0^t f(t)\, dt \right\} = \frac{1}{s} \bar{f}(s)}$$

*Q.) Find laplace of $L\{t \cos at\}$

Multiplication by t^n

$$L\{f(t)\} = \bar{f}(s)$$

$$\boxed{L\{t^n f(t)\} = \frac{d^n \bar{f}(s)}{ds^n} (-1)^n}$$

Division by t :-

$$\boxed{L\left\{ \frac{f(t)}{t} \right\} = \int_s^\infty \bar{f}(s)\, ds}$$

*Sol^n

$$L\{t \cos at\} = (-1) \frac{d \bar{f}(s)}{ds}$$

$f(t) = \cos at$

$$L\{f(t)\} \Rightarrow \bar{f}(s) = \frac{s}{s^2+a^2}$$

$$\Rightarrow (-1) \frac{d\left(\frac{s}{s^2+a^2}\right)}{ds} = \frac{(-1)(s^2+a^2) \cdot 1 - s \cdot 2s}{(s^2+a^2)^2} = \frac{s^2-a^2}{(s^2+a^2)^2}$$

Ans

Q) $L\{t e^{-t}\sin 3t\}$

$$f(t) = \sin 3t$$

$$\bar{f}(s) = \frac{3}{s^2+9}$$

$$L\{e^{-t}\sin 3t\} = \frac{3}{(s+1)^2+9}$$

$$L\{t e^{-t}\sin 3t\} = (-1)\frac{d}{ds}\left(\frac{3}{(s+1)^2+9}\right)$$

$$= (-1)\frac{d}{ds}\left\{\frac{3}{s^2+1+2s+9}\right\} = (-1)\frac{d}{ds}\left\{\frac{3}{s^2+2s+10}\right\}$$

$$= (-1) - \frac{2s+2}{(s^2+2s+10)^2}$$

$$= \frac{3(2s+2)}{s^2+2s+10} = \frac{6s+6}{(s^2+2s+10)} \text{ Ans.}$$

Q) $L\{t^3 e^{-3t}\}$

$$f(t) = e^{-3t}$$

$$\bar{f}(s) = \frac{1}{s+3}$$

$$L\{t^3 e^{-3t}\} = (-1)^3 \frac{d^3}{ds^3}\left(\frac{1}{s+3}\right)$$

∵ we use change of base trick rather than derivative.

$$f(t) = t^3$$

$$\bar{f}(s) = \frac{3!}{s^4} = \frac{3!}{(s+3)^4} \text{ Ans.}$$

Q) $L\{e^{-t}\sin^2 t\}$

$\sin^2 t = \frac{1-\cos 2t}{2}$

$$= L\{e^{-t}\tfrac{1}{2}\} - L\left\{e^{-t}\frac{\cos 2t}{2}\right\}$$

$$\boxed{\frac{1}{2}\frac{1}{(s+1)} - \frac{1}{2}\frac{s}{(s+1)^2+4}} \text{ Ans.}$$

Q.) $L\{e^{4t}\sin 2t\cos t\}$

$\sin(A+B)+\sin(A-B) = 2\sin A\cos B.$

$\frac{1}{2}\left[L\{e^{4t}\sin 3t\}+L\{e^{4t}\sin t\}\right]$

$\frac{1}{2}\left[\frac{3}{(s-4)^2+9}+\frac{1}{(s-4)^2+1}\right]$

Q.) $L\{f(t)\}=?$

$$f(t)=\begin{cases}1; & 0<t<1\\ t; & 1<t<2\\ 0, & t>2\end{cases}$$

$$\int_0^1 e^{-st}\cdot 1\,dt+\int_1^2 e^{-st}t\,dt+\int_2^\infty e^{-st}\cdot 0\,dt$$

Q.) $L\{\cosh at\sin at\}$

$\cosh at=\frac{e^{at}+e^{-at}}{2}$

$\sinh at=\frac{e^{at}-e^{-at}}{2}$

$L\left\{\frac{e^{at}\sin at}{2}\right\}$

$+L\left\{\frac{e^{-at}\sin at}{2}\right\}$

$\frac{1}{2}\frac{a}{(s-a)^2+a^2}+\frac{1}{2}\frac{a}{(s+a)^2+a^2}$ = Ans.

$\frac{s}{}$

Q.) $\int_0^\infty t e^{-2t}\sin 3t\,dt$

$f(t)=t\sin 3t$

$\boxed{s=2}$

In such cases put the value of s=

$L\{t\sin 3t\}=\frac{6s}{(s^2+9)^2}$

$=\frac{6\times 2}{(2^2+9)^2}=\frac{12}{(4+9)^2}=\frac{12}{169}$ = Ans.

Q.) $L\left\{t\int_0^t \frac{e^{-t}\sin t}{t}\,dt\right\}$

$f(t) = \sin t$

$f(s) = \int_s^\infty \frac{1}{s^2+1}\,ds$

$= \tan^{-1}s\Big|_s^\infty$

$= \tan^{-1}\infty - \tan^{-1}s$

$= \pi/2 - \tan^{-1}s = \cot^{-1}s$

$\left[\frac{-d}{ds}\,\frac{1}{s}\cot^{-1}(s+1)\right]$

$$= -\left\{\frac{-\frac{1}{1+(s+1)^2}\cdot 1\cdot s + \cot^{-1}(s+1)\cdot 1}{s^2}\right\}$$

$$= \frac{\frac{s}{1+s^2+2s+1} - \cot^{-1}(s+1)}{s^2}$$

$$= \frac{1}{s^2}\left[\frac{s}{s^2+2s+2} - \cot^{-1}(s+1)\right]$$ = Ans.

Q.) $L\left\{\frac{\cos at - \cos bt}{t}\right\}$

$L\left\{\frac{\cos at}{t}\right\} - L\left\{\frac{\cos bt}{t}\right\}$

$\int_s^\infty \frac{s}{s^2+a^2}\,ds - \int_s^\infty \frac{s}{s^2+b^2}\,ds$

$s^2+a^2 = t$
$2s\,ds = dt$

$\frac{1}{2}\ln|s^2+a^2|\Big|_s^\infty - \frac{1}{2}\ln|s^2+b^2|\Big|_s^\infty$

$\frac{1}{2}$ = power

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)^{1/2}$

$t\ln\sqrt{\frac{s^2+a^2}{s^2+b^2}}\Big|_s^\infty$

$$\ln\sqrt{\frac{1+(a/s)^2}{1+(b/s)^2}}\Bigg|_s^\infty$$

$$\ln\sqrt{\frac{1+0}{1+0}} - \ln\sqrt{\frac{s^2+a^2}{s^2+b^2}}$$

$$= \quad -\ln\sqrt{\frac{s^2+a^2}{s^2+b^2}}$$

$L\,f(t)$

$\int_0^\infty e^{-st} f(t)\,dt$

Q.) $L\{|t-1| + |t+1|\}$

breaking pt is -1

$$L\left\{\int_0^1 -(t-1)dt + \int_1^\infty (t-1)dt\right\} + \int_0^\infty e^{-st}(t+1)dt$$

! Modulus funct^n =

$t-1=0$, $t+1=0$

$t=1$, $t=-1$

we have to see the breaking pt whether it is +ve or −ve

i.e $\underline{>0<}$

$$\left\{\int_0^1 e^{-st}-(t+1)dt + \qquad + \frac{1}{s^2} + \frac{1}{s}\right.$$

$\int_1^\infty e^{-st}(t-1)dt$

** first we check the function is defined in interval $(0-\infty)$, if not then break acc. to limit, then find the breaking pt., acc apply formula ~~of~~ laplace or definition of laplace.

$$\Rightarrow \left\{\int_0^1 e^{-st}(1-t)dt + \int_1^\infty e^{-st}(t-1)dt\right\} + \frac{1}{s^2} + \frac{1}{s}$$

Inverse Laplace Transformation:—

$$L^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

$$L^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{(n-1)!}$$

$$L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin at$$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$$

$$L^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{1}{a}\sinh at$$

$$L^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$$

$$L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{1}{2a} t \sin at$$

$$L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3}(\sin at - at\cos at)$$

Q.) $L^{-1}\left\{\frac{1}{s^2+s}\right\}$

$$L^{-1}\left\{\frac{1}{s^2+s}\right\}$$

$$L^{-1}\left\{\frac{1}{s(s+1)}\right\}$$

$$L^{-1}\left\{\frac{1}{s} - \frac{1}{(s+1)}\right\}$$

$$1 - e^{-t}$$ Ans.

Q.) $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+2}$

$$4s+5 = A(s-1)(s+2) + B(s+2) + C(s-1)^2$$

$$A = \tfrac{1}{3},\ B = 3,\ C = -\tfrac{1}{3}$$

$$= \frac{1/3}{s-1} + \frac{3}{(s-1)^2} + \frac{-1/3}{s+2}$$

$$= \frac{1}{3}e^{t} + 3\cdot e^{t}\, t - \frac{1}{3}e^{-2t}$$

$L^{-1}[\bar{f}(s-a)] = f(t)\cdot e^{at}$

Q) $L^{-1}\left\{\frac{(s+2)^2}{(s^2+4s+8)^2}\right\}$

$\cancel{L^{-1}\left\{\frac{(s+2)^2}{((s+2)^2)^2}\right\}}$

$\cancel{L^{-1}\left\{\frac{1}{(s+2)^2}\right\}}$

$\cancel{e^{-2t}\, t}$

$$L^{-1}\left\{\frac{(s+2)^2}{((s+2)^2+2^2)^2}\right\}$$

e^{-2t}

$$L^{-1}\ \frac{s\cdot s}{(s^2+2^2)^2}$$

$$L^{-1}\left[s\ \frac{1}{2a}\, t\sin at\right]$$

$\frac{df}{dt}(t)$

$$e^{2t}\,\frac{1}{4}\left\{\frac{\cos 2t}{2}\cdot t + \sin 2t\cdot 1\right]$$

$$= \frac{e^{-2t}}{4}\left\{\frac{t\cos 2t}{2} + \sin 2t\right]$$ Ans.

(-ve) sign is used

no (-ve) sign is used

$L\{t\, f(t)\} = (-1)\frac{d}{ds}\bar{f}(s)$	$L^{-1}\{s\,\bar{f}(s)\} = \frac{df}{dt}(t)$
$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds$	$L^{-1}\left\{\frac{\bar{f}(s)}{s}\right\} = \int_0^t f(t)\,dt$
$L[e^{at}f(t)] = \bar{f}(s-a)$	$L^{-1}\{\bar{f}(s-a)\} = e^{at}f(t)$
$f(t)\to\bar{f}(s)$	$\bar{f}(s)\longrightarrow f(t)$

Q) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$

$$L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{3}{s^2}\right\} + L^{-1}\left\{\frac{4}{s^3}\right\}$$

$$1 - 3t + \cancel{4}^{\,2}\frac{t^2}{\cancel{2}}$$

$$1 - 3t + 2t^2$$ Ans.

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Q.) $L\left(\frac{1-\cos 3t}{t}\right)$

$= L\{1/t\} - L\left\{\frac{\cos 3t}{t}\right\}$

$= L\left\{\frac{1}{t}\left(1/s - \frac{s}{s^2+9}\right)\right\}$

$= \int_s^\infty \left(1/s - \frac{s}{s^2+9}\right) ds$

$= \left|\ln s\right|_s^\infty - \frac{1}{2}\ln(s^2+9)\Big|_s^\infty$; $s^2+9 = t$

$= \ln \frac{s}{(s^2+9)^{1/2}}\Big|_s^\infty$

$= \ln \frac{s}{s\sqrt{1+9/s^2}}\Big|_s^\infty$

$= \ln 1 - \ln \frac{s}{\sqrt{s^2+9}} = -\ln \frac{s}{\sqrt{s^2+9}}$ Ans

Q.) $L\left[\int_0^t\int_0^t\int_0^t (t\sin t)\,dt\,dt\,dt\right]$

$f(s) = \frac{1}{s^2+1}$

$t f(s) = \cancel{\ln|s^2+1|\Big|_s^\infty} = (-1)\frac{-1}{(s^2+1)^2}\cdot\frac{2s}{\cancel{2}}$

with all integrals $= \frac{2s}{(s^2+1)^2}\cdot\frac{1}{s}\cdot\frac{1}{s}\cdot\frac{1}{s}$

$= \frac{2}{s^2(s^2+1)^2}$ Ans.

Q.) $\int_0^\infty e^{-t} t \sin^2 t \, dt$

$f(t) = t \sin^2 t$

$= t\left(\frac{1-\cos 2t}{2}\right)$

$t = \left(\frac{1}{2s} - \frac{s}{2(s^2+4)}\right)$

$$\boxed{\begin{aligned} &= \ln s^{1/2} - \ln(s^2+4)^{1/2} \Big|_s^\infty \\ &= \frac{1}{2}\ln\left(\frac{s}{\sqrt{s^2+4}}\right)^{1/2}\Big|_s^\infty \\ &= \frac{1}{2}\ln \frac{1}{\sqrt{1+4/s^2}}\Big|_s^\infty \\ &= \frac{1}{2}\left\{\ln 1 - \ln \frac{s}{\sqrt{s^2+4}}\right\} \\ &= -\frac{1}{2}\ln \frac{s}{\sqrt{s^2+4}} \end{aligned}}$$

$\frac{}{2s^2} + \frac{4-s^2}{2(s^2+4)^2}$

$\frac{1}{2} + \frac{1}{2}\cdot\frac{3}{25} = \frac{28}{50}$

Q.) $L\left\{e^{-t}\int_0^t \frac{\sin t}{t}\,dt\right\}$

$\int_s^\infty \frac{1}{s^2+1}\,ds = \tan^{-1}s \Big|_s^\infty$

$\cancel{-\frac{1}{(s^2+1)^2}\cdot 2s\Big|_s^\infty}$ $= \frac{\cot^{-1}s}{s}$

$\cancel{\frac{2s}{(s^2+1)^2}}$ $= \frac{\cot^{-1}(s+1)}{(s+1)}$ Ans.

Q.) $L\{2^t\}$

$e^{\ln 2^t} = 2^t$

$L\{e^{t\ln 2}\}$

$= \frac{1}{s-\ln 2}$ Ans

$e^{\ln x} = x$

$e^{\ln 2^t} = 2^t$

Q.) $L^{-1}\left\{\frac{s}{s^4+4a^4}\right\}$

s^4+4a^4

$(s^2)^2+(2a^2)^2$

$(s^2+2a^2)^2-2\cdot s^2\cdot 2a^2$ | $a^2+b^2=(a+b)^2-2ab$

$(s^2+2a^2)^2-4a^2s^2$

$(s^2+2a^2)^2-(2as)^2$

$(s^2+2a^2+2as)(s^2+2a^2-2as)$

$L^{-1}\left\{\frac{s}{(s^2+2a^2+2as)(s^2+2a^2-2as)}\right\}$

$A=0 \quad C=0$

$B=\frac{-1}{4a} \quad D=\frac{1}{4a}$

$$\frac{\frac{-1}{4a}}{s^2+2a^2+2as}+\frac{1/4a}{s^2+2a^2-2as}$$

$s^2+2as+a^2+a^2$

$(s+a)^2+(a^2)$

$L^{-1}\left\{\frac{-1/4a}{(s+a)^2+(a)^2}\right\}$

$=\frac{1}{a}\sin at\cdot e^{-at}$

Q.) $L^{-1}\left\{\frac{1}{S(S+a)^3}\right\}$

$f(s) = \frac{1}{(S+a)^3}$

$\frac{1}{(S-a)} = \frac{t^{n-1}}{(n-1)!}e^{at}$

$f(t) = e^{-at}\frac{t^2}{2}$

$f(t) = \int_0^t e^{-at}\frac{t^2}{2}\,dt$

$\left\{\frac{t^2}{2}\,\frac{e^{-at}}{-a} - \int \frac{2t}{2}\cdot\frac{e^{-at}}{-a}\,dt\right\}_0^t$

$= -\frac{t^2 e^{-at}}{2a}$.

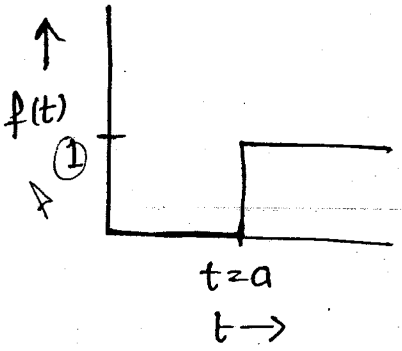
Unit step function :-

$u(t-a) = f(t) = \begin{cases} 0, & t<a \\ 1, & t \geq a \end{cases}$

at which step unit step is provided

if some terms remain alone use proper L

$L\{u(t-a)\} = \dfrac{e^{-as}}{s}$ (A)



Second Shifting Property :-

$L[f(t)\, u(t-a)] = e^{-as} \bar{f}(s)$ for trignometric functn

$L[f(t-a)\, u(t-a)] = e^{-as} \bar{f}(s)$ for algebraic functn

Q.) $\begin{cases} 0 & 0 \leq t < 1 \\ t-1 & 1 \leq t < 2 \\ 1 & t \geq 2 \end{cases}$ find Laplace.

$f(t)[u(t-0) - u(t-1)] + f(t)[u(t-1) - u(t-2)]$

$+ f(t)[u(t-2)]$

$0 + (t-1)[u(t-1) - u(t-2)] + 1[u(t-2)]$

$\cancel{t\,u(t-1) - t\,u(t-2) - u(t-1) + u(t-2)} + u(t-2)$

$(t-1)\, u(t-1) - (t-1)\, u(t-2) + u(t-2)$

$f(t) = t$
$a = -1$
$f(s) = e^{-as} \bar{f}(s)$

$L\{t(t-1)\, u(t-1)\}$

$\dfrac{1}{s^2} \cdot e^{-s} + u(t-2)(-t+1+1)$

$\dfrac{1}{s^2} \cdot e^{-s} - (t-2)\, u(t-2)$

$$= \frac{1}{s^2}\cdot e^{-s} - \frac{1}{s^2} e^{-2s}$$ ← Ans.

Q.) $f(t) = \begin{cases} t-1 & 1<t<2 \\ 3-t & 2<t<3 \end{cases}$

$$(t-1)\left[u(t-1) \cancel{(t-2)} - u(t-2)\right] + (3-t)\left[u(t-2) - u(t-3)\right]$$

$$(t-1)u(t-1) - u(t-2)\,\cancel{(t-3+1)}\,\{t-3+t-1\}\,\{2t-4\}\,2(t-2) + u(t-3)(t-3)$$

$$\frac{1}{s^2} e^{-s} - \frac{1}{s^2} e^{-2s} + \frac{1}{s^2} e^{-3s}$$

Q.) $L\{e^{-t}(1-u(t-2))\}$

$$L\{e^{-t} - e^{-t} u(t-2)\}$$; unit Stepp fun $(t-2)$

$$\frac{1}{s+1} - L\{e^{-t-2+2} u(t-2)\}$$

$$\frac{1}{s+1} - L\{e^{-(t-2)}\cdot e^{-2}\, u(t-2)\}$$; e^{-2} is constant so we take it out of laplace

$$\frac{1}{s+1} - e^{-2}\{e^{-(t-2)} u(t-2)\}$$; Yaha humara funct^n e^{-t} by dropping unit

$$\frac{1}{s+1} - e^{-2}\frac{1}{s+1} e^{-2s}$$ ← Ans.

$$L^{-1}\{\bar{f}(s)e^{-as}\} = f(t-a)u(t-a)$$

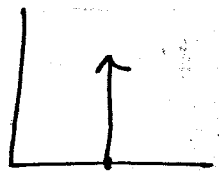
Q) $L^{-1}\left\{\underbrace{\frac{s}{s^2-\omega^2}}_{\bar{f}(s)}\cdot e^{-as}\right\} = \cosh(\omega)(t-a)\; u(t-a)$ ← Ans.

Unit Impulse function :-

Area under the curve should be 1

$\boxed{\lim_{A \to 0} \left(A \cdot \frac{1}{A}\right) = 1}$

$$\delta(t-a) = \begin{cases} 0, & t \neq a \\ \infty, & t = a \end{cases}$$



$$L\{\delta(t-a)\} = e^{-as}$$

Case of unit impulse theorem

↳ Sampling Theorem :-

$$\int_0^\infty f(t)\,\delta(t-a)\,dt \Rightarrow f(a)$$

Q.) $\int_0^\infty \sin 2t \, \delta(t - \pi/4)\,dt$

$$\sin 2(\pi/4) = \sin \pi/2 = 1$$

Q.) $L\left\{\frac{\delta(t-a)}{t}\right\}$

$$L\left\{\frac{e^{-as}}{t}\right\}$$

$$\int_s^\infty e^{-as}\,ds$$

$$\left.\frac{e^{-as}}{-a}\right|_s^\infty$$

$$-\frac{e^{-\infty}}{a} - \frac{e^{-as}}{(-a)}$$

$$0 + \frac{e^{-as}}{a}$$

Ans

open laplace.

$$\int_0^\infty e^{-st} \frac{\delta(t-a)}{t}$$

Compare with sampling.

put $t = a$

$$= \frac{e^{-as}}{a}$$

Ans

Periodic function

$$f(t) \rightarrow T$$

$$f(t+T) = f(t)$$

check

$$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

Final Value Theorem :-

*

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s f(s)$$

Initial Value Theorem :-

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s f(s)$$

Q.) $L^{-1}\left\{\frac{3s+1}{s^3+4s^2+(k-3)s}\right\}$ $\lim_{t\to\infty} f(t) = 1$. Find value of $k = ?$

$$\lim_{s\to 0} s \cdot \frac{3s+1}{s^3+4s^2+(k-3)s} = 1$$

$$3s^2+s = s^3+4s^2+ks-3s$$

$$4s = s^3+s^2+ks$$

$$s\to 0 \qquad s^2+s+(k-4) = 0$$

$$k = 4 \text{ Ans}$$

Q) Find $L^{-1}\left\{\ln \frac{S+1}{S-1}\right\}$

~~if we dont know the ans~~ let suppose ans $f(t)$ and multiply it with

$$L^{-1}\left\{\ln \frac{S+1}{S-1}\right\} = t\,f(t)$$

$$\ln S+1 - \ln S-1$$

; when multiplication = diffi

$$= L^{-1}\left\{\frac{1}{S+1} - \frac{1}{S-1}\right\} = t\,f(t)$$

$$= \frac{e^{-t} - e^{t}}{t} = f(t)$$

Q:) $y'' + 2y' + y(t) = \delta(t)$, $y(0) = -2$, $y'(0) = 0$

$$L\{y''\} = L\{2y'\} + L(y) = L(\delta(t))$$

$$S^2 y(s) - \cancel{S}\, S\overset{-2}{\overline{y(0)}} - \overset{0}{\overline{y'(0)}}$$

$$+ 2(sy(s) - \overset{-2}{y(0)}) + y(s) = 1$$

| $e^{as} =$ $a=0$
$e^0 = 1$

$$y(s)\,(S^2 + 2S + 1) + 2S + 4 = 1$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-2S-3}{S^2+2S+1}\right\}$$

$$= L^{-1}\left\{\frac{-2S}{(S+1)^2}\right\} - 3L^{-1}\left\{\frac{1}{(S+1)^2}\right\}$$

$$= -2\frac{d}{dt}\, t\cdot e^{-t} - 3te^{-t}$$

$$= -2\{$$

Differential Equation :–

An eq^n which ~~contains independent~~ variables & dependent variable & diff. coeff. of dependent variable w.r.t independent variable.

order of DE

The highest order derivative present in the eq^n is the order of DE.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

order = 2

Degree of DE

The power of the highest order derivative is the degree of DE.

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 0$$

Can be determine only when there is no fraction in power or –ve.

degree of DE = 3

Linear D.E :–

The DE in which the power of each variable is 1, and the dependent variable and the diff^n coeff are not multiplied together.

$$x^1 + x\left(\frac{dy}{dx}\right)^1 = 0$$

$$x^1 + x\left(\frac{dy}{dx}\right)^1 = \sqrt{1+x^2}$$

⇒ The variables for DE are y, dy/dx $(\ldots d^n y/dx^n)$

⇒ The power of variable should not be -ve & not in fraction

⇒ For the diffⁿ eqn which contains some another functⁿ then degree of DE is not defined.

$$\frac{dy}{dx} = \sin y$$

* order (derⁿ), degree (power of high derⁿ), linearity

	order	degree	linearity
Q) $(y')^3 - 4y' + y = 3e^x$	1	3	N.L
2.) $(y'')^3 + 7(y')^4 = 3\sin x$	2	3	N.L
~~3.) $(y'')^2 + a^2x = 0$~~	2	2	N.L
4.) $-3y''' + 7y'' + 4y' - \ln x = 0$	3	1	L
5.) $\sqrt{1+(y')^2} = 4x$	1	2	N.L
6.) $[1+(y')^2]^{2/3} = y''$	2	3	N.L
$y'' - \sqrt{y'} = 0$	2	2	N.L
$\sqrt{1+x^2} = y'$	1	1	L.

Solution of D.E :-

Variable Separation Method

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln y = \ln x + \ln c$$

$$\ln y = \ln xc \quad \Rightarrow y/x = c$$

Homogeneous Method :-

for homogenous eqn check ~~power~~ degree is same.

degree ~~of~~ homogenity is zero, only then this method is applicable.

$\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{x^2-y^2}{(x^2+y^2)}$$

put $\boxed{y = vx}$

diffn $\frac{dy}{dx} = v \cdot 1 + x\frac{dv}{dx}$

$$x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{x^2+v^2x^2} = \frac{1-v^2}{1+v^2} - v$$

$$x = vy$$

$$x\frac{dv}{dx} = \frac{1-v^2-v+v^3}{1+v^2}$$

$$\boxed{\frac{1+v^2}{(v^3+v^2+v-1)}\,dv = \frac{-dx}{x}}$$

Integrating factor Method :-

$$\frac{dy}{dx} + Py = Q$$ P, Q are f(x)

$$I.F = e^{\int P dx}$$

Sol^n $$y(I.F) = \int Q.(I.F)\,dx + c$$

Bernoulli's Method :-

$$\frac{dy}{dx} + Py = Qy^n$$

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q$$

$P(y^{1-n}) = z$

$y^{1-n} = z$

$(1-n)\,y^{\cancel{1-n-1}}\frac{dy}{dx} = \frac{dz}{dx}$

$\frac{1}{y^n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dz}{dx}$

$$\frac{1}{1-n}\frac{dz}{dx} + Pz = Q$$

$$\boxed{\frac{dz}{dx} + P(1-n)z = Q(1-n)}$$

Now again, $$I.F = e^{\int P(1-n)dx}$$

$$\cancel{y}\; z.(I.F) = \int Q\,(I.F)\,dx + C$$

or

$$\boxed{\frac{dz}{dx} + P'z = Q'}$$

Q.) $(x^2 - ay)\,dx = (ax - y^2)\,dy.$

$\cancel{x^2dx - aydx = axdy - y^2dy}$

$$\frac{dy}{dx} = \frac{x^2 - ay}{ax - y^2}$$

$$ax\,dy - y^2dy = x^2dx - ay\,dx$$

$$\int x^2 dx + \int y^2 dy = a\int [x dy + y dx] \qquad d(x,y)$$

$$\frac{x^3}{3} + \frac{y^3}{3} = a\int d(xy) + c$$

$$\frac{x^3}{3} + \frac{y^3}{3} = a(xy) + c$$

Q.) $y' = e^{3x+y}$

$$\frac{dy}{dx} = e^{3x} \cdot e^{y}$$

$$e^{-y} dy = e^{3x} dx$$

Q.) $y\sqrt{1+x^2}\, dy + x\sqrt{1+y^2}\, dx = 0$

$$y\sqrt{1+x^2}\, dy = -x\sqrt{1+y^2}\, dx$$

$$\frac{y\, dy}{\sqrt{1+y^2}} = \frac{-x}{\sqrt{1+x^2}}\, dx \qquad 1+y^2 = t$$

$$\frac{1}{2}\frac{(\sqrt{1+y^2})^{1/2}}{1/2} = -\frac{1}{2}\frac{(\sqrt{1+x^2})^{1/2}}{1/2} + c$$

Q.) $(\cos x + \sin x)dy + (\cos x - \sin x)dx = 0$

$$dy = \frac{(\sin x - \cos x)}{(\cos x + \sin x)}\, dx$$

$$y = \ln|\cos x + \sin x|$$

Q.> $(x^2+y^2)\,dx = 2xy\,dy$.

homoginity is zero.

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Q> A spherical napthlene ball exposed to the atmosphere looses volume at a rate proportional to its instantaneous surface area. Due to evaporation, if the initial dia of ball is 2cm & diameter reduces to 1cm, After 3 months, the ball completely evaporates in ?

i> 6 mnths ii> 3 mnths iii> 9 mnths iv> 12 mnths.

$$\frac{dV}{dt} \propto -A$$

$$\boxed{\frac{dV}{dt} = -k\, 4\pi r^2}$$

when
$t=0$, $d = 2cm$
$t = 3$ mnths, $d = 1cm$

$\because$ $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi r^2 \cdot 3\frac{dr}{dt}$$

$$= 4\pi r^2 \frac{dr}{dt}$$

$$\cancel{4\pi r^2}\frac{dr}{dt} = -k\,\cancel{4\pi r^2}$$

$dr = -k\,dt$

~~$\frac{1}{r} - 0.5 = -k$~~

$r = -kt + c$

$1 = c$

$0.5 = -3k + 1$

$\frac{-0.5}{-3} = k$

$0 = -\frac{0.5}{3} \times t + 1$

$r = 0$
$t = ?$

$-1 = +\frac{0.5}{3} \times t$

$3/0.5 = t$

$t = 6$ mnts.

Q.> A body initially at 60°C, cools down to 40°C in 15 mins, when kept in air at a temp of 25°C. What will be the temp of the body after 30 mins.

$T_1 = 60°$ $\Rightarrow t = 0$

$T_2 = 40°$ $\Rightarrow t = 15$

$\frac{dT}{dt} \alpha -(T - T_{air})$

$$\boxed{\frac{dT}{dt} = -k(T-25)}$$

$\boxed{T = 31.41}$ Ans

$dT = -k(T-25)\,dt$

$\frac{dT}{-kT + 25k} = dt$

$\ln|T-25| = -k\{t + c\}$

$k = 0.056$.

$C = -63.24$.

Q.) Represents family of $-3y\frac{dy}{dx} + 2x = 0$:

i.) Circles   ii.) Parabolas   iii.) hyperbola   iv.) Elipse.

$$3y\,dy = -2x\,dx$$

$$\frac{3y^2}{2} = -x^2 + c$$

$$\frac{y^2}{\sqrt{2/3}} + \frac{x^2}{1^2} = (\sqrt{2})^2$$

$y^2 = 4ax$, $x^2 = 4ay$	Parabola
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Ellipse
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Hyperbola
$(x-h)^2 + (y-k)^2 = r^2$ OR $x^2 + y^2 + 2gx + 2fy + c = 0$	Circle.
$(y - y_1) = m(x - x_1)$	line.
$xy = c$	Rectangular hyperbola

Q.) $\frac{dy}{dx} = y^2$, $y(0) = 1$

It is bounded in the interval

i) $\mathbb{R}$ ii.) $[-\infty, 1]$ ✓ iii.) $x > 1$, $x < 1$ iv) $[-2, 2)$

after solving $\boxed{y = \frac{-1}{x-1}}$ $\boxed{x \neq 1}$

Exact D.E

DE can be written in $d(x,y)=0$ form is exact D.E. (total derivative form)

$$\boxed{Mdx + Ndy = 0}$$

where M & N are funct^n of x & y.

$$\left.\frac{\partial M}{\partial y}\right|_x = \left.\frac{\partial N}{\partial x}\right|_y$$

; this condition should be checked.

Sol^n

$$\boxed{\int M dx + \int N_y dy = C}$$

iss waale N ko dekhenge; w/o x

terms of N that don't contain x.

$N_y = x^2 + y^3 = y^3$

$N_y = x^2 y = 0$

Q:> $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$

$M = x^3 + 3xy^2$

$N = y^3$

$$\cancel{\frac{x^4}{4}} + \cancel{\frac{3x^2}{2}y^2} +$$

$$\left.\frac{\partial M}{\partial y}\right|_x = 6xy$$

Yeh waala N chahiye, saari term consider karni h.

$$\left.\frac{\partial N}{\partial x}\right|_y = 6xy.$$

| The eq^n is exact.

$$\boxed{\frac{x^4}{4} + \frac{3x^2}{2}y^2 + \frac{y^4}{4} = C}$$ Ans.

Not exact

* * *

$$f(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$$

$$I.F = e^{\int f(x)dx}$$

$$f(y) = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$$

$$I.F. = e^{\int f(y)dy}$$

factor which make the eq^n exact is known as I.F

Q) $2\sin y^2 \, dx + xy \cos y^2 \, dy = 0$

$y(2) = \sqrt{\pi/2}$

$$\left.\frac{\partial M}{\partial y}\right|_x = \cancel{2 \cdot 2\sin y \cos y} \cdot 2 \cdot \cos y^2 \cdot 2y.$$

$$\left.\frac{\partial N}{\partial x}\right|_y = y\cos y^2$$

$$\left.\frac{\partial M}{\partial y}\right|_x \neq \left.\frac{\partial N}{\partial x}\right|_y \Rightarrow \text{EQ is not Exact.}$$

$$\frac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = \frac{4y\cos y^2 - y\cos y^2}{xy\cos y^2} = \frac{3\cancel{y\cos y^2}}{x\cancel{y\cos y^2}} = \frac{3}{x} = f(x)$$

$$I.F = e^{\int f(x)dx} = e^{\int 3/x \, dx}$$

$$= e^{3\ln x} = e^{\ln x^3} = x^3$$

Multiply x^3 with question.

$$2x^3 \sin y^2 dx + x^3 \cdot x y \cos y^2 dy = 0$$

$$2x^3 \sin y^2 dx + x^4 \cdot y \cos y^2 dy = 0$$

$$\left.\frac{\partial M}{\partial y}\right|_x = 2\cos^2 y \cdot 2y$$

$$\left.\frac{\partial N}{\partial x}\right|_y = 4.$$

$$\int 2x^3 \sin^2 y \, dx + \int 0 \, dy = c$$

$$2\sin y^2 \frac{x^4}{4} = c$$

Linear D.E of II^{nd} order :- (wid constant. coeff.)

general form of linear DE of II^{nd} order

$$a_1 \frac{d^2y}{dx^2} + a_2 \frac{dy}{dx} + a_3 y = X$$

Constant
zero
any f(x)

Take $\boxed{\frac{d}{dx} = D}$

operator

$$a_1 D^2 y + a_2 D y + a_3 y = X$$

$$\boxed{y(\underbrace{a_1 D^2 + a_2 D + a_3}_{\text{quadratic eq}^n \text{ in } D}) = X}$$

$$\boxed{f(D)\,y = X}$$

Sol^n = C.F + P.I

C.F ↓ Complementry funct^n

P.I ↓ Particular Integral.

If $X = 0$, Sol^n = C.F

$X \neq 0$, sol^n = C.F + P.I

For finding C.F :-

Step 1 :- $\boxed{f(D) = 0}$ Auxiliary eq^n.

Step 2 :- then we find roots. $(m_1, m_2, m_3 \ldots)$ depends on degree.

Case I :- When roots are real & unequal.

$$\boxed{C.F = C_1 e^{m_1 x} + C_2 e^{m_2 x}}$$

Case II :- roots are real & equal

$$\boxed{C\cdot F = (C_1 x + C_2)e^{mx}}$$; $m_1 = m_2 = m$ say.

Case III :- roots are imaginary / complex conjugate
$m_1 = \alpha + i\beta$
$m_2 = \alpha - i\beta$

$$\boxed{C\cdot F = e^{\alpha x}\left[C_1\cos\beta x + C_2 \sin\beta x\right]}$$

Q.) $D^2y + Dy - 6y = 0$. Find the solⁿ of DE

$(D^2 + D - 6)y = 0$ Aux eqⁿ

$D^2 + 3D - 2D - 6$
$D(D+3) - 2(D+3)$
$(D-2)(D+3)$
$D = 2, -3$

$y(D^2+D-6) = 0$
$D^2 + D - 6 = 0$
$D = \dfrac{-1 \pm \sqrt{1+24}}{2}$
$= 4/2, -6/2$, $2, -3$

$C\cdot F = C_1 e^{2x} + C_2 e^{-3x}$

Q.) $D^2y + 2Dy + 17y = 0$

$f(D)y = 0$

$(D^2 + 2D + 17)y = 0$

$D^2 + 2D + 17 = 0$

$D = \dfrac{-2 \pm \sqrt{4 - 68}}{2} = \dfrac{-2 \pm \sqrt{-64}}{2} = \dfrac{-2 \pm 8i}{2}$

$CF = e^{-x}\left[C_1 \cos 4x + C_2 \sin 4x\right]$

Finding the Particular Integral :-

$$P.I = \frac{X}{f(D)}$$

$$C.F \Rightarrow f(D) = 0$$

Case I :- $DX = e^{ax}$

$$P.I = \frac{e^{ax}}{f(D)}$$

Replace $D \Leftrightarrow a$

$$= \frac{e^{ax}}{f(a)} \quad f(a) \neq 0$$

if $f(a) = 0$

$$P.I = x\left[\frac{e^{ax}}{f'(D)}\right]$$

$$= x\left[\frac{e^{ax}}{f'(a)}\right] \quad f'(a) \neq 0$$

& if $f'(a) = 0$

$$P.I = x^2\left[\frac{e^{ax}}{f''(D)}\right]$$

and so on.

Case II :- $X = \sin(ax+b)$ or $\cos(ax+b)$

$$P.I = \frac{\sin(ax+b)}{f(D^2)} \Rightarrow f(-a^2) \neq 0$$

$$D^2 \Leftrightarrow -(a^2)$$

If $f(-(a)^2) = 0$

$$= x\left[\frac{\sin(ax+b)}{f'(D^2)}\right]$$

Case - III $\quad X = x^m$

$$P.I = \frac{x^m}{f(D)}$$

$$= x^m [f(D)^{-1}]$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2 + \frac{n(n-1)(n-2)}{\lfloor 3} x^3 + \dots$$

Q.) $\quad f(D) = D^2 + 3D + 3$

$$= 3\left[1 + \frac{D^2+3D}{3}\right]$$

Case - IV. $\quad X = e^{ax} \cdot V$

$$P.I = \frac{e^{ax} \cdot V}{f(D)}$$

$$= \frac{e^{ax} V}{f(D+a)} \Rightarrow e^{ax}\left[\frac{V}{f(D+a)}\right]$$

Q.) $\quad D^2y + 6Dy + 9y = 5e^{3x}$

$$(D^2 + 6D + 9) = 0$$

$$D^2 + 3D + 3D + 9 = 0$$

$$(D+3)(D+3)$$

$$D = -3, -3$$

$$C.F = (C_1 x + C_2) e^{-3x}$$

$$P.I = 5 \frac{e^{3x}}{f(D \to a)}$$

$$= 5 \frac{e^{3x}}{9+6\cdot 3+9}$$

$$= 5 \frac{e^{3x}}{36}$$

Q.7

$$P.I = \frac{\sin 3x}{f(D^2 \to -a^2)}$$

$$= \frac{\sin 3x}{-9+6D+9}$$

$$= \frac{\sin 3x}{6D}$$

$$= \frac{1}{6} \frac{\cos 3x}{3}$$

$$P.I = \frac{\sin 4x}{-16+6D+9}$$

$$= \frac{\sin 4x}{6D-7} \times \frac{6D+7}{6D+7}$$

$$= \frac{\sin 4x \, (6D+7)}{36D^2-49}$$

$$= \frac{(6D+7)(\sin 4x)}{36\cdot(-16)-49)}$$

$$= \frac{1}{625}(6\cos 4x \cdot 4 + 7\sin 4x)$$

$$= \frac{1}{625}(24\cos 4x + 7\sin 4x)$$

Q.) $P.I = \frac{x^2}{f(D)}$

$$= \frac{x^2}{D^2+6D+9}$$

$$= \frac{x^2}{9}\left[1+\frac{D^2+6D}{9}\right]^{-1}$$

$$= \frac{x^2}{9}\left[1 - \frac{D^2+6D}{9} + \frac{(-2)(-1)(D^2+6D)^2}{\lfloor 2 \; 81}\right]$$

$$= \frac{1}{9}\left[1 - \frac{D^2+6D}{9} + \frac{D^4+12D^3+36D^2}{81}\right]x^2$$

$$= \frac{1}{9}\left[x^2 + \frac{2+6\cdot 2x}{9} + \frac{36}{81}\times 2\right]$$

$$= \frac{1}{9}\left[x^2 + \frac{2}{9} + \frac{12x}{9} + \frac{\cancel{72}8}{\cancel{81}9}\right]$$

$$= \frac{1}{9}\left[x^2 + \frac{12}{9}x + \frac{10}{9}\right]$$

Q.) $D^2y+6Dy+9y = e^{2x}\sin 2x$

$$= e^{2x}\left[\frac{\sin 2x}{f(D+a)}\right]$$

$$= e^{2x}\frac{\sin 2x}{(D+2)^2+6(D+2)+9} = e^{2x}\frac{\sin 2x}{D^2+4D+4+6D+12+9}$$

$$= e^{2x}\frac{\sin 2x}{D^2+10D+25}$$

$$= e^{2x} \frac{\sin 2x}{-4+10D+25}$$

$$= e^{2x} \frac{\sin 2x}{10D+21} \frac{10D-21}{10D-21}$$

$$= e^{2x} \frac{(\sin 2x)\ (10D-21)}{100D^2 - 441}$$

$$= e^{2x} \frac{10.\cos 2x.2 - 21\sin 2x}{-400-441}$$

$$= -\frac{e^{2x}}{841} (20\cos 2x - 21\sin 2x)$$

$$= \frac{e^{2x}}{841} (21\sin 2x - 20\cos 2x)$$

Q.) P.I $\quad e^{2x}\left[\frac{x^2}{D^2+10D+25}\right]$

$$= e^{2x} \left[\frac{1}{25\left[1+\frac{D^2+10D}{25}\right]}\right] x^2$$

$$= \frac{e^{2x}}{25}\left[1+\frac{D^2+10D}{25}\right]^{-1} x^2$$

$$= \frac{e^{2x}}{25}\left[1 - \left(\frac{D^2+10D}{25}\right) + \frac{D^4+20D^3+100D^2}{625}\right] x^2$$

$$= \frac{e^{2x}}{25}\left[x^2 - \frac{2}{25} - \frac{20x}{25} + \frac{\overset{20}{\cancel{100}} \times 2}{\cancel{625}\,25}\right]$$

$$= \frac{e^{2x}}{25}\left[x^2 - \frac{4}{5}x + \frac{38}{25}\right]$$

Case V $\quad X =$ any funct^n

$$\frac{X}{D-a} = e^{ax}\int e^{-ax} X\,dx$$

Q) $D^2y + 6Dy + 9y = \sec x$

$$P.I = \frac{\sec x}{D^2+6D+10}$$

$$D = \frac{-6 \pm \sqrt{36-4\times 10}}{2} = \cancel{-\frac{6}{2} = -3, -3}$$

$$= \frac{-6 \pm 2i}{2} = -3+i \text{ or } -3-i$$

$$= \frac{\sec x}{(D-(-3+i))\,(D-(-3-i))}$$

$=$

Method of Variation of parameter :-

1.) Calculate C.F

say $C.F = C_1 y_1 + C_2 y_2$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Wronskian

$\Downarrow$ constant

$$P.I = -y_1 \int \frac{y_2 X}{W}\,dx + y_2 \int \frac{y_1 X}{W}\,dx$$

Q.) $D^2 y + 4y = \tan 2x$

$$D^2 + 4 = 0$$
$$D^2 = -4$$
$$D = \pm 2i$$

$$C.F = e\,[C_1 \cos 2x + C_2 \sin 2x]$$

$\Rightarrow y_1 = \cos 2x \quad , \quad y_2 = \sin 2x$

$y_1' = -2\sin 2x \quad , \quad y_2' = 2\cos 2x$

$$\Rightarrow 2\cos^2 2x - (-2\sin^2 2x)$$

$$W = 2$$

$$P.I = -\cos 2x \int \frac{\sin 2x \cdot \tan 2x}{2}\,dx + \sin 2x \int \frac{\cos 2x \cdot \tan 2x}{2}\,dx$$

$$= -\cos 2x \int \frac{\sin^2 2x}{2\cos 2x}\,dx + \sin 2x \int \frac{\sin 2x}{2}\,dx$$

$$= -\frac{\cos 2x}{2}\left\{ \int \left(\frac{1}{\cos 2x} - \frac{\cos^2 2x}{\cos 2x} \right) dx \right\} + \frac{\sin 2x}{2} \int \sin 2x\,dx$$

$$= \left\{ -\frac{\cos 2x}{2} \left| \frac{\sec 2x \cdot \tan 2x}{2} - \ln \left| \frac{\sec 2x + \tan 2x}{2} \right| \right\} \right.$$

$$+ \left[\frac{\sin 2x}{2} \cdot \frac{\cos 2x}{2} \right\} + c$$

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Q.) $y = e^{ax}$

$$S = \frac{dy}{dx} + \frac{d^2y}{dx^2} + \cdots\cdots + \frac{d^ny}{dx^n}$$

$\boxed{\lim_{n\to\infty} S = 2y}$ Find a ?

$$S = ae^{ax} + a^2e^{ax} + \cdots a^ne^{ax}$$

$$= ae^{ax}\left[1 + a + a^2 + \cdots + a^{n-1}\right]$$

$CR = a$ Common Ratio

$$= \frac{a(1-r^n)}{1-r} \qquad |r| < 1$$

$$|r| > 1$$

$$= \frac{a(r^n - 1)}{r-1}$$

$$S = \frac{1(a^n - 1)}{a-1}$$

$$\lim_{n\to\infty} S = 2y$$

$$S = \frac{1(1-a^n)}{1-a}$$

a^∞ = infinite $\because a > 1$

a^∞ = finite $\because a < 1$

So value of a^∞ approaches zero.

$$S = \cancel{e^{ax}} \cdot \frac{1(1-a^n)}{1-a} = 2e^{ax}$$

$a^\infty = 0$

$$\frac{1}{1-a} = 2$$

$\cancel{1} + \cancel{a} = (\cancel{1}+a)\,2$

$-\cancel{a^2 \ 2a\ 1}$

$$\frac{1}{2} = 1 - a$$

$$a = \tfrac{1}{2}$$

$a = \tfrac{2}{3}$

Q) The d.E for the variation of amt of salt in a tank with tym T is given by $\frac{dx}{dt} + \frac{x}{20} = 10$.

Determine the tym in min, at which amt of salt increases to 100 kg. If there is no salt present initially.

$t = 0, \; x = 0$

$x = 200(1 - e^{-t/20})$

=

$\frac{dx}{dt} = 10 - \frac{x}{20}$

$x = 10x -$

find t, when $x = 100$ kg.

$100 = 200(1 - e^{-t/20})$

$\frac{1}{2} = (1 - e^{-t/20})$

$+\frac{1}{2} = +e^{-t/20}$

$\ln \frac{1}{2} = -t/20$

$\ln(\frac{1}{2})^{-1} = t/20$

$t = 20 \ln 2$

Q)

Euler Cauchy Homogeneous Eqⁿ (D.E):-

$a_1 d^2y$

$$x^2\frac{d^2y}{dx^2} + \underbrace{x\frac{dy}{dx}}_{Dy} + ay = X$$

$D = d/dt$

$\boxed{x = e^t}$ $\boxed{\ln x = t}$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$= \frac{1}{x}\left(\frac{dy}{dt}\right) D$$

$$x\frac{dy}{dx} = Dy$$

$$\underline{x\frac{dy}{dx} = Dy}$$

$$\underline{x^2\frac{d^2y}{dx^2} = D(D-1)y}$$

$D(D-1)$

Q.) $x^2 d^2y/dx^2 + x\,dy/dx + y = \ln x$

$D(D-1)y + Dy + y = \ln x$

$(D^2 - D + D + 1)y = \cancel{\ln x}\ t$

$D^2+1 = t$

$D^2 = -1$

$D = \pm i$

C.F = $C_1 \cos \cancel{x}\,t + C_2 \sin \cancel{x}\,t$

P.I = $\dfrac{t}{D^2+1}$

$= t\,(1+D^2)^{-1}$

$= (1-D^2)\,t$

$= t$

$= \ln x$

$y = C_1 \cos \ln x + C_2 \sin \ln x + \ln x$

Legendre's Homogenous Eqn :- (DE)

$$(ax+b)^2 \frac{d^2y}{dx^2} + (ax+b)\frac{dy}{dx} + y = X$$

$\boxed{ax+b = e^t}$ $\boxed{t = \ln|ax+b|}$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = Dy\,\frac{a}{ax+b}$$

$$(ax+b)\frac{dy}{dx} = aDy$$

$$(ax+b)^2\frac{d^2y}{dx^2} = a^2D(D-1)y$$

Q.) $(3x+4)^2 \frac{d^2y}{dx^2} + (3x+4)\frac{dy}{dx} + y = \cancel{\sin}(3x+4)$

$$3^2 D(D-1)y + 3Dy + y = \cancel{\sin}(e^t)$$

$$(9D^2 - 9D + 3D + 1)y = e^t$$

$$(9D^2 - 6D + 1)y = e^t$$

$$\cancel{D(3D-2) = 0}$$

$$\cancel{D = 0,\ 2/3}$$

$$\cancel{C.F = c_1 + c_2 e^{2/3x}}$$

$$\frac{+6 \pm \sqrt{36-36}}{2}$$

$$= 3;\ 3$$

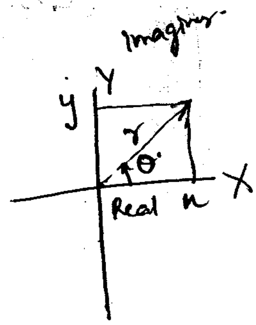
$$C.F = (c_1 + c_2 t)e^{3t}$$

$$P.I = \frac{e^t}{9D^2-6D+1} = \frac{e^t}{4}$$

$$y = \left(c_1 + c_2(\ln|ax+b|)\right)e^{3\ln(ax+b)} + \frac{e^{\ln|ax+b|}}{4}$$

Complex Number

A no. of the form $x+iy$ where x & y are real no. $i = \sqrt{-1}$ is called a complex no.



Cartesian form complex variable $\boxed{z = x+iy} = re^{i\theta}$

Real part $(z) = x$

Imaginary Part $(z) = y$

Conjugate $\bar{z} = x - iy$

Modulus

$$r = |z| = \sqrt{x^2+y^2}$$

Cartesian form $\left\{ \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array} \right\}$ Polar form.

$$r = \sqrt{x^2+y^2}$$

argument → $\boxed{\theta = \tan^{-1} y/x}$

Polar form

$$\boxed{z = r(\cos\theta + i\sin\theta)}$$

r = modulus

θ = amplitude or argument of the complex number.

& the value of amplitude b/w $-\pi$ to π is known as the principale value of argument.

Complex no. can be represented lyk a vector also. b'cz, it has both magnitude & direction. But it is not a vector as it don't follow vector law.

Q.) $z = \dfrac{(3-\sqrt{2}i)^2}{1+2i}$. Represent in form of complex no & find modulus.

$$= \frac{9 + 2(-1) - 6\sqrt{2}i}{1+2i}$$

$$= \frac{47 - 6\sqrt{2}i}{1+2i} \times \frac{1-2i}{1+2i}$$

$$= \frac{(11-6\sqrt{2}i)(1-2i)}{1-4(-1)} = \frac{47 - \cancel{14}i - 6\sqrt{2}i + 12\sqrt{2}i^2}{5}$$

$(x+iy)$

$$= \frac{7-12\sqrt{2}}{5} - i\left(\frac{14+6\sqrt{2}}{5}\right)$$

$$= \frac{49 + 144\cdot2 - \cancel{168\sqrt{2}} + 196 + 36\cdot2 + \cancel{168\sqrt{2}}}{25}$$

$$= \frac{605}{25}$$

$$= \frac{11\sqrt{5}}{5}$$

Q.) $z = -1+i$, write this in polar form

Sol^n

$$r = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2}$$

$$\theta = \tan^{-1}\frac{1}{-1} = \tan^{-1}(-1)$$

$$\pi - \theta = 3\pi/4$$

$$Z = \sqrt{2}\left(\cos 3\pi/4 + i \sin 3\pi/4\right)$$

∵ first see $\tan(1)$ $\theta = \pi/4$

$-1 \Rightarrow$

see in which quadrant $\tan\theta = -1$

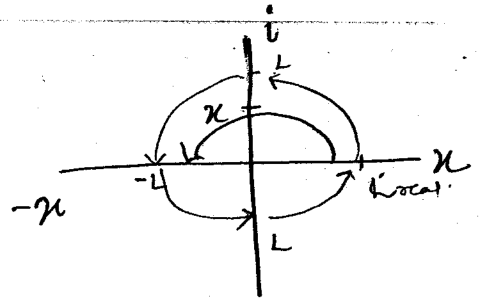
it is in 2, 4.

$\pi - \theta = 3\pi/4$

$2\pi - \theta = 7\pi/4$

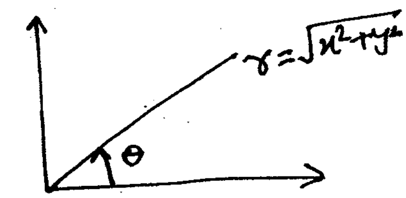
Now by observing complex no. find the value of θ as θ lt in II.

Geometrical Representation of i



i is an operation, which when multiplied to any real number makes it imaginary and rotates its direction through a right angle on the complex plane.

Complex no. is <u>not a vector</u>, but it can be <u>represented as</u> a vector <u>bcz</u> it has a <u>direction</u> as well as <u>magnitude</u>.



Q.) Represent this in polar form $z = 1 - \cos\alpha + i\sin\alpha$

$$r = \sqrt{1 + \cos^2\alpha - 2\cos\alpha + \sin^2\alpha}$$

$$= \sqrt{2(1-\cos\alpha)}$$

$$r = \sqrt{2}\sin\alpha/2$$

$$\theta = \tan^{-1}\left(\cancel{\frac{\sin^2\alpha/2}{\sin\alpha/2}}\right)\left(\frac{\sin\alpha}{1-\cos\alpha}\right)$$

$$= \frac{\cancel{2}\,\cancel{\sin\alpha/2}\cdot\cos\alpha/2}{\cancel{2}\,\cancel{\sin^2\alpha/2}} = \cot\alpha/2$$

$$\theta = \tan^{-1}\left(\tan\left(\pi/4 - \alpha/2\right)\right)$$

$$\theta = \pi/2 - \alpha/2$$

Exponential form of complex number.

$z = x + iy$

$$e^x = 1 + \frac{x}{\underline{|1}} + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \cdots\cdots$$

$$e^z = 1 + \frac{z}{\underline{|1}} + \frac{z^2}{\underline{|2}} + \frac{z^3}{\underline{|3}} + \cdots\cdots$$

$$e^{iy} = 1 + \frac{iy}{\underline{|1}} + \frac{(iy)^2}{\underline{|2}} + \frac{(iy)^3}{\underline{|3}} + \cdots\cdots$$

$$= \left(1 - \frac{y^2}{\underline{|2}} + \frac{y^4}{\underline{|4}} + \cdots\right) + i\left(y - \frac{y^3}{\underline{|3}} + \frac{y^5}{\underline{|5}} - \cdots\right)$$

$$\boxed{e^{iy} = \cos y + i\sin y}$$

$z = x + iy.$

$$\boxed{e^z = e^x \cdot e^{iy} = e^x(\cos y + i\sin y)}$$

$e^{i\theta} = \cos\theta + i\sin\theta$; Euler's theorem

$\theta = 0, \quad e^{i\theta} = 1$

$\theta = \pi/2, \quad e^{i\theta} = i$

$\theta = \pi, \quad e^{i\theta} = -1.$

$\theta = 2\pi, \quad e^{i\theta} = 1$

$\theta = 3\pi/2, \quad e^{i\theta} = -i$

$$\boxed{\begin{array}{l} 1 \longrightarrow e^{2\pi i} \\ i \longrightarrow e^{i\pi/2} \\ -1 \longrightarrow e^{i\pi} \\ -i \longrightarrow e^{i3\pi/2} \end{array}}$$

Logarithmic form of a Complex Number :-

$$\ln z = \ln(x+iy)$$

$$\ln(x+iy) = \ln(re^{i\theta}).$$ | $e^{i\theta} = \cos\theta + i\sin\theta$

$$= \ln r + \ln e^{i\theta}$$

$$= \ln r + i\theta$$ | $\ln e = 1$

$$\boxed{\ln z = \tfrac{1}{2}\ln(x^2+y^2) + i\tan^{-1}\tfrac{y}{x}}$$

Q.) Find the value of complex no of $\ln(-5)$

$$\ln(-1\times 5) = \ln(-1) + \ln 5$$

$$= \ln(e^{\pi i}) + \ln 5$$

$$\boxed{\ln(-5) = \pi i + \ln 5}$$ Ans

$$\ln(-5) = (2n+1)\pi i + \ln 5$$ Ans | 2n+1 is a general form.

Q.) Find the real values of x and y, so that $(-3+ix^2y)$ and (x^2+y+4i) represent complex conjugate numbers.

$$\bar{z} = x - iy$$

$$z = -3 + ix^2y \Rightarrow \bar{z} = -3 - ix^2y$$

$$\bar{z} = x^2+y+4i$$

$$\Rightarrow x^2+y+4i = -3 - ix^2y$$

$$x^2+y = -3$$

$$x^2y = -4$$

$$y = -4/x^2$$

| $x^2 - 4/x^2 = -3$

$x^4 - 4 = -3x^2$

$x^4 + 3x^2 = 4$

$x^2(x^2+3) = 4$

$$-3 - ix^2y = x^2 + y + 4i$$

$$x^2 = \frac{-3 \pm \sqrt{9+16}}{2}$$

$$x^2 = \frac{-3 \pm 5}{2}$$

$$x^2 = 1, \cancel{-4}$$

$$x = \pm 1$$

$$e^z \cdot 1 = e^z \cdot e^{2n\pi i}$$

$$e^{2n\pi i} = \underbrace{\cos 2n\pi i}_{1} + i \underbrace{\sin 2n\pi i}_{0}$$

Exponential form of a complex function is multi-valued function.

$$\ln z + 0 = \ln z + \ln 1$$

$$= \ln z + \ln e^{2n\pi i}$$

$$\boxed{\ln z = \ln z + 2n\pi i}$$

Logarithm form of a complex function is multi-valued function

Complex function

If for each value of complex variable z, in a given region R; we have one or more values of w, then w is said to be a complex function of z and written as

$$w = f(z) \rightarrow x+iy$$

$$u + iv = f$$

$$f(x,y) \quad f(x,y)$$

For the derivative of complex function to exist the necessary and sufficient conditions.

→ All the derivative in the region R must be continuous.

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$$

$$\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

→ Functⁿ satisfy Cauchy-Reimann Eqⁿ (C-R eqⁿ.):

$$\boxed{\begin{aligned}\frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x}\end{aligned}}$$

$$\frac{dw}{dz} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$= \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

Analytic functions:-

A functⁿ $f(z)$ which is single valued and posses a unique derivative w.r.t z at all the pts of a region R is called an analytic function of z in that region.

Aug 20,14

Cauchy's Theorem

Pole. A pt. at which an analytic functⁿ is not analytic or its derivative doesn't exists. Also known as Singular functⁿ

If the functⁿ $f(z)$ is analytic at each & every point of a region R; then the integral ~~$f(z)dz$~~

$$\boxed{\oint_R f(z)\,dz = 0}$$

Extension of Cauchy's Theorem.

$$\oint_R f(z)\,dz = \oint_{C_1} f(z)\,dz$$

$$\oint_R f(z)\,dz = \oint_{C_1} f(z)\,dz + \oint_{C_2} f(z)\,dz$$

$$\boxed{\oint_C f(z)dz = 2\pi i \operatorname{Res} f(c)}$$

Residue :-

The coeff. of $(z-a)^{-1}$ in the expansion of laurent series around a singular pt. is called the residue at that point.

$$= a_1(z-a) + a_2(z-a)^2 + a_3(z-a)^3 + \dots$$
$$+ a_{-1}(z-a)^{-1} + a_{-2}(z-a)^{-2} + \dots$$

$$\boxed{\operatorname{Res} f(a) = a_{-1}}$$

$(z-a)^{-1}$ coeff. will be the Residue.

Residue Theorem :-

$$\operatorname{Res} f(a) = \lim_{z\to a} \frac{1}{\lfloor n-1} \left[\frac{d^{n-1}}{dz^{n-1}} (z-a)^n f(z) \right]$$ of order n.

$$\boxed{\operatorname{Res} f(a) = \lim_{z\to a} (z-a) f(z)}$$

→ Simple pole

$$\oint_R f(z)dz = \oint_{C_1} f(z)dz + \oint_{C_2} f(z)dz + \dots + \oint_{C_n} f(z)dz$$
$$= 2\pi i \operatorname{Res}(C_1) + 2\pi i \operatorname{Res}(C_2) + \dots + 2\pi i \operatorname{Res}(C_n)$$
$$= 2\pi i \left[\operatorname{Res}(C_1) + \operatorname{Res}(C_2) + \dots + \operatorname{Res}(C_n) \right]$$

\#\#

$$\boxed{\oint_R f(z)dz = 2\pi i \,[\text{sum of Residues}]}$$

Q.) $f(z) = \dfrac{1}{(z+2)^2(z-2)^2}$. find value of residues.

$f(z)$ has two poles at $z = -2, 2$. Both are of order 2

$$\text{Res } f(-2) = \lim_{z\to -2} \frac{1}{\underline{|2-1}} \frac{d}{dz} (z+2)^2 \frac{1}{(z+2)^2(z-2)^2}$$

$$= \lim_{z\to -2} \frac{1}{\underline{|1}} \frac{d}{dz} \frac{1}{(z-2)^2}$$

$$= \lim_{z\to -2} \frac{-2}{(z-2)^3} = \frac{-2}{(-4)^3} = \frac{2}{64} = 1/32$$

$$\text{Res } f(2) = \lim_{z\to 2} \frac{1}{\underline{|2-1}} \frac{d}{dz} (z-2)^2 \frac{1}{(z+2)^2(z-2)^2}$$

$$= \lim_{z\to 2} \frac{1}{\underline{|1}} \frac{d}{dz} \frac{1}{(z+2)^2}$$

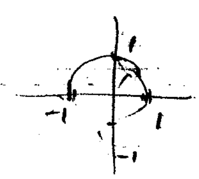
$$= \lim_{z\to 2} \frac{-2}{(z+2)^3}$$

$$= \frac{-2}{(4)^3} = -1/32$$

Q.) find value of $\oint_R f(z) = \dfrac{1}{(z+2)^2(z-2)^2}$

w/o R, any! can't be find out

* Default $|z|=1$



$$= 2\pi i \left(1/32 + (-1/32) \right)$$

$$= 0$$

Q.) $f(z) = \dfrac{1-2z}{z(z-1)(z-2)}$

$$\text{Res } f(0) = \lim_{z\to 0} z \cdot \frac{1-2z}{z(z-1)(z-2)}$$

$$= \frac{1}{-1 \cdot -2} = 1/2$$

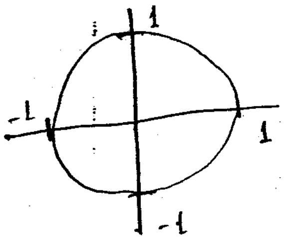
$$\text{Res } f(1) = \lim_{z\to 1} (z-1) \frac{1-2z}{z(z-1)(z-2)}$$

$$= \frac{-1}{1\cdot(-1)} = 1$$

$$\text{Res of } f(2) = \lim_{z\to 2} (z-2) \frac{1-2z}{z(z-1)(z-2)}$$

$$= \frac{-3}{2\cdot 1} = -3/2$$

$\because$ Region is not given, so by default $|z| = 1$



$$= 2\pi i \left[\text{Res}(0) + \text{Res}(1)\right]$$

$$= 2\pi i \left[\tfrac{1}{2} + 1 \right]$$

$$= 2\pi i \cdot 3/2$$

$$= 3\pi i$$

We include only those Residue which lie inside the Region or over the region.

Q.) $f(z) = C_2 + C_1 z^{-1}$

$$\oint \left(\frac{1+f(z)}{z}\right) dz$$

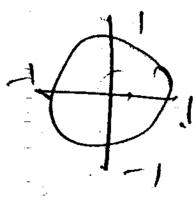
$$\frac{1 + C_2 + C_1/z}{z} = \frac{z + zC_2 + C_1}{z^2}$$

$z = 0$ of order 2.

$$\text{Res. } f(0) = \lim_{z\to 0} \frac{d}{dz} (z-0)^2 \frac{z+zC_2+C_1}{z^2}$$

$$= \lim_{z\to 0} (1 + C_2)$$

$$= 1 + C_2$$



$R \Rightarrow |z| = 1$

$$\oint \left(\frac{1+f(z)}{z}\right) dz = 2\pi i\,(\text{Res}(0)) \Rightarrow 2\pi i\,(1+C_2)$$

Q $\oint \frac{1}{z^2+4}\, dz \quad , \quad |z-i| = 2$

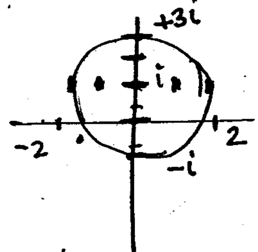
$z = \pm 2i$

$\text{Res}\, f(2i) = \lim_{z \to 2i}$

$|z-i| = 2$

$|z-a| = r$

$r = 2, \; a = (0, i)$



$\text{Res}\, f(2i) = \lim_{z \to 2i} \frac{d}{dz}(z-2i) \cdot \frac{1}{(z-2i)(z+2i)}$

$= \lim_{z \to 2i} \frac{-2 \cdot 1}{(z+2i)^2}$

$= \frac{-2}{+16 i^2} = -1/8$

$= 2\pi i \left[1/8 \right]$

$= \frac{\pi i}{4}$

$\lim_{z \to 2i} \frac{1}{(z+2i)}$

for first order no diffⁿ.

$\frac{1}{4i}$

$\oint \frac{1}{z^2+4}\, dz = 2\pi i \left(\frac{1}{4i} \right)$

$= \frac{\pi i}{2}$

Q.) i^i . find value ?

$= \left(e^{i\pi/2}\right)^i = e^{i^2 \pi/2} = e^{-\pi/2}$

Q.) $\oint f(z) = \frac{\cos z}{z}$

$z = 0$

$\text{Res}(0) = \lim_{z \to 0} (z-0) \frac{\cos z}{z}$

$= \cos 0$

$= 1$

$\oint f(z)\, dz = 2\pi i (1)$

$= 2\pi i$

Q.) An analytic funct^n of complex variable $z = x+iy$ is expressed as $f(z) = u+iv$, where u & v are funct^n of x & y. If $u = xy$, find the value of v. or find the harmonic conjugate (calculate v = ?)

$$\left.\frac{\partial u}{\partial x}\right|_y = y = \left.\frac{\partial v}{\partial y}\right|_x \Rightarrow \int \partial v = \int y\,dy$$

$$v = \frac{y^2}{2} + f(x)$$

In partial integration $f(x)$ is added

$$\cancel{\left.\frac{\partial u}{\partial y}\right|_x = x}$$

$$\left.\frac{\partial v}{\partial x}\right|_y = f'(x)$$

$$\left.\frac{\partial u}{\partial y}\right|_x = x = -\left.\frac{\partial v}{\partial x}\right|_y$$

$$-x = \left.\frac{\partial v}{\partial x}\right|_y = f'(x)$$

$$f'(x) = -x$$

integrating (total)

$$f(x) = -\frac{x^2}{2} + c$$

$$v = \frac{y^2}{2} + \left(-\frac{x^2}{2}\right) + c$$

$$= \frac{y^2}{2} - \frac{x^2}{2} + c$$

Q.) $u = 3x^2 - 3y^2$

$$\left.\frac{\partial u}{\partial x}\right|_y = 6x = \left.\frac{\partial v}{\partial y}\right|_x \Rightarrow \int \partial v = \int 6x\,dy$$

$$v = 6xy + f(x) \quad \left|\ \left.\frac{\partial v}{\partial x}\right|_y = 6y + f'(x)\right.$$

$$\left.\frac{\partial u}{\partial y}\right|_x = -6y = -\left.\frac{\partial v}{\partial x}\right|_y = +6y + f'(x) \Rightarrow f'(x) = 0$$

$$f(x) = c$$

$$v = 6xy + c$$

Q.) $\oint \frac{z^3-6}{3z-i} dz$ Ans = $\frac{2\pi}{81}, -4\pi i$

$$\lim_{z\to i/3} (z-i/3) \frac{z^3-6}{3(z-i/3)}$$

$$= \frac{(i/3)^3-6}{3} = \frac{-i/27-6}{3} = \frac{-i}{81}-2$$

$$= 2\pi i\left[-\frac{i}{81}-2\right]$$

$$= \frac{2\pi}{81} - 4\pi i$$

Q.) $I = \oint \sec z \, dz$

✓ a.) $I = 0$, Singularity set $= \phi$

b.) $I = 0$, Singularity set $= \pm(2n+1)\pi/2$, $n = 0, 1, 2, \ldots$

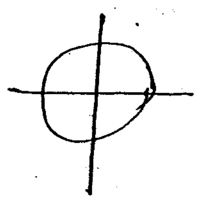
c.) $I = \pi/2$, Singularity set $= \pm n\pi$

d.) None of these.

$$\oint \frac{1}{\cos z} dz$$

$\cos z = 0$

$z = \pi/2, 3\pi/2, -\pi/2, -\frac{3\pi}{2}, \ldots$



Inside Region, there is no singularity.

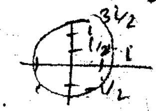
Q.) $\oint \frac{\cos 2\pi z}{(2z-1)(z-3)} dz$ find integral. $2\pi i/5$

Q.) $X(z) = \frac{z}{(z-a)^2}$, Residue of $X(z)z^{n-1}$ at $z=a$ is.

na^{n-1} ist deri

3.) $\oint \frac{dz}{1+z^2} = \pi$ $|z-i/2|^2=1$

$a=(0,1/2)\ r=1$



$|z-i/2|=1$

4.) $\oint f(s)\,ds$

$|s|=3$, where $f(s)=\frac{(3s+4)}{(s+1)(s+2)}$ $s \Rightarrow$ Complex Number.

Res 1 2 $= 6\pi i$

5.) $\oint \frac{\sin z}{z\cos z}\,dz$ $|z|=2$ $=0$

6.) $\oint \frac{z^2}{(z-1)^2(z+2)}\,dz$ $= \cancel{-27\pi i/8}\ 2\pi i$

$|z|=2$

7.) $\oint \frac{z^3}{(z-1)^4(z-2)(z-3)}\,dz$ $|z|=2.5$ $= -27i\pi/8$

8.) $\int \frac{e^z}{\cos \pi z}\,dz = -4i\sinh 1/2$

9.) $\oint \tan z\,dz$ $\left(\frac{\sin}{\cos}\right)$ $\cos z=0$ $|z|=2$ $\lim_{z\to\pi/2}(z-\pi/2)\frac{\sin z}{\cos z}$ $\lim_{h\to 0} h\frac{\sin(\pi/2+h)}{\cos(\pi/2+h)}$ $-\sin h$ $= 4\pi i$

10.) $\oint \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z+2)}\,dz$ $|z|=3$ $= 4\pi(\pi+1)i$

$= 2\ (-)$

11.) $u=\cos x\cosh y$. Find V. $-\sin x\sinh y + c$

12.) Residue of $\frac{1}{(z^2+a^2)^{n+1}}$ at $z=ia$ is $\underline{\cosec 3+3+c}$

$\text{Res}\, f(ia) = \lim_{z\to ia}\frac{d^n}{dz^n}(z-ia)^{n+1}\frac{1}{\{(z+ia)(z-ia)\}^{n+1}}$

$= \lim_{z\to ia}\frac{1}{\lfloor n+1-1}\frac{d^n}{dz^n}\cancel{(z-ia)^{n+1}}\frac{1}{\cancel{(z-ia)^{n+1}}(z+ia)^{n+1}}$

$= \lim_{z\to ia}\frac{1}{\lfloor n}\frac{d^n}{dz^n}\frac{1}{(z+ia)^{n+1}}$

$$= \lim_{z \to ia} \frac{1}{\lfloor n} \frac{\{-(n+1)\}\{-(n+2)\} \cdots \{-(n+n)\}}{(z+ia)^{n+n+1}}$$

$$= \lim_{z \to ia} \frac{1}{\lfloor n} \frac{(-1)^n \{(n+1)(n+2) \cdots (n+n)\}}{(z+ia)^{2n+1}} \cdot \frac{\lfloor n}{\lfloor n}$$

$$= \boxed{\frac{\lfloor 2n}{(\lfloor n)^2} \frac{1}{(2ia)^{n+1}} (-1)^n}$$

$\because$ for most cases n is even so $(-1)^n = 1$.

$$\boxed{\frac{\lfloor 2n}{(\lfloor n)^2} \frac{1}{(2ia)^{n+1}}}$$ Ans.

Milney- Thomson Method :-

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \quad \text{———} *$$

or

$$\frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}$$

first find derivative in terms of x, y)

$u = \text{-----}$

$$\frac{\partial u}{\partial x} = f(\cancel{x}^{z}, y)$$

$$\frac{\partial u}{\partial y} = f(x, y)$$

Replace $x \Rightarrow z$, & $y \Rightarrow 0$

put the value in eq $(*)$

then integrate and find $f(z)$

Q.) $v = \dfrac{x-y}{x^2+y^2}$. find funct^n in terms of z .

$$\frac{\partial v}{\partial y} = \frac{(x^2+y^2)(-1) - (x-y)2y}{(x^2+y^2)^2} = \frac{-x^2 \cancel{+y^2} - 2xy \cancel{+2y^2}}{(x^2+y^2)^2}$$

$$= \cancel{\frac{-x^2+2xy}{}} -x^2+y^2-2xy$$

$$= \frac{-x^2+y^2+2xy}{(x^2+y^2)^2} = \cancel{\frac{-x(x+2y)}{(x^2+y^2)^2}}$$

$$\frac{\partial v}{\partial x} = \frac{(x^2+y^2)(1) - (x-y)2x}{(x^2+y^2)^2} = \frac{x^2+y^2-2x^2+2xy}{(x^2+y^2)^2}$$

$$= \frac{-x^2+y^2+2xy}{(x^2+y^2)^2}$$

$$\left.\frac{\partial v}{\partial y}\right|_{(z,0)} = \frac{-z^2}{z^4} = -\frac{1}{z^2}$$

$$\left.\frac{\partial v}{\partial x}\right|_{(z,0)} = \frac{-z^2}{z^4} = -\frac{1}{z^2}$$

$$f'(z) = \frac{-1}{z^2} - i\left(\frac{1}{z^2}\right)$$

$$= -\frac{1}{z^2}(1+i)$$

$$f(z) = (-1)\frac{z^{-2+1}}{-2+1}(1+i) = \frac{1+i}{z} + c$$

Q.) if $v = 2xy$. find funct^h in terms of z $= z^2+c$

Q.) If $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ then find funct^h in terms of z.

$\cot z + c$

Q.) $\oint_c \frac{f(z)}{dz} = e^{1/z}$

$= 1 + \frac{1}{z} + \frac{1}{\lfloor 2} \frac{1}{z^2} + \dots$

Res $= (1)$

$\oint_c f(z)\,dz = 2\pi i\,(1)$

$= 2\pi i$

$(z-0)^{-1}$
$= 1$
. as region $|z| = 1$

Q.) $\oint z^4 e^{1/z} dz$

$= z^4 + \frac{z^4}{z} + \frac{z^4}{2 z^2} + \frac{z^4}{\lfloor 3 z^3} + \frac{z^4}{\lfloor 4 z^4} + \frac{z^4}{\lfloor 5 z^5} + \dots$

in this term ~~coeff~~ power of $e^{y} = 1/z$

Res $= \frac{1}{\lfloor 5}$

$\frac{1}{3 \times 2}$

$\oint z^4 e^{1/z} dz = 2\pi i \times \frac{1}{5 \times 4 \times 3 \times 2}$

$= \frac{\pi i}{60}$

Q.) $\oint f(z) = z^3 \cos\left(\frac{1}{z-2}\right)$ | $(z-2) = h$

* All question of $\frac{1}{z}$ form alwaz use expansion instead of integ

$(2+h)^3 \cos\frac{1}{h}$

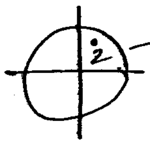
$= \left[1 - \left(\frac{1}{h}\right)^2 \cdot \frac{1}{\lfloor 2} + \left(\frac{1}{h}\right)^4 \cdot \frac{1}{\lfloor 4} - \left(\frac{1}{h}\right)^6 \frac{1}{\lfloor 6} + \dots\right]$

$= 2^3 + h^3 + 3 \cdot 2^2 \cdot h + 3 \cdot 2 \cdot h^2$

$= 8 + h^3 + 12h + 6h^2$

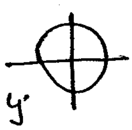
Aug 22,14

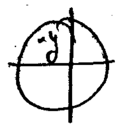
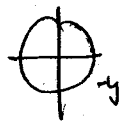
~~VECTORS~~

Q.) 

$\rightarrow |z| = 1$, $z = x+iy$.

$\frac{1}{2} = y$

a.) 

b.) 
c.) 
d.) 

Q.) $\oint \frac{z-3}{z^2+2z+5}\,dz$ find integral for.

with a.) $|z| = 1$, b.) $|z+1-i| = 2$, c.) $|z+1+i| = 2$

Q.) Chang CR eq^n into polar co-ordinates.

VECTORS

A quantity which have magnitude as well as direction is known as a vector. generally denoted by small letters with an arrow on its head.

$$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$$

Modulus of Vector :-

$$|\vec{a}| = \sqrt{x^2+y^2+z^2}$$

Unit Vector:-

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

Unit vector is a vector of unit magnitude which represents only the direction.

Direction cosine

The cosine of the angle made by a vector with the +ve direction of x, y & z axis are known as direction cosines. Let α, β, γ represents the direction, then the direction cosine will be represented by

$$\cos\alpha = l$$
$$\cos\beta = m$$
$$\cos\gamma = n$$

$$\boxed{l^2+m^2+n^2 = 1}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

** The components of the unit vectors are direction cosines.

$$\frac{x}{\sqrt{x^2+y^2+z^2}} = l \qquad \frac{z}{\sqrt{x^2+y^2+z^2}} = n$$

$$\frac{y}{\sqrt{x^2+y^2+z^2}} = m$$

Direction Ratio

Three smallest nos. denoted by a, b, c which when divided by direction cosines make their ratio equal, —

$$\boxed{\frac{l}{a} = \frac{m}{b} = \frac{n}{c}}$$

*** The components of a vector are direction ratio.

$a^2+b^2+c^2 \neq 1$ | These are random nos.

* Direction cosines are unique but direction ratios are not unique.

Equal Vectors:-

Vectors having the same length (magnitude) and same direction. | In other words components are same

Like and Unlike Vectors:-

Vectors having the same directn are like vectors and having different directn are unlike vectors irrespective of there magnitude.

Co-linear Vectors:-

Two ore more vectors are said to be colinear if they are parallel or anti-parallel to the same straigt line irrespective of their magnitude and direction.

$$\boxed{\vec{a} = \lambda\vec{b}}$$

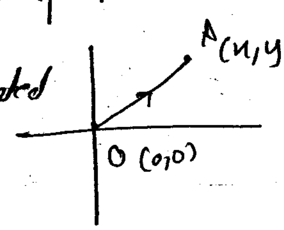
λ = +ve parallel
λ = -ve anti-parallel

$3\hat{i}+2\hat{j}+5\hat{k}$
$6\hat{i}+4\hat{j}+10\hat{k}$ $\boxed{2\vec{a}=\vec{b}}$

Position Vectors:-

Let O be any fixed pt, and A be any other pt. then the vector OA is called as the position vector of A.

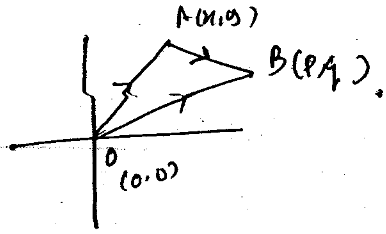
Every pt. in a cartesian plane can be represented lyk a vector and that will be a position vector.



$\vec{OA} = (x-0)\hat{i}+(y-0)\hat{j}$

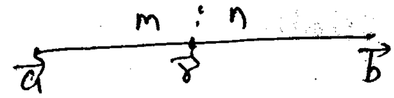
$$\vec{OA} = (x-0)\hat{i} + (y-0)\hat{j}$$

$$\vec{AB} = (p-x)\hat{i} + (q-y)\hat{j}$$



Section formula

If there is a line joining a & b and r is pt which divides the line in the ratio m : n.



$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

if it divides externally $$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

Mid pt. formula :-

Let there is a line segment joining pts a & b and r be the mid pt.

$$\vec{r} = \frac{\vec{a}+\vec{b}}{2}$$

Scalar dot product of two vectors :-

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

Sc Sc Sc

geometrically, it represents the projection of any vector on any other vector.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos\theta$$
$$= 1 \times 1 \times 1 = 1$$

$$\boxed{\hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = \hat{i}\cdot\hat{i} = 1}$$

$$\boxed{\begin{aligned}\vec{a}\cdot\vec{a} &= |\vec{a}|\,|\vec{a}|\cos 0 \\ &= |\vec{a}|^2\end{aligned}}$$

$$\hat{i}\cdot\hat{j} = |\hat{i}| \times |\hat{j}| \cos\theta \rightarrow \pi/2$$
$$= 1 \times 1 \times 0 = 0$$

$$\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$$

$$\vec{b}\cdot\vec{a} = |\vec{b}|\,|\vec{a}|\cos\theta$$

$$\vec{a}\cdot\vec{b} = |\vec{b}|\,|\vec{a}|\cos\theta$$

$$\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$$

Multiplication

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\boxed{\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3}$$

; $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
; $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

*** Dot pdt reduces the order by 2

Order of a vector.

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

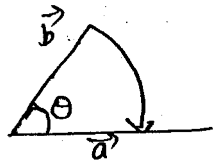
order 1 1

1·1

order. = 2

Geometrical Meaning of Dot pdt :-

The dot pdt is used to find the projection of of vector to find the vector.



$$\vec{a}\cdot\vec{b} = a(b\cos\theta)$$

Projection of vector on any other vector :-

⇒ Projection of vector b on vector a

$$\vec{b} \text{ on } \vec{a} = \vec{b}\cdot\hat{a}$$
$$\vec{a} \text{ on } \vec{b} = \vec{a}\cdot\hat{b}$$

⇒ If two vectors are $\perp^r$, then

$$\boxed{\vec{a}\cdot\vec{b} = 0}$$

Cross pdt or Vector pdt

$$\vec{a} \times \vec{B} = \underbrace{|\vec{a}|}_{sc} \underbrace{|\vec{b}|}_{sc} \underbrace{\sin\theta}_{sc} \cdot \underbrace{\hat{n}}_{vec}$$

if we need a vector which is $\perp^r$ to both A & B we go for cross pdt

$\hat{n}$ is a unit vector which is $\perp^r$ both A & B

$$\hat{i} \times \hat{j} = |\hat{i}|\,|\hat{j}| \sin 90^\circ \hat{k}$$

$$\boxed{\hat{i} \times \hat{j} = \hat{k}}$$

$$\boxed{\begin{aligned} \hat{j} \times \hat{k} &= \hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} \end{aligned}}$$

$$\boxed{\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})}$$

If two rows are changed in matrix then value of determinant is changed.

$$\hat{i} \times \hat{i} = |\hat{i}|\,|\hat{i}| \sin\theta\, \hat{n}$$

$$\boxed{\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}}$$

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Vectors are || or anti-||

$$\boxed{\vec{a} \times \vec{b} = 0}$$

if vectors are parallel (collinear law)

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda}$$

Cross pdt reduces the order by 1.

Geometrically meaning of Cross pdt

Geometrically the cross pdt represents the area of a parallelogram. let $\vec{a} \times \vec{b}$ be the adjacent sides of the parallelogram. Then the area of parallelogram is given by $|\vec{a} \times \vec{b}|$.

In the same way cross pdt is used to find the area of Δ. That $\vec{a}$ & $\vec{b}$ be the two adjacent side of a triangle, then the area of triangle is $\frac{1}{2}|\vec{a} \times \vec{b}|$

Scalar Triple pdt :—

$$\vec{a}\cdot(\vec{b} \times \vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}]$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Geometrically it represents the volume of a parallelopipe.

If the 3-vectors are co-plainar then the scalar three pdt is 0.

Q.) Find the values of x, y & z so that 2 vectors $x\hat{i} + 2\hat{j} + y\hat{k} =$ $3\hat{i} + z\hat{j} + 5\hat{k}$ ~~for~~ are equal.

$$x = 3,\ y = 5,\ z = 2$$

Q.) For a vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$. Find the values of direction ratios & direction cosines

$$DR = 1, 1, -2$$

$$DC = \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}.$$

Q.) Find the value of A for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + a\hat{j} + 3\hat{k}$

are parallel & $\perp^r$.

<u>for parallel</u>

$$\frac{3}{1} = \frac{2}{a}$$

$$a = 2/3$$

<u>for $\perp^r$</u>

$$3\cdot1 + 2\cdot a + 9\cdot3 = 0$$

$$2a = 30$$

$$a = 15.$$

Q.) If $\vec{a} = 4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b} = \hat{i}-4\hat{j}+5\hat{k}$, $\vec{c} = 3\hat{i}+\hat{j}-\hat{k}$. Then find the vector $\vec{d}$ which $\perp^r$ to both $\vec{a}$ & $\vec{b}$, and $\vec{d}\cdot\vec{c} = 21$.

$$\vec{d}\cdot\vec{a} = 0$$
$$\vec{d}\cdot\vec{b} = 0$$

$$\vec{d} = a\hat{i}+b\hat{j}+c\hat{k}$$

$$4a+5b-c = 0$$
$$a-4b+5c = 0$$
$$3a+b-c = 21$$

$$d = 7(\hat{i}+\hat{j}-\hat{k})$$

Q.) If $\alpha = 3\hat{i}-\hat{j}$, $\beta = 2\hat{i}+\hat{j}-3\hat{k}$, Then express β in the form of $\vec{\beta_1}+\vec{\beta_2}$ When $\vec{\beta_1} \parallel \alpha$, $\vec{\beta_2} \perp^r \alpha$.

$$3a_2 - b_2 = 0$$
$$\frac{3}{a_1} = \frac{-1}{b_1}$$
$$3b_1 = -a_1$$
$$3a_2 + a_1 = 2$$
$$-b_2 + 3b_1 = 1$$

$$\beta_1 = \lambda\alpha$$

$$\beta_2 = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

$$\vec{\beta_2}\cdot\vec{\alpha} = 3b_1 - b_2 = 0$$

$$\vec{\beta} = \vec{\beta_1}+\vec{\beta_2}$$

$$2\hat{i}+\hat{j}-3\hat{k} = 3\lambda\hat{i} - \lambda\hat{j} + b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

$$2 = 3\lambda + b_1 \qquad -3 = b_3$$
$$1 = -\lambda + b_2$$

Q.) For what value of λ, $2\hat{i}-\hat{j}+\lambda\hat{k}$, $\hat{i}-\hat{j}+2\hat{k}$ & $3\hat{i}-\hat{j}+2\hat{k}$ are Co-planer.

Q.) If $\hat{i}+\hat{j}+\hat{k}$ is equally inclied to the axis, then the value of direction angles are. → $\alpha=\beta=\gamma \Rightarrow l^2+m^2+n^2=1$

Q.) A girls walks 4 km towards west then she walk 3 km in a dirth 30°E of North and stops. Determine the girls displacement from her initial pt of departure.

Q.) If a unit vector A makes angle $\pi/3$ with the $\hat{i}$, $\pi/4$ wid $\hat{j}$ & acute angle with $\hat{k}$ then find the components of the vector A and the value of acute angle.

Aug 25, 14

Gradient of a Scalar field

$\nabla \rightarrow$ Del or Nabla

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\phi = xyz$$

$$\boxed{\nabla\phi = yz\hat{i} + xz\hat{j} + xy\hat{k}}$$

gradient of any scalar field gives the normal vector of that field.

$|\nabla\phi|$ = magnitude of vector.

Divergence:–

$$\boxed{\nabla\cdot\vec{F} = \frac{\partial}{\partial x}F_1 + \frac{\partial}{\partial y}F_2 + \frac{\partial F_3}{\partial z}}$$

$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ $F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

gives Rate of outflow of any quantity per unit area, per unit tym.

* $\boxed{\nabla\cdot\vec{F} = 0}$ is known as Solenoidal condition.

* $\boxed{\nabla\cdot\vec{V} = 0}$ if the fluid is incompressible

velocity

Curl of vector field:–

Curl of a vector is a vector.
→ geometrically curl is a measure of angular velocity at any pt.

$\boxed{\vec{\Omega} = \frac{1}{2}\,\text{curl}\,\vec{V}}$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ $F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

Physical curl determines rate of rotation or angular velocity.

* $\boxed{\nabla\times\vec{F} = 0}$ vector is irrotational.

* $\boxed{\frac{1}{2}(\nabla\times\vec{V}) = \text{Angular Velocity.}}$

* Curl of velocity is called Vorticity. $(\nabla\times\vec{V})$

Laplacian Operator:–

defined as the dot pdt of grad with grad of any scalar field

It is the divergence of gradient of a scalar pdt.

$\phi(x,y,z)$

$$(\nabla\cdot\nabla\phi)$$

$$(\nabla^2\phi)$$

Laplacian Operator

$\boxed{\text{Laplacian op.}\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$ $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

* $\boxed{\nabla^2\phi = 0}$ then funct^n is harmonic

Finding the accel^n from the velocity vector:—

$$\vec{V} = U\hat{i} + V\hat{j} + W\hat{k}$$
$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$
$$a_x = u\partial u/\partial x + v\,\partial u/\partial y + w\,\partial u/\partial z + \partial u/\partial t$$
$$a_y = u\,\partial v/\partial x + v\partial v/\partial y + w\,\partial v/\partial z + \partial v/\partial t$$
$$a_z = u\,\partial w/\partial x + v\,\partial w/\partial y + w\partial w/\partial z + \partial w/\partial t$$

velocity chang w.r.t to tym & accel^n

Directional Derivative: - defined as a vector drawn outward to a surface in the direction of vector $\vec{a}$,

$$\boxed{DD = \hat{a}\cdot(\nabla\phi)}$$
$$= (l \quad m \quad n)\left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}\right)\phi$$
$$= \left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y} + n\frac{\partial}{\partial z}\right)\phi$$

Q.> A field is given by $(3x^2y - y^3z^3)$. Find the gradient of this field at pt $(1,1,1)$

$$\nabla\phi = 6xy\,\hat{i} + (3x^2 - 3y^2z^3)\hat{j} + (-3y^3z^2)\hat{k}$$
$$\nabla\phi_{(1,1,1)} = 6\hat{i} + -3\hat{k}$$
$$|\nabla\phi| = \sqrt{36+9} = \sqrt{45}$$

dir^n derivative in dir^n $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\hat{a} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$
$$\hat{a}\cdot(\nabla\phi) = \frac{1}{\sqrt{3}}\cancel{(\hat{i}+\hat{j}+\hat{k})}\cdot(6 - 3)$$
$$= 3/\sqrt{3}$$

Q.) Find gradient for the field xy^2+yz^3 at pt $(2,-1,1)$

$$\nabla\phi = y^2\hat{i} + (2xy+z^3)\hat{j} + (3yz^2)\hat{k}$$

$$\nabla\phi_{(2,-1,1)} = \hat{i} - 3\hat{j} - 3\hat{k}$$

$$|\nabla\phi| = \sqrt{1^2+9^2+9^2} = \sqrt{19}$$

find the dirth derivative in the dir^n $\parallel$ to line $\frac{x-5}{3} = \frac{y-4}{2} = \frac{z-3}{1}$

$$\vec{a} = 3\hat{i}+2\hat{j}+\hat{k}$$

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

! if two lines are $\parallel$ then there dir^n ratio are equal & a,b,c are the dir^n

$$\hat{a} = \frac{1}{\sqrt{14}}(3\hat{i}+2\hat{j}+\hat{k})$$

$$\hat{a}\cdot(\nabla\phi) = \frac{1}{\sqrt{14}}(3-6-3) = -6/\sqrt{14}$$

Q.) h $2x^2+3y^2+z^2$. Find gradient & modulus.

$$\nabla\phi = 4x\hat{i} + 6y\hat{j} + 2z\hat{k}$$

$$\nabla\phi_{(2,1,3)} = 8\hat{i} + 6\hat{j} + 6\hat{k}$$

$$|\nabla\phi| = \sqrt{64+36+36} = \sqrt{136}.$$

find D.D in the direction of a vector $\perp^r$ this plane.

$$3(x-4)+2(x-3)+(z-3)=0$$

$$a(x-x_1)+b(y-y_1)+c(z_1-z_1)=0$$

! general eq^n of plane a,b,c are direct ratio of normal vector, (x_1,y_1,z_1) are the pt through which line pass

$$\vec{a} = 3\hat{i}+2\hat{j}+\hat{k}$$

$$\hat{a} = \frac{1}{\sqrt{14}}(3\hat{i}+2\hat{j}+\hat{k})$$

$$\hat{a}(\nabla\phi) = \frac{1}{\sqrt{14}}(24+12+6) = 42/\sqrt{14}$$

Q) If the field is given by $3x^2\hat{i} + 5xy^2\hat{j} + xyz^3\hat{k}$. find div at (1,2,3)

$$\nabla\cdot\vec{F} = 6x\hat{i} + 10xy\hat{j} + 3xyz^2\hat{k}$$

$$\nabla\phi_{(1,2,3)} = 6\hat{i} + 20\hat{j} + \cancel{54}\hat{k}$$

$$\nabla\cdot\vec{F}_{(1,2,3)} = 6+20+54 = 80$$

Q) find the curl of this vector field.

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & 5xy^2 & xyz^3 \end{vmatrix}$$

$$= \hat{i}\,[xz^3 - 0] - \hat{j}\,[yz^3 - 0] + \hat{k}\,[5y^2 - 0]$$

$$\nabla\times\vec{F}\Big|_{(1,2,3)} = 27\hat{i} - 54\hat{j} + 20\hat{k}$$

Q) Vector field is given by $\vec{F} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ and find the value of a if the vector is solenoid.

$$\cancel{0 = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+3y & y-2z & x+az \end{vmatrix}}$$

for solenoidal condition

$$\cancel{0 = [0-1]\hat{i} - \hat{j}\,[1-0] + \hat{k}\,[0+3}$$

$$0 = \nabla\cdot\vec{F} = \frac{\partial}{\partial x}(x+3y)\hat{i} + \frac{\partial}{\partial y}(y-2z)\hat{j} + \frac{\partial}{\partial z}(x+az)\hat{k}$$

$$= 1 + 1 + a = 0$$

$$= \boxed{a = -2}$$

Q) If a vector field is given by $\vec{F} = 3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$. Find div.

$$\nabla\cdot\vec{F} = 3z^2 + 2x - 3yz^2$$

$$\nabla\cdot\vec{F}(1,1,1) = 3+2-3$$
$$= 2$$

Laplacian.

$$\nabla^2 F = 3z + 2y + 2x - 2yz + 0$$

Q for any $\vec{r}$ = find div?

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$| \; 1+1+1 = 3$

$$\boxed{\nabla\cdot\vec{r} = 3}$$

find curl?

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= 0$$

$$\boxed{\begin{array}{l} \nabla\cdot\vec{r} = 3 \\ \nabla\times\vec{r} = 0 \end{array}}$$

Q.) For the velocity vector $\vec{V} = 2xy\hat{i} - x^2z\hat{j}$. Find the vorticity vector.

$$\text{Curl}\,\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy & -x^2z & 0 \end{vmatrix}$$

$$= \hat{i}\left[0 - (-x^2)\right] - \hat{j}\left[0-0\right] + \hat{k}\left[-2xz - 2x\right]$$

$$= x^2\hat{i} - (2xz+2x)\hat{k}$$

$$\text{Curl}\,\vec{V}_{(1,1,1)} = \hat{i} - 4\hat{k}$$

angular velocity $= \frac{1}{2}(\text{curl}\,\vec{v})$

$= \frac{i - 4\hat{k}}{2}$

Q→ $\phi = x^2 + 3y^2 + 2z^2$

Find D.D at pt $(1,2,-1)$. In the direction of a line joining the pts $(2,-2,3)$ & $(3,-3,5)$

$(1,-1,2)$

$(1,-1,2)$ DD $= \frac{1}{\sqrt{6}}(2) - \frac{1}{\sqrt{6}}(12) + \frac{2}{\sqrt{6}}(-4)$

$= \frac{2-12-8}{\sqrt{6}} = \frac{-18}{\sqrt{6}}$

$\hat{a} = \frac{1}{\sqrt{6}}$

apply formula.

Q→ If a vector field is given by $\vec{F} = x^2y\hat{i} + xyz\hat{j} + yz^3\hat{k}$. Then find div of curl of this field.

$[\nabla \cdot (\nabla \times \vec{F})] = \text{div}(\text{curl}\,\vec{F})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & xyz & yz^3 \end{vmatrix}$$

$= \cancel{i[0-0] - j[0-0] + \hat{k}[0-0]}$

0

$= i[z^3 - 2xyz] - j[0-0] + \hat{k}[y^2z - x^2]$

$= (z^3 - 2xyz)\hat{i} + (y^2z - x^2)\hat{k}$

div (curl $\vec{F}$) $= -2yz + 2yz$

$= 0$.

$$(\text{curl grad } \phi) = (\nabla \times (\nabla\phi))$$

$$\nabla\phi = x^2 + 2xyz + 3yz^2 = 2x\hat{i} + (2x+4xy+3z^2)\hat{j} + 6yz)\hat{k}$$

$$\text{curl}(\nabla\phi) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x & 2x+4xy+3z^2 & 6yz \end{vmatrix} = 0$$

$$\boxed{\begin{aligned} \nabla\cdot(\nabla\times\vec{F}) &= \text{div}(\text{curl}\vec{F}) = 0 \\ (\text{curl grad } \phi) &= (\nabla\times(\nabla\phi)) = 0 \end{aligned}}$$

Q.) For the unit sphere find the normal vector at (1, 1, 1)

$$\phi = x^2+y^2+z^2 = 1$$

$$\nabla\phi = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\nabla\phi_{(1,1,1)} = 2(\hat{i}+\hat{j}+\hat{k})$$

Q.) Directional derivative of the field at $\ln(x^2+z^2)$ at pt (4,0) in the dir^n of a line parallel to ($x=-z$) is ?

Sol^n

$$\nabla\phi = \ln(x^2+z^2)$$

$\parallel$ line ($x=z$)

$$a=1, b=0, c=-1$$

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{i}-\hat{k})(\nabla\phi)$$

$$\nabla\phi = \frac{2x}{x^2+z^2} + 0 + \frac{2z}{x^2+z^2}$$

$m = 1/2\sqrt{2}$

$$\nabla\phi_{(4,0)} = \frac{1}{2}\hat{i}$$

$$D.D = \hat{a}(\nabla\phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\hat{i}\cdot\hat{i} + 0 + 0\right)$$

$$= \frac{1}{2\sqrt{2}}$$

Q> For an incompressible flow the velocity vector is $\vec{V} = 2(x+y)\hat{i} + 3(y+z)\hat{j} + \frac{1}{2}\hat{k}$ then the z-component of this velocity vector at the boundary condition ($V_z = 0$ at $z=0$) will be?

for incompressible

$$\nabla \cdot \vec{V} = 0$$

$$2 + 3 + \frac{\partial V_z}{\partial z} = 0$$

$$\int_0^{V_z} \partial V_z = -\int_0^z 5\, \partial z$$

$$V_z = -5z$$

Q.) Find D.D $f = x^{2/3} + y^{2/3}$, at the line $x = y$ at $(8,8)$

$\therefore a = 1,\ b = -1,\ c = 0$

$$\hat{a} = \frac{1}{\sqrt{2}} (\hat{i} - \hat{j}) (\nabla \phi)$$

$\frac{1}{3} - 1$

$$= \frac{1}{\sqrt{2}} (\hat{i} - \hat{j}) \left(\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\frac{2}{3} 8^{-1/3} + \frac{2}{3} 8^{-1/3} \right)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{2}{3} = \frac{\sqrt{2}}{3} \text{ Ans.}$$

$\frac{\sqrt{2}}{3}$ Ans.

Q.) Find the derivative of max^m directional derivative for the field at Pt $P(1,4,3)$ in the dir^n of line PQ where $Q(5,0,4)$

$$\phi = x^2 - y^2 + 2z^2$$

$$\nabla \phi = 2x - 2y + 4z$$

$(1,2,3)$

$$= 2 - 4 + 12$$

$$|\nabla \phi| = \sqrt{4 + 16 + 144} = \sqrt{164}$$

$a = 4,\ b = -2,\ c = 1$

normal vector is max^m d.d.

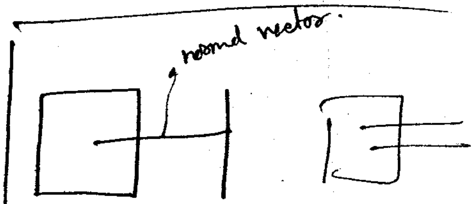
greatest

fastest

Q.) $5x^2y - 5y^2z + 2.5xz^2$. Find D.D at $(1,1,1)$ in the directⁿ of a line $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$ such that line is ~~⊥~~ ∥ plane.

$2(x-5) + 3(y-4) + (z-3) = 0$

$Ans = \frac{5}{2}\sqrt{69}$.



* if two lines are ∥ then there d.r are same

* ⊥ pt of vector & d.r = 0, if two lines are ⊥r

Q.) A steady flow film of an incompressible fluid is given by

$$\vec{V} = (Ax + By)\hat{i} + Ay\hat{j}$$

$A = B = 1\,s^{-1}$. x & y are in m.

Find the magnitude of accelⁿ at the pt $(1,2)$ in m/sec^2.

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$ = . $u = Ax + By = x + y$

$v = -Ay = -y$

$w = 0$

$a_x = (x+y)1 + (-y)1$

$a_{x(1,2)} = 3 - 2$

$= 1$

$a_y = 0 + (-y)(-1) + 0$

$a_{y(1,2)} = 2$

$\vec{a} = \hat{i} + 2\hat{j}$

$|\vec{a}| = \sqrt{5}\ m/sec^2$

Q.) $\phi = ax^2y - y^3$

Find the value of a, if the funct^n is harmonic

$$\nabla^2\phi = 0$$

$$2ay + 6y = 0$$

$$(6 + 2a)y = 0$$

3 Ans

Q.) The temp field in a body varies acc to eq^n $T(x,y) = x^3 + 4xy$.
The dir^n of the fastest variation of the temp at (1, 0) is given by.

$(0.6\hat{i} + 0.8\hat{j})$ Ans.

$$\nabla\phi = 3x^2 + 4x$$

$$\nabla\phi_{(1,0)} = 7$$

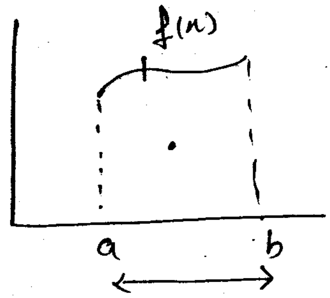
$$|\nabla\phi| = \sqrt{9+16} = 5$$

Aug 26, 14

LINE INTEGRAL

The integral which is to be evaluated over a line is known as line integral.

$$\int_a^b f(x)\,dx$$



for vector funct^n

$$\oint_C \vec{F}\cdot d\vec{r}$$

$$\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\boxed{\int_C \vec{F}\cdot d\vec{r} = \int_C F_1 dx + F_2 dy + F_3 dz}$$

Q-> $\vec{F} = (x^2-y^3)\hat{i} + (xy)\hat{j}$ and curve C is the arc of the curve $y=x^3$ from the pts $(0,0)$ & $(2,8)$

$$\int_C \vec{F}\cdot d\vec{r} = \oint_C (x^2-y^2)\,dx + (xy)\,dy$$

| $y=x^3$ (given)
| $dy = 3x^2dx$

$$= \int_0^2 \left((x^2-x^6)dx + x\cdot x^3\cdot 3x^2 dx\right)$$

$$= \frac{x^3}{3} - \frac{x^7}{7} + \frac{3x^7}{7}\Big|_0^2$$

$$= \frac{x^3}{3} + \frac{2x^7}{7}\Big|_0^2$$

$$= \cancel{\frac{8}{3}}\ \cancel{A}\ 2 \quad = 39.23$$

Q.) if $\vec{a} = t\hat{i} + 3\hat{j} + 2t\hat{k}$
$\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$
$\vec{c} = 3\hat{i} + t\hat{j} - \hat{k}$

a, b, c are vectors given. Find $\int_1^2 [\vec{a}\,\vec{b}\,\vec{c}]\,dt$

Ans = 0

$[\vec{a}\vec{b}\vec{c}]$ = scalar triple pdt.

$\int_1^2 \{\vec{a}\cdot(\vec{b}\times\vec{c})\}\,dt$ determinant

$\hat{i} \quad \hat{j} \quad \hat{k}$

Q.) $\vec{F} = \dfrac{\hat{i}y - \hat{j}x}{x^2+y^2}$, Curve $\rightarrow x^2+y^2 = 1$. find $\oint_c \vec{F}\cdot d\vec{r}$

$\int \left(\dfrac{y}{x^2+y^2}\right)dx - \left(\dfrac{x}{x^2+y^2}\right)dy$

$\dfrac{y}{1-\cancel{y^2}+\cancel{y^2}}\,dx - \dfrac{\sqrt{1-y^2}}{1-y^2+y^2}\,dy$

$x^2+y^2 = 1$
$x^2 = 1-y^2$
$x = \sqrt{1-y^2}$
$2x\,dx = 1-2y\,dy$
$dx = \dfrac{1-2y\,dy}{2x}$

change to parametric form

$\int y\,dx - x\,dy$

$\int_0^{2\pi} \sin\theta\,(-\sin\theta)\,d\theta - \cos\theta\,(\cos\theta)\,d\theta$

$x = \cos\theta$
$y = \sin\theta$
$dx = -\sin\theta\,d\theta$

$\cancel{-\sin^2\theta\,d\theta - \cos^2\theta\,d\theta}$
$\cancel{-2\sin\theta\cos\theta + 2\cos\theta\sin\theta}$

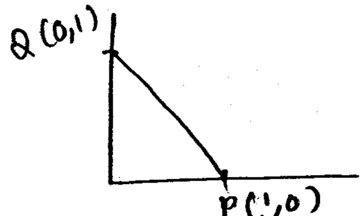
$$= -\int_0^{2\pi} d\theta.$$

$$= -\theta\Big|_0^{2\pi}$$

$$= -2\pi$$

Q.) consider the pts P & Q in x-y plane P(1,0), Q(0,1)

Find $2\int_P^Q (x\,dy + y\,dx)$ over the line PQ.



Sol^n

$$2\int_1^0 -x\,dx + (1-x)\,dx$$

$$(y-0) = \frac{1-0}{0-1}(x-1)$$

$$y = -x+1$$

$$x+y=1$$

$$dx+dy=0$$

$$dx - dy.$$

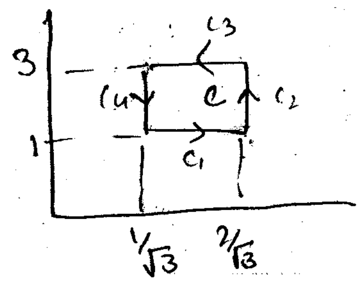
$$= 2\left|\frac{-x^2}{2} + x - \frac{x^2}{2}\right|_1^0$$

$$2\,|\,-x^2+x$$

$$= 2(-1+1)$$

$$2\times 0 = 0$$

Q) If $\vec{A} = xy\hat{i} + x^2\hat{j}$. Find the value of $\oint \vec{A}\cdot d\vec{r}$ over a rectangle



$$\oint_C \vec{A}\cdot d\vec{r} = \oint_C \underbrace{xy\,dx + x^2 dy}_{*}$$

$$= \oint_{C_1} * + \oint_{C_2} * + \oint_{C_3} * + \oint_{C_4} *$$

$$= \oint_{1/\sqrt{3}}^{2/\sqrt{3}} x\,dx + \tfrac{4}{3}\oint_1^3 dy + 3\oint_{2/\sqrt{3}}^{1/\sqrt{3}} x\,dx + \tfrac{1}{3}\oint_3^1 dy$$

$$= \frac{x^2}{2}\Big|_{1/\sqrt{3}}^{2/\sqrt{3}} + \tfrac{4}{3}\,y\Big|_1^3 + 3\,\frac{x^2}{2}\Big|_{2/\sqrt{3}}^{1/\sqrt{3}} + \tfrac{1}{3}\,y\Big|_3^1 \qquad \Big|\frac{3}{2\sqrt{3}}$$

$$\frac{12+}{3} - \frac{7}{3}$$

$$\frac{4}{2\sqrt{3}} - \frac{1}{2\sqrt{3}} + \frac{4}{3}\cdot 3 - \frac{4}{3} + \frac{3}{2\sqrt{3}} - \frac{6}{2\sqrt{3}} + \frac{1}{3}\cdot 1 - \frac{3}{3}$$

$$\frac{1}{2}\left(\frac{4}{2\cdot 3}-\frac{1}{2\cdot 3}\right)+\frac{4}{3}(3-1)+\frac{3}{2}\left(\frac{1}{3}-\frac{4}{3}\right)+\frac{1}{3}(1-3)$$

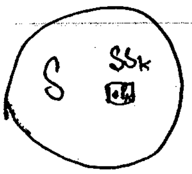
$$\cancel{\frac{1}{2}+\frac{8}{3}-\frac{3}{2}-\frac{2}{3}}$$

$$-1+2$$

$$=1$$

Ans $=1$

Surface Integral :—



$\vec{F}(x_k\, y_k\, z_k)$

$\oint_S \vec{F}(x_k\, y_k\, z_k)\, dS_k$

$n \to \infty$
$\delta S_k \to 0$

For vectors.

$$\oint_S \vec{F}\cdot\hat{n}\, ds$$

$$\oint_S \vec{F}\, \vec{ds}$$

$$ds = \left|\frac{dx\cdot dy}{\hat{n}\cdot\hat{k}}\right|$$

Q.7 The $\vec{F} = yz\hat{i} + 2x\hat{j} + xy\hat{k}$. The surface is given to be a part of Sphere $x^2+y^2+z^2=1$, which lies in the first octant. Then the value of $\iint \vec{F}\cdot\hat{n}\, ds$ will be.

$$\phi = x^2+y^2+z^2-1$$

$$\nabla\phi = 2x\hat{i}+2y\hat{j}+2z\hat{k}$$

$$|\nabla\phi| = \sqrt{2x^2+2y^2+2z^2} = \sqrt{4(1)} = \sqrt{4} = 2$$

$$\hat{n} = x\hat{i}+y\hat{j}+z\hat{k}$$

$$\vec{F}\cdot\hat{n} = 3xyz$$

$$|\hat{n}\cdot\hat{k}| = z$$

$$\iint \frac{3xyz}{2}\, dx\, dy.$$

$\cancel{\frac{3x^2y^2}{4}}$

$3\cancel{\cos}\cos\theta \cancel{\sin\theta\,(-\sin\theta)\,d\theta}\ \cancel{d}$

$$\int_0^{\pi/2}\int_0^1 3r\cos\theta\sin\theta\, dr\, d\theta$$

$$\int_0^{\pi/2} 3\cos\theta\sin\theta \cdot \frac{1}{2}\, d\theta$$

$$\frac{3}{4}\ \sin 2\theta\, d\theta \Big|_0^{\pi/2}$$

$$\frac{3}{4}\ \frac{\cos 2\theta}{2}\Big|_0^{\pi/2}$$

$$\frac{3}{8}(0-1) = -3/8.$$

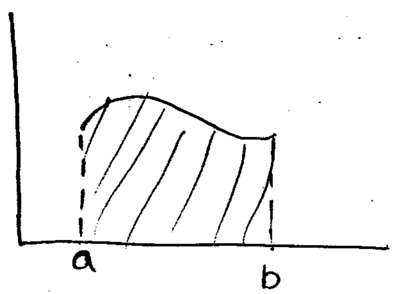
$\because$ z doesn't lie in 1st octant.

$x = \cos\theta$
$y = \sin\theta$
$dx =$
$\theta \to 0, \pi/2$
$x \to 0, 1$

$\boxed{dx\,dy = r\,dr\,d\theta}$

Ans $= 3/8$

Stoke's Theorem :— Random shape

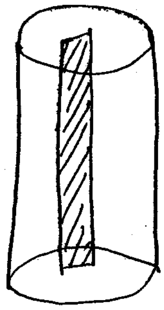


Stoke Theorem relate line integral to surface Integral

$$\oint \vec{F}\cdot d\vec{r} = \int_S \int (\nabla\times\vec{F})\,\hat{n}\, ds$$

Gauss- Divergence Theorem :— for perfect shape

relates volume integral with surface integral.



$$\iiint_V \nabla\cdot\vec{F}\, dv = \iint_S \vec{F}\cdot\hat{n}\, dS$$

$\hat{n}$ = normal vector to the surface S

Q) $I = \iint \vec{r}\cdot d\vec{s}$. determine the value of I if s is the surface of a sphere of radius r

$$I = \iint \vec{r}\cdot d\vec{s}$$

$$= \iint \vec{r}\cdot\hat{n}\, ds$$

$$= \iiint_V \underbrace{\nabla\cdot\vec{r}}_{3}\, dV$$

$$= 3\iiint dV$$

| $dV = dx\,dy\,dz$.

$$= 3\,\frac{4}{3}\pi R^3$$

Q) The value of the surface integral $\vec{r} = x\hat{i}$ $\iint_S (x\hat{i}+y\hat{j})\cdot\hat{n}\,dA$ over the surface of cube having side a.

$$\iint_S (x\hat{i}+y\hat{j})\cdot\hat{n}\,dA = \iiint_V \nabla\cdot(x\hat{i}+y\hat{j})\,dV$$

$$= 2\iiint dV$$

$$= 2a^3$$

Green's Theorem:- relates line integral with surface integral

$$\oint_C \vec{F}\cdot d\vec{r} = \oint_C F_1\,dx + F_2\,dy$$

$$= \oint_C M\,dx + N\,dy$$

$$\boxed{\oint_C M\,dx + N\,dy = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dx\,dy}$$

Q) $I = \dfrac{y\,dx - x\,dy}{x^2+y^2}$, $x^2+y^2=1$

$$\oint_C I = \oint_C y\,dx - x\,dy$$

$M = y \quad N = -x$.

$$= \iint \left(\frac{\partial(-x)}{\partial x} - \frac{\partial y}{\partial y}\right) dx\,dy$$

$$= \iint (-1-1)\,dx\,dy\,.$$

$$= -2\iint \underbrace{dx\,dy}_{=ds} \qquad \Big|\; dx\,dy = r\,dr\,d\theta$$

$$= \cancel{-2s}$$

$$= -2\int_0^{2\pi}\int_0^1 r\,dr\,d\theta$$

$$= -2\int_0^{2\pi} \frac{r^2}{2}\Big|_0^1 d\theta$$

$$= -2\int_0^{2\pi} \frac{1}{2}\,d\theta$$

$$= -2\cdot\frac{1}{2}\,\theta\Big|_0^{2\pi}$$

$$= -2\pi$$

Q) A fluid element has the velocity $\vec{V} = -y^2x\,\hat{i} + 2yx^2\,\hat{j}$. The motion at $(\frac{1}{\sqrt{2}}, 1)$ is

i) rotational & incompressible
ii) irrotational & incompressible
iii) irrotational & compressible
iv) rotational & compressible

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y^2x & 2yx^2 & 0 \end{vmatrix}$$

$$= \hat{i}\,(0-0) + \hat{j}\,(0-0) + \hat{k}\,(4xy \mp 2yx)$$

$$= \cancel{4xy+2yx}\; 8xy\,\hat{k}\Big|_{\frac{1}{\sqrt{2}},1} = \sqrt{2}\,\hat{k}$$

$\neq 0$

rotational.

$$\nabla\cdot\vec{F} = -y^2 + 2x^2$$

$$= -1 + 2\cdot\frac{1}{2} \qquad 1-1=0$$

incompressible.

Numerical Methods

1.) Sol. of D.E

2.) " " Integral.

3.) " " Transcedental Eq^n

4.) " " Algebraic Eq^n.

Solution of Transcedental Eq^n:-

Any eq^n $\boxed{f(x) = 0}$

f(x) can be any funct^n.

Bisection Method:- (A.M)

⇒ Find the two values a, b such that f(a) = −ve & f(b) = +ve. Then acc. to bisection method, the root can be calculated by AM of a & b

$$x_1 = \frac{a+b}{2}$$

⇒ if $f(x_1) = 0$, then x_1 is the desired root.

→ if $f(x_1) = +ve$, then replace the +ve root.

$$x_2 = \frac{a+x_1}{2}$$

⇒ if $f(x_1) = -ve$, then replace the −ve root.

$$x_2 = \frac{x_1+b}{2}$$

and the procedure continue till we find the root.

Q.) $f(x) = x^3 - 4x - 9$. find root btw 2 & 3

$f(2) = -9$

$f(3) = 6$

$x_1 = \frac{2+3}{2} = 2.5$.

$f(2.5) = -3.375$

$\cancel{x_2 = \frac{2.5+3}{2}} = 2.75$

$f(2.75) = 0.796.$

$x_3 = \frac{2.5+2.75}{2} = 2.625$

$f(2.625) = -1.412$

$x_4 = 2.625 + 2.75$

$= 2.6875$

$f(2.6875) = -0.339.$

$x_5 = \frac{2.6875+2.75}{2}$

$= 2.71875$

$f(2.71875) = 0.2209$

$x_6 = 2.703125$ Ans.

Q.) $f(x) = x\sin x - 1$. Find the root btw 1 & 1.5, find the 5th value upto 4 decimal places. Calc. (rad).

$f(1) = \cancel{+0.9828}\ -0.1585$

$f(1.5) = 0.49624$

$x_1 = \frac{1+1.5}{2} = 1.25$

$f(1.25) = 0.18623$

$x_2 = \frac{1+1.25}{2} = 1.125$

$f(1.125) = 0.0180$

$x_3 = \frac{1+1.125}{2} = 1.0625$

$f(1.0625) = -0.0718$

$$\cancel{x_4 =} \frac{1.0625 + 1.125}{2} = 1.09375$$

$f(1.09375) = -0.02836$

$$x_5 = \frac{1.09375 + 1.125}{2} = \underline{1.109375}$$

$f(x_5) = -0.00664.$

Aug 27, 14

Regula - Falsi Method (Chord)

$(y - y_1) = m(x - x_1)$

$$y - f(x_0) = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}(x - x_0)$$

$$0 - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_0)$$

$$\boxed{x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)}$$

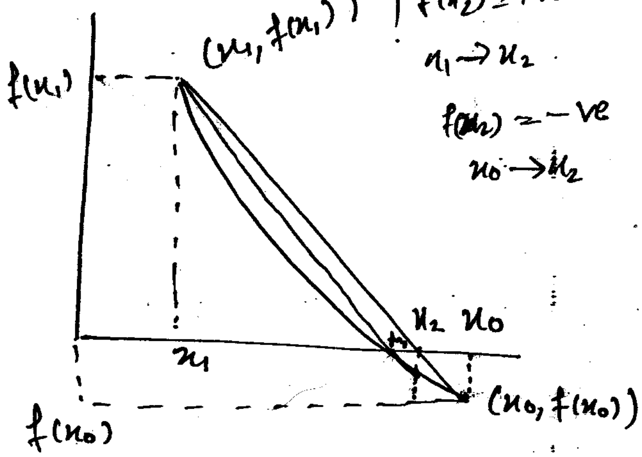
x_2

$f(x_2) = +ve$

$x_1 \to x_2$

$f(x_2) = -ve$

$x_0 \to x_2$



Q.) $x^3 - 2x - 5 = 0$. b/w roots 2, 3

$f(2) = -1$

$f(3) = 16$

$$x_1 = 2 - \frac{(3 - 2)}{16 - (-1)} \cdot (-1)$$

$$= 2 + \frac{1}{17} = 2.05882.$$

$f(x_1) = -0.3908$

$\cancel{x_2 = 2 - \frac{2.05882 - 2}{}}$

$$x_2 = 2.05882 - \frac{3 - 2.05882}{16 - (-0.3908)} \cdot (-0.3908)$$

$$= \cancel{2.03637} \; 2.08126$$

$$f(x_2) = \cancel{-0.62821} \; 0.14724$$

$$\cancel{x_3 = 2.03637} - \frac{3 - 2.03637}{16 - (-0.62821)} (-0.62821)$$

$$\cancel{=} \; \mathrm{P}$$

$$x_3 = 2.08126 - \frac{3 - 2.08126}{16 - (-0.14724)} (-0.14724)$$

$$= 2.089637$$

$$f(x_3) = -0.054693$$

$$x_4 = 2.089637 - \frac{3 - 2.089637}{16 - (-0.054693)} \times (-0.054693)$$

$$= 2.092738$$

$$f(x_4) = -0.020217$$

$$x_5 = 2.092738 - \frac{3(2.092738)}{16 - (-0.020217)} \times (-0.020217)$$

$$= 2.096498$$

$$f(x_5) = 0.021754$$

$$x_6 = 2.092738 - \frac{2.096498 - (2.092738)}{0.021754 - (-0.020217)} (-0.020217)$$

$$= 2.09469$$

Secant Method :-

~~Same~~ as Regula Falsi

$x_0 \rightarrow x_1$

$x_1 \rightarrow x_2$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_2)} f(x_0)$$

Q.) Find the $\sqrt[4]{32}$ by Regular-Falsi method. up to 3 stages.

pts 2 & 3.

$32^{1/4} = x$

2.3349

$32^{\cancel{1}\times 4} = x^4$

$x^4 - 32 = 0$

$x_0 \quad f(2) = -16$

$x_1 \quad f(3) = 49$

$$x_2 = 2 - \frac{3-2}{49-(-16)}(-16)$$

$= 2.24615$

$f(x_1) = -6.54588$

$$x_2 = \cancel{2.24615} \; 3 - \frac{2.24615 - 3}{(-6.54588)-(+49)} \times (+49)$$

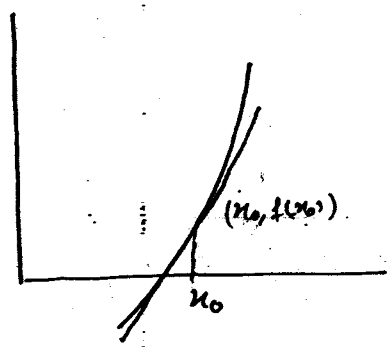
Newton Rapshon Method:—

(tangent)

(fastest method of all)

$$(y - y_1) = m(x - x_1)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



$$\boxed{y - f(x_0) = f'(x_0)(x - x_0)}$$

Recurrence

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

N.R. Algorithm

Q.) $3x - \cos x - 1$, find fourth root. $\underline{x_0 = 0}$

$f'(x) = 3 + \sin x$

$f(0) = -2$

$x_0 = 1 \quad f(1) = 1.45969$

$$x_1 = 1 - \frac{1.45969}{3.84147}$$

$= 0.62001$

$f(x_1) = 0.046185$

$f'(x_1) = 3.58104$

$$= 0.62001 - \frac{''}{3.58104}$$

$x_2 = 0.60711$

$x_3 = 0.60711 - \underline{0.000402}$

$x_4 = 0.60699$

**

Can be used
→ to solve/find real as well as complex root.
& also use to
→ solve both algebraic & transcendental eqn

$$x_{n+1} = x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n}$$

$$\boxed{x_{n+1} = \frac{\cos x_n + x_n \sin x_n + 1}{3 + \sin x_n}}$$

Newton Rapshon iterative formula.

Q> Find the N·R iterative formula for the reciprocal of a natural no.

; y not we write $x = \frac{1}{N}$, Here we have to find reciprocal of natural no....

$$N = 1/x$$

$$f(x) = 1/x - N$$

$$f'(x) = -\frac{1}{x^2}$$

$$x_{n+1} = x_n - \frac{1/x_n - N}{-1/x_n^2}$$

$$= x_n - \frac{\frac{1 - Nx_n}{x_n}}{-1/x_n^2}$$

$$= x_n + (1 - Nx_n)x_n$$

$$\boxed{x_{n+1} = 2x_n - Nx_n^2}$$

$$\frac{1}{N} \rightarrow x_n(2 - Nx_n)$$

Q> Find N-R iterative formula for the square root of any nat. no.

$$\sqrt{N} = x$$

$$N = x^2$$

$$f(x) = x^2 - N$$

$$x_{n+1} = x_n - \frac{x_n^2 - N}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 - N}{2x_n} = \frac{x_n^2 - N}{2x_n}$$

$$\boxed{x_{n+1} = \frac{x_n^2 - N}{2x_n}}$$

$$\sqrt{N} \rightarrow \frac{x_n}{2} + \frac{N}{2x_n}$$

Q.) Find the N-R iterative formula for reciprocal of sq. root of N.

$$x = \frac{1}{\sqrt{N}}$$

$$\sqrt{N} = 1/x$$

$$N = 1/x^2$$

$$f(x) = 1/x^2 - N$$

Here we don't do $x^2 - \frac{1}{N}$ as we have to remove the total complexity of N, upto $x^2 = \frac{1}{N}$ we have removed the square root complexity and still we have to remove the reciprocal complexity.

$$x_{n+1} = x_n - \frac{\frac{1 - Nx_n^2}{x_n^2}}{-2/x_n^3}$$

$$= x_n - \frac{(1 - Nx_n^2)x_n}{-2}$$

$$= \frac{2x_n + x_n - Nx_n^3}{2}$$

$$\boxed{x_{n+1} = \frac{3x_n - Nx_n^3}{2}}$$

$$\frac{1}{\sqrt{N}} \rightarrow \frac{3x_n - Nx_n^3}{2}$$

Q.) Find N-R Iterative formula for k^{th} root of N

$$x = \sqrt[k]{N}$$

$$N = x^k$$

$$f(x) = x^k - N$$

$$x_{n+1} = x_n - \frac{x_n^k - N}{kx_n^{k-1}} = \frac{kx_n^k - x_n^k - N}{kx_n^{k-1}}$$

$$x_{n+1} = \frac{(k-1)x_n^k - N}{\quad}$$

$$\boxed{x_{n+1} = \frac{(k-1)x_n^k + N}{k x_n^{k-1}}}$$

$$\sqrt[k]{N} = \frac{(k-1)}{k} x_n + \frac{N}{k x_n^{k-1}}$$

$\boxed{}$ ⇒ Iterative formula.

═══ → Reccurence formula.

Q.) Find Reccurence formula for $\sqrt[3]{24}$.

$$\sqrt[3]{24} = \frac{(3-1)}{3} x_n + \frac{24}{3 x_n^{3-1}}$$

$$= \frac{2}{3} x_n + \frac{8}{x_n^2}$$

Q.) Find the Reccurence formula Reciprocal of $\sqrt{50}$

$$\frac{1}{\sqrt{50}} \rightarrow \frac{3x_n - 50x_n^3}{2}$$

Q.) for $N=7$, $x_0=0.2$ first two iteration of reciprocal of 7 will be.

$$x_{n+1} = x_n(2-7x_n)$$

$$x_1 = 0.2(2-7\times0.2)$$

$$= 0.12$$

$$x_2 = 0.12(2-7\times0.12)$$

$$= 0.12(2-0.84)$$

$$= 0.12(1.16) = 1.392$$

Q> Find reccurence formula for the eqn $e^x - 1$

$$\cancel{f(x) = e^x - 1}$$

$$x_{n+1} = x_n - \frac{e^{x_n} - 1}{e^{x_n}}$$

$$= \frac{(x_n - 1)e^{x_n} + 1}{e^{x_n}}$$

Start intial guess as -1, find scnd iteration value.

$$x_1 = -1 - \frac{e^{-1} - 1}{e^{-1}}$$

$$= 0.71828$$

$$x_2 = +0.71828 - \frac{e^{0.71828} - 1}{e^{0.71828}}$$

$$= 0.205$$

Q> Find reccurence formula for $x^2 - 117 = 0$

$$f(x) = x^2 - 117$$

$$x_{n+1} = x_n - \frac{x_n^2 - 117}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + 117}{2x_n}$$

$$= \frac{x_n^2 + 117}{2x_n}$$

Q> for the $\sqrt{N}$, N.R. iterative formula $\sqrt{N} = \frac{x_n}{2} + \frac{N}{2x_n}$. This iterative formula is used to calculate the

i> N^2 ii> $1/N$ iii> $\sqrt{N}$ iv> $\log N$

$$x_{n+1} = \frac{x_n}{2} + \frac{N}{2x_n}$$

for any pt of convergence

* $x_{n+1} = x_n = \alpha$

$$x_{n+1} = x_n = \alpha$$

$$\alpha = \frac{\alpha}{2} + \frac{N}{2\alpha}$$

$$\alpha^2 = N$$

$$\boxed{\alpha = \sqrt{N}}$$

Q.) If force-position (Regula-Falsi) method intial guess is required true or False ?

⇒ True

Q.) N-R Method give

a) exact solⁿ b) approx. solⁿ c) Real roots ~~d)~~ b&c both.

Q.) $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$

$x_0 = 0.5$, the series conver to

~~i)~~ 1.5 ii) $\sqrt{2}$ iii) 1.6 iv) 1.4

$$\alpha = \frac{\alpha}{2} + \frac{9}{8\alpha}$$

$$\frac{1}{2}\alpha = \frac{9}{8\alpha}$$

$$8\alpha^2 = 18$$

$$\alpha^2 = 18/8$$

$$= 1.5$$

Solution of Linear Simultaneous Eqns :—

Gauss Elimination Method :— (Back Substitution Method)

$$a_1x + a_2y + a_3z = d_1$$
$$\boxed{b_1x} + b_2y + b_3z = d_2$$
eliminate
$$\boxed{c_1x} + c_2y + c_3z = d_3$$

$$a_1 \times \frac{-b_1}{a_1} \quad , \quad a_1 \times \left(\frac{-c_1}{a_1}\right)$$

$R_2 \to R_1 - \frac{b_1}{a_1} + R_2$
$R_3 \to R_3 + R_1 \times \left(\frac{-c_1}{a_1}\right)$

$$a_1x + a_2y + a_3z = d_1$$
$$b_2'y + b_3'z = d_2'$$
$$c_2'y + c_3'z = d_3'$$

$$b_2' \times \frac{-c_2'}{b_2'}$$

$R_3 \to R_3 + R_2 \times \left(-\frac{c_2'}{b_2'}\right)$

$$a_1x + a_2y + a_3z = d_1$$
$$b_2'y + b_3'z = d_3'$$
$$c_3''z = d_3''$$

Pivotting

i) Partial Pivotting :—

Numerically largest coeff of x is choosen from all the eqns and brought ~~one~~ as the first pivot by interchanging the first eqn with that eqn having the largest coeff. of x. and, lly in the second 1.
this op is continued till we arrive at ~~the~~ eqn with single variable.

ii) Complete Pivotting :—

If we are not keen abt the elimination of x, y, z in a specified order then we choose at each stage, The numerically largest coeff. of the entire system is known as complete pivotting.

Q.) $x + 4y - z = -5$ Find the value of first & second pivot

$x + y - 6z = -12$ by partial & complete pivotting!

$3x - y - z = 4$

Soln

$3x - y - z = 14$

$4y - z = -5$

$-6z = -12$

$z = 2$

$y = -3/4$

$3x = 16 + -3/4$

$= \frac{64-3}{4} = 61/4$

$x = \frac{61}{12}$

$3x - y - z = 14$

$-13y + 2z = 19$

$-4y + 17z = 40$

$-1\ 2\ -12$

$-1+3$

$4+18$

$-1+18$

$4+36$

Partial:

$$\begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -6 \\ 1 & 4 & -1 \end{bmatrix}$$

I^{st} pivot $= 3$

$$\begin{bmatrix} 3 & -1 & -1 \\ 0 & +4/3 & \\ 0 & 13/3 & \end{bmatrix}$$

II^{nd} Pivot $= 13/3$

Complete

$$\begin{bmatrix} 1 & 1 & -6 \\ 3 & -1 & -1 \\ 1 & 4 & -1 \end{bmatrix}$$

| Numerically largest

$R_2 + \frac{R_1}{-6}$

$R_3 + \frac{R_1}{-6}$

I^{st} pivot $= -6$

$$\begin{bmatrix} 1 & 1 & -6 \\ 17/6 & -7/6 & 0 \\ -5/6 & 23/6 & 0 \end{bmatrix}$$

II^{nd} Pivot $= 23/6$

Q.) $20x + y - 2z = d_1$

$3x + 20y - z = d_2$

$2x - 3y + 20z = d_3$

Partial pivot or complete pivot.

3×20

$$\begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix}$$

Ist pivot = 20

$R_2 - R_1 \times \frac{3}{20}$

$R_3 - R_1 \times \frac{2}{20}$

$20R_2 - 3R_1$

$R_3 - \frac{R_1}{10}$

$$\begin{bmatrix} 20 & 1 & -2 \\ 0 & \frac{397}{20} & - \\ 0 & -3.1 & \end{bmatrix}$$

IInd pivot

$$\begin{bmatrix} 20 & & \\ 0 & \frac{397}{20} & -\frac{14}{20} \\ 0 & -\frac{31}{10} & \frac{406}{20} \end{bmatrix}$$

Gauss-Jordan :-

When we eliminate any variable, we eliminate from all of the rest eqn-

~~Aug~~ Sep 01, 14

Jacobi's Method:-

$\boxed{x_0 = y_0 = z_0 = 0}$

$a_1 x + b_1 y + c_1 z = d_1 \Rightarrow x_1 = \dfrac{d_1 - b_1 y_0 - c_1 z_0}{a_1} = \dfrac{d_1}{a_1}$

$a_2 x + b_2 y + c_2 z = d_2 \Rightarrow y_1 = \dfrac{d_2 - a_2 x_0 - c_2 z_0}{b_2} = \dfrac{d_2}{b_2}$

$a_3 x + b_3 y + c_3 z = d_3 \Rightarrow z_1 = \dfrac{d_3 - a_3 x_0\, b_3 y_0}{c_3} = \dfrac{d_3}{c_3}.$

2.7 $20x + y - 2z = 3$ $\qquad x_3 + y_3 + z_3 = ?$

$3x + 20y - z = 5$

$2x - 3y + 20z = 7$

$x_1 = 3/20$

$y_1 = 5/20$

$z_1 = 7/20$

$x_2 = \dfrac{3 - 5/20 + 2 \cdot 7/20}{20}$

$= \dfrac{3 - 19/20}{20} = \dfrac{51}{20} \times \dfrac{1}{20} = \dfrac{51}{400}$

$y_2 = \dfrac{5 - 3 \cdot 3/20 + 7/20}{20} = \dfrac{98}{400}$

$z_2 = \dfrac{7 - 2 \cdot 3/20 + 3 \cdot 5/20}{20} = \dfrac{131}{400}$

$$x_3 = \frac{3 - \frac{98}{400} + 2 \cdot \frac{131}{400}}{20} = \frac{1036}{8000} = 0.1295$$

$$y_3 = \frac{5 - 3 \cdot \frac{51}{400} + \frac{131}{400}}{20} = \frac{1978}{8000} = 0.2472$$

$$z_3 = \frac{7 - 2 \cdot \frac{51}{400} + 3 \cdot \frac{98}{400}}{20} = \frac{2992}{8000} = 0.374$$

Gauss seidal

latest value

Q.) same quest second value.

$$x_1 = 3/20$$

$$y_1 = \frac{5 - 3 \times \frac{3}{20}}{20} = 0.2275$$

$$z_1 = \frac{7 - 2 \times 3/20 + 3 \times 0.2275}{20} = 0.369125$$

$$x_2 = \frac{3 - 0.2275 + 2 \times 0.369125}{20} = 0.1755$$

$$y_2 = \frac{5 - 3 \times 0.1755 + 0.369125}{20} = 0.242125$$

$$z_2 = \frac{7 - 2 \times 0.1755 + 3 \times 0.242125}{20} = 0.36876$$

Rate of convergence of siedal method is almost twice the rate of convergence of jacobi's Method.

Solution of DE :-

Euler's Method :-

$$y_{i+1} = y_i + h f(x_i, y_i)$$

$$\frac{dy}{dx} = f(x,y)$$

$h \rightarrow$ step.

Q) $\frac{dy}{dx} = x+y$, $\quad y(0) = 1$, $\quad y(1) = ?$

$x=0 \qquad x=1.$

x	y	$f(x,y)$
0	1	1
0.1	1.1	1.2
⋮	⋮	⋮

$y_{i+1} = 1 + 0.1(1) = 1.1$

$1.1 + 0.1(1.2) =$

⋮

$x_0 = 0 \qquad y_0 = 1 \qquad , h = 0.1 ,$

$$y_1 = y_0 + h f(x, y)$$
$$= 1 + 0.1(1)$$
$$= 2.1$$

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 2.1 + 0.1(2.2)$$
$$= 2.32$$

$$y_3 = 2.32 + 0.1(0.2 + 2.32)$$
$$= 2.572$$

$$y_4 = 2.572 + 0.1(0.3 + 2.572)$$
$$= 2.8592$$

$$y_5 = 2.8592 + 0.1\,(0.4 + 2.8592)$$
$$= 3.18512$$

$y(1) = 0.318$

$$y_6 = 3.18512 + 0.1\,(0.5 + 3.18512)$$
$$= 3.55$$

Modified Euler Method / predictor-corrector Method / Backward Euler Method / Implicit Method

$$\boxed{y_{i+1} = y_i + h f(x_{i+1}, y_{i+1})}$$

Q.) $\dfrac{dy}{dx} = 0.25y^2$

$y(0) = 1$

$y(1) = ?$ $\quad h = 1$

$\cancel{y_{i+1}} =$

$$y_1 = y_0 + 1\,(0.25\,y_1^2)$$
$$y_1 = \cancel{1.25}\ 1 + 0.25y_1^2$$
$$y_1 - 0.25y_1^2 = 1$$
$$0.25y_1^2 - y_1 + 1 = 0$$
$$y_1 = +\frac{1}{0.5} = 2$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\frac{1 \pm \sqrt{1-1}}{2\times 0.25}$$

Runge-Kutta Method :-

$\dfrac{dy}{dx} = f(x,y) \qquad y(0) = 1\,,\ y(1) = ?$

$$y_1 = y_0 + k$$

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$k_1 = hf(x_0, y_0)$

$k_2 = hf\cancel{(x_0+\tfrac{1}{2}h, y_0+\tfrac{1}{2}k_1)}$

$k_3 = hf(x_0+\tfrac{1}{2}h, y_0+\tfrac{1}{2}k_2)$ $\qquad k = \dfrac{k_1+2k_2+2k_3+k_4}{6}$

$k_4 = hf(x_0+h, y_0+k_3)$

Q.) $\dfrac{dy}{dx} = x+y$, $\qquad y(0)=1$, $y(1)=?$ $\qquad \cancel{y_1 = y_0+k}$

$x_0 = 0$

$k_1 = 1$

$k_2 = 1(\tfrac{1}{2}, 1.5)$

$= 2$

$k_3 = 1(0.5, 1+$

$= 2.5$

$k_4 = 1(1, 3.5)$

$= 4.5$

$\dfrac{1+4+5+4.5}{6} = 2.4166$

$y_1 = y_0 + k$

$y_1 = 3.4166$

Solution of Integrals:-

i) Trapezoidal Rule

$$\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\dots+y_{n-1})\right]$$

$$h = \frac{b-a}{n} = \frac{2-0}{10} = 0.2$$

Q-> x^2,

x	0	0.2	0.4	0.6	0.8	~~0~~1.0	~~1~~.2	1.4	1.6	1.8	2
$f(x)$	0	0.04	0.16	0.36	0.64	1	1.44	1.96	2.56	3.24	4

$$f(x)dx = \frac{h}{2}\left[(y_0+y_n) + \cancel{\tfrac{h}{2}}(y_1+y_2 \dots)\right]$$

$$= \frac{0.2}{2}\left[(0+4)+2(11.4)\right]$$

$$= 2.68.$$

$$\int_0^2 x^2 dx = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3} = 2.666$$

Exact − true = Error

Error = Exact − Approx

$\varepsilon < 0.0314$ $\qquad \varepsilon =$

To apply the trapezoidal rule the intervals must be divided into even no. of sub-intervals.

Simpson's 1/3rd Rule :-

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\Big[(y_0+y_n)+2(y_2+y_4+y_6+\dots+y_{n-2})$$
$$+4(y_1+y_3+y_5+\dots y_{n-1})\Big]$$

Q7 $\int_0^2 x^2 dx$ = n,4

$h = \frac{2-0}{4} = 0.5$

~~f ex~~

x	0	0.5	1	1.5	2
$f(x)$	0	0.25	1	2.25	4
	y_0	y_1	y_2	y_3	y_4

$$= \frac{0.5}{3}\left[(0+4)+2(1)+4(0.25+2.25)\right]$$

$$= 2.6666$$

No. of intervals must be even to apply this Rule.

Simpsons 3/8th Rule

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+\dots+y_{n-1})+2(y_3+y_6+y_9\dots+y_{n-3})\right]$$

To apply this Rule the no. of intervals must be divided into a multiple of 3.

$h = \frac{2-0}{6} = 0.333$

x	0	0.33	0.66	0.99	1.32	1.65	1.98
$f(x)$	0	0.11	0.4356	0.9801	1.7424	2.7225	3.9204
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\frac{3\times 0.33}{8}\left[3.9204+3(0.11+0.4356+1.7424+2.7225)+2(0.9801)\right]$$

Order of a Numerical Method:-

It is the way of quantifying the extent of error. The higher the order the lesser will be the error.

for ex:-

Simpson's $\frac{1}{3}$rd rule is fifth order method, while trapezoidal rule is 3rd order method.

Q.) $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$. Calculate the value simpson's $\frac{1}{3}$rd Rule if step size be $\pi/12$.

Ans = 1.1877

Q.) $\int_0^1 e^{-x^2} dx$ Simpson $\frac{1}{3}$rd Rule $h = 0.1$

Ans = 0.7468.

Q.) A second degree polynomial f(x) has values of 1, 4, 15 at $x = 0,1,2$ resp. ~~The integral $\int_0^2 f(x)dx$ is~~

$\int_0^2 f(x)dx$ is to be evaluated by trapezoidal rule wid help of given data. Find the value of error.

Sol.) $\cancel{n/2 = \frac{\pi/2 - 0}{\pi/12} = 6}$

x	0	1	2
f_x	1	4	15

, $\int_0^2 f(x)dx = \frac{1}{2}\left[(1+15)+2\times4\right]$

$\Rightarrow \frac{24}{2} = 12$

$f(x) = ax^2 + bx + c$

$x = 0$
$f(x) = 1$

$\cancel{a+b+}c = 1$

$x = 1$
$a + b + c = 4$

$a + b = 3$

$x = 2$

$4a + 2b + c = 15$

$2a + 2(3) + 1 = 15$

$2a = 8$
$\underline{\underline{a = 4}}$

$b = -1$
$c = 1$

$$\int_0^2 (4x^2 - x + 1)\, dx$$

$$\frac{4x^3}{3} - \frac{x^2}{2} + x \Big|_0^2$$

$$\frac{4 \times 8}{3} - \cancel{\frac{4}{2}} + \cancel{2}$$

10.666.

$\boxed{12 - 10.666 = 1.333}$

Error $= 10.666 - 12$

$= -1.333$

Estimation of nature of roots in the algebraic Eq^n :-

1) If the coeff of all the power of x are +ve, then the eq^n will not have a +ve root. (real.)

$$\underbrace{x^7+x^5+4x^4+2x^3+x+1=0}$$

roots is not +ve.

2.) If the coeff of even powers of x are of one sign and the coeff. of odd powers of x are of opposite sign. Then the eq^n cann't have a (-ve) root.

$$\underbrace{-x^7-x^5+4x^4-2x^3-x+1=0}$$

3.) If the eq^n contains only even powers of x and coeff are all of same sign. Then the eq^n cann't have a real root.

$$\underbrace{x^4+x^2+1=0}$$

4.) If the eq^n contains only odd powers of x and coeff are all of same sign then the eq^n cann't have a real root except the $x=0$,

$$x^5+x^3+x=0$$

$$\underbrace{x}_{\text{real}}\underbrace{(x^4+x^2+1)}_{\text{not real}}=0$$

5.) For any eq^n if $a+ib$ is a root then, $a-ib$ must be the root and vice-versa. (complex pairs)

6.) For an eq^n $a+\sqrt{b}$ is a root then, $a-\sqrt{b}$ must be root (Surd pair)

7.) If in eq^n $f(x)=0$, the max^m no. of real +ve root is always less than or equal to no. of sign changes in $f(x)$.

$f(x) = x^7 + x^6 - x^5 + x^4 + x^3 - x^2 - 1 = 0$

at max 3 real +ve root be there, it may be 2 or 1.

8) Max^m no. of real -ve roots is alwaz less than or equals to the no. of sign changes in $f(-x) =$

$f(-x) = -x^7 + x^6 + x^5 - x^4 - x^3 + x^2 + 1 = 0$

at max 2 real -ve roots.

Q.) $6x^4 - 13x^3 - 35x^2 - x + 3$, $(2-\sqrt{3})$ is the roots of eq^n. Calculate other roots

$6x^3(x-1) - 7x^2(x-5) - x + 3$

$-\frac{1}{3}, -\frac{3}{2},$

$ax^2 + bx + c =$

$\Rightarrow (x-a)(x-b)$

$(x-(2+\sqrt{3})), (x-(2-\sqrt{3}))$

$(x-2-\sqrt{3})(x-2+\sqrt{3})$

$(x^2 - 4x + 1) =$

$(6x^2 + 11x + 3) = ?$

Q.) $x^9 + 5x^5 + x^3 + 7x + 2$. Find the min no. of complex roots.

$9 - 5$

$= 4$

Sep 02,14

PROBABILITY

1> Sample space

Set of all possible outcomes of a random experiment.

dice sample pt

{1, 2, 3, 4, 5, 6} sample space

coin

{H, T}

2> Sample point

Each outcome of an experiment is known as sample point.

Random Experiment :-

An experiment in which, ~~each~~ in each trial the outcome is not unique but may be any 1, of the possible outcomes when performed under identical conditions.

Q.) A coin is tossed, if the result is head, a die is thrown. If the dice shows up an even no. The die is thrown again. What is the sample space and no. of sample pt. for this experiment.

H, T

H → H1 ✓

T ✓ H2 H21 H22 H23 H24 H25 H26

H3 ✓ — 18 ✓

H4 H41 — — — — — — H46

H5 ✓

H6 H61 — — — — — H66

✓ = sample pt

all in bracket is a sample space.

no. of samp pt (✓) = 18

Sample Space {T, H1, H3, H5, 18}

Collection of sample pt is known as sample space.

Q.) Consider the experiment in which a coin is tossed, repeatedly (again & again) untill a head comes up. Determine the sample space for this experiment.

H, TH, TTH, TTTH, . . .

sample pt = ∞

Samp space = { }

Events :-

Each sub-set of a sample space is called an event.

$S = \{1, 2, 3, 4, 5, 6\}$

event of getting $A = \{1, 3, 5\}$ odd no, $B = \{2, 4, 6\}$ even no, $C = \{2, 3, 5\}$ prime no

⇒ The event E of a sample space is said to have a chord if the outcome say ω of the experiment is such that $\omega \in E$

Types of Events

Null Event An event having no sample point. (a no. >6, will not have any sample pt.)

Sure Event

The event which is sure to occur. (a no < 7 will occur)

Equally likely Events :- when there is no preference of one event over the other.

When we don't expect the happening of one event in preferance to the other.

Mutually exclusive Events :-

Two events associated wid an experiment are said to be mutually exclusive, if both the events can not occur simultaneously in same trial. (either i will get a odd no or even no. both event can't occur simultaneously)

$\boxed{n(A \cap B) = \phi} \neq 0$

Algebra of events :-

⇒ Event A OR B — $A \cup B$

At least one of them / A & B

⇒ ~~Event A and B~~ — $A \cap B$

⇒ Complimentry Events A', A^c, all the elements of sample space which are not in A. They are also known as Mutually exclusive event

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$A^c = \{2, 4, 6\}$

⇒ $\boxed{A \cup A^c = S}$

$\boxed{A \cap A^c = \phi}$

⇒ Event A but not B — $(A - B)$, all the elements of A, which are not in B

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$ $B = \{3, 5\}$

$A - B = \{1\}$

$B - A = \{\phi\}$

Probability of an Event E P(E) =

Defined as the ratio of no of elements in E divided by no of elements in S. Where E = Event & S = sample space.

It is also defined as the ratio of no. of favarable case to the total no. of cases.

$$1 \geq P(E) = \frac{n(E)}{n(S)}$$

$\boxed{\Sigma P(E) = 1}$

$(A \cup A^c) = 0$

$P(A \cup A^c) = 1$

$\Rightarrow P(A) + P(A') = 1$

ODDs in favour of an event E :-

It is defined as the ratio of no. of favourable case to the no. of unfavourable cases.

ODDs against of an event E :-

Defined as the ratio of no. of unfavourable case to the favourable no. of cases.

E → Exactly 1 Head.

HH
HT ✓
TH ✓
TT

Q.) Find the probability of getting the sum a prime no. when two dies are thrown together.

(1,1) (6,6)
2 12

2	3	5	7	11
(1,1)	(1,2)	(1,4)	(1,6)	(6,5)
	(2,1)	(2,3)	(2,5)	(5,6)
		(3,2)	(3,4)	
		(4,1)	(4,3)	
			(5,2)	
			(6,1)	

$P(E) = 15/36 = 5/12$

HHH]— 3H　　　　0T

HHT
HTH]—2H　　　　1T
THH

TTH
THT]— 1H　　　　2T
HTT

TTT]— 0H　　　　3T

* If coin is tossed for n time, the no. of sample space is 2^n

* This method is only n only applied for the case of homogenity
If there is coin, coin only; ball than ball only.

Q:) 3 coins are tossed once, find the prob of getting

1:) 3 heads
2:) Exactly 2 Heads
3:) Atleast (कम से कम) 2 heads
4:) ~~Atmost~~ Atmost (ज्यादा से ज्यादा) 2 heads
5:) No heads
6:) Exactly two tail
7:) A head on the 1st pt
8:) ~~Att~~ At least 3 Tail.

HHH
HHT
HTH
THH
TTH
THT
HTT
TTT

1.) 3 heads = $1/8$

2:) Exactly 2 Heads = $3/8$

3:) Atleast 2 H = $4/8 = 1/2$

4:) At most 2 H. = $7/8$

5:) No head. = $1/8$

6:) Exactly two tail = $3/8$

7:) Head on the 1st pt = $4/8 = 1/2$

8:) At least 3 Tail = $1/8$

**

At least = kam se kam
At most = Jada se Jada

CARDS CONCEPT

52

- 26 Black
  - 13 club
    - 1 Jack
    - 1 Queen
    - 1 King
    - 1 Ace
  - 13 Spade
- 26 Red
  - 13 ~~Red~~ Heart
  - 13 Diamond

Suit

4 KING
4 QUEEN
4 JACK
] FACE CARDS

Q 1 card is drawn from well shuffled deck of 52 cards if each outcome is equally likely. Calculate the prob

1.) a heart
2.) Not a heart
3.) A black card
4.) Not a black card
5.) Not an ace
6.) An ace of space.

1.) A heart = $13/52$

2.) = $39/52$

3.) = $26/52$

4.) = $26/52$

5.) = $48/52$

6.) = ~~4/52~~ $1/52$

- Jab tk experiment chlta : (X)
- jab stop krteh to (+)

$$\boxed{\begin{matrix}\cancel{5W, 7R} \\ 5B\end{matrix}}$$

* One by one

13 balls >

$$\frac{5}{17} \times \frac{7}{16} \times \frac{5}{15}$$

* two balls together.

3 → 2W, 1B

$$\frac{{}^5C_2 \times {}^5C_1}{{}^{17}C_3}$$

$$\boxed{{}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}}$$

${}^{100}C_{8}$ ${}^{100}C_{?}$

$${}^nC_r = {}^nC_{n-r}$$

$$\boxed{{}^nC_n = 1} \quad \boxed{{}^nC_0 = 1}$$

$${}^7C_2 = \frac{7 \times 6}{2 \times 1}$$

$$\boxed{{}^nC_1 = {}^nC_{n-1} = n}$$

Q.) A bag contains 3W, 4R, 5B balls, 2 balls are draw 1 by 1, wid/out replacement. Find Proplo that of the 2 draw ball 1 is white & other is black.

$$\boxed{\begin{matrix}3W, 4R \\ 5B\end{matrix}}$$

$$\left[\frac{5}{12} \times \frac{3}{11}\right] + \left[\frac{3}{12} \times \frac{5}{11}\right] = \frac{30}{144}$$

⇒ W/o replacement is by default whether its mentioned or not & if the case is wid replacement, it must be mentioned in the question.

$$\left(\frac{5}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{5}{11}\right) = \frac{18}{44}$$

B W   W B

$$\frac{{}^3C_1 \times {}^5C_1}{{}^{12}C_2}$$

⇒ If the balls are drawn one by one, we use fractional method. or if the balls r drawn together then we will apply the combination method.

$$\frac{15}{132} + \frac{15}{132} = \frac{30}{132}$$

Q.) From a bag containing 12 balls of same size of which 6R, 4B, 2W. 3 balls are drawn. What is Prob.

1) All d balls are blue.
2) Balls are of diff colr.
3) None of the balls is blue.
4)

$$\boxed{\text{*** If mentioned only then one by one otherwise by default drawn together}}$$

$\boxed{6R, 4B, 2W}$

i.) $\frac{^4C_3}{^{12}C_3}$

ii.) $\frac{^6C_1 \times {^4C_1} \times {^2C_1}}{^{12}C_3}$

$\underbrace{C_1 \times C_1 \times C_1}_{3\cdot \text{ comp.}}$

iii.) $\frac{^8C_3}{^{12}C_3}$

Q.) A committe of 5 principle is to be selected from a group of 6 gents and 8 lady. If the selection is made randomly. Find the prob. there are 3 lady & 2 gent principal.

(Case of selection)

6G 8L

$$\frac{^6C_2 \times {^8C_3}}{^{14}C_5}$$

Q.) What is the prob. that a leap yr. selected at random will have 53 sunday.

$\frac{366}{7} = 52$ week + 2 days

$= 52$ Sunday $\times$ MT, TW, WT, TF, FS, SS, SM

(Sure event, $P = 1$)

$= 1 \times 2/7$

53 Sunday. $= 2/7$

Q) 4-digit nos are formed by using the digits 1, 2, 3, 4 & 5, w/o repeating any digit. Find prob that a no. chosen at random is an odd no.

Total Sample space

1, 2, 3, 4, 5

Total no. of choices available to fill → $2 \times 3 \times 4 \times 5$

120

~~Total~~ favorable

$2 \times 3 \times 4 \times 3$

$= 72$

$$P(E) = \frac{72}{120}.$$

Q.) Find the prob that a random arrangement of letter of the word UNIVERSITY, two 'I' don't come together.

n letter can arrange themselves in $\underline{|n}$ ways.

total no. of letter $= \dfrac{\underline{|10}}{\underline{|2}}$! 'I' repeated

UNIVERSITY

$$\frac{\underline{|10}}{\underline{|2}}$$

(II) UNVERSTY

$\underbrace{(II)}_{1}$ $\underbrace{\text{UNVERSTY}}_{8}$

$$\frac{\underline{|9}}{\underline{|1}} \times \frac{2}{\underline{|2}}$$ repeat case

$$\frac{\dfrac{\underline{|9}}{\underline{|1}}}{\dfrac{\underline{|10}}{\underline{|2}}} = \frac{\underline{|9} \times 2}{\underline{|10}} = \frac{\underline{|9} \times 2}{10\,\underline{|9}} = \frac{1}{5}$$

Q> Calculate prob, vowel occur together from the letter word ARRANGEMENT.

$$\text{total no} = \frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2 \lfloor 2}$$

$\underbrace{(AAEE)}_{1}\underbrace{RRNGMNT}_{7}$

favorable $\frac{\lfloor 8}{\lfloor 2 \times \lfloor 2} \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2}$

$$= \frac{\lfloor 8 \; \lfloor 4}{\lfloor 11}$$

Consonent are together. AAEE(RRNGNMT)

Q> A party of n persons sit at round table. Find the odds against two specified individuals sitting next to each other.

* N persons sitting on a row can arrange themselves by $\lfloor n$ ways and n persons sitting on a round table can arrange themselves by $\lfloor n-1$ no. of ways.

Total arrangement $= \lfloor n-1$

n

$\underbrace{(2)}_{1} \underbrace{(n-2)}_{n-2}$

total $(n-1)$

favourable arrange $\lfloor n-2 \times \lfloor 2$

$$P(E) = \frac{\lfloor n-2 \times \lfloor 2}{\lfloor n-1} = \frac{\lfloor 2}{(n-1)}$$

$$= \frac{2}{n-1}$$

Sep 03, 14

Q.) Two cards are drawn from a pack of 52 card. find the prob that either both are red or both are kings.

Q.)

3R
3B

3 are drawn
find $P(2R \,\&\, 1B$ or $1R$ or $2B)$

Sol^n

$$\frac{{}^3C_2 \times {}^3C_1}{{}^6C_3} + \frac{{}^3C_1 \times {}^3C_2}{{}^6C_3}$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Sol^n (i)

$$\frac{\overset{\text{red}}{{}^{26}C_2} + \overset{\text{king}}{{}^4C_2} - {}^2C_2 \; \text{king of red}}{{}^{52}C_2}$$

$$\frac{\frac{\lfloor 26}{\lfloor 24 \lfloor 2} + \frac{\lfloor 4}{\lfloor 2 \lfloor 2} - \frac{\lfloor 2}{\lfloor 0 \lfloor 2}}{\frac{\lfloor 52}{\lfloor 50 \lfloor 2}} = \frac{\frac{26 \times 25}{2} + \frac{4 \times 3}{2} - 1}{\frac{52 \times 51}{2}}$$

Condition Probability :— $P(A/B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Q.) A die is thrown twice and sum of the no. appearing is observed to be 7. What is the probability that no. 2 has appeared atleast once.
A

$$= \frac{2/36}{6/36}$$

$$= 1/3$$

7
(1,6) ×
(2,5) ✓
(3,4) ×
(4,3) ×
(5,2) ✓
(6,1) ×

* we will never calculate A alone we will ch $A \cap B$

Q.) In a school there are 1000 students out of which 430 are girls. It is known that out of 430, 10% of girls studied in cls 12th. What is the probab that a student choosen at random studies in cls 12, given that choosen student is a girl.

$430 \times \frac{10}{100} = 43$

$$\frac{43/1000}{430/1000} = \frac{43}{430} = \frac{1}{10}$$

$\frac{1}{10} = 0.1$

Q.) A coin is tossed, if the coins shows head, it is tossed again, bt if it shows tail, then a die is thrown. Find the prob of the event that the die shows a no. > 4, given that there is atleast 1T.

H ⟶ HH $\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

T ⟶ HT $\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

T1, T2 T3 T4 T5 T6

$\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{12}$ $\frac{1}{12} + \frac{1}{12}$

$$P(B) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{4} = \frac{6}{12} + \frac{1}{4} = \frac{3}{4}$$

$$= \frac{1/6}{3/4} = \frac{2}{9}$$

Q) A card is drawn from a pack of 52 cards and then a second card is drawn w/o replacement. What is the probs that both the cards are Queens.

$$\frac{4}{52}\times\frac{3}{51}$$

Q) A bag contains 5W, 8B, 3 balls are drawn together w/o replacement 2 times. Find the prob that the first draw results 3W, second draw results 3 black.

$$\frac{{}^5C_3}{{}^{13}C_3}\times\frac{{}^8C_3}{{}^{10}C_3}$$

Q) 3 cards are drawn successively w/o replacement from a pack of 52 cards. Find the prob. that 1st two cards are kings and 3 is an ace.

$$\frac{4}{52}\times\frac{3}{51}\times\frac{4}{50}$$

Independent Events:—

$$P(A\cap B) = P(A)\cdot P(B)$$

Q) A problem is maths is ~~given~~ given to 3 students A, B, C.

whose chances of solving it are

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

what is the probab ~~not~~ will be solved.

$$P(A) = \frac{1}{2}\ ,\ P(B) = \frac{1}{3}\ ,\ P(C) = \frac{1}{4}$$

$$P(\bar{B}) = \frac{2}{3}\ ,\ P(\bar{C}) = \frac{3}{4}$$

$P(\bar{A}) = 1-\frac{1}{2} = \frac{1}{2}$ $\quad \bar{A}BC + AB\bar{C} + A\bar{B}C + AB\bar{C},$

$P(B) =$ $\quad A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C$

$P(C) =$ $\quad \bar{A}\bar{B}\bar{C} = 1$

$P(B) = 2/3$, $P(C) = 3/4$

Q) Rahul speaks truth ~~may~~ 5 times out of 6 & Neha speaks truth 9 times of 10. Find the prob. that they contradict each other in stating the fact.

$P(A) = 5/6 \Rightarrow P(\bar{A}) = 1/6$

$P(B) = 9/10$

$P(\bar{A}) = 1/6$

$P(\bar{B}) = 1/10$

$$\left[\underset{R\,T}{\frac{5}{6}} \times \underset{N \times T}{\frac{1}{10}} + \underset{R \times T}{\frac{1}{6}} \times \underset{N\,T}{\frac{9}{10}}\right]$$

Q) 2 balls drawn at random from a bag containing 2W, 3R, 5G, 4B, two balls are drawn one by one w/o replacement. Find the probs that both balls are of diff. colors.

2W. 3R 5G 4B

$$1 - \left(\frac{2}{14} \times \frac{1}{13} + \frac{3}{14} \times \frac{3}{13} \times \frac{4}{13} + \frac{4}{13} \times \frac{3}{13}\right)$$

Q) ~~A & B a pointwe~~ A & B throw a coin alternatively til one of them heard ~~A circumstance~~ win the game. Find the prob of wining of A.

H + TTH + TTTTH

$$\frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots$$

$$S = \frac{a}{1-r} = \frac{1/2}{1 - (1/2)^2} = \frac{1/2}{1 - 1/4} = \frac{1/2}{3/4} = 2/3$$

Q.) 2 prsn throw a pair of diece altnatively ~~beginning~~ begining wi A.

Find ~~the prob that B gets~~ doublet and wins the game befor A gets a total of 9.

$$P(A) = 6/36$$

$$P(B) = 4/36$$

$$\cancel{AB + \bar{A}\bar{B}AB + \bar{A}\bar{B}\bar{A}\bar{B}AB + \cdots}$$

Q.) A bag contain 2W, 4B balls & anothe bag ~~ab~~ contains 6W & 4 B Ball.

One bag is choosen at random from the selected bag. 1 ball is drawn. find the ~~dra~~ prob that the ball drawn is white.

$$\frac{1}{2} \times \frac{2}{6} + \frac{1}{2} \times \frac{6}{10}$$

Q.) A bag contain 10W, 3B & anothe mag contains 3/5 laten.

Two balls are drawn from the bag & put into ~~and then ball is drawn~~ & then ball is drawn from 2nd. Find prob that the ball drawn is white.

$$\frac{^{10}C_2}{^{13}C_2} \times \frac{5}{10} + \frac{^{3}C_2}{^{13}C_2} \times \frac{3}{10}$$

2W 1B 2B 1W

$$\frac{^{10}C_1 \times {^{3}C_1}}{^{13}C_2} \times \frac{4}{10}$$

1W 1B 1W

Q.) A coin is baised, so that $P(H)=2/3$ & $P(T)=1/3$ is tossed. If head appears then a no. is selected at random from the no 1 to 9, If tail appears then a no. is selected at random from 1 to 5. Find the prob of getting an even no.

$$\frac{8}{27} + \frac{2}{15}$$

Heven   Teven

$\frac{2}{3}\times\frac{1}{9} = \frac{2}{27}$ $\frac{1}{3}\times\frac{1}{5} = \frac{1}{15}$

H1	H7	T1
✓ H2	H8 ✓	T2 ✓
H3	H9	T3
✓ H4		T4 ✓
H5		T5
✓ H6		

Bayes Theorem :- (Reverse Probability Case)

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)\cdot P(A/E_n)}$$

Q.) Bag 1 contains 2W & 3R & Bag 2 contains 4W & 5R, 1 ball is drawn at random from 1 of the bag and found to be red. Find the probab that it was drawn from the bag 2.

E = Event which is asked bag.

$P(E_1) = 1/2$

$P(E_2) = 1/2$

$P(A/E_1) = 3/5$

$P(A/E_2) = 5/9$

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{5}{9}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{5}{9}} = \frac{\frac{5}{18}}{\frac{3}{10}+\frac{5}{18}} = \frac{5}{\cancel{18}}\times\frac{\cancel{180}^{10}}{3\times18+50}$$

Q:) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the prob. that it is actually a 6.

$$P(E_1) = 1/6$$
$$P(E_2) = 5/6$$
$$P(A/E_1) = 3/4$$
$$P(A/E_2) = 1/4$$

$$P(E_1/A) =$$

Q:) Given 3-identical box 1,2,3. Each contains 2 coins, In box 1 both coins are of gold, box 2 - silver, box 3 - 1 gold & 1 silver. A person chooses the box at random and takes out the ~~gym~~ coin, if the coin is gold. What is the prob that the other coin in box is also a gold.

$$P(E_1) = 1/3 \quad , P(E_2) = 1/3 \quad , P(E_3) = 1/3$$
$$P(A/E_1) = 1 \quad , \quad P(E_2) = 0 \quad , \quad P(A/E_3) = 1/2$$

$$P(E_1/A) = \frac{1/3 \cdot 1}{1/3 \cdot 1 + 1/3 \cdot 0 + 1/3 \cdot 1/2} = \frac{1/3}{1/2} = 2/3$$

Q:) A doctor is to visit a patient ~~from~~, from the past experience it is known that if he come by train, bus, scooter or by any other means of transport. Prop. $(T) = 3/10$, $(B) = 1/5$, $(S) = 1/10$, $(O) = 2/5$. The prob that he will be late are $1/4$, $1/3$, $1/12$. If he comes by train, bus & scooter resp. But if he comes by any other means of transport he will not be late. When he arrive he is late what is the probab that he comes by train.

$P(E_1) = 3/10$ $\quad$ $P(A/E_1) = 1/4$

$P(E_2) = 1/5$ $\quad$ $P(A/E_2) = 1/3$

$P(E_3) = 1/10$ $\quad$ $P(A/E_3) = 1/12$

$P(E_4) = 2/5$ $\quad$ $P(A/E_4) = 0$

$$P(E_1/A) = \frac{3/10 \cdot 1/4}{1/3 \cdot 1}$$

$1/4$

Q.) A card from a 52 pack of cards is lost. From the remaining cards of the pack 2 cards are drawn and are found to be diamonds. Find the prob. that missing card is diamond.

$P(E_1) = 13/52$ $\quad$ $P(A/E_1) = 2/12 = \frac{^{12}C_2}{^{51}C_2}$

$P(E_2) = 39/52$ $\quad$ $P(A/E_2) = \frac{^{13}C_2}{^{51}C_2}$

$P(E_1/A) =$

Q.) Suppose a girl throws a die, if she gets a 5-6 she tosses coin 3 times and ~~notes~~ no. of heads. If she gets 1,2,3,4 she tosses coin once and ~~note~~ notes wether it is head or tail. If she obtained exactly 1 H, what is the probab that he will throw 1, 2, 3, 4 ~~told~~ the die

8/11 Ans.

$P(E_1) = 4/6$ $\quad$ $P(A/E_1) = 1/2$

$P(E_2) = 2/6$ $\quad$ $P(A/E_1) = 3/8$

$P(E_1/A)$

Binomial Distribution

Bernaulli's Trial

The trials of random experiment are called Bernaulli's trial; if they satisfy following condition.

⇒ There should be a finite no. of trials.
⇒ The trial should be independent.
⇒ Each trial should have exactly two outcomes. Namely success & failure.
⇒ Prob. of success remains the same in each trial.

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

P = Prob. of success
q = " " failure
n = no of trial
r = desired no. of success

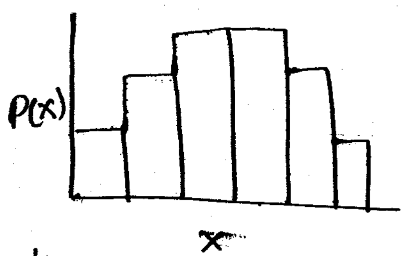
$\left\{ \boxed{p+q=1} \right.$

$B(n, P)$

☆ If n trials constitute an experiment, then the experiment is repeated 'N' times then the frequency of 'r' no. of successes is given by.

$$N \cdot P(X=r)$$

* If p=q in binomial distribution, then the binomial distribution is symmetrical.



* If p≠q; then the histogram will be unsymmetrical.

* The mean of the binomial distribution is np

variance " " " " " npq

<u>Poission Distribution</u>:- It is a distribution related to the events which are extremly ~~too~~ rare.

np = Finite

$\left.\begin{matrix} n = \infty \\ p \to 0 \end{matrix}\right\rangle$ finite.

No. of death by horse kick in army camp.

$$\boxed{P(x=r) = \frac{e^{-\lambda}\lambda^r}{\lfloor r}}$$

$$\boxed{\lambda = np}$$

* <u>Mean & Variance</u> of Poission Dist = $\underline{\lambda}$

* Poission Dist is +vely <u>Skewed</u>

Normal Distribution:-

It is a limiting form of binomial distribution in which n is very large but p is not very small.

$$P(x=x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

* The mean & Variance of N.D is same as that B.D

$n \to \infty$
$p \not\to 0$ (not too low.)

$\Rightarrow \mu = np$ Mean
$\Rightarrow \sigma = \sqrt{npq}$ std. deviation
$\Rightarrow \sigma^2 = npq$. Variance

$$\boxed{\frac{x-\mu}{\sigma} = Z}$$

Z = Normal Variate
x = Binomial Variate.

* The N.D is symmetrical alway.

* The skewness of N.D is zero. and the relation b/w mean & variance

$$\boxed{\mu = \frac{4}{5}\sigma}$$

Q> A die is thrown 6 times, getting an odd no. is success. What is prob?

1> 5 success   ii> Atmost ~~w~~ 5 success

~~ii> Atleast 5 "~~   iv> No success.

$n = 6 \qquad p = 3/6 = 1/2 \qquad q = 1/2$

i> $P(X=5) = {}^6C_5 (1/2)^5 (1/2)^1$

ii> ~~$P(X \geq 5) = P(X=5) + P(X=6)$~~

iii> $P(X \leq 5) = 1 - P(X=6)$

iv> $P(X=0) =$

Q.> Find the change of getting sum of 9 atleast twice in 10th thrown with two die

$P(X \geq 2) = 1 - [P(X=0) + (P(X=1))]$

9
3 6
4, 5
5, 4
6 3

$p = 4/36 \quad , \quad q = 32/36$

Q> An unbaised die are ~~drawn~~ thrown again and again untill 3 6 are obtaind. Find the ~~ab~~ propb of obtaining 3rd six in the sixth row of a da.

_ _ _ _ _ 6

$$ {}^5C_2 (1/6)^2 (5/6)^3 \times 1 $$

Q.) Prob of man hiting a target is 1/2, how many times he would fire So that prob of heating the target is atleast 1 which ~~who~~ is more than 90%.

$P(s) = 1/2$

$r = 1$

$$ P(X \geq 1) > 90\% $$

$$ 1 - P(X=0) > 0.9 $$

$$ 1 - {}^nC_0 (1/2)^0 (1/2)^n > 0.9 $$

$$ 1 - (1/2)^n > 0.9 $$

Q> If the prob of bad rxn from a certain rxn is 0.001. Determine the chance that at 2000 melted ~~out of two~~ more than 2 get broken.

Chance = 2000
r = 2

$n = 2000 \,, \quad p = 0.001$

$$\boxed{np = \lambda = 2}$$

$$P(X>2) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - \left[\frac{e^{-2}2^0}{\lfloor 0} + \frac{e^{-2}2^1}{\lfloor 1} + \frac{e^{-2}2^2}{\lfloor 2}\right]$$

$$= 1 - [e^{-2}(1+2+2)] = \boxed{1 - 5/e^2} \text{ Ans}$$

Q> 6 dies are thrown 729 times. How many times do you accept atleast 3 dice to show 5 & 6.

n = 6
N = 729

233 Ans

$n = 6$

$p = 2/6 = 1/3$

$q = 2/3$

$$729 \times P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

Q> A survey 200 families, each having 4 children was conducted. In how many families do u ~~accept~~ expect 3 boys and girls ~~and given~~ equally ~~probable~~ boy & girls.

N = 200

$n = 4$

$p = 1/2 \,, \quad q = 1/2$

$$200 \times P(X=3) = {}^4C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^1$$

$=$

Mean

★ if $x_1\, x_2\, x_3 \ldots x_n$ are diff nos.

$$\boxed{\bar{x} = \mu = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}}$$

sum of observation / no. of observation

C.I	freq. f_i	x_i (mid value of class Interval)	$f_i x_i$
0 – 10	5	5	
10 – 20	10	15	
20 – 30	15	25	
30 – 40	7	35	
40 – 50	9	45	
	$n = \Sigma f_i$		$\Sigma f_i x_i$

$$\boxed{\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}}$$

for group data. ★

C.I	f_i	x_i	$d_i = x_i - A$	$f_i d_i$	$u_i = \frac{d_i}{h}$
		5	-20		-2
		15	-10		-1
		25 = A	+0		0
		35	10		1
		45	20	$\Sigma f_i d_i$	2
	Σf_i				

A = Assumed mean.

h = width.

$u_i = \frac{x_i - A}{h} = \frac{d_i}{h}$

$$\boxed{\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}}$$

Shortcut Method.

$$\boxed{\bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h}$$

Step deviation Method.

Median

If the values of a variable are arranged in the accending order. ~~Th~~ Then the ~~the~~ median is the middle term, if the no. of terms are odd. Median is the mean (avg) of two middle terms, if the no. of terms are even.

(for raw data ↑)

(for group data)

C.I	fre(f_i)	C.F
0 - 10	5	5
10 - 20	10	15
20 - 30	15	30
30 - 40	7	37
40 - 50	9	46
	46	

median class $= \frac{46}{2} = \frac{\Sigma f_i}{2} = 23$

$$\text{Median} = L + \left(\frac{\frac{\Sigma f_i}{2} - C}{f}\right) \times h$$

$$= 20 + \left(\frac{\frac{46}{2} - 15}{15}\right) \times 10$$

$$= 25.33$$

N = summation of frequency.

L = lower limit of median class.

h = width of C.I.

C = cumulative freq. preciding the median class.

f = frequency of median class

Mode

Mode is the value of variable which occur most frequently. i.e value no' with the max^m frequency

ex → 2,2, 3,3,3, 4,4,4,4, 5,5,5,9,9.

2 - 2
3 - 3
4 - 4 Mode
5 - 3
9 - 2

for group data

see in which, have max^m frequency
↓
⟹ Modal class

C.I	fre
0-10	5
10-20	10
20-30	15
30-40	7
40-50	9

Modal class is the class which have the max^m frequency.

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

f_1 = frequency of modal class (15)

f_0 = " " just preceeding modal class (10).

f_2 = " " just ~~proceeding~~ Succeeding modal class (greater, (7)

Measure of dispersion :-

Variance

Standard deviation :-

It is computed as the square root of the mean of the square of diff. of the variate from there mean.

$$\boxed{\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}}$$

$$\boxed{\sigma^2 = \text{Variance}}$$

Coefficient of Variation :- (cov)

It is defined as the ratio of std. deviation. to the mean.

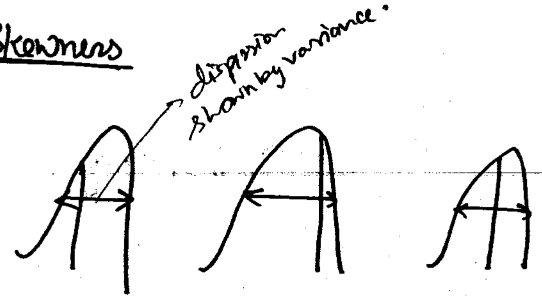
$$\boxed{\text{COV} = \frac{\sigma}{\bar{x}}}$$

It is used to compare the dispersion of diff. data. And the data whose COV is more is said to be more dispersed.

* For a symmetrical distribution $\boxed{\text{mean} = \text{mode} = \text{median}}$

* For unsymmetrical distribution $\boxed{\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}}$

Skewness



+ve　　　-ve　　　0

for the +vely skewed data　　Mode > Median > Mean

-vely skewed data

0 skewness　　Mean = Mode = Median.