GATEFLIX

PLANT DESIGN AND ECONOMICS

For

CHEMICAL ENGINEERING

PLANT DESIGN AND ECONOMICS

SYLLABUS

Principles of process economics and cost estimation including depreciation and total annualized cost, cost indices, rate of return, payback period, discounted cash flow, optimization in process design and sizing of chemical engineering equipments such as compressors, heat exchangers, multistage contactors.

ANALYSIS OF GATE PAPERS

Exam Year	Total
2001	6
2002	9
2003	6
2004	3
2005	5
2006	6
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6. MECHANICAL DESIGN OF EQUIPMENTS

1 PROCESS ECONOMICS

1.1 INTRODUCTION

Chemical plants are built to make a profit and an estimate of the investment required and the cost of production are needed before the profitability of a project can be assessed. The design engineer, by analyses of costs and profits, attempts to predict whether capital should be invested in a particular project. After the investment is made, records must be maintained to check on the actual financial results. These records are kept and interpreted by accountants. The design engineer, of course, hopes that the original predictions will agree with the facts reported by the accountant.

The purpose of accounting is to record and analyze any financial transactions that have an influence on the utility of capital. Accounts of expenses, income, assets, liabilities, and similar item are maintained. These records can be of considerable value to the engineer, since they indicate where errors were made in past estimates and give information that can be used in future evaluations. Thus, the reason why the design engineer should be acquainted with accounting procedures is obvious.

Statements showing the financial condition of the business concern are prepared periodically from the ledger accounts. These statements are presented in the form of balance sheets and income statements.

1.1.1 BALANCE SHEET

It shows the financial condition of the business at a particular time.

1.1.2 INCOME STATEMENT

It is a record of the financial gain or loss of the organization over a given period of time.

1.1.3 ASSET

Asset may be defined as anything of value, such as cash, land, equipment, raw materials, finished products, or any type of property.

Assets are commonly divided into the classifications of current, fixed, and miscellaneous.

CURRENT ASSETS

Represent capital which can readily be converted into cash. Examples would be accounts receivable, inventories, cash, and marketable securities. These are liquid *assets*.

FIXED ASSETS

It cannot be converted into immediate cash. Such as land, buildings, and equipment.

MISCELLANEOUS ASSETS

Deferred charges, other investments, notes and accounts due after 1 year, and similar items are ordinarily listed as miscellaneous assets.

A fundamental relationship in accounting can be written as

Assets = equities

Equities can be divided into two general classes as follows:

PROPRIETORSHIP

The claims of the concern or person who owns the asset.

LIABILITIES

The claims of anyone other than the owner.

The term **proprietorship** is often referred to as **net worth** or simply as **ownership** or **capital.**

Therefore, assets can be written as:

Assets = liabilities + proprietorship

The meaning of this basic equation can be illustrated by the following simple example. Five students have gone together and purchased a second hand automobile worth $1000. Because they did not have the necessary $1000 they borrowed $400 from one of their parents. Therefore, as far as the students are concerned, the value of their asset is $1000, their proprietorship is $600, and their liability is $400.

Modern balance sheets often use the general term **liabilities** in place of **equities. Current liabilities** are grouped together and include all liabilities such as accounts payable, debts, and tax accruals due within 12 months of the balance sheet date. The **net working capital** of a company can be obtained directly from the balance sheet as the difference between current assets and current liabilities.

Income-sheet accounts of all income and expense items, such as sales, purchases, depreciation, wages, salaries, taxes, and insurance, are maintained, and these accounts are summarized periodically in **income statements.** The terms **gross income** or **gross revenue** used by accountants refers to the total amount of capital received as a result of the sale of goods or service. **Net income** or **net revenue** is the total profit remaining after deducting all costs, including taxes.

1.1.4 DEBITS AND CREDITS

Debit entry represents an addition to an account, while a **Credit** entry represents a deduction from an account. In more precise terms, a **debit** entry is one which increases the assets or decreases the equities, and a **credit** entry is one which decreases the assets or increases the equities.

1.1.5 COST ACCOUNTING METHODS

Cost accounting is the determination and analysis of the cost of producing a product or rendering a service. This is exactly what the designer engineer does when estimating costs for a particular plant or process, and cost estimation is one type of cost accounting.

2 COST ESTIMATION

There are many types of cost used in process industries, some of them are as follows:

2.1 FIXED AND WORKING CAPITAL INVESTMENT

2.1.1 FIXED CAPITAL

The total cost of the plant ready for start-up. It is the cost paid to the contractors. It includes the cost of:

- Design, and other engineering and construction supervision.
- All items of equipment and their installation.
- All piping, instrumentation and control systems.
- Buildings and structures.
- Auxiliary facilities, such as utilities, land and civil engineering work.

It is a once-only cost that is not recovered at the end of the project life, other than the scrap value. Fixed capital investment is divided into two categories as:

2.1.1.1 MANUFACTURING FIXED-CAPITAL INVESTMENT

It represents the capital necessary for the installed process equipment with all auxiliaries that are needed for complete process operation. Expenses for piping, instruments, insulation, foundations, and site preparation are typical examples of costs included in the manufacturing fixed-capital investment.

2.1.1.2 NONMANUFACTURING FIXED-CAPITAL INVESTMENT

Fixed capital required for construction overhead and for all plant components that are not directly related to the process operation is designated as the nonmanufacturing fixed-capital investment. These plant components include the land, processing buildings, administrative, and other offices, warehouses, laboratories, transportation, shipping, and receiving facilities, utility and waste-disposal facilities, shops, and other permanent parts of the plant. The construction overhead cost consists of field-office and supervision expenses, home-office expenses, engineering expenses, miscellaneous construction costs, contractor's fees and contingencies.

2.1.2 WORKING CAPITAL

The additional investment needed, over and above the fixed capital, to start the plant up and operate it to the point when income is earned. It includes the cost of:

- Start-up.
- Initial catalyst charges.
- Raw materials and intermediates in the process.
- Finished product inventories.
- Funds to cover outstanding accounts from customers.
- Most of the working capital is recovered at the end of the project. The total investment needed for a project is the sum of the fixed and working capital.

Working capital can vary from as low as 5 per cent of the fixed capital for a simple, single-product, process, with little or no finished product storage; to as high as 30 percent for a process producing a diverse range of product grades for a sophisticated market, such as synthetic fibers.

$$\begin{pmatrix}\textbf{Working}\\ \textbf{Capital}\end{pmatrix}=\begin{pmatrix}\textbf{5 to 30\%}\\ \textbf{Fixed Capital}\end{pmatrix}$$

$$\begin{pmatrix}\textbf{Total Capital}\\ \textbf{Investment}\end{pmatrix}=\begin{pmatrix}\textbf{Fixed}\\ \textbf{Capital}\end{pmatrix}+\begin{pmatrix}\textbf{Working}\\ \textbf{Capital}\end{pmatrix}$$

2.2 TYPES OF CAPITAL COST ESTIMATES

Following five categories represent the accuracy range and designation normally used for design purposes:

- Order-of-magnitude estimate (ratio estimate) based on similar previous cost data; probable accuracy of estimate over ± 30 percent.
- Study estimate (factored estimate) based on knowledge of major items of equipment; probable accuracy of estimate up to ± 30 percent.
- Preliminary estimate (budget authorization estimate; scope estimate) based on sufficient data to permit the estimate to be budgeted; probable accuracy of estimate within ± 20 percent.
- Definitive estimate (project control estimate) based on almost complete data but before completion of drawings and specifications; probable accuracy of estimate within ± 10 percent.
- Detailed estimate (contractor's estimate) based on complete engineering drawings, specifications, and site surveys; probable accuracy of estimate within ± 5 percent.

2.3 COST INDEX

The method usually used to update historical cost data makes use of published cost indices. These relate present costs to past costs, and are based on data for labour, material and energy costs published in government statistical digests.

$$\begin{pmatrix}\text{present}\\ \text{cost}\end{pmatrix}=\begin{pmatrix}\text{original}\\ \text{cost}\end{pmatrix}\times\left(\frac{\text{Cost index of present year}}{\text{Cost index of original year}}\right)$$

The common indexes permit fairly accurate estimates if the time period involved is less than 10 years.

2.4 ESTIMATING EQUIPMENT COSTS BY SCALING

An approximate estimate of the capital cost of a project can be obtained from knowledge of the cost of earlier projects using the same manufacturing process. This method can be used prior to the preparation of the flow-sheets to get a quick estimate of the investment likely to be required.

The capital cost of a project is related to capacity by the equation:

$$C_2 = C_1 \times \left(\frac{Q_2}{Q_1} \right)^n$$

Where, C_2= capital cost of the project with capacity Q_2,

C_1= capital cost of the project with capacity Q_1.

The value of the index n is traditionally taken as 0.6; the well-known **six-tenths rule**.

Many different types of cost indexes are published regularly. Some of these can be used for estimating equipment costs; others apply specifically to labour, construction, materials, or other specialized fields. The most common of the

2.5 ESTIMATION OF TOTAL PRODUCT COST

Total product costs are commonly calculated on one of three bases:

- Daily basis,
- Unit-of-product basis,
- Annual basis.

The annual cost basis is probably the best choice for estimation of total cost because

a) The effect of seasonal variations is smoothed out,

b) Plant on-stream time or equipment operating factor is considered,

c) It permits more-rapid calculation of operating costs at less than full capacity,

d) It provides a convenient way of considering infrequently occurring but large expenses such as annual turnaround costs in a refinery.

Total product cost is the sum of the manufacturing cost and general expenses.

$$\begin{pmatrix}\textbf{Total Product}\\\textbf{Cost}\end{pmatrix} = \begin{pmatrix}\textbf{Manufacturing}\\\textbf{Cost}\end{pmatrix} + \begin{pmatrix}\textbf{General}\\\textbf{Expenses}\end{pmatrix}$$

2.4.1 MANUFACTURING COSTS

All expenses directly connected with the manufacturing operation or the physical equipment of a process plant itself are included in the manufacturing costs.

These expenses, as considered here, are divided into three classifications as follows:

A) Direct production costs (DPC),

B) Fixed charges (FC),

C) Plant-overhead costs (POC).

Manufacturing Cost = DPC + FC + POC

2.4.1.1 DIRECT PRODUCTION COSTS

It includes expenses directly associated with the manufacturing operation. This type of cost involves expenditures for raw materials (including transportation, unloading, etc.,); direct operating labor; supervisory and clerical labor directly connected with the manufacturing operation; plant maintenance and repairs; operating supplies; power; utilities; royalties; and catalysts.

2.4.1.2 FIXED CHARGES

Fixed charges are expenses which remain practically constant from year to year and do not vary widely with changes in production rate. Depreciation, property taxes, insurance, and rent require expenditures that can be classified as fixed charges.

2.4.1.3 PLANT-OVERHEAD COSTS

Plant-overhead costs are for hospital and medical services; general plant maintenance and overhead; safety services; payroll overhead including pensions, vacation allowances, social security, and life insurance; packaging, restaurant and recreation facilities, salvage services, control laboratories, property protection, plant superintendence, warehouse and storage facilities, and special employee benefits.

2.4.2 GENERAL EXPENSES

In addition to the manufacturing costs, other general expenses are involved in any company's operations. These general expenses may be classified as

a) Administrative expenses,

b) Distribution and marketing expenses,

c) Research and development expenses,

d) Financing expenses,

e) Gross-earnings expenses.

2.4.2.1 ADMINISTRATIVEEXPENSES

Administrative expenses include costs for executive and clerical wages, office supplies, engineering and legal expenses, upkeep on office buildings, and general communications.

2.4.2.2 DISTRIBUTION AND MARKETING EXPENSES

These are costs incurred in the process of selling and distributing the various products. These costs include expenditures for materials handling, containers, shipping, sales offices, salesmen, technical sales service, and advertising.

2.4.2.3 RESEARCH AND DEVELOPMENT EXPENSES

These expenses are incurred by any progressive concern which wishes to remain in a competitive industrial position. These costs are for salaries, wages, special equipment, research facilities, and consultant fees related to developing new ideas or improved processes.

2.4.2.4 FINANCING EXPENSES

These expenses include the extra costs involved in procuring the money necessary for the capital investment. Financing expense is usually limited to interest on borrowed money, and this expense is sometimes listed as a fixed charge.

2.4.2.5 GROSS-EARNINGS EXPENSES

These expenses based on income-tax laws. These expenses are a direct function of the gross earnings made by all the various interests held by the particular company. Because these costs depend on the company-wide picture, they are often not included in predesigned or preliminary cost-estimation figures for a single plant, and the probable returns are reported as the gross earnings obtainable with the given plant design.

Ex2.1. A heat exchanger of area 10 m^2 costed Rs.50, 000 in the year 1985. What is the estimated cost of a 15 m^2 exchanger in 1987? Assume that the cost index in 1985 was 270 and in 1987 it is 320.

Solution:
From six-tenths-factor rule,

Cost of heat exchange of size 15 m^2 in 1985 = $50000 \times (15/10)^{0.6}$

$= \text{Rs. } 63771.23$

Relation for cost index

$$\begin{pmatrix}\text{Present}\\ \text{cost}\end{pmatrix} = \begin{pmatrix}\text{original}\\ \text{cost}\end{pmatrix} \times \left(\frac{\begin{matrix}\text{index value at}\\ \text{present time}\end{matrix}}{\begin{matrix}\text{index value at time}\\ \text{original cost was obained}\end{matrix}} \right)$$

Therefore,

$$\begin{pmatrix}\text{Cost of heat exchanger}\\ \text{of size 15 m}^2 \text{ in 1987}\end{pmatrix} = 63771.23 \times \left(\frac{320}{270}\right)$$

$$= Rs.75580.71$$

Ex2.2. Capital cost of fluid-processing plant, if the delivered costs of equipment of a fluid processing plant is Rs 4×10^6, what is the capital cost of the plant?

Solution:
Lang multiplication factors for estimation of fixed-capital investment or total capital investment.
Factor x delivered equipment cost = fixed-capital investment or total capital investment.

Type of plant	Factor for	
	Fixed-capital investment	Total capital investment
Solid-processing plant	3.9	4.6
Solid-fluid-processing plant	4.1	4.9
Fluid-processing plant	4.8	5.7

From the above Table,

Capital cost of the plant = $5.7\times(4\times10^6)$

$= 22.8\times10^6$ Rs

Ex2.3.The annual production costs for a plant are Rs.36.4 lakhs, while the sum of the annual fixed charges, overhead costs and general expenses are Rs. 26.0 lakhs. What is the break-even point, in units of production per year if the total annual sales are Rs.72.S lakhs and the product sells at RS.520 per unit?

Solution:

$$\begin{pmatrix}\text{Number of} \\ \text{units produced}\end{pmatrix} = \frac{72.8\times10^5}{520} = 14000$$

$$\begin{pmatrix}\text{Production} \\ \text{cost per unit}\end{pmatrix} = \frac{36.4\times10^5}{14000} = 260 \text{ Rs}$$

Break-even point,

Total annual sales = total product cost

The number of units at break-even point, then

$520n = 260n + 26\times10^5$

$g, n = 10000.$

GATE QUESTIONS

Q.1 For a solid processing plant, the delivered equipment cost is Rs. 10 lakhs. Using Lang multiplication method, the total capital investment, in lakhs of rupees, is

[GATE-2005]

(A) 46 (B) 57
(C) 100 (D) 200

Q.2 The cost of a drum dryer is Rs. 10 lakhs. The cost of a drum dryer with double the surface area in lakhs of rupees is

[GATE-2005]

(A) 2×10 (B) $3^{0.6} \times 10$
(C) $5^{0.6} \times 10$ (D) $2^{0.6} \times 10$

Q.3 The cost of a distillation column in the year 2000 is x rupees. What is the cost of the column in rupees in the year 2010 given the cost indices for the years 2000 and 2010 are 480 and 520 respectively?

[GATE-2005]

(A) $(520/480)^2$ x
(B) (480/520) x
(C) (520/480) x
(D) $(520/480)^{0.6}$x

Q.4 In a desalination plant, an evaporator of area 200 m^2 was purchased in 1996 at a cost of \$3, 00,000. In 2002, another evaporator of area 50 m^2 was added. What was the cost of the second evaporator (in \$)? Assume that the cost of evaporators scales as $(\text{capacity})^{0.54}$. The Marshall and Swift index was 1048.5 in 1996 and 1116.9 in 2002.

[GATE-2006]

(A) 1,30,500 (B) 1,39,100
(C) 1,41,900 (D) 1,51,200

Q.5 The purchase cost of a heat exchanger of 20 m^2 area was Rs. 500000 in 2006. What will be the estimated cost (in Rs. to the nearest integer) of a similar heat exchanger of 50 m^2 area in the year 2013? Assume the six-tenths factor rule for scaling and the cost index for 2006 as 430.2. The projected cost index for the year 2013 is 512.6. _____

[GATE-2005]

Q.6 Two design options for a distillation system are being compared based on the total annual cost. Information available is as follows:

	Option P	Option Q
Installed cost of the system (Rs in lakhs)	150	120
Cost of cooling water for condenser (Rs in lakhs/year)	6	8
Cost of steam for reboiler (Rs in lakhs/year)	16	20

The annual fixed charge amounts to 12 % of the installed cost. Based on the above information, what is the total annual cost (Rs in lakhs /year) of the better option?

[GATE-2015]

(A) 40 (B) 42.4

(C) 92 (D) 128

Q.7 In the year 2005, the cost of a shell and tube heat exchanger with 68 m^2 heat transfer area was Rs. 10.6 lakh. Chemical Engineering index for cost in 2005 was 509.4 and now the index is 575.4. Based on index of 0.6 for capacity scaling, the present cost (in Lakhs of rupees) of a similar heat exchanger having 100 m^2 heat transfer area is estimated to be **[GATE-2018]**

(A) 17.94 (B)19.94

(C)20.94 (D)22.94

ANSWER KEY:

1	2	3	4	5	6	7							
(a)	(d)	(c)	(d)	(1032386.23)	(a)	(a)							

EXPLANATIONS

Q.1 (A)

Lang multiplication factor is used for estimation of fixed capital investment or total capital investment

Fixed-capital investment or total capital investment. = Lang multiplication factor X delivered equipment cost

Lang multiplication factor for different cases:

Type of plant	Fixed-capital investment	Total capital investment
Solid processing plant	3 .9	4 .6
Solid fluid-processing plant	4 .1	4 .9
Fluid-processing plant	4 .8	5 .7

Therefore for this problem Lang multiplication factor will be 4.6 and the total capital investment will be 46 lakhs

Q.2 (D)

Q.3 (C)

To predict the present cost data from the cost data of previous years

$$\text{Cost in 2010} = \text{Cost in 2000} \times \frac{\text{Cost index in 2010}}{\text{Cost index in 2000}}$$

Therefore, Cost in 2010= x $\left(\frac{520}{480}\right)$

Q.4 (D)

Q.5 (1032386.23)

Cost of Heat exchanger of 20 m^3,

C_1= Rs. 500000 in year 2006

Cost Index of Heat exchanger in year 2006,

CI_1 = 430.2

Cost Index of Heat exchanger in year 2013,

CI_2 = 512.6

$$\left(\begin{array}{c}\text{cost of heat}\\ \text{exchanger of}\\ 20\text{ m}^3\text{ in year 2013}\end{array}\right) = \left(\frac{\begin{array}{c}\text{cost index}\\ \text{of year 2013}\end{array}}{\begin{array}{c}\text{cost index}\\ \text{of year 2006}\end{array}}\right) \times \left(\begin{array}{c}\text{cost of}\\ \text{year 2006}\end{array}\right)$$

$$\left(\begin{array}{c}\text{cost of heat exchanger}\\ \text{of 20 m}^3\text{ in year 2013}\end{array}\right) = \frac{512.6}{430.2} \times 500000$$

$$= \text{Rs. } 595769.4096$$

Now, by 6 / 10^{th} rule,

$$\left(\begin{array}{c}\text{cost of heat}\\ \text{exchanger of}\\ 20\text{ m}^3\text{ in year 2013}\end{array}\right) = \left(\frac{\begin{array}{c}\text{capacity of new}\\ \text{heat exchanger}\end{array}}{\begin{array}{c}\text{capacity of old}\\ \text{heat exchanger}\end{array}}\right)^{0.6} \times \left(\begin{array}{c}\text{cost of old heat}\\ \text{exchanger year 2013}\end{array}\right)$$

$$= \left(\frac{50}{20}\right)^{0.6} \times 595769.4096$$

$$= \text{Rs. } 1032386.235$$

The answer range is between Rs. 1010000 – 1050000.

Q.6 (A)

Q.7 (A)

$$E_2 = E_1 \times \left[\frac{100}{68}\right]^{0.6} = 12.61 \times (1.2603) = 15.88 \text{Lakhs}$$

$$\left[\begin{matrix}\text{Current (or) Present} \\ \text{cost of Equipment 2 } (CE_2)\end{matrix}\right] = \frac{\text{current cost of index}}{\text{Previous cost of Index}} \times E_2$$

$$= \frac{575.4}{509.4} \times 15.88$$

$$CE_2 = 17.938 \text{ Lakhs}$$

3 INTEREST

Interest is the money returned to the owners of capital for use of their capital.

3.1 TYPES OF INTEREST

On the basis of accounting the interests can be divided as follows:

3.1.1 SIMPLE INTEREST

In economic terminology, the amount of capital on which interest is paid is designated as the principal and rate of interest is defined as the amount of interest earned by a unit of principal in a unit of time. The time unit is usually taken as one year.

If *P* represents the principal, *n* the number of time units or interest periods and ***i*** the interest rate based on the length of one interest period, the amount of simple interest *Z* during *n* interest periods is

$$Z = P \times i \times n$$

The principal must be repaid eventually; therefore, the entire amount S of principal plus simple interest due after *n* *i*nterest periods is

$$S = P + Z = P\,(1 + i \times n)$$

3.1.1.1 ORDINARY AND EXACT SIMPLE INTEREST

The time unit used to determine the number of interest periods is usually 1 year, and the interest rate is expressed on a yearly basis. When an interest period of less than 1 year is involved, the ordinary way to determine simple interest is to assume the year consists of twelve 30-day months, or 360 days. The exact method accounts for the fact that there are 365 days in a normal year. Thus, if the interest rate is expressed on the regular yearly basis and *d* represents the number of days in an interest period, the following relationships apply:

$$\text{Ordinary simple interest} = P \times i \times \frac{d}{360}$$

$$\text{Exact simple interest} = P \times i \times \frac{d}{365}$$

3.1.2 COMPOUND INTEREST

If the interest were paid at the end of each time unit, the receiver could put this money to use for earning additional returns. **Compound interest** takes this factor into account by stipulating that interest is due regularly at the end of each interest period.

The total amount of principal plus compounded interest due after n interest periods and designated as

$$S = P(1+i)^n$$

The term $(1+i)^n$ is commonly referred to as the **discrete single-payment compound-amount factor.**

3.1.3 NOMINAL & EFFECTIVE INTEREST RATES

It is desirable to express the exact interest rate based on the original principal and the convenient time unit of 1 year. A rate of this type is known as the effective interest rate.

S represents the total amount of principal plus interest due after n periods at the periodic interest rate i. Let r be the nominal interest rate under conditions where there are m conversions or interest periods per year.

Then the interest rate based on the length of one interest period is r/m, and the amount S after 1 year is

$$\mathbf{S_{after\ 1\ year} = P\left(1+\frac{r}{m}\right)^m}$$

The effective interest rate as $\boldsymbol{i_{eff}}$, the amount S after 1 year can be expressed in an alternate form as

$$\mathbf{S_{after\ 1\ year} = P(1+i_{eff})}$$

Therefore, by combining above both the equation, we get expression for effective interest rate in terms of nominal interest rate as

$$\textbf{Effective annual interest rate} = \mathbf{i_{eff}} = \mathbf{\left(1+\frac{r}{m}\right)^m - 1}$$

Where r is the nominal interest rate,

m is the interest period per year

For e.g.

1. Semi-Annually -(m=2)

2. Quarterly - (m=4)

3.1.4 CONTINUOUS INTEREST

The effective annual interest rate $\boldsymbol{i_{eff}}$ which is the conventional interest rate that most executives comprehend, is expressed in terms of the nominal interest rate $\boldsymbol{r}$ compounded continuously as

$$\mathbf{i_{eff} = e^r - 1}$$

Continuous interest compounding at a nominal annual interest rate of r the amount S and initial principal P will compound to in n years is as

$$\mathbf{S = Pe^{rn}}$$

3.2 PRESENT WORTH AND DISCOUNT

The present worth (or present value) of a future amount is the present principal which must be deposited at a given interest rate to yield the desired amount at some future date.

For compound interest:

$$\text{Present worth}(P) = \frac{S}{(1+i)^n}$$

For continuous interest:

$$\text{Present worth}\left(\text{P}\right)=\frac{\text{S}}{\text{er}^{\text{n}}}$$

3.3 ANNUITIES

An annuity is a series of equal payments occurring at equal time intervals.

Payments of this type can be used to pay off a debt, accumulate a desired amount of capital, or receive a lump sum of capital that is due in periodic installments as in some life-insurance plan

3.3.1 ORDINARY ANNUITY

The common type of annuity involves payments which occur at the end of each interest period. This is known as an ordinary annuity. Interest is paid on all accumulated amounts, and the interest is compounded each payment period.

An annuity term is the time from the beginning of the first payment period to the end of the last payment period. The amount of an annuity is the sum of all the payments plus interest if allowed to accumulate at a definite rate of interest from the time of initial payment to the end of the annuity term.

3.3.2 RELATION BETWEEN AMOUNT OF ORDINARY ANNUITY AND THE PERIODIC PAYMENTS

Let R represent the uniform periodic payment made during n discrete periods in an ordinary annuity. The interest rate based on the payment period is i, and S is the amount of the annuity.

$$S=R\frac{(1+i)^{n}-1}{i}$$

3.4 CONTINUOUS CASH FLOW AND INTEREST COMPOUNDING

Let r represent the nominal interest rate with m conversions or interest periods per year so that $i = r/m$ and the total number of interest periods in n years is mn. With m annuity payments per year, let R represent the total of all ordinary annuity payments occurring regularly and uniformly throughout the year so that R/m is the uniform annuity payment at the end of each period.

$$S = R\frac{(e^{rn}-1)}{r}$$

3.5 PRESENT WORTH OF AN ANNUITY

The present worth of an annuity is defined as the principal which would have to be invested at the present time at compound interest rate I to yield a total amount at the end of the annuity term equal to the amount of the annuity. Let P represent the present worth of an ordinary annuity.

For compound interest:

$$\boldsymbol{S = R\frac{(1+i)^{n}-1}{i(1+i)^{n}}}$$

For continuous interest:

$$S = R\frac{(e^{rn} - 1)}{re^{rn}}$$

Ex3.1. A bond matures after five years and has a maturity value of Rs.1000, If interest rate is 12% what is the present worth of the bond?

Solution:

$$P = S\frac{1}{(1+i)^n}$$

Where i = compound interest rate.

Therefore, Present worth of the bond

$$= 1000\frac{1}{(1+0.12)^5} = \text{Rs.}567.43$$

3.6 PERPETUITIESAND CAPITALIZED COSTS

Perpetuity is an annuity in which the periodic payments continue indefinitely. This type of annuity is of particular interest to engineers, for in some cases they may desire to determine a total cost for a piece of equipment or other asset under conditions which permit the asset to be replaced perpetually without considering inflation or deflation.

If perpetuation is to occur, the amount *S* accumulated after *n* periods minus the cost for the replacement must equal the present worth *P*. Therefore, letting C_R represent the replacement

Cost then present worth given as

$$P = \frac{C_R}{(1+i)^n - 1}$$

The capitalized cost is defined as the original cost of the equipment plus the present value of the renewable perpetuity. Designating *K* as the capitalized cost and C_V as the original cost of the equipment then capitalized cost of the equipment is given as:

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

Ex3.2. An equipment costs Rs.1,70,000 and will have a scrap value of Rs.25, 000 at the end of its useful life of 10 years. If the interest is compounded at 10% per year, what are the cost of replacement, the present worth and the capitalized cost?

Solution:

If cost of equipment remains at constant value, then

Cost of replacement C_R = equipment cost at present time –scrap value

Therefore

$$C_R = 170000 - 25000 = Rs.1,45,000$$

Present worth P is related to C_R as

$$P = \frac{C_R}{(1+i)^n - 1}$$

Therefore

$$P = \frac{145000}{(1+0.1)^{10} - 1} = Rs.90,981.$$

Capitalized cost K is related to C_R and original cost of equipment C_V as:

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

Therefore

$$K = 170000 + 90981 = Rs.2,60,981.$$

Ex3.3. A new equipment made of material A costs, post installation, Rs.3,00,000 and is expected to have a scrap value of 10% of this cost at the end of a useful life of 10 years. Similar equipment made of material B cost Rs.1,50,000, but is likely to have no scrap value. Assume that both types of equipment could be replaced at a cost that is 20% more than the original value. On the basis of equal capitalized cost for both types of equipment, estimate what should be the useful life for equipment made of material B. The company has to pay an annual interest on the investment at a rate of 15%.

Solution:

Capitalized cost K is related to cost of replacement C_R and original cost of equipment C_V

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

Therefore capitalized cost of equipment made of material A

$$K_A = 300000 + \frac{1.2\times 300000 - 0.1\times 300000}{(1+0.15)^n - 1} = Rs.4,08,355.$$

Capitalized cost of equipment made of material B is given by

$$K_B = 150000 + \left(\frac{1.2\times 150000}{(1+0.15)^n - 1}\times 21\right)$$
$$+\left(1.2 - 0.1\times 3001.2\right)$$

Since it is given that K_B and K_A are to be equal,

$$408355 = 150000 + \frac{1.2\times 150000}{(1+0.15)^n - 1}$$

Solving, n = 3.78 years.

Therefore, the useful life of equipment made of material B should be 3.87 years.

Ex3.4. A heat exchanger with an initial investment of Rs.300, 000 has a 6 years life. How much can be spent on an improved design which has a life of 12 years and is expected to save Rs.10, 000 per year?
Annual compound rate = 8%
Solution:

$$\text{Capitalized cost } = \frac{C_R(1+i)^n}{(1+i)^n - 1} + V_s$$

Where C_R = cost of replacement

V_s =salvage value at the end of useful life

n = estimated useful life of equipment

i = interest rate

For the exchanger with 6 years life:

$$\text{Capitalized Cost} = \frac{300000(1+0.8)^6}{(1+0.08)^6 - 1} = Rs.811183$$

Present worth (P) of savings from the exchanger with 12year life:

$$\text{P} = R\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$= 10000\frac{(1+0.08)^{12} - 1}{0.08(1+0.08)^{12}} = Rs.75361$$

Capitalized cost of the improved exchanger with 12year life shall be equal to the sum of the present worth of its savings and the capitalized cost of exchanger with 6 year life.

$$75361 + 811183 = \frac{C_R(1+0.08)^{12}}{(1+0.08)^{12} - 1}$$

Solving, C_R = Rs. 534485.

Therefore an amount of Rs. 534485 shall be invested for the improved design.

3.7 TAXES AND INSURANCE

3.7.1 TYPES OF TAXES

Taxes may be classified into three types:

- Property taxes,
- Excise taxes,
- Income taxes.

3.7.1.1 PROPERTY TAXES

Local governments usually have jurisdiction over property taxes, which are commonly charged on a county basis. In addition to these, individual cities and towns may have special property taxes for industrial concerns located within the city limits. Property taxes vary widely from one locality to another, but the average annual amount of these charges is 1 to 4 percent of the assessed valuation. Taxes of this type are referred to as direct since they must be paid directly by the particular concern and cannot be passed on as such to the consumer.

3.7.1.2 EXCISE TAXES

Excise taxes are levied by Federal and state governments. Federal excise taxes include charges for import customs duties, transfer of stocks and bonds, and a large number of other similar items. Manufacturers' and retailers' excise taxes are levied by Federal and state governments on the sale of many products such as gasoline and alcoholic beverages. Taxes of this type are often referred to as indirect since they can be passed on to the consumer. Many business concerns must also pay excise taxes for the privilege of carrying on a business or manufacturing enterprise in their particular localities.

3.7.1.3 INCOME TAXES

Income taxes are based on gross earnings, which are defined as the difference between total income and total product cost. Revenue from income taxes is an important source of capital for both Federal and state governments. National and state laws are the basis for these levies, and the laws change from year to year. State income taxes vary from one state to another and are a function of the gross earnings for individual concerns. Depending on the particular state and the existing laws, state income taxes may range from 0 to 5 percent or more of gross earnings.

3.8 INSURANCE

The annual insurance cost for ordinary industrial concerns is approximately 1 percent of the capital investment. Insurance costs may represent only a small fraction of total costs, it is necessary to consider insurance requirements carefully to make certain the economic operation of a plant is protected against emergencies or unforeseen developments.

Following are the types of insurance:

- Fire insurance and similar emergency coverage on buildings, equipment, and all other owned, used, or stored property. Included in this category would be losses caused by lightning, wind- or hailstorms, floods, automobile accidents, explosions, earthquakes, and similar occurrences.

- Public-liability insurance, including bodily injury and property loss or damage, on all operations such as those involving automobiles, elevators, attractive nuisances, bailee's charges, aviation products, or any company function carried on at a location away from the plant premises.
- Business-interruption insurance. The loss of income due to a business interruption caused by a fire or other emergency may far exceed any loss in property. Consequently, insurance against a business interruption of this type should be given careful consideration.
- Power-plant, machinery, and special-operations hazards.
- Workmen's-compensation insurance.
- Marine and transportation insurance on all property in transit.
- Comprehensive crime coverage.
- Employee-benefit insurance, including life, hospitalization, accident, health, personal property, and pension plans.

Ex.3.5. A new piece of completely installed equipment costs Rs 12,000 and will have a scrap value of 2000 at the end of its useful life if useful life period is 10 years and the interest is compounded of 6%per year. What is the capitalized cost of equipment?

Solution:

Capitalized cost= $p + \dfrac{(p-s)}{(1+i)^n - 1}$

$$= 12000 + \frac{(12000-2000)}{(1+0.06)^{10}-1}$$

Capitalized cost = 24645

Ex.3.6.A sale contract signed by a chemical manufacture is expected to generate a net cash flow of Rs 4.00,000 per year at the end of each year for a period of three years. The applicable discount rate is 15%. The net present worth of the total cash flow is?

Solution:

$$S = R\left[\frac{(C+i)^n - 1}{i(1+i)^n}\right]$$

$$S=4,00,000\left[\frac{(1.15)^3-1}{0.15\times(1.15^3}\right]=9,13,290$$

S= Rs. 913290

Ex3.7. For an interest rate of 2% per month, the effective semiannual rate is closest to:

Solution: In this example, the i on the left-hand side of the effective interest rate equation will have units of semiannual periods. Therefore, the r must have units of semiannual periods (i.e., 12% per six months) and m must be the number of times interest is compounded per semiannual period, 6 in this example.

$i=(1 + 0.12 / 6)^6 - 1$

i = 12.62 %.

GATE QUESTIONS

Q.1 An investment of Rs. 1000 is carrying an interest of 10% compounded quarterly. The value of the investment at the end of five years will be.

[GATE-2001]

(A) $1000\left(1+\frac{0.1}{4}\right)^{20}$

(B) $1000(1+0.10)^{20}$

(C) $1000\left(1+\frac{0.1}{4}\right)^{5}$

(D) $1000\left(1+\frac{0.1}{2}\right)^{5}$

Q.2 If an amount R is paid at the end of every year for n years, then the net present value of the annuity at an interest rate of i is

[GATE-2002]

(A) $R\left[\frac{(1+i)^n-1}{i}\right]$

(B) $R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$

(C) $R\left[(1+i)^n\right]$

(D) $\frac{R}{(1+i)^n}$

Q.3 Two pumps under consideration for installation at a plant have the following capital investment and salvage values.

Pump A : C_1= Rs 40,000 , C_{sal} = Rs 3900 . Pump B: C_I = Rs50,000 ,C_{sal} = 20,000. Using capitalized cost, determine what should be the common life of the pumps for both to be competitive (economically equivalent). Interest rate is 10% per annum. Maintenance and operational cost are negligible.

[GATE-2003]

(A) 3 years (B) 5 years

(C) 6 years (D) 8 years

Q.4 A reactor has been installed at a cost of Rs. 50,000 and is expected to have a working life of 10 years with a scrap value of Rs. 10,000. The capitalized cost (in Rs.) of the reactor based on an annual compound interest rate of 5% is

[GATE-2008]

(A) 1,13,600 (B) 42,000

(C) 52,500 (D) 10,500

Q.5 A reactor needs to be lined with a corrosion resistant lining. One type of lining cost Rs.5 lakhs and is expected to last for 2 years. Another type of lining lasts for 3 years. If both choices have to be equally economical, with the effective interest rate being 18%, compounded annually, the price one should pay for the second type of lining is

[GATE-2010]

(A) Rs. 6.1 lakhs (B) Rs. 6.5 lakhs

(C) Rs. 6.9 lakhs (D) Rs. 7.6 lakhs

Q.6 A continuous fractionators system is being designed. The following cost figures are estimated for a reflux ratio of 1.4. The annualized fixed charge is 15 % of the fixed cost. The total annualized cost (in Rs.) is **[GATE-2011]**

(A) 10.8×10^6 (B) 13.35×10^6

(C) 15.9×10^6 (D) 3.15×10^6

Q.7 Heat integration is planned in a process plant at an investment Rs 2×10^6. This would result in a net energy savings of 20 GJ per year. If the nominal rate of interest is 15 % and the plant life is 3 years, then the breakeven cost of energy, in Rs. per GJ (adjusted to the nearest hundred), is

[GATE-2012]

(A) 33500 (B) 43800
(C) 54200 (D) 65400

Q.8 A cash flow of Rs. 12,000 per year is received at the end of each year (uniform periodic payment) for 7 consecutive years. The rate of interest is 9% per year compounded annually. The present worth (in Rs.) of such cash flow at time zero is ________. **[GATE-2014]**

Q.9 A bond has a maturity value of 20, 000 Rupees at the end of 4 years. The interest is compounded at the rate of 5% per year. The initial investment to be made, rounded off to the nearest integer, is ______Rupees. **[GATE-2017]**

Q.10 A furnace installed at a cost of Rs.24 Lakh is expected to serve its useful life of 5 years. Salvage value of the furnace is Rs. 8 Lakh. The interest rate compounded annually is 8% . The estimated capitalized cost (in Lakhs of rupees) is

[GATE-2018]

(A) 30 (B) 34.09

(C) 34.9 (D) 58.09

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10				
(b)	(b)	(b)	(a)	(c)	(a)	(b)	(60395.43)	(16454)	(d)				

EXPLANATIONS

Q.1 (B)

$n = 4 \times 5$

(Q Interest compound quarterly for five years) I = 10% (given)

The value of the investment at the end of five year = $1000\,(1+0.1)^{20}$

Q.2 (B)

If an amount R is paid at the end of every year for n year, then the net present value of the annuity at an interest rate of I is

$$R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$$

Q.3 (B)

$$C_1-\frac{C_{sal(1)}}{(1+i)^n}=C_2-\frac{C_{sal(2)}}{(1+i)^n}$$

$$40000-\frac{3900}{1.1^n}=50000-\frac{20000}{1.1^n}$$

$$\frac{20000-3900}{1.1^n}=10000$$

$$1.1^n=\frac{16100}{10000}$$

$$1.1^n=1.61$$

$$n=4.99\simeq 5\ \text{years}$$

Q.4 (A)

We know that

$$K = Cp+\frac{C_R}{(1+i)^n-1}$$

$$K= 50000+\frac{10000}{(1+0.5)^{10}-1}$$

K = 113600

Q.5 (C)

Given $n_1 = 2,$ and $n_1 = 3$

Equal capitalized cost in both cases

$$\frac{5(1+i)^{n_1}}{(1+i)^{n_1}-1}=\frac{x(1+i)^{n_2}}{(1+i)^{n_2}-1}$$

x = 6.9 lakhs

Q.6 (A)

Q.7 (B)

We know capital recovery

$$(A / P, \%, n) = \frac{p\times i(1+i)^n}{\left[(1+i)^n-1\right]}$$

A = Annual cash flow for

P = Present worth

$\therefore\ P = Rs.\,2\times10^6, i = 0.15,$

$n = 3$ yrs

$$A=\frac{2\times10^6\times(1+0.15)^3\times0.15}{\left[(1+0.15)^3-1\right]}$$

$= Rs.\,875953.93$

Annual savings = 20 GJ energy

Let Cost of energy = Rs. P / GJ

For break-even point

20 P = 875953.93

P **=** 43797.696 **=** Rs. 43800

Q.8 (60395.43)

We know that

$$\text{Present worth } P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$P = 12000\left[\frac{(1.09)^7 - 1}{0.09\times(1.09)^7}\right] = 60395.43$$

Q.9 (16454)

Maturity value = 20,000. Bond life = 4 years, Interest rate = 5 %

Amount of investment to be made can be found from present worth formula

$$= 20000/(1+0.05)^4 = 16454.04 = 16454 \text{ Rs}$$

Q.10 (D)

Capitalize cost of E_{quip}

$$K = C_v + \frac{C_R}{(1+i)^n - 1}$$

$$K = 24 + \frac{16}{(1+0.008)^5 - 1} = 24 + 34.091$$

$$K = 58.09 \text{ Lakhs}$$

4 DEPRICIATION

4.1 DEPRECIATION

Equipment, buildings, and other material objects comprising a manufacturing plant require an initial investment which must be written off as a manufacturing expense. In order to write off this cost, a decrease in value is assumed to occur throughout the usual life of the material possessions. This decrease in value is designated as depreciation.

4.2 TYPES OF DEPRECIATION

There are following types of depreciation:

4.2.1 PHYSICAL DEPRECIATION

Physical depreciation is the term given to the measure of the decrease in value due to changes in the physical aspects of the property. Wear and tear, corrosion, accidents, and deterioration due to age or the elements are all causes of physical depreciation.

4.2.2 FUNCTIONAL DEPRECIATION

In this type of depreciation, the serviceability of the property is reduced because of physical changes. Depreciation due to all other causes is known as functional depreciation.

4.3 BASIC TERMS

4.3.1 DEPLETION

Capacity loss due to materials actually consumed is measured as depletion.

Depletion cost equals the initial cost times the ratio of amount of material used to original amount of material purchased. This type of depreciation is particularly applicable to natural resources, such as stands of timber or mineral and oil deposits.

4.3.2 SERVICE LIFE

The period during which the use of a property is economically feasible is known as the service life of the property. Both physical and functional depreciation are taken into consideration in determining service life.

4.3.3 SALVAGE VALUE

Salvage value is the net amount of money obtainable from the sale of used property over and above any charges involved in removal and sale. If a property is capable of further service, its salvage value may be high. This is not necessarily true, however, because other factors, such as location of the property, existing price levels, market supply and demand, and difficulty of dismantling, may have an effect. The term salvage value implies that the asset can give some type of further service and is worth more than merely its scrap or junk value.

4.3.4 PRESENT VALUE

The present value of an asset may be defined as the value of the asset in its condition at the time of valuation.

4.4 METHODS FOR DETERMINING DEPRECIATION

Depreciation accounting methods may be divided into two classes:

1) Arbitrary methods giving no consideration to interest costs.

   a) Straight-line method

   b) Declining-balance method

   c) Sum-of-the-years-digits method.

2) Methods taking into account interest on the investment.

   a) Sinking-fund method

   b) Present-worth method.

4.4.1 STRAIGHT-LINE METHOD

In the straight-line method for determining depreciation, it is assumed that the value of the property decreases linearly with time. Equal amounts are charged for depreciation each year throughout the entire service life of the property. The annual depreciation cost may be expressed in equation form as follows:

$$\mathbf{d = \frac{V - V_s}{n}}$$

Where, d= annual depreciation, Rs/year

V= original value of the property at start of the service-life period, completely installed and ready for use,

V_s= salvage value of property at end of service life,

n = service life, years

The asset value (or book value) of the equipment at any time during the service life may be determined from the following equation:

$$\mathbf{V_a = V - a\ d}$$

Where, V_a = asset or book value, a = the number of years in actual use.

4.4.2 DECLINING-BALANCE (OR FIXED PERCENTAGE) METHOD

When the declining-balance method is used, the annual depreciation cost is a fixed percentage of the property value at the beginning of the particular year. The fixed-percentage (or declining-balance) factor remains constant throughout the entire service life of the property, while the annual cost for depreciation is different each year. Under these conditions, the depreciation cost for the first year of the property's life is V_f, where f represents the fixed-percentage factor.

At the end of the first year:

$$\mathbf{Asset\ value = V_a = V(1 - f)}$$

At the end of n years (i.e., at the end of service life):

$$\mathbf{V_n = V(1 - \mathit{f})^n = V_s}$$

$$f = 1 - \left(\frac{V_s}{V}\right)^{1/n}$$

4.4.3 SUM-OF-THE-YEARS-DIGITS METHOD

The sum-of-the-years digits method is an arbitrary process for determining depreciation which gives results similar to those obtained by the declining-balance method. Larger costs for depreciation are allotted during the early-life years than during the later years. This method has the advantage of permitting the asset value to decrease to zero or a given salvage value at the end of the service life.

In the application of the sum-of-the-years-digits method, the annual depreciation is based on the number of service-life years remaining and the sum of the arithmetic series of numbers from 1 to *n*, where *n* represents the total service life. The yearly depreciation factor is the number of useful service-life years remaining divided by the sum of the arithmetic series. This factor times the total depreciable value at the start of the service life gives the annual depreciation cost. Equation for determining annual depreciation is given by:

$$d_a = Depreciation\ for\ year = \frac{(n - a + 1)}{\sum_1^n a}(V - V_s)$$

$$d_a = \frac{2(n - a + 1)}{n(n + 1)}(V - V_s)$$

4.4.4 SINKING-FUND METHOD

The use of compound interest is involved in the sinking-fund method. It is assumed that the basic purpose of depreciation allowances is to accumulate a sufficient fund to provide for the recovery of the original capital invested in the property. An ordinary annuity plan is set up wherein a constant amount of money should theoretically be set aside each year. At the end of the service life, the sum of all the deposits plus accrued interest must equal the total amount of depreciation.

Let, i = Annual interest rate expressed as a fraction

R= Uniform annual payments made at end of each year (this is the annual depreciation cost),

$V - V_s$ = Total amount of the annuity accumulated in an estimated service life of n years (original value of property minus salvage value at end of service life)The amount accumulated in the fund after $\boldsymbol{a}$ years of useful life must be equal to the total amount of depreciation up to that time. This is the same as the difference between the original value of the property $\mathbf{V}$ at the start of the service life and the asset value $\mathbf{V_s}$ at the end of $\boldsymbol{a}$ years.

Total amount of depreciation after $\boldsymbol{a}$ years = $V - V_a$

$$V_a = V - (V - V_s)\frac{(1+i)^a - 1}{(1+i)^n - 1}$$

4.-LINE, SUM-OF-THE-YEARS-DIGITS AND DECLINING-BALANCE METHODS FOR DETERMINING DEPRECIATION

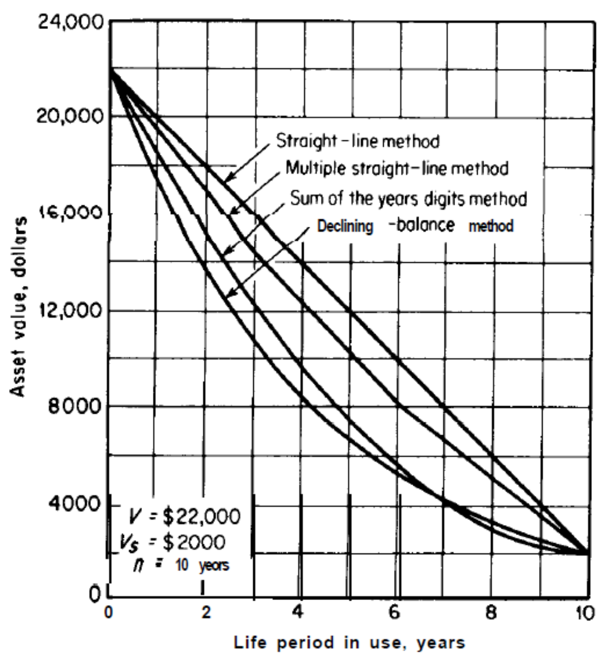


Figure: **Comparison of straight-line, multiple straight-line, sum-of-the-years-digits, and declining-balance methods for determining depreciation.**

Ex4.1. The plant of a chemical company has an initial worth of Rs.50 lakhs, and an estimated salvage value of Rs.2 lakhs in a service life of 8 years.

(a) Given a choice between the straight-line and declining-balance methods of depreciation which method would you recommend to save tax and why?

(b) Estimate the book value of the plant at the end of 4 years for each of the two methods of depreciation.

Solution:

(a) Declining-balance method of depreciation is to be chosen, as this method permits greater depreciation allowances in the early life of the property than in the latter life.

(b) Book value at the end of 4 years:

Straight line method:

The annual depreciation cost d is given as:

$$d = \frac{V - V_s}{n}$$

Where V = original value of the property at the start of the service-life period

V_S = salvage value of property at the end of service life

n= service life years

The asset value Va (or book value) of the equipment at any time during the service life is given as:

$$V_a = V - ad$$

Where a= number of years in actual use.

Given: V=Rs. 50 lakhs; V_s = Rs. 2 lakhs; n = 8 years; and a = 4 years.

Therefore,

$$d = \frac{50-2}{8} = \text{Rs.6 lakhs}$$

And $V_a = 50 - 4\times6$ = Rs. 26 lakhs

Declining balance method:

The asset value V_a is given by the relation:

$V_a = V(1 - f)^a$

Where f represents the fixed-percentage factor:

$$f = 1 - \left(\frac{V_S}{V}\right)^{1/n}$$

Therefore,

$$f = 1 - \left(\frac{2}{50}\right)^{1/8} = 0.3313$$

And $V_a = 50 \times (1\text{-}0.3313)^4 = \text{Rs.}10 \text{ lakhs}$,

Ex4.2. A compressor has an installed cost of Rs 24,000 and a 15 year estimated life. The salvage value of the compressor is zero at the end of 15 years. The compressor value after depreciation by the double declining method at the end of 10 years is?

Solution: We know that,

$$f = \frac{2}{n} = \frac{2}{15} = 0.133$$

$$V_n = V(1\text{-f})^n$$

$$= 24,000\ (1\text{-}0.133)^{10}$$

$$= 5760$$

GATE QUESTIONS

Q.1 P is the investment made on an equipment, S is its salvage value at n is the life of the equipment in years. The depreciation for the m^{th} year by the Sum-of –Years –Digits method will be

[GATE-2001]

(A) $\frac{P-S}{n}$ (B) $1-\left(\frac{P}{S}\right)^{\frac{1}{m}}$

(C) $\frac{m}{n}(P-S)$ (D) $\frac{2(n-m+1)}{n(n+1)}(P-S)$

Q.2 A company has a depreciable investment of Rs.36400 which is depreciated in equal installments in two years. Assume that the tax rate is 50% and the interest rate is 10%. The net present value of the tax that the company would have saved if it had depreciated 2/3 of the investment in the first year and the rest in the second year, is

[GATE-2001]

(A)0 (B)250

(C) 375 (D)500

Common Data Questions 3 – 4

A process has fixed capital investment of Rs. 150 lakhs, working capital of Rs. 30 lakhs and salvage value zero. Annual revenues from sales are Rs. 250 lakhs , manufacturing costs 145 lakhs and other expenses 10% of the revenue. Assume project life span of 11 years , tax life of 5 years and interest rate to be 10%. Tax rate is 40% and straight line depreciation i.e. 20% per year is applicable

[GATE-2001]

Q.3 Discounted value (to present time) of the profit before tax (for the total plant life period) in rupees is

(A) 228 lakhs (B) 400 lakhs

(C) 520 lakhs (D) 660 lakhs

Q.4 Discounted value of the depreciation benefit over the tax life in rupees is

(A) 12 lakhs (B) 24 lakhs

(C) 46 lakhs (D) 60 lakhs

Q.5 The original value of an equipment is Rs. 10000/-. The salvage value is Rs. 500/- at the end of its useful life period of 5 years. What is the asset value in rupees after two years by textbook declining balance method?

[GATE-2005]

(A) 3025/- (B) 4010/-

(C) 5020/- (D) 6050/-

Q.6 A pump has an installed cost of Rs. 40,000 and a 10-year estimated life. The salvage value of the pump is zero at the end of 10 years. The pump value (in rupees) after depreciation by the double declining balance method, at the end of 6 years is

[GATE-2007]

(A) 4295 (B) 10486
(C) 21257 (D) 37600

Q.7 A column costs Rs. 5.0 lakhs and has a useful life of 10 years. Using the double declining balance depreciation method, the book value of the unit at the end of five years (in lakhs of Rs.) is

[GATE-2009]

(A) 1.21 (B) 1.31
(C) 1.64 (D) 2.05

Q.8 A process plant has a life of 7 years and its salvage value is 30 %. For what MINIMUM fixed – percentage factor will the depreciation amount for the second year, calculated by declining balance method be EQUAL to that calculated by the straight line depreciation method?

[GATE-2011]

(A) 0.1 (B) 0.113
(C) 0.527 (D) 0.88

ANSWER KEY:

1	2	3	4	5	6	7	8						
(d)	(b)	(c)	(c)	(a)	(b)	(c)	(b)						

EXPLANATIONS

Q.1 (D)

Depreciation for the m^{th} year by sum of year digit method

$$=\frac{(P-S)\times(n-m+1)}{1+2+3+....n}=\frac{(P-S)\,(n-m+1)}{\frac{n(n+1)}{2}}$$

$$=\frac{2(n-m+1)}{n(n+1)}(P-S)$$

Q.2 (B)

For depreciation by first method

Tax saved in first year = 0.5 × 18200 = 9100

Tax saved in the second year = 0.5 × 18200 = 9100

For depreciation by the second method

$$\begin{pmatrix}\text{Tax saved in}\\ \text{the first year}\end{pmatrix}=\frac{2}{3}\times 36400\times 0.5$$

$$=12133.33$$

$$\text{Tax saved in the second year} = \frac{1}{3}\times 36400\times 0.5$$

$$= 6066.66$$

As compared to the first method amount of tax saved in the second method each year is

First year = 12133.33 -9100 = 3033.33

Second year = 6066.66 – 9100 = -3033.33

Net present value of the tax saved =3033.33/(1+0.10)-3033.33/(1 + 0.10)^2

= 250

Q.3 (C)

$$\frac{P}{A}=\left(P/A,\ i\ \%,\ n\right)=\frac{(1+i)^n-1}{i(1+i)^n}$$

$$=\frac{(1+0.1)^{11}-1}{0.1(1+0.1)^{11}}=6.495$$

Fixed capital investment = 150 lakhs

Annual revenue = 250 lakhs/year

Profit before tax = 250 – (145+0.1×250)

= 80 Lakhs /year

Discounted value of profit before tax

= 80 × 6.495 = 519.60 Lakhs

Q.4 (C)

Depreciation benefit = 30 × 0.4 = 12

Discounted value of this amount for a tax life of 5 years is approximately 46 lakhs.

Q.5 (A)

Q.6 (B)

We know that

$$f=\frac{2}{n}=\frac{2}{10}=0.2$$

Now,

$$V_a = V(1-f)^n$$
$$= 40000(1-.2)^6$$
$$=10485.76$$
$$=10486$$

Q.7 (C)

Given: Total Cost $= 500000$

Useful life $= 10$ years

Salvage value $=$ Nil

Annual depreciation $\dfrac{500000 - 0}{10} = 50,000$

Depreciation rate $= \dfrac{50000}{500000} \times 100 = 10\%$

Double decline rate(f) = 20%

Value at the end of 5 year,

$$V_a = (1 - f)^a V$$
$$= 500,000(1 - .2)^5$$
$$= 163840; 1.64 \text{ lakh}$$

Q.8 (B)

$$C(1-f)f = 0.1C$$
$$f - f^2 = 0.1$$

or $f^2 - f + 0.1 = 0$

or $f = \dfrac{1 \pm \sqrt{1 - 4\times1\times0.1}}{2} = \dfrac{1 \pm \sqrt{0.6}}{2}$

$= \dfrac{1 \pm 0.7745}{2} = \dfrac{1+0.7745}{2}$ or $\dfrac{1-0.7745}{2}$

= 0.88/or 0.113

minimum fixed % factor = 0.113

5 PROFITABILITY ANALYSIS & OPTIMIZATION

The most commonly used methods for profitability evaluation can be categorized as:

1. Rate of return on investment

2. Discounted cash flow based on full-life performance

3. Net present worth

4. Capitalized costs

5. Payout period

5.1 RATE OF RETURN ON INVESTMENT

In engineering economic studies, rate of return on investment is ordinarily expressed on an annual percentage basis. The yearly profit divided by the total initial investment necessary represents the fractional return, and this fraction times 100 is the standard percent return on investment.

Profit is defined as the difference between income and expense. Therefore, profit is a function of the quantity of goods or services produced and the selling price. The amount of profit is also affected by the economic efficiency of the operation, and increased profits can be obtained by use of effective methods which reduce operating expenses. To obtain reliable estimates of investment returns, it is necessary to make accurate predictions of profits and the required investment. To determine the profit, estimates must be made of direct production costs, fixed charges including depreciation, plant overhead costs, and general expenses. Profits may be expressed on a before-tax or after-tax basis, but the conditions should be indicated. Both working capital and fixed capital should be considered in determining the total investment.

5.2 DISCOUNTED CASH FLOW

The method of approach for a profitability evaluation by discounted cash flow takes into account the time value of money and is based on the amount of the investment that is unreturned at the end of each year during the estimated life of the project. A trial-and-error procedure is used to establish a rate of return which can be applied to yearly cash flow so that the original investment is reduced to zero (or to salvage and land value plus working-capital investment) during the project life. Thus, the rate of return by this method is equivalent to the maximum interest rate (normally, after taxes) at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal.

To illustrate the basic principles involved in discounted-cash-flow calculations and the meaning of rate of return based on discounted cash flow, consider the case of a proposed project for which the following data apply:

Initial FCI = Rs.100000

Working-capital investment = Rs.10000

Service life = 5 years

V_s at end of service life = Rs.10000

Designate the discounted-cash-flow rate of return as *i*. This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just break even at the end of the service life.

At the end of five years, the cash flow to the project, compounded on the basis of end-of-year income, will be

$(Rs.30000)(1 + i)^4 + (Rs.31,000)(1 + i)^3 + (Rs.36,000)(1 + i)^2 + (Rs.40,000)(1 + i) + Rs.43,000 = S$

The symbol *S* represents the future worth of the proceeds to the project and must just equal the future worth of the initial investment compounded at an interest rate *i* corrected for salvage value and working capital. Thus,

$$S = (Rs.110000)(1 + i)^5 - Rs.10000 - Rs.10000$$

Combining above both the equation and solving by trial and error for *i* gives ***i*** = 0.207, or the discounted-cash-flow rate of return is 20.7 percent.

Ex.5.1. For a project having a life of ten years the following cash flow pattern is expected:

End of year	Net cash flow(Rs.)
0	-50,00,000
1-10	20,00,000
10	-1,50,00,000

If the expected interest rate is 20 percent, what is your recommendation about implementing the project?

Solution: At the end of years, the net cash flow S to the project, computed on the basis of end-of-year income, will be

$$S = -5\times10^6(1+0.2)^{10} + 2\times10^6\Big[(1+0.2)^5 + (1+0.2)^8 + (1+0.2)^7 + (1+0.2)^6(1+0.2)^5 + (1+0.2)^4 + (1+0.2)^3 + (1+0.2)^2 + (1+0.2) + 2\times10^6 - 15\times10^6$$

$$S = -30.96\times10^6 + (2\times10^6)(24.96) + 2\times10^6 - 15\times10^6$$

$$S = Rs\ 5.96\times10^6$$

5.3 CAPITALIZED COSTS

Capitalized cost related to investment represents the amount of money that must be available initially to purchase the equipment and simultaneously provide sufficient funds for interest accumulation to permit perpetual replacement of the equipment.

$$K = C_V + \frac{C_R}{(1+i)^n - 1} = \frac{C_R(1+i)^n}{(1+i)^n - 1} + V_s$$

Where, K = capitalized cost

C_V= original cost of equipment

C_R = replacement cost

V_S= salvage value at end of estimated useful life

n = estimated useful life of equipment

i= interest rate

5.4 PAYOUT PERIOD

Pay out period or payout time is defined as the minimum length of time theoretically necessary to recover the original capital investment in the form of cash flow to the project based on total income minus all costs except depreciation.

Generally, for this method, original capital investment means only the original, depreciable, fixed-capital investment, and interest effects are neglected.

With considering no interest:

Payout period in years

$$= \frac{\text{Depreciable fixed Capital investment}}{\text{average profit/yr} + \text{avg. depreciation/yr}}$$

Ex5.2.Fixed capital investment for a chemical plant is Rs 80 million with an estimated useful life of 10 years and a salvage value of Rs 10 million The rate of interest is 15 % tax is 25% of the annual taxable income in the first year of operation, the income from sales is Rs30 Million What is the rate of return on investment?

Solution:

We know that

$$\text{Rate of return} = \frac{\text{yearly profit}}{\text{Total initial investment}} \times 100$$

Yearly profit = 30 – 30×0.25=22.5

$$\text{Rate of return} = \frac{22.5}{80} \times 100 = \underline{28\%}$$

Ex5.3.A column costs of Rs10 lakhs and has a useful life of 15 years. Using the double declining balance depreciation method, the book value of the unit at the end of 7 years is?

Solution:

$$\text{Annual depreciation} = \frac{10,00,000}{15} = 66,666.67$$

$$\text{Depreciation rate} = \frac{66,666.67}{10,00,000} \times 100 = 6.67$$

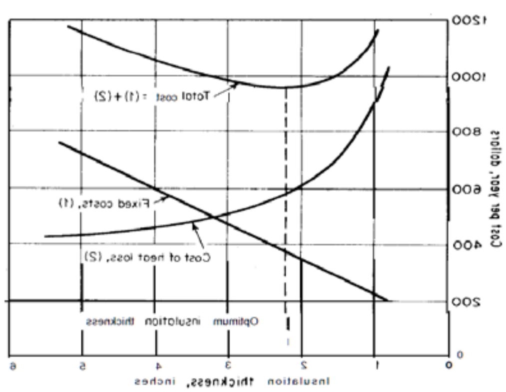
Double decline rate (f) =13.33%

$$V_n = V(1-f)^a$$
$$= 10,00,000(1-0.1333)^7$$
$$= 3.67 \text{ lakhs}$$

5.5 OPTIMIZATION IN DESIGN

An optimum economic design could be based on conditions giving the least cost per unit of time or the maximum profit per unit of production. When one design variable is changed, it is often found that some costs increase and others decrease. Under these conditions, the total cost may go through a minimum at one value of the particular design variable, and this value would be considered as an optimum.

In this simple case, the problem is to determine the optimum thickness of insulation for a given steam-pipe installation. As the insulation thickness is increased, the annual fixed costs increase, the cost of heat loss decreases, and all other costs remain constant. Therefore, as shown in Fig., the sum of the costs must go through a minimum at the optimum insulation thickness.



5.6 GENERAL PROCEDURE FOR DETERMINING OPTIMUM CONDITIONS

The first step in the development of an optimum design is to determine what factor is to be optimized. Typical factors would be total cost per unit of production or per unit of time, profit, amount of final product per unit of time, and percent conversion. Once the basis is determined, it is necessary to develop relationships showing how the different variables involved affect the chosen factor. Finally, these relationships are combined graphically or analytically to give the desired optimum conditions.

5.6.1 PROCEDURE WITH ONE VARIABLE

Consider the example presented in Fig. as shown above, where it is necessary to obtain the insulation thickness which gives the least total cost. The primary variable involved is the thickness of the insulation, and relationships can be developed showing how this variable affects all costs.

The two cost relationships obtained might be expressed in a simplified form similar to the following:

Fixed charges = $\Phi(x) = ax + b$

Cost of heat loss = $\Phi^{i}(x) = c/x + d$

Total variable cost = $C_T = \Phi(x) + \Phi^{i}(x) = \Phi^{ii}(x) = ax + b + c/x + d$

Where a, *b*,c, and *d*are constants and *x* is the common variable (insulation thickness).

The optimum value can be found analytically by merely setting the derivative of C_T with respect to x equal to zero and solving for x.

$$\frac{dC_T}{dx} = a - \frac{c}{x^2} = 0$$

$$x = \left(\frac{c}{a}\right)^{1/2}$$

If the factor being optimized (C_T) does not attain a usable maximum or minimum value, the solution for the dependent variable will indicate this condition by giving an impossible result, such as infinity, zero, or the square root of a negative number.

The value of x shown in above equation occurs at an optimum point or a point of inflection. The second derivative of above equation evaluated at the given point, indicates if the value occurs at a minimum (second derivative74 greater than zero), maximum (second derivative less than zero), or point of inflection (second derivative equal to zero). An alternative method for determining the type of point involved is to calculate values of the factor being optimized at points slightly greater and slightly smaller than the optimum value of the dependent variable.

$$\frac{d^2C_T}{dx^2} = \frac{2c}{x^3}$$

5.6.2 PROCEDURE WITH TWO OR MORE VARIABLE

When two or more independent variables affect the factor being optimized, the procedure for determining the optimum conditions may become rather tedious; however, the general approach is the same as when only one variable is involved. Consider the case in which the total cost for a given operation is a function of the two independent variables x and y.

$C_T = \Phi^{iii}(x, y)$

By analyzing all the costs involved and reducing the resulting relationships to a simple form, the following function might be found for above equation:

$$C_T = ax + \frac{b}{xy} + cy + d$$

$$\frac{dC_T}{dx} = a - \frac{b}{x^2y}$$

$$\frac{dC_T}{dy} = c - \frac{b}{xy^2}$$

At the optimum conditions, both of these derivatives must be equal to zero.

Both the above derivatives are set equal to zero and optimum value of variable x and y can be found.

Ex.5.4 Cost of a pipeline of exotic material is estimated as $3D^{1.5}$ Rs/m. where D id the diameter in mm. The annual maintenance cost is estimated as 10% of the total capital annual operating cost of the pipe is given as follows:

Annual operating cost =

$$3\times10^{15} / D^5 \text{Rs}/(\text{m.year})$$

The most economic pipe diameter based on the least annual cost approach to the multiple of 10mm.

Estimated amortization period is 10 year.

Solution:

Annual cost C_r of pipeline is related to the pipe diameter as

$$C_r = \frac{3D^{1.5}}{10} + 0.1 \times \frac{3\times10^{15}}{D^5}$$

$$= 0.3D^{1.5} + \frac{3\times10^{14}}{D^5}$$

Equating this expression with respect to D,

$$\frac{dC_T}{dD} = 0.45D^{0.5} - \frac{15\times10^{14}}{D^6}$$

Total annual cost is abstained by equating dC_T/dD to zero. Therefore,

D =240 mm.

The nearest multiple of 10 mm:

Annual cost of pipeline of D =240mm

$$=0.6\times240^{1.5}+0.1\times\frac{3\times10^{15}}{240^5} = Rs.2607$$

Annual cost of pipeline of D=250 mm

$$0.6\times250^{1.5}+0.1\times\frac{3\times10^{15}}{250^5} = Rs.2678$$

Cost economic pipe diameter = 240mm.

GATE QUESTIONS

Q.1 An investment of Rs. 100 lakhs is to be made for construction of a plant which will take two years to start production. The annual profit from operation of the plant is Rs.20 lakhs. What will be the payback time?

[GATE-2001]

(A) 5 years (B) 7 years

(C) 12 years (D) 10 years

Q.2 The total investment in a project is Rs. 10 Lakhs and the annual profit is Rs. 1.5 lakhs. If the project life is 10 years, then the simple rate of return on investment is

[GATE-2002]

(A) 15% (B) 10%

(C) 1.5% (D) 150%

Common Data Questions 3 – 4

Fixed capital investment for a chemical plant is Rs. 40 million with an estimated useful life of 6 years and a salvage of Rs. 4 million. The rate of interest is 15%. Tax is 25% of the annual taxable income. In the first year of operation, the income from sales is Rs. 20 million and manufacturing expenses are Rs. 5 million. The plant depreciates on a straight line basis.

[GATE-2004]

Q.3 The rate of return on investment is given by

(A) 50 (B) 37.5

(C) 32 (D) 20

Q.4 The net present value (NPV) in million Rs. at the start and at the end of the first year of operation is respectively given by

(A) zero and -28.9 (B) -40 and -28.9

(C) -40 and 12.75 (D) zero and 12.75

Q.5 The depreciable fixed cost is Rs. 100 lakhs. The average profit per year is Rs. 15 lakhs. The average depreciation cost per year is Rs. 10 lakhs. What is the payout period in years, if there is no interest charge?

[GATE-2005]

(A) 8 (B) 4

(C) 10 (D) 20

Q.6 Due to a 20% drop in the product selling price, the pay-back period of a new plant increased to 1.5 times of that estimated initially, the production cost and the production rate remaining unchanged. If the production cost is Cp and the new selling price is Cs, then Cp/Cs is

[GATE-2006]

(A) 0.2 (B) 0.4

(C) 0.5 (D) 0.6

Q.7 Obtain the optimal diameter of a cylindrical storage vessel of volume V. The curved shell costs Cs (in \$/m²), and the flat top and bottom plates cost Cp (in \$./m²)

[GATE-2006]

(A) $D = \frac{C_S}{C_P}\left[\frac{4V}{\pi}\right]^{\frac{1}{3}}$ (B) $D = \left[\frac{8V}{\pi}\frac{C_S}{C_P}\right]^{\frac{1}{3}}$

(C) $D = \left[V\frac{C_S}{C_P}\right]^{\frac{1}{3}}$ (D) $D = \left[\frac{4V}{\pi}\frac{C_S}{C_P}\right]^{\frac{1}{3}}$

Q.8 A sale contract signed by a chemical manufacturer is expected to generate a net cash flow of \$ 2,50,000 per year at the end of each year for a period of three years. The applicable discount rate (interest rate) in 10%. The net present worth of the total cash flow is \$.

[GATE-2006]

(A) 7,50,000 (B) 6,83,750
(C) 6,21,500 (D) 3,32,750

Q.9 For the case of single lump-sum capital expenditure of Rs. 10 crores which generates a constant annual cash flow of Rs. 2 crores in each subsequent year, the payback period (in years), if the scrap value of the capital outlay is zero is

[GATE-2008]

(A) 10 (B) 20
(C) 1 (D) 5

Q.10 The relation between capital rate of return ratio (CRR), net present value (NPV) and maximum cumulative expenditure (MCE) is

[GATE-2008]

(A) $CRR = \frac{NPV}{MCE}$

(B) $CRR = \frac{MCE}{NPV}$

(C) $CRR = NPV \times MCE$

(D) $CRR = \frac{MCE}{NPV + MCE}$

Q.11 The total fixed cost of a chemical plant is Rs. 10.0 lakhs; the internal rate of return is 15% and the annual operating cost is Rs. 2.0 lakhs. The annualized cost of the plant (in lakhs of Rs.) is

[GATE-2009]

(A) 1.8 (B) 2.6
(C) 3.5 (D) 4.3

Common Data Questions 12 and 13:

A plant produces phenol. The variable cost in rupees per ton of phenol is related to the plant capacity P (in tons/day) as 45,000 + 5P. The fixed charges are Rs. 100,000 per day. The selling price of phenol is Rs. 50,000 per ton.

[GATE-2010]

Q.12 The optimal plant capacity (in tons per day) for minimum cost per ton of phenol, is

(A) 101 (B) 141
(C) 283 (D) 422

Q.13 The break-even capacity in tonnes per day, is

(A) 50 (B) 40
(C) 30 (D) 20

Q.14 A batch reactor produces 1×10^5 kg of a product per year. The total batch time (in hours) of the reactor is $k\sqrt{P_B}$, where P_B is the product per batch in kg and k = 1.0 $h/\sqrt{kg}$. The operating cost of the reactor is Rs. $200/h$. The total annual fixed charges are Rs. 340 $\times P_B$ and the annual raw material cost is Rs. 2×10^6. The optimum size (in kg) of each batch (adjusted to the nearest integer) is

[GATE-2012]

(A) 748 (B)873
(C) 953 (D) 1148

Q.15 A plant manufactures compressors at the rate of N units/day. The daily fixed charges are Rs. 20000 and the variable cost per compressor is Rs. 500 + 0.2 $N^{1.3}$. The selling price per compressor is Rs. 1000. The number of compressors to be manufactured, to the nearest integer, in order to maximize the daily profit is ____

[GATE-2013]

Q.16 A polymer plant with a production capacity of 10,000 tons per year has an overall yield of 70%, on mass basis (kg of product per kg of raw material). The raw material costs Rs. 50,000 per ton. A process modification is proposed to increase the overall yield to 75% with an investment of Rs.12.5 crore. In how many years can the invested amount be recovered with the additional profit? _____

[GATE-2014]

Q.17 The cost of two independent process variable f_1 and f_2 affects the total cost C_T (in lakhs of rupees) of the process as per the following function:

$$C_T = 100f_1 + \frac{1000}{f_1 f_2} + 20f_2^2 + 50$$

The lowest total cost C_T, in lakhs of rupees (up to one decimal place), is ____

[GATE-2015]

Q.18 A proposed chemical plant is estimated to have a fixed capital (FC) of Rs.24 crores. Assuming other costs to be small, the total investment may be taken as to be same as FC. After commissioning (at t=0 years), the annual profit before tax is Rs. 10 crores/year (at the end of each year) and the expected life of the plant is 10 years. The tax rate is 40% per year and a linear depreciation is allowed at 10% per year. The salvage value is zero, If the annual interest rate is 12%, the NPV (net present value or worth) of the project in crores of rupees (up to one decimal place) is _________. **[GATE-2015]**

Q.19 Terms used in engineering economics have standard definitions and interpretations. Which one of the following statements is **INCORRECT?**

[GATE-2016]

(A) The profitability measure 'return on investment' does not consider the time value of money

(B) A cost index is an index value for a given time showing the cost at that time relative to a certain base time

(C) The 'six-tenths factor rule' is used to estimate the cost of equipment from the cost of similar equipment with a different capacity

(D) Payback period is calculated based on the payback time for the sum of the fixed and the working capital investment

Q.20 A vertical cylindrical tank with a flat roof and bottom is to be constructed for storing 150 m^3 of ethylene glycol. The cost of material and fabrication for the tank wall is Rs. 6000 per m^2 and the same for the roof and the tank bottom are Rs 2000 and Rs 4000 per m^2, respectively. The cost of accessories, piping and instruments can be taken as 100% of the cost of the wall. 10% of the volume of the tank needs to be kept free as vapor space above the liquid storage. What is the optimum diameter (in m) for the tank? **[GATE-2016]**

(A) 3.5 (B) 3.9
(C) 7.5 (D) 7.8

Q.21 The total cost (C_T) of an equipment in terms of operating variables x and y is

$$C_T = 2x + \frac{12000}{xy} + y + 5$$

The optimum value of C_T rounded to 1 decimal place, is _____ **[GATE-2017]**

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(a)	(a)	(b)	(b)	(b)	(b)	(d)	(c	(d)	(a)	(c)	(b)	(d)	(c)
15	**16**	**17**	**18**	**19**	**20**	**21**							
(217)	(12.5×10^7)	(572.8)	(15.32)	(d)	(d)	(91.5)							

EXPLANATIONS

Q.1 (A)

$$\text{Payback period} = \frac{\text{Fixed capital investment}}{\text{Annual profit}}$$

$$= \frac{100}{20} = 5\text{years}$$

Q.2 (A)

$$\text{simple rate of return} = \frac{\text{Profit}}{\text{Investment}} \times 100$$

$$= \frac{1.5}{10} \times 100 = 15\%$$

Q.3 (B)

$$\begin{pmatrix}\text{Rate of return}\\ \text{on investment}\end{pmatrix} = \frac{\text{Yearly profit}}{\text{Total initial investment}} \times 100$$

Yearly profit = Income from sales − expenses

$$= 20 - 20 \times \frac{25}{100} = 15 \text{ million}$$

$$\text{Rate of return} = \frac{15}{40} \times 100 = 37.5\%$$

Q.4 (B)

Q.5 (B)

100 lakhs/ (15+10) lakhs = 4 yrs

Q.6 (B)

$$t_1 = \frac{i}{x - C_p},\ t_2 = \frac{i}{C_S - C_P}$$

given $x - 0.2x, = C_S$

$$\Rightarrow x = \frac{C_S}{0.8}$$

$$\Rightarrow \frac{1}{1.5} = \frac{C_s - C_p}{\frac{C_s}{0.8} - C_p}$$

$$\Rightarrow \frac{C_p}{C_s} = \frac{1}{2} = 0.5$$

Q.7 (D)

$$\text{Cost} = C_s \pi D h + C_p \times 2 \times \pi \frac{D^2}{4}$$

$$V = \frac{\pi D^2 h}{4}$$

$$h = \frac{4V}{\pi D^2}$$

$$C = \frac{C_s \pi D 4V}{\pi D^2} + \frac{2\pi D^2 C_p}{4}$$

$$\frac{dC}{dD} = -\frac{C_s 4V}{D^2} + \pi D C_p$$

$$D^3 = \frac{4VC_s}{\pi C_p}$$

$$D = \left(\frac{4VC_s}{\pi C_p}\right)^{1/3}$$

Q.8 (C)

Q.9 (D)

For the case of single lump sum capital expenditure of Rs. 10 crores which generates a constant annual cash flow of Rs. 2 cores in each subsequent year the payback period (in years if the scarp value of the capital outlay is zero is

= 5 (10 crores/2 crores)

Q.10 (A)

Q.11 (C)

Q.12 (B)

Let C = cost per ton phenol

$$C\left(\frac{Rs}{ton}\right)=\underbrace{100,000\frac{Rs}{day}/\frac{1}{p}\frac{day}{ton}}_{\text{Fixed Cost}}$$

$$+\underbrace{(45000+5p)\frac{Rs}{ton}}_{\text{Variable Cost}}$$

$$\therefore \quad C = \frac{100000}{p}+45,000+5p$$

For optional plant capacity $\frac{dC}{dp}=0$

$$\frac{dC}{dp}=-\frac{100,000}{p^2}+5$$

$$0 = -\frac{100,000}{p^2}+5$$

or $\frac{100,000}{p^2}=5$

or $p^2=20,000$

or p = 141.42³141

Q.13 (D)

Selling price equation

$$C_s=50000\frac{Rs}{ton}\times p\frac{ton}{day} = 50000p\frac{Rs}{day}$$

Production price (C_p) equation

$$C_p=100000\frac{Rs}{day} + (45000+5p)\frac{Rs}{ton}\times p\frac{ton}{day}$$

Fixed cost variable cost

$$C_p=100000 + (45000+5P)P$$

At Break even point $C_s=C_P$

$$50000P=100000+(45000+5P)P$$

$$= 100000+45000P+5P^2$$

or $5p^2-5000p+100000=0$

or $p^2-1000p+20,000=0$

or $p = \frac{1000\pm\sqrt{1000^2-4\times 20,000}}{2}$

$$=\frac{1000+959.163}{2}$$

P = 979.58 and 20.41

We will choose he lower value, is at P = 20 production cost seller cost

Q.14 (C)

Basis = 1 year

Production capacity = 1×10^5 kg

No. of batches produced

$$= 1\times 10^5 \text{ kg product x } \frac{1}{P_B}\frac{\text{batch}}{\text{kg product}}$$

$$=\frac{1\times 10^5}{P_B}\text{ batches}$$

Batch type (time required to process 1 batch)

$$=k\sqrt{P_B}\,\frac{h}{batch}\left[k=1.0\frac{h}{\sqrt{\text{kg product}}}\right]$$

$\therefore$ Total time of production

$$=\frac{1\times 10^5}{P_B}(\text{batches})\times k\sqrt{P_B}\,\frac{h}{batch}$$

$$=\frac{k\times 10^5}{\sqrt{P_B}}\,h$$

Operating cost of the reactor= Rs. 200 / h

Thus, total cost of operation in 1 year

$$=\frac{k\times 10^5}{\sqrt{P_B}}\,h\times 200\frac{Rs.}{h}$$

$$=Rs.\frac{k\times 2\times 10^7}{\sqrt{P_B}}$$

⇒ Annual fixed charges = Rs. 340 P_B

⇒ Annual raw material cost=Rs. 2×10^6

Total cost of production in 1 year (for P_B kg product / batch)

$$T = 2\times10^6 + 340\,P_B + \frac{k\times2\times10^7}{\sqrt{P_B}} \qquad k=1.0$$

For optimum size in kg of each batch (i.e. minimum production cost), $\frac{\partial T}{\partial P} = 0$

$$\frac{\partial T}{\partial P_B} = 0 + 340 + \left(2\times10^7\right)\left(-\frac{1}{2}\right)P_B^{-3/2} = 0$$

P_B = 952.83 kg

Q.15 (217)

Total unit produces = N units / day

Total Cost = Total Fixed Charges + Total Variable Cost

= 20000 + N × (500 + 0.2 × $N^{1.3}$)

Total Profit = Total Selling Price–Total Cost

= N × 1000 – (20000+ N × (500+ 0.2 × $N^{1.3}$))

For Maximum Profit,

$$\frac{\partial}{\partial N}(\text{Total Profit}) = 0$$

Thus,

$$\frac{\partial}{\partial N}\left[1000\times N - \left(200000 + N\times\left(500+0.2\times N^{1.3}\right)\right)\right] = 0$$

$$\Rightarrow 500\times N - 2000 - 0.2\times N^{2.3} = 0$$

$$\Rightarrow N = 216.54 \approx \underline{217\ units}$$

The answer range is between 215 – 220.

Q.16 (2.625)

We have

Let number of years = n

Total product = 10,000 n

Raw material used $= \frac{10,000\,n}{0.7}$

Total cost of Raw material$= \frac{10,000\,n}{0.7}\times 50,000$

from question

$$\frac{50,000\times10,000\,n}{0.7} - \frac{50,000\times10,000\,n}{0.75} = 12.5\times10^7$$

Solving, n = 2.625 years.

Q.17 (572.8)

We have

$$\text{Given } C_T = 100f_1 + \frac{1000}{f_1f_2}20f_2^2 + 50$$

$$\frac{\partial C_T}{\partial f_1} = P = 100 = \frac{1000}{f_2f_1^2}$$

$$\text{and } \frac{\partial C_T}{\partial f_2} = q = \frac{-1000}{f_2^2f_1} + 40f_2$$

For maxima of minima P = 0, q = 0

$$100 = \frac{1000}{f_2f_1^2} \text{ and } \frac{1000}{f_1f_2^2} = 40f_2$$

$$\Rightarrow f_1^2f_2 = 10 \ldots\ldots\ldots\ldots(1)$$

$$\Rightarrow f_1f_2^3 = 25 \ldots\ldots\ldots\ldots(II)$$

Dividing equation (ii) by equation (1)

(ii)/(i)

$$\Rightarrow \frac{f_1f_2^3}{f_1^2f_2} = 2.5$$

$$\Rightarrow \frac{f_2^2}{f_1} = 2.5 \Rightarrow f_1 = \frac{f_2^2}{2.5}$$

From (1)

$$\Rightarrow \frac{f_2^4}{(2.5)^2}f_2 = 10$$

$$\Rightarrow f_2^5 = 62.5 \Rightarrow f_2 = 2.2865$$

$$\text{so, } f_1 = \frac{(2.2865)^2}{2.5} = 2.091$$

$$f_1 = 2.091,\ f_2 = 2.2865$$

Q.18 (15.32)

FCI = 24 crores

Depreciation = 0.10 x 24 = 2.4 crores

Taxable income = 10-2.4 =7.6 crores

Profit after tax = 7.6 x 0.6 = 4.56 crores

Annual cash flow = 4.56 + 2.4 = 6.96

NPV = In – Out

=Discounted value of cash flows–initial investment

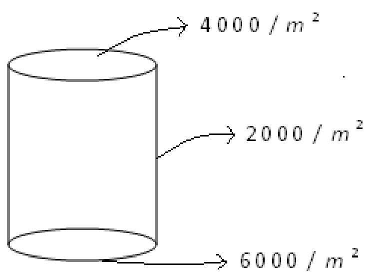
= 39.32 – 24

= 15.32

Q.19 (D)

Q.20 (D)

Given that



volume of liquid =150 m^3

$$x = 150 + \frac{10}{100} \times [10\% \text{ of the vapor space}]$$

$x = 166.6m^3 = v$

$$C_T = \frac{\pi d^2}{4}(2000+4000) + \pi dh\,(6000) + \frac{10}{100} \times 6000\,(\pi dh)$$

$$V = \frac{\pi d^2 h}{4} \quad h = \frac{166.6}{\left(\frac{\pi d^2}{4}\right)}$$

$$C_T = 6000\frac{\pi d^2}{4} + \pi d\left(\frac{166.6}{\frac{\pi d^2}{4}}\right)(6600)$$

$$\frac{d(C_T)}{d(d)} = \frac{6000\,\pi(2)d}{4} + \frac{4\,(166.6)\,(6600)}{d^2}$$

$$3000\pi d = \frac{4\,(166.6)\,(6600)}{d^2}$$

$$d^3 = \frac{4\,(166.6)\,(6600)}{3000\,\pi}$$

$$d^3 = 466.3$$

$$d = 7.75\ m$$

Q.21 (91.5)

The total cost of an equipment in terms of the operating variables x and y is

$$C_T = 2x + \frac{1200}{xy} + y + 5$$

To find optimum value of C_T

$$\frac{\partial C_T}{\partial x} = 0 \Rightarrow 2 - \frac{1200}{x^2 y} = 0$$

$$2x^2 y = 1200 \text{ -------} \rightarrow (1)$$

and

$$\frac{\partial C_T}{\partial y} = 0 \Rightarrow 1 - \frac{1200}{xy^2} = 0$$

$$xy^2 = 1200 \text{ ---------} \rightarrow (2)$$

Solving equation (1) and (2) yields

= 14.4 and y = 28.8

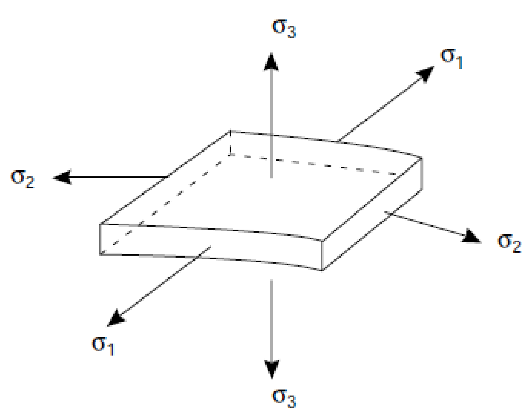
Hence the optimum value of C_T is 91.5

6 MECHANICAL DESIGN OF EQUIPMENTS

6.1 Classification of pressure vessels

For the purposes of design and analysis, pressure vessels are sub-divided into two classes depending on the ratio of the wall thickness to vessel diameter: thin-walled vessels, with a thickness ratio of less than 1 : 10; and thick-walled above this ratio.

The principal stresses acting at a point in the wall of a vessel, due to a pressure load, are shown in Figure. If the wall is thin, the radial stress σ_3 will be small and can be neglected in comparison with the other stresses, and the longitudinal and circumferential stresses σ_1 and σ_2 can be taken as constant over the wall thickness. In a thick wall, the magnitude of the radial stress will be significant, and the circumferential stress will vary across the wall. The majority of the vessels used in the chemical and allied industries are classified as thin-walled vessels.



Let P = pressure,

t = thickness of shell,

σ_1 = the meridional (longitudinal) stress, the stress acting along a meridian,

σ_2 = the circumferential or tangential stress, the stress acting along parallel circles (often called the hoop stress),

r1 = the meridional radius of curvature,

r2 = circumferential radius of curvature.

For a cylinder $\sigma_1 = \frac{PD}{4t}$ and $\sigma_2 = \frac{PD}{2t}$

For a sphere $\sigma_1 = \sigma_2 = \frac{PD}{4t}$

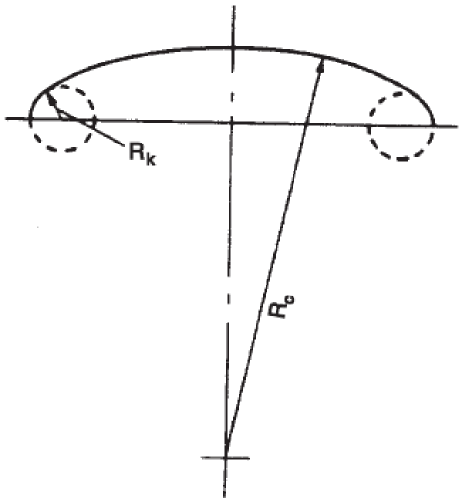
6.2 Torispherical heads

A torispherical shape, which is often used as the end closure of cylindrical vessels, is formed from part of a torus and part of a sphere. The shape is close to that of an ellipse but is easier and cheaper to fabricate. In Figure Rk is the knuckle radius (the radius of the torus) and R_c the crown radius (the radius of the sphere).

For the spherical portion $\sigma_1 = \sigma_2 = \frac{PR_c}{2t}$

For the torus $\sigma_1 = \frac{PR_k}{4t}$

The ratio of the knuckle radius to crown radius should be made not less than 6/100 to avoid buckling. The stress will be higher in the torus section than the spherical section.



6.3 Welded joint efficiency, and construction categories

The strength of a welded joint will depend on the type of joint and the quality of the welding. The soundness of welds is checked by visual inspection and by non-destructive testing (radiography). The possible lower strength of a welded joint compared with the virgin plate is usually allowed for in design by multiplying the allowable design stress for the material by a "welded joint factor" J. The value of the joint factor used in design will depend on the type of joint and amount of radiography required by the design code.

Taking the factor as 1.0 implies that the joint is equally as strong as the virgin plate; this is achieved by radiographing the complete weld length, and cutting out and remaking any defects. The use of lower joint factors in design, though saving costs on radiography, will result in a thicker, heavier, vessel, and the designer must balance any cost savings on inspection and fabrication against the increased cost of materials.

6.4 Corrosion allowance

The "corrosion allowance" is the additional thickness of metal added to allow for material lost by corrosion and erosion, or scaling. The allowance to be used should be agreed between the customer and manufacturer. Corrosion is a complex phenomenon, and it is not possible to give specific rules for the estimation of the corrosion allowance required for all circumstances. The allowance should be based on experience with the material of construction under similar service conditions to those for the proposed design. For carbon and low-alloy steels, where severe corrosion is not expected, a minimum allowance of 2.0 mm should be used; where more severe conditions are anticipated this should be increased to 4.0 mm. Most design codes and standards specify a minimum allowance of 1.0 mm.

6.5 THE DESIGN OF THIN-WALLED VESSELS UNDER INTERNAL PRESSURE

For a cylindrical shell the minimum thickness required to resist internal pressure can be determined from equation; the cylindrical stress will be the greater of the two principal stresses.

If D_i is internal diameter and e is the minimum thickness required, the mean diameter will be $(D_i + e)$ substituting this for D in equation,

$$e = \frac{P_i(D_i + e)}{2f}$$

Where, f is the design stress and P_i the internal pressure.

Rearranging gives:

$$e = \frac{P_i D_i}{2f - P_i}$$

An equation for the minimum thickness of a sphere can be obtained from equation

$$e = \frac{P_i D_i}{4f - P_i}$$

If a welded joint factor is used

$$e = \frac{P_i D_i}{2fJ - P_i}$$

$$e = \frac{P_i D_i}{4fJ - P_i}$$

Where, J is the joint factor

6.6 Heads and closures

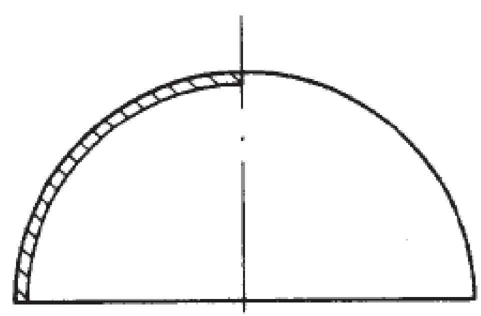
The ends of a cylindrical vessel are closed by heads of various shapes. The principal Types used are:

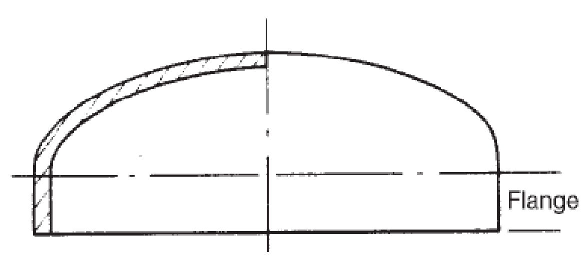
1. Flat plates and formed flat heads.
2. Hemispherical heads
3. Ellipsoidal heads.
4. Torispherical heads

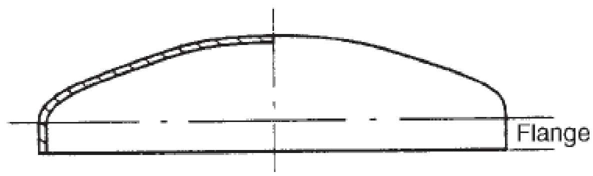
Hemispherical, ellipsoidal and torispherical heads are collectively referred to as domed heads. They are formed by pressing or spinning; large diameters are fabricated from formed sections. Torispherical heads are often referred to as dished ends.

Flat plates are used as covers for manways, and as the channel covers of heat exchangers. Formed flat ends, known as "flange-only" ends, are manufactured by turning over a flange with a small radius on a flat plate. "Flange-only" heads are the cheapest type of formed head to manufacture, but their use is limited to low-pressure and small-diameter vessels. Standard torispherical heads (dished ends) are the most commonly used end closure for vessels up to operating pressures of 15 bar. They can be used for higher pressures, but above 10 bar their cost should be compared with that of an equivalent ellipsoidal head.

Above 15 bar an ellipsoidal head will usually prove to be the most economical closuret ouse.A hemispherical head is the strongest shape; capable of resisting about twice the pressure of a torispherical head of the same thickness. The cost of forming a hemispherical head will, however, be higher than that for a shallow torispherical head. Hemispherical heads are used for high pressures.





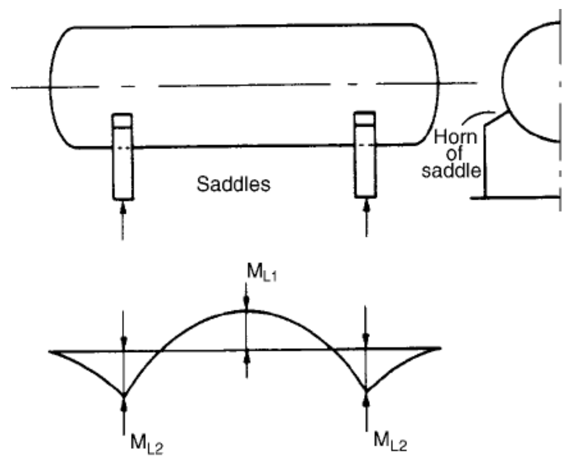


6.7 VESSEL SUPPORTS

The method used to support a vessel will depend on the size, shape, and weight of the vessel; the design temperature and pressure; the vessel location and arrangement; and the internal and external fittings and attachments. Horizontal vessels are usually mounted on two saddle supports Skirt supports are used for tall, vertical columns Brackets, or lugs, are used for all types of vessel. The supports must be designed to carry the weight of the vessel and contents, and any superimposed loads, such as wind loads. Supports will impose localised loads on the vessel wall, and the design must be checked to ensure that the resulting stress concentrations are below the maximum allowable design stress. Supports should be designed to allow easy access to the vessel and fittings for inspection and maintenance.

6.7.1 Saddle supports

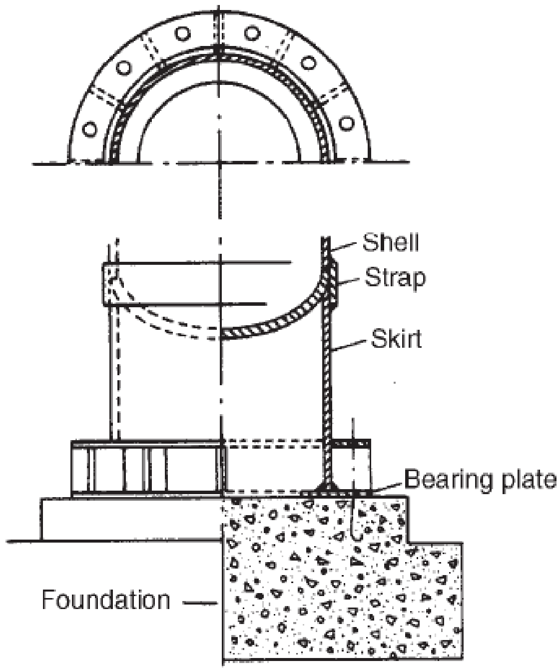
Though saddles are the most commonly used support for horizontal cylindrical vessels, legs can be used for small vessels. A horizontal vessel will normally be supported at two cross-sections; if more than two saddles are used the distribution of the loading is uncertain.
A vessel supported on two saddles can be considered as a simply supported beam, with an essentially uniform load, and the distribution of longitudinal axial bending moment will be as shown in Figure. Maxima occur at the supports and at mid-span. The theoretical optimum position of the supports to give the least maximum bending moment will be the position at which the maxima at the supports and at mid-span are equal in magnitude.

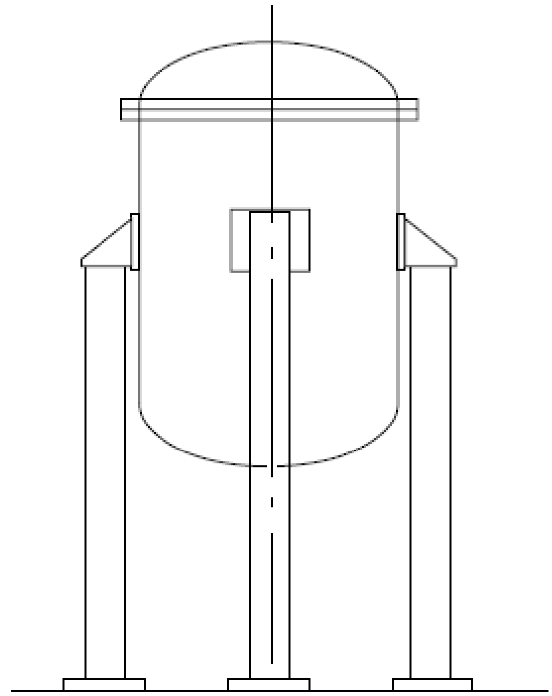


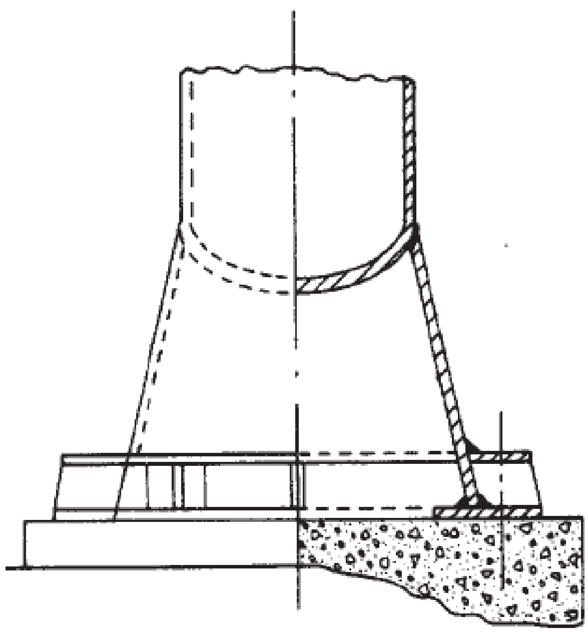
For a uniformly loaded beam the position will be at 21 per cent of the span, in from each end. The saddle supports for a vessel will usually be located nearer the ends than this value, to make use of the stiffening effect of the ends.

6.7.2 Skirt supports

A skirt support consists of a cylindrical or conical shell welded to the base of the vessel. A flange at the bottom of the skirt transmits the load to the foundations. Typical designs are shown in Figure. Openings must be provided in the skirt for access and for any connecting pipes; the openings are normally reinforced.
Skirt supports are recommended for vertical vessels as they do not impose concentrated loads on the vessel shell; they are particularly suitable for use with tall columns subject to wind loading.







6.7.3 Bracket supports

Brackets, or lugs, can be used to support vertical vessels. The bracket may rest on the building structural steel work, or the vessel may be supported on legs;

The main load carried by the brackets will be the weight of the vessel and contents; in addition the bracket must be designed to resist the load due to any bending moment due to wind, or other loads. If the bending moment is likely to be significant skirt supports should be considered in preference to bracket supports. The point of support, at which the reaction acts, should be made as close to the vessel wall as possible; allowing for the thickness of any insulation. Methods for estimating the magnitude of the stresses induced in the vessel.

6.8 HEAT-EXCHANGER TUBE-PLATES

The tube-plates (tube-sheets) in shell and tube heat exchangers support the tubes, and separate the shell and tube side fluid. One side is subject to the shell side pressure and the other the tube-side pressure. The plates must be designed to support the maximum differential pressure that is likely to occur. Radial and tangential bending stresses will be induced in the plate by the pressure load and, for fixed-head exchangers, by the load due to the differential expansion of the shell and tubes. A tube-plate is essentially a perforated plate with an unperforated rim, supported at its periphery. The tube holes weaken the plate and reduce its flexual rigidity. The equations developed for the stress analysis of unperforated plates can be used for perforated plates by substituting "virtual" (effective) values for the elastic constants E and v, in place of the normal values for the plate material. The virtual elastic constants

E0 and v0 are functions of the plate ligament efficiency. The ligament efficiency of a perforated plate is defined as:

$$\lambda = \frac{p_h - d_h}{p_h}$$

Where, p_h = hole pitch and

d_h = hole diameter

The "ligament" is the material between the holes (that which holds the holes together). Ina tube-plate the presence of the tubes strengthens the plate, and this is taken into account when calculating the ligament efficiency by using the inside diameter of the tubes in place of the hole diameter.

GATE QUESTIONS

Q.1 A perforated plate has holes of diameter d_h arranged in a pitch p_h. Each hole has a tube of internal diameter d_t passing through it. The ligament efficiency is given by

[GATE-2001]

(A) $\frac{p_h - d_h}{p_h}$ (B) $\frac{p_h - d_h d_t/2}{p_h}$

(C) $\frac{p_h - d_t}{p_h}$ (D) $\frac{p_h - d_t}{d_h}$

Q.2 A cylindrical pressure vessel of volume 6π m^3 has to be designed to withstand a maximum internal pressure of 10 atm. The allowable design stress of the material is 125 N/mm^2 and corrosion allowance is 2 mm. The thickness of the vessel for a length/diameter ratio of 3 will be close to

[GATE-2002]

(A) 5mm (B) 6mm
(C) 8mm (D) 10mm

Q.3 Standard pipes of different schedule numbers and standard tubes of different BWG numbers are available in the market. For a pipe / tube of a given nominal diameter, which one of the following statements is TRUE?

[GATE-2016]

(A) Wall thickness increases with increase in both the schedule number and the BWG number
(B) Wall thickness increases with increase in the schedule number and decreases with increase in the BWG number
(C) Wall thickness decreases with increase in both the schedule number and the BWG number
(D) Neither the schedule number, nor the BWG number has any relation to wall thickness

ANSWER KEY:

1	2	3												
(c)	(d)	(b)												

EXPLANATIONS

Q.1 (C)

When a shell or drum is drilled for tubes in a line parallel to the axis of the shell the efficiency of the ligament between the tube holes is

$$\frac{P_h - d_h}{P_h}$$

where, P_h = Pitch of tube holes

d_h =Diameter of holes, But in the presence of the tubes the hole diameter becomes equal to the internal diameter of the tubes .

Q.2 (D)

Given $\frac{L}{D} = 3$ Volume $= 6\pi m^3$

Corrosion allowance = 2 mm

Now, volume = 6π $\frac{\pi D^2}{4} \times L = 6\pi$

$\frac{\pi D^2}{4} \times 3D = 6\pi$ $\because \frac{L}{D} = 3(\text{given})$

$D^3 = 8$ $D = 2m$

So, L = 3 x D = 3 x 2 = 6 m

$\sigma = \frac{Pr}{t}$ $\therefore \quad t = \frac{Pd}{2\sigma}$

Thickness of vessel = t + corrosion allowance

$$= \frac{Pd}{2\sigma} + 2(\text{mm}) = \frac{10^6 \times 2}{2 \times 125 \times 10^6} \times 10^3 + 2 \text{ mm}$$

$$= 8 + 2 = 10 \text{ mm}$$

Q.3 (B)